

# Critical behavior in one dimension: Unconventional pairing, phase separation, BEC-BCS crossover, and magnetic Lifshitz transition

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(Received 11 November 2016; published 13 March 2017)

We study the superconducting properties of population-imbalanced ultracold Fermi mixtures in one-dimensional optical lattices that can be effectively described by the spin-imbalanced attractive Hubbard model in the presence of a Zeeman magnetic field. We use the mean-field theory approach to obtain the ground-state phase diagrams including some unconventional superconducting phases such as the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase, and the  $\eta$  phase (an extremal case of the FFLO phase), both for the case of a fixed chemical potential and for a fixed number of particles. It allows us to determine optimal regimes for the FFLO phase as well as  $\eta$ -pairing stability. We also investigate the evolution from the weak coupling (BCS-like limit) to the strong coupling limit of tightly bound local pairs (BEC) with increasing attraction, at  $T = 0$ . Finally, the obtained results show that in spite of the occurrence of the Lifshitz transition induced by an external magnetic field, the superconducting state can still exist in the system, at higher magnetic field values.

DOI: [10.1103/PhysRevA.95.033613](https://doi.org/10.1103/PhysRevA.95.033613)

## I. INTRODUCTION

The immense development of experimental techniques in cold atomic Fermi gases in previous years has opened new avenues for research of strongly correlated systems in condensed matter physics and beyond. The ability to control the interactions via Feshbach resonances [1] sets new perspectives for experimental realization and studies of many different unconventional systems, such as spin-polarized superfluidity (with population imbalance), superconductivity with nontrivial Cooper pairing, Bose-Fermi mixtures or mixtures of fermions with unequal masses [2–5].

There are indications that the properties of unconventional superconductors place them between two regimes: BCS and BEC [6–10]. The evolution from the weak attraction (BCS-like) to the strong attraction (BEC-like) limit takes place when the interaction is increased or the particle concentration is decreased at moderate fixed attraction. According to the Leggett criterion [11], the Bose regime begins when the chemical potential  $\mu$  drops below the lower band edge. The possibility of controlling population imbalance has motivated attempts to understand the BCS-BEC crossover phase diagrams in the presence of spin polarization [9,12].

Currently, the unconventional superconductivity with a nontrivial Cooper pairing lays down one of the most important directions of studies in the theory of condensed matter [13] and ultracold quantum gases [14,15]. In the presence of a Zeeman magnetic field, the densities of states are different for

the particles with spin down and spin up. In the case of ultracold Fermi gases, paramagnetic effects are introduced artificially by population imbalance producing a mismatch between the Fermi surfaces. At strong imbalance, in the weak coupling regime, superfluidity is destroyed and undergoes a first-order phase transition to the polarized normal state at a universal critical magnetic field  $h_P = \Delta_0/\sqrt{2} \approx 0.707\Delta_0$ . The latter is called the Chandrasekhar-Clogston (CC) limit or Pauli limit [16,17], where  $\Delta_0$  is the gap at zero temperature in the absence of external field. Rather recently, a behavior in accordance with the CC limit has been observed in population imbalanced atomic Fermi gases [18,19].

In the weak coupling limit, at a large difference in the occupation number (or at a strong magnetic field), states with nontrivial Cooper pairing can exist. An example of such pairing is the formation of Cooper pairs across the spin-split Fermi surface with nonzero total momentum ( $\mathbf{k} \uparrow, -\mathbf{k} + \mathbf{Q} \downarrow$ ), leading to the so-called Fulde-Ferrell-Larkin-Ovchinnikov [20,21] (FFLO) state. Solid-state experiments typically involve highly anisotropic materials—made up either of weakly coupled two-dimensional (2D) planes or one-dimensional (1D) wires [13]. The potential candidates for finding the FFLO phase are heavy fermions [13,22–25], organic [26–28] or iron-based superconductors [29–31]. These systems are characterized by a discontinuous phase transition from the superconducting to the normal state in the regime of low temperatures. However, it is still unclear in which range of parameters the FFLO phase is stable. Moreover, the observation of this type of superconductivity is very difficult because of the very strong destructive influence of the orbital (diamagnetic) effect.

For instance, some calculations indicate that if a FFLO phase exists in three-dimensional (3D) trapped gases, it will occupy a very small volume in parameter space [32–34]. Another kind of pairing and phase coherence that can appear is

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the spatially homogeneous spin-polarized superfluidity (called breached pair state or Sarma phase [35]), which has a gapless spectrum for the majority spin species.

Quite recently, the Rice University experimental group [32] predicted the FFLO phase appearance in ultracold lattice gases. The experimental setup allows one to investigate imbalanced quantum Fermi gases ( $N_\downarrow \neq N_\uparrow$ ) by trapping the two lowest hyperfine levels of the  $^6\text{Li}$  ground state in quasi-1D geometries [36]. Similar experiments have been performed for the mass-imbalanced mixtures of  $^6\text{Li}$  and  $^{40}\text{K}$  atoms [37–40].

Also theoretical analyses suggest a possibility of a realization of the FFLO phase in optical lattices [19,41,42]. The existence of nonzero total momentum Cooper pairs leads to a spontaneous symmetry breaking of the order parameter in real space [13]. It is manifested by sign change of the superconducting order parameter as well as the occurrence of nodal lines in real space. The spatial profile and the number of the nodal lines depend on the magnetic field [43]. The same behavior of the order parameter can be observed in ultracold fermionic gases in parabolic or toroidal traps. In the former case there can occur oscillations of the order parameter in the radial direction [42,44,45], whereas in the latter case the breaking of the rotational symmetry can result in oscillations of the order parameter depending on the angle [46,47].

For one-dimensional two-component Fermi atomic gases in a magnetic trap the exact thermodynamic Bethe ansatz [48] solution shows that (in some range of magnetic field and in the strong coupling limit) a mixed phase with the two-shell structure with a partially polarized superfluid core surrounded (analogous to the FFLO phase) by either a fully paired or fully polarized phases occurs in the ground state [49,50]. Similar situation has been found in a case of the one-component trapped gas [51]. Moreover, the FFLO phase occurs at all nonzero partial polarization for any attractive interaction, whereas all of the phase transitions are continuous [49–51]. Theoretical investigations predict that the FFLO state can be also realized in a case of the mass-imbalance fermionic system [52–55]. There has been work on exact numerical studies [quantum Monte Carlo (QMC) simulations and density matrix renormalization group (DMRG)] of the 1D attractive Hubbard model with population-imbalanced fermions [55–65], suggesting that the FFLO state is stable in one dimension. Indeed, the instability of the normal state with respect to FFLO is due to a Fermi surface “nesting” which is enhanced in one dimension [66].

Motivated by the experimental feasibility of such systems with ultracold gases loaded on a quasi-1D lattice, we study the unconventional superfluid phases of the attractive Hubbard model (AHM) ( $U < 0$ ), in the presence of an external magnetic field. We show that with increasing magnetic field, the system evolves from the BCS-type superconducting state to the FFLO phase (where the Cooper pairs have nonzero total momentum  $\mathbf{Q}$ ). In an extremal case, this momentum  $\mathbf{Q}$  can lie on the vertex of the first Brillouin zone (FBZ) [67–69] and the so-called  $\eta$  phase emerges. It should be stressed that the Hubbard model on a bipartite (alternate) lattice has been rigorously proved to have  $\eta$  states as eigenstates [70]. Moreover,  $\eta$  pairing has been found as a mechanism of superconductivity in a large class of models of strongly correlated electron systems (extended Hubbard models) [71].

We obtain the magnetic field vs chemical potential as well as vs filling (i.e.,  $h - \mu$  and  $h - n$ , respectively) phase diagrams for several values of the on-site pairing interactions. Therefore, the results of our analysis can be compared to experimental results where the filling or particle concentrations can be fully controlled and measured. We find a topological quantum phase transition, of the Lifshitz type, in the ground-state phase diagrams. As a consequence of this transition, there is a change of the Fermi surface (FS) topology due to the variation of the Fermi energy and/or the band structure.

The paper is organized as follows. In Sec. II, we introduce the main theoretical model for the system under study, the attractive Hubbard model in a Zeeman magnetic field and we briefly discuss the mean-field method. Section III presents numerical results and their discussion: the  $h - \mu$  as well as  $h - n$  phase diagrams in the weak coupling limit (Sec. III A), the BCS-BEC crossover analysis and magnetic Lifshitz transition (Sec. III B). We conclude in Sec. IV with a brief summary of the obtained results and an outlook.

## II. MODEL AND METHOD

We study an  $s$ -wave superconductor on a one-dimensional lattice, described by the AHM ( $U < 0$ ) in a magnetic field which in real space takes the form,

$$\hat{\mathcal{H}} = \sum_{(i,j)\sigma} (-t - (\mu + \sigma h)\delta_{ij}) \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}, \quad (1)$$

where  $t$  is the nearest-neighbor hopping,  $\sigma = \uparrow, \downarrow$  the spin index,  $U$  the on-site attraction, and  $\mu$  is the chemical potential;  $h$  is a Zeeman field, which originates from an external magnetic field (in  $g\mu_B/2$  units) or from a population imbalance in the context of the cold atomic Fermi gases with  $\mu = (\mu_\uparrow + \mu_\downarrow)/2$  and  $h = (\mu_\uparrow - \mu_\downarrow)/2$ , where  $\mu_\sigma$  is the chemical potential of atoms with (pseudo)spin  $\sigma$ . The second term can be decoupled using the mean-field approximation,

$$\hat{n}_{i\uparrow} \hat{n}_{i\downarrow} = \Delta_i^* \hat{c}_{i\downarrow} \hat{c}_{i\uparrow} + \Delta_i \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger - |\Delta_i|^2, \quad (2)$$

where  $\Delta_i = \langle \hat{c}_{i\downarrow} \hat{c}_{i\uparrow} \rangle$  is defined as the superconducting order parameter (SOP). Then, the mean-field Hamiltonian in real space takes the form,

$$\hat{\mathcal{H}}^{\text{MF}} = \sum_{(i,j)\sigma} (-t - (\mu + \sigma h)\delta_{ij}) \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_i (\Delta_i^* \hat{c}_{i\downarrow} \hat{c}_{i\uparrow} + \text{H.c.}) - U \sum_i |\Delta_i|^2. \quad (3)$$

Without loss of generality, we can write down the SOP as  $\Delta_i = \Delta_0 \exp(i\mathbf{Q} \cdot \mathbf{R}_i)$ , where  $\Delta_0$  is the spatially oscillating amplitude and  $\mathbf{Q}$  is the total momentum of the Cooper pair.

Transforming Hamiltonian (1) to the reciprocal space, one obtains

$$\hat{\mathcal{H}}^{\text{MF}} = \sum_{k\sigma} E_{k\sigma} \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + U \sum_k (\Delta_0^* \hat{c}_{-k+\mathbf{Q}\downarrow} \hat{c}_{k\uparrow} + \text{H.c.}) - UN|\Delta_0|^2. \quad (4)$$

In the one-dimensional lattice case, the dispersion relation is given by  $E_{k\sigma} = -2t \cos(k_x) - (\mu + \sigma h)$ . Using the Nambu notation, the Hamiltonian (4) can be rewritten in a matrix

form,  $\hat{\mathcal{H}}^{\text{MF}} = \sum_k \hat{\Phi}_k^\dagger \mathbb{H}_k \hat{\Phi}_k$ , with

$$\mathbb{H}_k = \begin{pmatrix} E_{k\uparrow} & U\Delta_0 \\ U\Delta_0^* & -E_{-k+Q\downarrow} \end{pmatrix}, \quad (5)$$

where  $\hat{\Phi}_k^\dagger = (\hat{c}_{k\uparrow}^\dagger, \hat{c}_{-k+Q\downarrow})$  are the Nambu spinors. Then, the eigenvalues  $\lambda_{k\pm}$  of  $\hat{\mathcal{H}}^{\text{MF}}$  are given by

$$\lambda_{k\pm} = \eta_k^- \pm \vartheta_k, \quad \eta_k^\pm = \frac{E_{k\uparrow} \pm E_{-k+Q\downarrow}}{2},$$

$$\vartheta_k = \sqrt{(\eta_k^+)^2 + U^2|\Delta_0|^2}. \quad (6)$$

The grand canonical potential defined by  $\Omega \equiv -k_B T \ln\{\text{Tr}[\exp(-\hat{\mathcal{H}}^{\text{MF}}/k_B T)]\}$  can be written as

$$\Omega = -k_B T \sum_{k, \alpha \in \pm} \ln \left( 1 + \exp \left( \frac{-\lambda_{k\alpha}}{k_B T} \right) \right) + \sum_k (E_{k\downarrow} - U|\Delta_0|^2), \quad (7)$$

while the particle number equation takes the form,

$$n \equiv \frac{-1}{N} \frac{\partial \Omega}{\partial \mu} = 1 + \frac{1}{N} \sum_k \frac{\eta_k^+}{\vartheta_k} (f(\lambda_{k,+}) - f(\lambda_{k,-})). \quad (8)$$

In above,  $f(x) = 1/(1 + \exp(x/k_B T))$  denotes the Fermi-Dirac distribution function. The ground state is found by a minimization of  $\Omega$  with respect to the SOP amplitude  $\Delta_0$  and momentum  $\mathbf{Q}$ , for fixed  $\mu$  and  $h$ , at a temperature  $T/t = 10^{-5}$  (effectively  $T = 0$ , nonzero value has been taken for numerical reasons). As mentioned above, the systems in which the FFLO phase can be realized are characterized by discontinuous phase transitions, which are associated with discontinuous changes of  $\Delta_0$  and/or  $\mathbf{Q}$ . As a consequence, the energy gap equation for a given phase, equivalent to one of the conditions of the energy minimization  $d\Omega/d\Delta_0 = 0$ , at fixed  $\mathbf{Q}$ , cannot be used for the phase boundary estimations. In this case, the procedure of the minimization of  $\Omega$  with respect to the SOP amplitude and all possible momenta  $\mathbf{Q}$  realized in the system is essential. Because of the unequivocal relation of the real space and reciprocal space (via the Fourier transform), the number of all possible  $\mathbf{Q}$  vectors in the lattice is equal to the number of lattice sites (given by  $N$ ). It is worth mentioning that  $\mathbf{Q}$  as well as  $\Delta_0$  change discontinuously [30] going from the BCS to the FFLO phase. To find the minimum of the energy of the system, one minimizes  $\Omega(\Delta_0)$  functions for  $N$  different  $\mathbf{Q}$  vectors. For simplicity, without loss of generality, numerical calculations have been performed in the lattice with  $N = 200$  sites and periodic boundary conditions, which makes the finite-size effects negligible [72]. To speed up the calculations, graphical cards have been used. We have proceeded according to the numerical procedure described in Ref. [31].

### III. NUMERICAL RESULTS AND DISCUSSION

In this section, we focus on the analysis of superconducting properties of ultracold atomic mixtures assuming a one-dimensional lattice geometry. Within the mean-field (BCS-Stoner) approach, we construct the phase diagrams in two ways: by fixing either the chemical potential ( $\mu$ ) or the particle

concentration ( $n$ ), and show the relevant differences resulting from these possibilities. The ground-state phase diagrams are obtained for a wide range of attractive interactions, i.e., for a weak and intermediate coupling (Sec. III A) and for the local pairs limit (BEC) (Sec. III B) by using the mean-field approximation. Notice that in the general case this approximation overestimates critical temperatures and can give only a qualitative description of the long-range order phases. However, it gives a fairly good account of the system in the ground state (at  $T = 0$ ) even in the strong coupling limit [6].

#### A. Superconducting properties of the AHM in the presence of a Zeeman magnetic field: Weak and intermediate coupling

In this subsection, we consider the ground-state phase diagrams in the weak and intermediate couplings. In the following, we set the hopping parameter as an energy unity, i.e.,  $t = 1$ .

In the weak coupling regime and in the absence of an external Zeeman field, the usual superconducting BCS-type  $s$ -wave state is stable (Fig. 1). As the magnetic field rises, superfluidity gets destroyed, at weak and intermediate couplings, due to paramagnetic effects or by population imbalance. Hence, the unpolarized BCS-like superconducting phase undergoes a first-order phase transition to the polarized normal state or to the FFLO phase. Rising higher the field and close to half-filling, the polarized FFLO- $\eta$ -pairing superconducting phase also undergoes a first-order phase transition to the normal state.

These two first-order phase transition lines were determined from the conditions:  $\Omega^{\text{BCS}} = \Omega^{\text{FFLO}}$ ,  $\Omega^{\text{BCS}} = \Omega^{\text{NO}}$ ,  $\Omega^{\text{FFLO}} = \Omega^{\text{NO}}$ , where  $\Omega^{\text{BCS}}$ ,  $\Omega^{\text{FFLO}}$  and  $\Omega^{\text{NO}}$  denote the grand canonical potential of the BCS ( $\Delta_0 \neq 0$ ,  $\mathbf{Q} = 0$ ,  $P = 0$ ), FFLO ( $\Delta_0 \neq 0$ ,  $\mathbf{Q} \neq 0$ ,  $P \neq 0$ ), and the normal ( $\Delta_0 = 0$ ,  $P \neq 0$ ) state, respectively, where  $P = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$  is the polarization. Then, these results have been mapped onto the case of fixed  $n$  (Fig. 2). Moreover, there is a special case of the FFLO phase, for which the Cooper pair momentum takes the value of the momentum on the FBZ vertex ( $|\mathbf{Q}| = \pi$ ). This case is called  $\eta$  pairing and is found in the phase diagrams as well. It is worth mentioning that we take into account in our analysis the Sarma phase, which is characterized by the spatially homogeneous order parameter, in the presence of nonzero polarization (i.e.,  $\Delta_0 \neq 0$ ,  $\mathbf{Q} = 0$ ,  $P \neq 0$ ). However, these solutions are unstable for the whole region of parameters.

*h vs  $\mu$  phase diagram.* Figure 1 shows the  $h - \mu$  phase diagrams at  $T = 0$ . These diagrams are symmetric with respect to the sign change of  $\mu$  or  $h$  due to the particle-hole symmetry. For the sake of clarity, we only show the range of  $\mu$  from 0 to 3 and  $h \geq 0$ . In this case (see Fig. 1), we find two types of superconducting phases: BCS and FFLO type. Note that inside the latter, above some magnetic field value identified by a red dashed line, we find the  $\eta$ -FFLO phase. The stability range of the BCS state as well as the  $\eta$  pairing depends on the value of the attractive interaction—both phases widen when increasing the attraction and the FFLO phase shrinks. Notice that obtained phase diagrams (Fig. 1) are in a qualitative agreement with the previous DMRG calculation performed for trapped spin-imbalanced Fermi gas, where partially polarized state

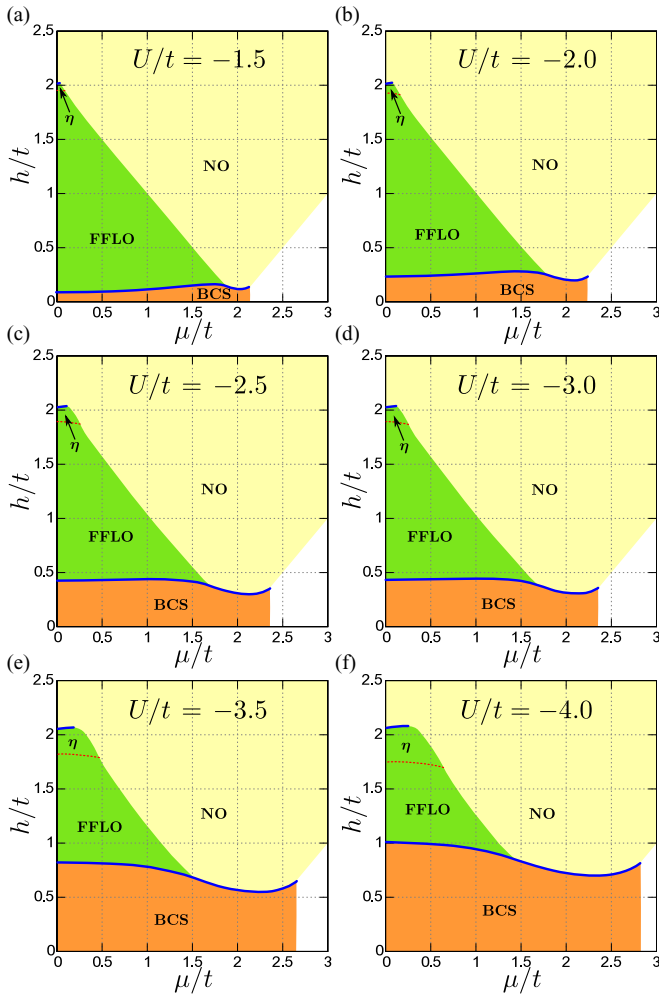


FIG. 1.  $h - \mu$  ground-state phase diagram for several values of the pairing interaction  $U$ . Labels are as follows: NO, normal phase; BCS, nonpolarized superconducting state with  $\mathbf{Q} = 0$ ; FFLO, polarized superconducting phase with  $\mathbf{Q} \neq 0$ . Additionally, within the FFLO phase, above the dashed red line there is a region where the  $\eta$  phase is distinguished. The solid blue lines indicate first-order phase transitions between different states. The white region indicates the empty state (or fully filled state, depending on the sign of the chemical potential).

(i.e., the FFLO state in the present paper) exists in a large range of the model parameters [62].

*Phase transitions.* We find that the phase transition from the BCS phase to the FFLO or NO state for  $h \neq 0$  is always of the first order (associated with a discontinuous change of the order parameters). At relatively high magnetic field and around half-filling ( $\mu \simeq 0$ ), we also obtain a first-order phase transition from the  $\eta$ -FFLO phase to the NO state (blue solid lines in Fig. 1), whereas the transition for larger  $\mu$  changes its nature into second order. On the other hand, the transitions from the FFLO phase to the NO state as well as between the BCS phase and the empty (full-filled) state are second-order ones (connected with a continuous change of the order parameters). It is important to emphasize that the first-order phase transitions are reflected by the existence of the phase separation (PS) regions in the  $h - n$  phase diagrams. The BCS

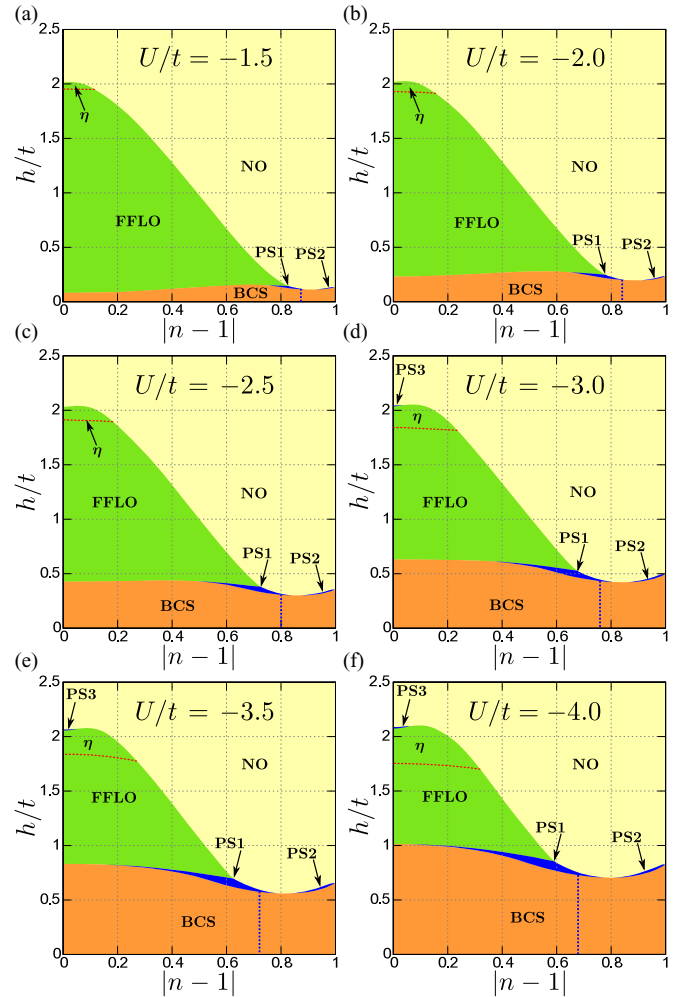


FIG. 2.  $h - n$  ground-state phase diagram for several values of the pairing interaction  $U$ . Labels are as follows: NO, normal phase; BCS and FFLO, superconducting states with  $\mathbf{Q} = 0$  and  $\mathbf{Q} \neq 0$ , respectively; PS1, phase separation region between BCS and FFLO phases or between BCS and NO phases (details in the text); PS2, phase separation region between BCS and NO phases; PS3, phase separation region between FFLO ( $\eta$  pairing) and NO phases. Within the FFLO phase the  $\eta$  state exists above the dashed red line.

boundary shows strong nonlinearities, especially around the BCS-BEC crossover point (for  $|\mu| \simeq 2$ ), while the boundary between the FFLO phase and the NO state changes in an approximately linear way with  $\mu$ .

Generally, the order of the phase transition between the FFLO and BCS phases is still under debate [73–79]. For 1D systems the studies of that problem within the framework of the Ginzburg-Landau theory show that, e.g., disorder can change the type of the phase transition [80]. A combination of the renormalization group and mean-field approximation for Fermi gases with attractive interaction gives second-order phase transition between uniform (BCS) and nonuniform (FFLO) superconducting states [81]. Moreover, studies of two-component Fermi atomic gases in a magnetic trap using the exact thermodynamic Bethe ansatz solution in the continuum model show that all phase transitions are continuous [49–51]. In such systems there has been shown that the phase separation

in a real space can occur which can be the source of other types of the phase transitions. The effective Ginsburg-Landau theory studies for quasi-2D  $d$ -wave superconductors by renormalization group analysis indicate that the transition from the FFLO to normal state is generically first order, even when the mean-field theory suggests a continuous transition [74].

However, it is crucial to emphasize that the validity of the mean-field results is generally limited, especially in the 1D case in which the pair fluctuations become increasingly important [82]. Moreover, the mean field can fail regarding the identification of the nature of the phase transition, although this approximation has been found to provide useful description in the weak and intermediate coupling, basing on the comparison with the exact or asymptotically exact Bethe ansatz for a free polarized Fermi gas [82]. Additionally, there is evidence that the mean-field description with a single-plane-wave FF-type order parameter does not predict the correct phase transition between BCS and FFLO states, although the self-consistent Bogoliubov–de Gennes formalism gives the results which are in good agreement with those from Bethe ansatz. Hence, it has been shown, for continuum models, that the first-order phase transition is merely an artifact of the single-plane-wave approximation for the order parameter [82]. This suggests that the same problem can appear for the system on the lattice, at least in the dilute limit, when both  $n \rightarrow 0$  and  $a \rightarrow 0$  ( $a$  is the lattice constant).

*$h$  vs  $n$  phase diagram and phase separations.* As mentioned above, there are relevant differences between the phase diagrams obtained for fixed chemical potential and fixed particle concentration. Figure 2 shows the dependence of the critical magnetic fields on the filling, for several attraction values. Here, due to the particle-hole symmetry, we only show the range of  $|n - 1|$  from 0 to 1. In contrast to the fixed chemical potential case, if the number of particles is fixed and  $n \neq 1$ , there are phase separated states present on the diagrams. The occurrence of the phase separated states for fixed concentration (so-called macroscopic phase separation) is associated with the first-order phase transitions occurring for fixed  $\mu$  [8,83]. We have found that the transition for fixed  $\mu$  between the FFLO and BCS phases, between the BCS and NO phases, and between the  $\eta$ -FFLO and NO phases (in some ranges of the model parameters) are discontinuous, hence, the corresponding phase separated states are presented on the diagrams as a function of  $n$ . One can distinguish three different phase separation regions in the  $h - |n - 1|$  phase diagram: PS1, the region of phase separation between the BCS and FFLO phases as well as between the BCS and NO phases (for  $h$  above and below, respectively, the points indicated by the arrow in Fig. 2), PS2, between the BCS and NO phases, and PS3, between the  $\eta$ -FFLO-pairing and NO states.

The phase diagrams in Fig. 2 show that the FFLO phase can be realized at relatively large doping. Similarly as in the case of the  $h - \mu$  phase diagram, the phase boundaries show strong nonlinearities in the BCS-BEC regime (small density of particles). The blue dashed vertical line indicates the critical value of  $n$  above which, according to the Leggett criterion, there is the BCS-BEC crossover at the large spin imbalance or Zeeman fields.

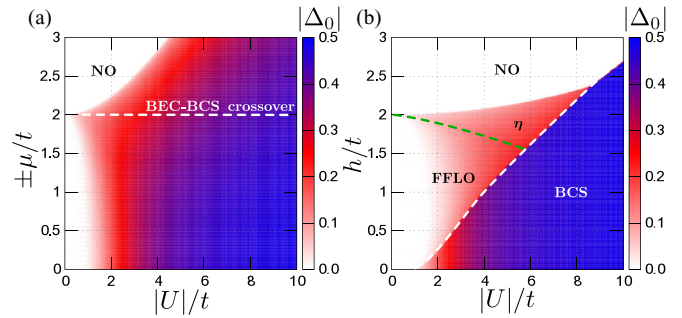


FIG. 3. The influence of the pairing  $U$  interaction on the BCS-BEC region for  $h = 0$  (a) and the magnetic Lifshitz transition region for  $\mu = 0$  (b); color-coded  $\Delta_0$  the amplitude of the order parameter.

*Role of the pairing  $U$  interaction.* The increasing of the pairing interaction  $U$  leads to the stabilization of some critical behaviors (Fig. 3). As it is known, in the strong coupling limit of AHM ( $h = 0$ ), the tightly bound local pairs of fermions behave as hard-core bosons and can exhibit a superfluid state similar to that of  $^4\text{He II}$  [6]. According to the Leggett criterion, the Bose regime begins when the chemical potential  $\mu$  drops below the lower band edge. In the case of a one-dimensional system, the band edges are at  $\pm\mu/t = 2$ . Figure 3(a) shows the  $\mu$  vs  $U$  phase diagram, at  $T = 0$  and  $h = 0$ . As one can notice, in the case of the strong coupling (larger values of  $U$ ), the superconducting phase exists above the band boundary (white dashed line). Above this line, one can speak about non-BCS behavior.

We observe similar critical behavior with an increasing Zeeman magnetic field [Fig. 3(b)]. Namely, at the critical point,  $U = 0$  and  $h/t = 2$ , the magnetic Lifshitz transition (MLT) [84] takes place. As has been mentioned above, at a nonzero Zeeman magnetic field, the population imbalance introduces a mismatch between the Fermi surfaces. Hence, effectively there are two Fermi surfaces in the system, one for the majority spin component and one for the minority spin component. However, above  $h/t = 2$  (the value of the band edge), one of the Fermi surfaces disappears. Therefore, one can observe a change in the FS topology. Strikingly, the superconducting phase can still survive above MLT and the increasing of  $U$  stabilizes the  $\eta$  phase [Fig. 3(b)]. The boundary between the FFLO and  $\eta$  phase is moved towards lower values of the magnetic field, which is clearly visible in the  $h - \mu$  as well as  $h - |n - 1|$  phase diagrams.

### B. BCS-BEC crossover and magnetic Lifshitz transition

In this subsection, we present results concerning the BCS-BEC crossover as well as the magnetic Lifshitz transition. Both possibilities can be simply shown by means of  $\mu_\uparrow$  vs  $\mu_\downarrow$  phase diagrams (Fig. 4), where  $\mu_\sigma = \mu + \sigma h$  is the effective chemical potential.

First, let us discuss the schematic phase diagram in Fig. 4(a). In the weak coupling limit, for  $|\mu_\sigma|/t \leq 2$  (the inside of the dashed black square), we have the BCS phase or FFLO, depending on the population imbalance. If  $\mu_\uparrow \simeq \mu_\downarrow$  (*i*), there is the unpolarized BCS phase, otherwise [for  $\mu_\uparrow \neq \mu_\downarrow$  (*ii*)] the FFLO state is stable. In Fig. 4(a), dashed blue lines indicate the schematic boundaries between the BCS phase and the FFLO

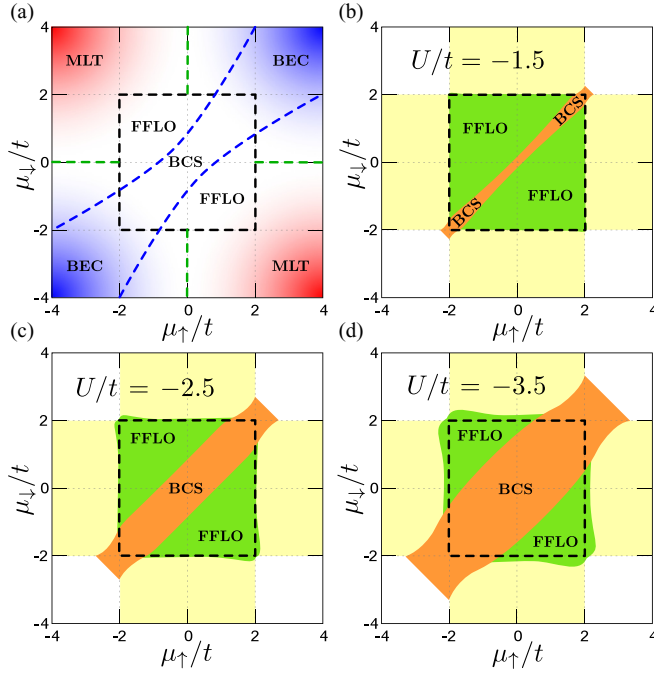


FIG. 4. Ground-state effective chemical potentials  $\mu_{\uparrow} = \mu + h$  vs  $\mu_{\downarrow} = \mu - h$  phase diagram. (a) A schematic diagram. The dashed black square indicates the band edges. The inside of the square shows possible regions of the BCS, FFLO, BEC state. Red region, magnetic Lifshitz transition (MLT) occurrence; blue region, the BCS-BEC crossover region. The dashed blue lines show the schematic boundaries of the BCS and FFLO phase occurrence. (b)–(d) Results for different values of the pairing interaction  $U$ .

state. In Figs. 4(b)–4(d), these boundaries are obtained from the minimization of the grand canonical potential with respect to the amplitude of the order parameter  $\Delta_0$  and the vector  $\mathbf{Q}$ .

With increasing  $U$ , when the effective chemical potentials drop below the lower band edge, there is the crossover to the tightly bound local pair region [BEC is the blue shaded area in Fig. 4(a)]. It takes place in the region of parameters for which  $\mu_{\uparrow} \simeq \mu_{\downarrow}$ , i.e., the polarization  $P$  of the system is low. However, it is worth emphasizing that the BCS-BEC crossover takes place both for a low particle concentration [ $\mu = (\mu_{\uparrow} + \mu_{\downarrow})/2 < -2t$ ] and for a low concentration of holes [ $\mu = (\mu_{\uparrow} + \mu_{\downarrow})/2 > 2t$ ]. This behavior is clearly visible in Figs. 4(b)–4(d) in which the orange area significantly exceeds the black dashed square. Therefore, one can speak of the crossover to the tightly bound local pairs.

In the weak coupling region, in the presence of a Zeeman magnetic field, there are two FS's in the system. If the system is strongly polarized, i.e.,  $\mu_{\uparrow} + \mu_{\downarrow} \sim 0$ , the magnetic Lifshitz transition can take place. In this case, one of the spin bands is fully filled or empty. However, the increase of the attractive interaction leads to the stabilization of the superconducting state, although there is only one FS in the system (see: Fig. 5). Pairing between particles with opposite spins is possible but the total momentum of pairs ( $\mathbf{Q}$ ) takes the maximum allowed value of momentum in the system (i.e., the vertex of the FBZ), in the presence of high polarization.

The phase diagrams for different values of pairing interaction  $U$  are shown in Figs. 4(b)–4(d). As it is clearly

visible, in the weak coupling limit ( $|U/t| \rightarrow 0$ ), the BCS phase ( $\mu_{\uparrow} \simeq \mu_{\downarrow}$ ) as well as the FFLO state ( $\mu_{\uparrow} \neq \mu_{\downarrow}$ ) are stable. However, the increasing  $U$  widens the range of occurrence of the BEC and MLT regions, which is clearly visible in Fig. 4(d). In the strong coupling limit, the chemical potential drops below the band edge, the Fermi surfaces disappear and the FFLO phase is unstable. In this regime, only the unpolarized superconducting state is realized.

#### IV. SUMMARY

We studied the superconducting properties of the spin-imbalanced attractive Hubbard model in the context of experiments with ultracold atomic Fermi mixtures with population imbalance in one-dimensional optical lattices. The ground-state phase diagrams were obtained for the cases of a fixed chemical potential and a fixed density (lattice filling) by using the mean-field approach for the 1D system. We found that the FFLO phase is stabilized for a wide range of atomic densities due to a Fermi surface nesting, which is enhanced in one dimension. Superconductivity is destroyed by the pair breaking in a very weak coupling regime. If the number of particles is fixed and  $n \neq 1$ , one can obtain two critical Zeeman magnetic fields (population imbalance), which limit the phase separation of the superconducting and the normal states.

At relatively high values of a Zeeman magnetic field, there is a region of the  $\eta$  pairing (within the FFLO phase). With an increasing attractive interaction, the  $\eta$  pairing is stabilized with respect to the FFLO state. Moreover, the  $\eta$  phase can be stable even above the magnetic Lifshitz transition (Fig. 5). As a consequence of this transition, there is a change of the Fermi surface topology due to the variation of the Fermi energy and/or the band structure. Our finding of an MLT in the spin-imbalanced AHM with the external field in

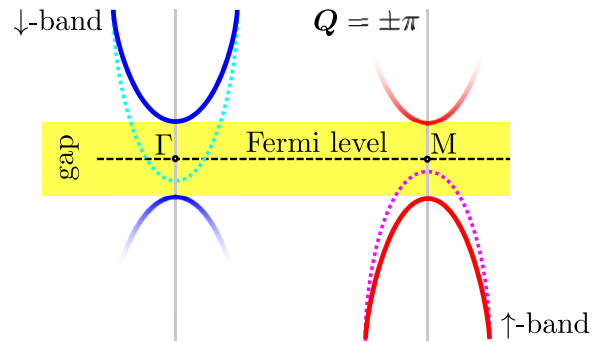


FIG. 5. A schematic illustration of the (quasi)particle band structure above the magnetic Lifshitz transition. The solid (dashed) lines indicate the quasiparticle (particle) bands in the superconducting (normal) state. At relatively high magnetic field and interaction, the  $\eta$  pairing can be realized (which is the superconducting state with total momentum of the Cooper pairs equaling  $M$  point of the first Brillouin zone). In this case, the magnetic field causes a splitting of bands with opposite spins and, hence, one of the bands can be fully filled (empty), whereas the top or bottom of the other band crosses the Fermi level. However, the existence of a strong pairing interaction (and the energy gap) leads to pairing and modifies the quasiparticle spectrum in a similar manner to that in the BCS-BEC crossover regime [85–87], at  $Q > 0$ .

one-dimensional lattice and determination of the stability of  $\eta$  pairing are reported here.

Hence, at  $T = 0$ , in the weak coupling regime and for fixed  $n$ , the following states have been found in the 1D system: at  $h \geq 0$ , the BCS state; for higher values of magnetic fields ( $h \neq 0$ ), the FFLO phase; at relatively high  $h \neq 0$ , the  $\eta$  pairing; three different PS regions. Phase separated states end at tricritical points. The NO phase is also present on the diagrams. In the present paper, we have not considered the magnetic orderings.

We have also investigated the ground-state BCS-BEC crossover diagrams for the AHM in the presence of a Zeeman magnetic field. We have observed that the FFLO phase is suppressed with increasing attraction, and the  $\eta$  pairing is favored as well as only the BCS-like phase in the strong coupling limit.

## ACKNOWLEDGMENTS

We thank Krzysztof Cichy, Tadeusz Domański, and Matteo Rizzi for careful reading of the manuscript, and valuable comments and discussions. This work was supported by the National Science Centre (NCN, Poland) under Grant No. UMO-2016/20/S/ST3/00274 (A.P.) and by the Large Infrastructures for Research, Experimental Development and Innovations project “IT4Innovations National Supercomputing Center–LM2015070” of the Czech Republic Ministry of Education, Youth and Sports. K.R. acknowledges support from CIBioFi and the Colombian Science, Technology and Innovation Foundation–COLCIENCIAS “Francisco José de Caldas” under Project No. 1106-712-49884 (Contract No. 264-2016) and General Royalties System (Fondo CTeI-SGR) under Contract No. BPIN 2013000100007.

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- [1] P. O. Fedichev, Yu. Kagan, G. V. Shlyapnikov, and J. T. M. Walraven, Influence of Nearly Resonant Light on the Scattering Length in Low-Temperature Atomic Gases, *Phys. Rev. Lett.* **77**, 2913 (1996).
- [2] C. H. Schunck, Y. Shin, A. Schirotzek, M. W. Zwierlein, and W. Ketterle, Pairing without superfluidity: The ground state of an imbalanced Fermi mixture, *Science* **316**, 867 (2007).
- [3] G. B. Partridge, W. Li, R. I. Kamar, Y.-a. Liao, and R. G. Hulet, Pairing and phase separation in a polarized Fermi gas, *Science* **311**, 503 (2006).
- [4] M. W. Zwierlein and W. Ketterle, Comment on “Pairing and phase separation in a polarized Fermi gas”, *Science* **314**, 54 (2006).
- [5] I. M. Georgescu, S. Ashhab, and F. Nori, Quantum simulation, *Rev. Mod. Phys.* **86**, 153 (2014).
- [6] R. Micnas, J. Ranninger, and S. Robaszkiewicz, Superconductivity in narrow-band systems with local nonretarded attractive interactions, *Rev. Mod. Phys.* **62**, 113 (1990).
- [7] T. Bourdel, L. Khaykovich, J. Cubizolles, J. Zhang, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, Experimental Study of the BEC-BCS Crossover Region in Lithium 6, *Phys. Rev. Lett.* **93**, 050401 (2004).
- [8] K. Kapcia, S. Robaszkiewicz, and R. Micnas, Phase separation in a lattice model of a superconductor with pair hopping, *J. Phys.: Condens. Matter* **24**, 215601 (2012).
- [9] A. Cichy and R. Micnas, The spin-imbalanced attractive Hubbard model in: Phase diagrams and BCS-BEC crossover at low filling, *Ann. Phys.* **347**, 207 (2014).
- [10] K. J. Kapcia, Superconductivity, metastability and magnetic field induced phase separation in the atomic limit of the Penson-Kolb-Hubbard model, *Acta Phys. Pol. A* **126**, A–53 (2014).
- [11] A. J. Leggett, Cooper pairing in spin-polarized Fermi systems, *J. Phys. Colloques* **41**, C7–19 (1980).
- [12] M. J. Wolak, B. Grémaud, R. T. Scalettar, and G. G. Batrouni, Pairing in a two-dimensional Fermi gas with population imbalance, *Phys. Rev. A* **86**, 023630 (2012).
- [13] Y. Matsuda and H. Shimahara, Fulde-Ferrell-Larkin-Ovchinnikov state in heavy fermion superconductors, *J. Phys. Soc. Jpn.* **76**, 051005 (2007).
- [14] H. Hu and X.-J. Liu, Mean-field phase diagrams of imbalanced Fermi gases near a Feshbach resonance, *Phys. Rev. A* **73**, 051603 (2006).
- [15] O. Dutta, M. Gajda, P. Hauke, M. Lewenstein, D.-S. Lühmann, B. A. Malomed, T. Sowiński, and J. Zakrzewski, Non-standard Hubbard models in optical lattices: A review, *Rep. Prog. Phys.* **78**, 066001 (2015).
- [16] B. S. Chandrasekhar, A note on the maximum critical field of high-field superconductors, *Appl. Phys. Lett.* **1**, 7 (1962).
- [17] A. M. Clogston, Upper Limit for the Critical Field in Hard Superconductors, *Phys. Rev. Lett.* **9**, 266 (1962).
- [18] M. W. Zwierlein, C. H. Schunck, A. Schirotzek, and W. Ketterle, Direct observation of the superfluid phase transition in ultracold Fermi gases, *Nature (London)* **442**, 54 (2006).
- [19] A. Cichy, K. Cichy, and T. P. Polak, Competition between Abelian and Zeeman magnetic field effects in a two-dimensional ultracold gas of fermions, *Ann. Phys.* **354**, 89 (2015).
- [20] P. Fulde and R. A. Ferrell, Superconductivity in a strong spin-exchange field, *Phys. Rev.* **135**, A550 (1964).
- [21] A. I. Larkin and Yu. N. Ovchinnikov, Nonuniform state of superconductors, *Zh. Eksp. Teor. Fiz.* **47**, 1136 (1964) [*Sov. Phys. JETP* **20**, 762 (1965)].
- [22] A. Bianchi, R. Movshovich, N. Oeschler, P. Gegenwart, F. Steglich, J. D. Thompson, P. G. Pagliuso, and J. L. Sarrao, First-Order Superconducting Phase Transition in CeCoIn<sub>5</sub>, *Phys. Rev. Lett.* **89**, 137002 (2002).
- [23] H. A. Radovan, N. A. Fortune, T. P. Murphy, S. T. Hannahs, E. C. Palm, S. W. Tozer, and D. Hall, Magnetic enhancement of superconductivity from electron spin domains, *Nature (London)* **425**, 51 (2003).
- [24] A. Bianchi, R. Movshovich, C. Capan, P. G. Pagliuso, and J. L. Sarrao, Possible Fulde-Ferrell-Larkin-Ovchinnikov Superconducting State in CeCoIn<sub>5</sub>, *Phys. Rev. Lett.* **91**, 187004 (2003).
- [25] M. Kenzelmann, Th. Strässle, Ch. Niedermayer, M. Sigrist, B. Padmanabhan, M. Zolliker, A. D. Bianchi, R. Movshovich, E. D. Bauer, J. L. Sarrao, and J. D. Thompson, Coupled superconducting and magnetic order in CeCoIn<sub>5</sub>, *Science* **321**, 1652 (2008).
- [26] S. Uji, T. Terashima, M. Nishimura, Y. Takahide, T. Konoike, K. Enomoto, H. Cui, H. Kobayashi, A. Kobayashi, H. Tanaka, M. Tokumoto, E. S. Choi, T. Tokumoto, D. Graf, and

- J. S. Brooks, Vortex Dynamics and the Fulde-Ferrell-Larkin-Ovchinnikov State in a Magnetic-Field-Induced Organic Superconductor, *Phys. Rev. Lett.* **97**, 157001 (2006).
- [27] R. Lortz, Y. Wang, A. Demuer, P. H. M. Böttger, B. Bergk, G. Zwicky, Y. Nakazawa, and J. Wosnitzer, Calorimetric Evidence for a Fulde-Ferrell-Larkin-Ovchinnikov Superconducting State in the Layered Organic Superconductor  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>, *Phys. Rev. Lett.* **99**, 187002 (2007).
- [28] H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, K. Miyagawa, K. Kanoda, and V. F. Mitrovic, Evidence of Andreev bound states as a hallmark of the FFLO phase in  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>, *Nat. Phys.* **10**, 928 (2014).
- [29] D. A. Zocco, K. Grube, F. Eilers, T. Wolf, and H. v. Löhneysen, Pauli-Limited Multiband Superconductivity in KFe<sub>2</sub>As<sub>2</sub>, *Phys. Rev. Lett.* **111**, 057007 (2013).
- [30] A. Ptok, Multiple phase transitions in Pauli-limited iron-based superconductors, *J. Phys.: Condens. Matter* **27**, 482001 (2015).
- [31] M. Januszewski, A. Ptok, D. Crivelli, and B. Gardas, GPU-based acceleration of free energy calculations in solid state physics, *Comput. Phys. Commun.* **192**, 220 (2015).
- [32] Y.-A. Liao, A. S. C. Rittner, T. Paprotta, W. Li, G. B. Partridge, R. G. Hulet, S. K. Baur, and E. J. Mueller, Spin-imbalance in a one-dimensional Fermi gas, *Nature (London)* **467**, 567 (2010).
- [33] J. P. A. Devreese, S. N. Klimin, and J. Tempere, Resonant enhancement of the Fulde-Ferrell-Larkin-Ovchinnikov state in three dimensions by a one-dimensional optical potential, *Phys. Rev. A* **83**, 013606 (2011).
- [34] J. Wang, Y. Che, L. Zhang, and Q. Chen, Enhancement effect of mass imbalance on Fulde-Ferrell-Larkin-Ovchinnikov type of pairing in Fermi-Fermi mixtures of ultracold quantum gases, *Sci. Rep.* **7**, 39783 (2017).
- [35] G. Sarma, On the influence of a uniform exchange field acting on the spins of the conduction electrons in a superconductor, *J. Phys. Chem. Solids* **24**, 1029 (1963).
- [36] E. Zhao and W. V. Liu, An effective field theory for one-dimensional polarized Fermi gases, *J. Low Temp. Phys.* **158**, 36 (2009).
- [37] E. Wille, F. M. Spiegelhalter, G. Kerner, D. Naik, A. Trenkwalder, G. Hendl, F. Schreck, R. Grimm, T. G. Tiecke, J. T. M. Walraven, S. J. J. M. F. Kokkelmans, E. Tiesinga, and P. S. Julienne, Exploring an Ultracold Fermi-Fermi Mixture: Interspecies Feshbach Resonances and Scattering Properties of <sup>6</sup>Li and <sup>40</sup>K, *Phys. Rev. Lett.* **100**, 053201 (2008).
- [38] A.-C. Voigt, M. Taglieber, L. Costa, T. Aoki, W. Wieser, T. W. Hänsch, and K. Dieckmann, Ultracold Heteronuclear Fermi-Fermi Molecules, *Phys. Rev. Lett.* **102**, 020405 (2009).
- [39] A. Trenkwalder, C. Kohstall, M. Zaccanti, D. Naik, A. I. Sidorov, F. Schreck, and R. Grimm, Hydrodynamic Expansion of a Strongly Interacting Fermi-Fermi Mixture, *Phys. Rev. Lett.* **106**, 115304 (2011).
- [40] M. Jag, M. Zaccanti, M. Cetina, R. S. Lous, F. Schreck, R. Grimm, D. S. Petrov, and J. Levinsen, Observation of a Strong Atom-Dimer Attraction in a Mass-Imbalanced Fermi-Fermi Mixture, *Phys. Rev. Lett.* **112**, 075302 (2014).
- [41] P. Rosenberg, S. Chiesa, and S. Zhang, FFLO order in ultra-cold atoms in three-dimensional optical lattices, *J. Phys.: Condens. Matter* **27**, 225601 (2015).
- [42] K. Machida, T. Mizushima, and M. Ichioka, Generic Phase Diagram of Fermion Superfluids with Population Imbalance, *Phys. Rev. Lett.* **97**, 120407 (2006).
- [43] H. Shimahara, Structure of the Fulde-Ferrell-Larkin-Ovchinnikov state in two-dimensional superconductors, *J. Phys. Soc. Jpn.* **67**, 736 (1998).
- [44] P. Castorina, M. Grasso, M. Oertel, M. Urban, and D. Zappalà, Nonstandard pairing in asymmetric trapped fermi gases, *Phys. Rev. A* **72**, 025601 (2005).
- [45] Y. Chen, Z. D. Wang, F. C. Zhang, and C. S. Ting, Exploring exotic superfluidity of polarized ultracold fermions in optical lattices, *Phys. Rev. B* **79**, 054512 (2009).
- [46] Y. Yanase, Angular Fulde-Ferrell-Larkin-Ovchinnikov state in cold fermion gases in a toroidal trap, *Phys. Rev. B* **80**, 220510 (2009).
- [47] A. Ptok, The Fulde-Ferrell-Larkin-Ovchinnikov state in quantum rings, *J. Supercond. Nov. Magn.* **25**, 1843 (2012).
- [48] X.-W. Guan, M. T. Batchelor, and Ch. Lee, Fermi gases in one dimension: From Bethe ansatz to experiments, *Rev. Mod. Phys.* **85**, 1633 (2013).
- [49] X. W. Guan, M. T. Batchelor, C. Lee, and M. Bortz, Phase transitions and pairing signature in strongly attractive Fermi atomic gases, *Phys. Rev. B* **76**, 085120 (2007).
- [50] G. Orso, Attractive Fermi Gases with Unequal Spin Populations in Highly Elongated Traps, *Phys. Rev. Lett.* **98**, 070402 (2007).
- [51] H. Hu, X.-J. Liu, and P. D. Drummond, Phase Diagram of a Strongly Interacting Polarized Fermi Gas in One Dimension, *Phys. Rev. Lett.* **98**, 070403 (2007).
- [52] W. V. Liu and F. Wilczek, Interior Gap Superfluidity, *Phys. Rev. Lett.* **90**, 047002 (2003).
- [53] S. Pahl and Z. Koinov, Phase diagram of a <sup>6</sup>Li-<sup>40</sup>K mixture in a square lattice, *J. Low Temp. Phys.* **176**, 113 (2014).
- [54] A. Hu, M. M. Maška, Ch. W. Clark, and J. K. Freericks, Robust finite-temperature disordered Mott-insulating phases in inhomogeneous Fermi-Fermi mixtures with density and mass imbalance, *Phys. Rev. A* **91**, 063624 (2015).
- [55] S. S. Chung and C. J. Bolech, Multiple phase separation in one-dimensional mixtures of mass- and population-imbalanced attractive Fermi gases, [arXiv:1612.03149](https://arxiv.org/abs/1612.03149).
- [56] A. E. Feiguin and F. Heidrich-Meisner, Pairing states of a polarized Fermi gas trapped in a one-dimensional optical lattice, *Phys. Rev. B* **76**, 220508 (2007).
- [57] G. G. Batrouni, M. H. Huntley, V. G. Rousseau, and R. T. Scalettar, Exact Numerical Study of Pair Formation with Imbalanced Fermion Populations, *Phys. Rev. Lett.* **100**, 116405 (2008).
- [58] A. Lüscher, R. M. Noack, and A. M. Läuchli, Fulde-Ferrell-Larkin-Ovchinnikov state in the one-dimensional attractive Hubbard model and its fingerprint in spatial noise correlations, *Phys. Rev. A* **78**, 013637 (2008).
- [59] M. Rizzi, M. Polini, M. A. Cazalilla, M. R. Bakhtiari, M. P. Tosi, and R. Fazio, Fulde-Ferrell-Larkin-Ovchinnikov pairing in one-dimensional optical lattices, *Phys. Rev. B* **77**, 245105 (2008).
- [60] M. Tezuka and M. Ueda, Density-Matrix Renormalization Group Study of Trapped Imbalanced Fermi Condensates, *Phys. Rev. Lett.* **100**, 110403 (2008).
- [61] E. Burovski, G. Orso, and T. Jolicoeur, Multiparticle Composites in Density-Imbalanced Quantum Fluids, *Phys. Rev. Lett.* **103**, 215301 (2009).
- [62] F. Heidrich-Meisner, G. Orso, and A. E. Feiguin, Phase separation of trapped spin-imbalanced Fermi gases in one-dimensional optical lattices, *Phys. Rev. A* **81**, 053602 (2010).



- [63] F. Heidrich-Meisner, A. E. Feiguin, U. Schollwöck, and W. Zwerger, BCS-BEC crossover and the disappearance of Fulde-Ferrell-Larkin-Ovchinnikov correlations in a spin-imbalanced one-dimensional Fermi gas, *Phys. Rev. A* **81**, 023629 (2010).
- [64] V. V. França, D. Hördlein, and A. Buchleitner, Fulde-Ferrell-Larkin-Ovchinnikov critical polarization in one-dimensional fermionic optical lattices, *Phys. Rev. A* **86**, 033622 (2012).
- [65] L. Riegger, G. Orso, and F. Heidrich-Meisner, Interaction quantum quenches in the one-dimensional Fermi-Hubbard model with spin imbalance, *Phys. Rev. A* **91**, 043623 (2015).
- [66] M. M. Parish, S. K. Baur, E. J. Mueller, and D. A. Huse, Quasi-One-Dimensional Polarized Fermi Superfluids, *Phys. Rev. Lett.* **99**, 250403 (2007).
- [67] M. Mierzejewski and M. M. Maška, Critical field in a superconductivity model with local pairs, *Phys. Rev. B* **69**, 054502 (2004).
- [68] A. Ptok and M. Mierzejewski, Superconductivity in the Penson-Kolb Model on a triangular lattice, *Acta Phys. Pol. A* **114**, 209 (2008).
- [69] A. Ptok, M. M. Maška, and M. Mierzejewski, The Fulde-Ferrell-Larkin-Ovchinnikov phase in the presence of pair hopping interaction, *J. Phys.: Condens. Matter* **21**, 295601 (2009).
- [70] C. N. Yang,  $\eta$  Pairing and Off-Diagonal Long-Range Order in a Hubbard Model, *Phys. Rev. Lett.* **63**, 2144 (1989).
- [71] J. de Boer, V. E. Korepin, and A. Schadschneider,  $\eta$  Pairing as a Mechanism of Superconductivity in Models of Strongly Correlated Electrons, *Phys. Rev. Lett.* **74**, 789 (1995).
- [72] A. Ptok and D. Crivelli, Influence of finite size effects on the Fulde-Ferrell-Larkin-Ovchinnikov state, *Commun. Comput. Phys.* **21**, 748 (2017).
- [73] R. Casalbuoni and G. Nardulli, Inhomogeneous superconductivity in condensed matter and QCD, *Rev. Mod. Phys.* **76**, 263 (2004).
- [74] D. Dalidovich and K. Yang, Fluctuation-Driven First-Order Transition in Pauli-Limited  $d$ -Wave Superconductors, *Phys. Rev. Lett.* **93**, 247002 (2004).
- [75] K. V. Samokhin and M. S. Mar'enko, Quantum fluctuations in Larkin-Ovchinnikov-Fulde-Ferrell superconductors, *Phys. Rev. B* **73**, 144502 (2006).
- [76] F. Konschelle, J. Cayssol, and A. I. Buzdin, Anomalous fluctuation regimes at FFLO transition, *Europhys. Lett.* **79**, 67001 (2007).
- [77] H. T. C. Stoof, K. B. Gubbels, and D. B. M. Dickerscheid, *Ultracold Quantum Fields* (Springer Netherlands, Amsterdam, 2009).
- [78] H. Caldas and M. A. Continentino, Quantum normal-to-inhomogeneous superconductor phase transition in nearly two-dimensional metals, *Phys. Rev. B* **86**, 144503 (2012).
- [79] M. Buchhold, B. Everest, M. Marcuzzi, I. Lesanovsky, and S. Diehl, Nonequilibrium effective field theory for absorbing state phase transitions in driven open quantum spin systems, *Phys. Rev. B* **95**, 014308 (2017).
- [80] D. F. Agterberg and K. Yang, The effect of impurities on Fulde-Ferrell-Larkin-Ovchinnikov superconductors, *J. Phys.: Condens. Matter* **13**, 9259 (2001).
- [81] K. Yang, Inhomogeneous superconducting state in quasi-one-dimensional systems, *Phys. Rev. B* **63**, 140511 (2001).
- [82] X.-J. Liu, H. Hu, and P. D. Drummond, Fulde-Ferrell-Larkin-Ovchinnikov states in one-dimensional spin-polarized ultracold atomic Fermi gases, *Phys. Rev. A* **76**, 043605 (2007).
- [83] K. J. Kapcia, W. R. Czart, and A. Ptok, Phase separation of superconducting phases in the Penson-Kolb-Hubbard model, *J. Phys. Soc. Jpn.* **85**, 044708 (2016).
- [84] A. Ptok, K. J. Kapcia, A. Cichy, A. M. Oleś, and P. Piekarz, Magnetic Lifshitz transition and its consequences in multi-band iron-based superconductors, *Sci. Rep.* **7**, 41979 (2017).
- [85] M. Cuoco and J. Ranninger, From an insulating to a superfluid pair-bond liquid, *Phys. Rev. B* **74**, 094511 (2006).
- [86] Y. Lubashevsky, E. Lahoud, K. Chashka, D. Podolsky, and A. Kanigel, Shallow pockets and very strong coupling superconductivity in  $\text{FeSe}_x\text{Te}_{1-x}$ , *Nat. Phys.* **8**, 309 (2012).
- [87] Y. L. Loh, M. Randeria, N. Trivedi, Ch.-Ch. Chang, and R. Scalettar, Superconductor-Insulator Transition and Fermi-Bose Crossovers, *Phys. Rev. X* **6**, 021029 (2016).