Quantum non-Markovian reservoirs of atomic condensates engineered via dipolar interactions

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We investigate the quantum dephasing dynamics of an impurity qubit immersed in a quasi-two-dimensional dipolar Bose-Einstein condensate whose collective excitations act as a reservoir for the qubit. We show that the properties of the environment are highly engineerable through the relative strength of the dipolar and contact interactions such that qubit's dephasing dynamics could be Markovian, weak non-Markovian, or even highly non-Markovian. It is also revealed that the appearance of the roton excitation is responsible for the highly non-Markovian dephasing dynamics. Since rotonlike dispersions also appear in condensates placed in cavities or with spin-orbit couplings, our results pave the way for searching for systems that are suitable environment engineering.

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I. INTRODUCTION

Open quantum systems always decohere through their couplings to the surrounding environments [1]. In the simplest situations, a decoherence process is described by the Markovian master equation of the Lindblad form [2], which assumes that information flows only from the system to the environment. However, in a non-Markovian process, information also flows from the environment back to the system such that the open quantum system recovers some of its lost memory. Recently, information flow has been successfully employed to quantify the non-Markovianity of quantum processes [3–7]. Of particular importance, non-Markovianity has been found to be an essential resource in applications such as detecting the characteristic properties of the environment [8,9], quantum metrology [10], continuous-variable quantum key distribution [11], energy transfer processes in photosynthetic complexes [12], and steady-state entanglement [13]. Although transitions from Markovian to non-Markovian dynamics have been experimentally realized in all-optical setups [14–17], they are generally difficult to achieve due to the large number of degrees of freedom of the environment.

Owing to their unprecedented controllability and low temperature, atomic Bose-Einstein condensates (BECs) are often referred to as the reservoirs suitable for engineering in both experimental [18–24] and theoretical [25–30] studies. Moreover, a qubit coupled to a BEC reservoir is described by a pure dephasing model [28–30], which is exactly solvable [1,31] and is considered to be an ideal test bed for investigating the quantum memory effects. In fact, Haikka *et al.* show that the information flow between the system and the BEC reservoir can be manipulated by engineering the control parameters of the BEC reservoir [29,30].

In this work, we study the quantum dynamics of an atomic impurity qubit immersed in a quasi-two-dimensional (quasi-2D) dipolar BEC. The system under consideration is described by a pure dephasing spin-boson model, in which the collective excitations of the BEC act as the reservoir to the qubit. We show that, by increasing the strength of the dipole-dipole interaction (DDI), the dephasing dynamics of the qubit changes from being Markovian to weak non-Markovian and eventually to highly non-Markovian. Remarkably, the non-Markovianity for the latter case diverges, in striking contrast to the small non-Markovianity realized in the nondipolar BECs [29,30]. We also analytically demonstrate that high non-Markovianity is associated with the roton softening of the excitation spectrum [32–35], which makes the density of states singular at one or two particular frequencies. Therefore, the dipolar BEC reservoir can be engineered from a Markovian one to highly non-Markovian monochromatic or bichromatic reservoirs. On the other hand, the non-Markovianity of the impurity qubit can also be used as a probe qubit [25–27,30,36,37] to detect the roton mode softening.

We note that dipolar BECs have been experimentally realized for atoms with large magnetic dipole moments [38–40]. Following the fast experimental developments in creating degenerate gases of polar molecules, dipolar BEC is also expected to be realized in gases of heteronuclear molecules [41–47]. Meanwhile, rotonlike dispersions are also found in atomic condensates placed inside an optical cavity [48] or with spin-orbit coupling [49]. Therefore, our results suggest that similar reservoir engineering can be realized in a wide range of BECs.

II. MODEL

As schematically shown in Fig. 1(a), we consider a single two-level atom immersed in a thermally equilibrated quasi-2D dipolar gas reservoir at temperature T. The qubit is confined in a harmonic trap $V_A(\mathbf{x}) = m_A\omega_A^2\mathbf{x}^2/2$ that is independent of the internal states, where m_A is the mass of the impurity and ω_A is the trap frequency. For $\hbar\omega_A \gg k_BT$, the spatial wave function of the qubit is the ground state of $V_A(\mathbf{x})$, i.e., $\varphi_A(\mathbf{x}) = \pi^{-3/4}\ell_A^{-3/2}\exp[-\mathbf{x}^2/(2\ell_A^2)]$, with $\ell_A = \sqrt{\hbar/(m_A\omega_A)}$. The Hamiltonian of the qubit is

$$\hat{H}_A = \hbar\Omega_A|e\rangle\langle e|,$$

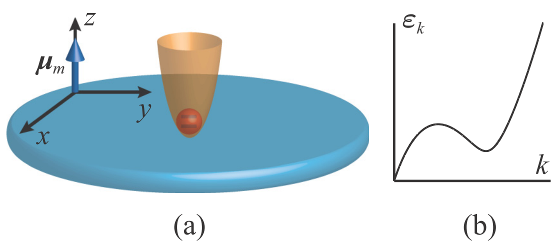

FIG. 1. Schematic diagrams of (a) a two-level atom immersed in a quasi-2D dipolar gas and (b) the typical roton spectrum of dipolar BECs.

where $\hbar\Omega_A$ is level splitting between the ground ($|g\rangle$) and excited ($|e\rangle$) states.

For the reservoir, we assume that each atom possesses a magnetic dipole moment μ_m that is polarized to the z direction. Therefore, two atoms interact via the potential $V^{(3D)}(\mathbf{x}-\mathbf{x}') = g_B\delta(\mathbf{x}-\mathbf{x}') + 3g_D(1-3\cos^2\theta)/(4\pi|\mathbf{x}-\mathbf{x}'|^3)$, where $g_B = 4\pi\hbar^2 a_B/m_B$ represents the contact interaction strength, with m_B being the mass of the reservoir atom and a_B being the s-wave scattering length; $g_D = \mu_0\mu_m^2/3$, with μ_0 being the permeability of vacuum; and θ is the polar angle of $\mathbf{x}-\mathbf{x}'$. Moreover, the gas is confined along the z axis by the potential $V_B(z) = m_B\omega_z^2 z^2/2$, where ω_z is the trap frequency. For sufficiently large ω_z, the motion of the atoms along the z axis is frozen to the ground state of $V_B(z)$, i.e., $\varphi_B(z) = \pi^{-1/4}\ell_B^{-1/2}\exp[-z^2/(2\ell_B^2)]$, with $\ell_B = \sqrt{\hbar/(m_B\omega_z)}$, which effectively reduces the reservoir into a quasi-2D one. Finally, for small T, we may assume that most of the reservoir atoms are condensed to the zero-momentum state with an area density n. Then, following Bogoliubov's method, the uncondensed atoms are described by the quasiparticle Hamiltonian

$$\hat{H}_B = \sum_{\mathbf{k}\neq 0} \varepsilon_{\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}, \tag{1}$$

where $\mathbf{k} \equiv (k_x,k_y)$, $\hat{b}_{\mathbf{k}}$ is the annihilation operator for the quasiparticle with wave vector $\mathbf{k}$, and the excitation energy is [34]

$$\varepsilon_{\mathbf{k}} = \tfrac{1}{2}\hbar\omega_z\sqrt{(\ell_B k)^4 + P(\ell_B k)^2[1+\chi\tilde{v}_D(\ell_B k)]}, \tag{2}$$

with $P = 8\sqrt{2\pi}\ell_B a_B n$ being a dimensionless parameter that measures the strength of the contact interaction, $\chi = g_D/g_B$ being the relative DDI interaction strength, and $\tilde{v}_D(x) = 2 - 3\sqrt{\pi/2}xe^{x^2/2}\mathrm{erfc}(x/\sqrt{2})$ being the Fourier transform of the effective 2D DDI. It is now well established that a sufficiently strong DDI would lead to the roton excitation and, eventually, the instability. In fact, for $P = 2$, roton excitation sets in when $\chi > \chi^* \simeq 4.23$. In addition, the condensate becomes unstable for $\chi > \chi^{**} = 5.67$. The typical roton spectrum is shown in Fig. 1(b).

For the qubit-reservoir coupling, we assume that the qubit undergoes s-wave collisions with reservoir atoms only when the qubit is in the excited state [27,29]. Let a_{AB} be the corresponding scattering length; the qubit-reservoir interaction Hamiltonian is then

$$\hat{H}_{AB} = \hbar\delta_e|e\rangle\langle e| + \hbar|e\rangle\langle e|\sum_{\mathbf{k}\neq 0} g_{\mathbf{k}}(\hat{b}_{\mathbf{k}} + \hat{b}_{\mathbf{k}}^\dagger), \tag{3}$$

where $\delta_e = 2\sqrt{\pi}\hbar n a_{AB}/[m_{AB}(\ell_A^2+\ell_B^2)^{1/2}]$ is the excited level shift due to the collision, $m_{AB} = m_A m_B/(m_A+m_B)$ is the reduced mass, and $g_{\mathbf{k}} = (nS)^{-1/2}\delta_e e^{-(\ell_A k)^2/4}\sqrt{E_{\mathbf{k}}/\varepsilon_{\mathbf{k}}}$ are the qubit-reservoir coupling parameters, with S being the area of the reservoir and $E_{\mathbf{k}} = \hbar^2\mathbf{k}^2/(2m_B)$ being the free-particle energy.

Now the total Hamiltonian, $\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_{AB}$, is

$$\hat{H} = \hbar(\Omega_A + \delta_e)|e\rangle\langle e| + \sum_{\mathbf{k}\neq 0}\varepsilon_{\mathbf{k}}\hat{b}_{\mathbf{k}}^\dagger\hat{b}_{\mathbf{k}} + \hbar|e\rangle\langle e|\sum_{\mathbf{k}\neq 0} g_{\mathbf{k}}(\hat{b}_{\mathbf{k}} + \hat{b}_{\mathbf{k}}^\dagger). \tag{4}$$

Since $\hat{H}_A$ commutes with $\hat{H}_{AB}$, the dynamics of the impurity qubit in the reservoir is purely dephasing. Namely, for the density matrix of the qubit $\rho^{(A)}$, the diagonal elements, $\rho_{gg}^{(A)}$ and $\rho_{ee}^{(A)}$, remain constants, and the off-diagonal elements evolve as

$$\left|\rho_{eg}^{(A)}(t)\right| = e^{-\gamma(t)}\left|\rho_{eg}^{(A)}(0)\right|, \tag{5}$$

with γ being the dephasing factor. To find $\gamma(t)$, we assume that the density matrix of the initial state is $\rho^{(T)}(0) = \rho^{(A)}(0)\otimes\rho^{(B)}$, where the density matrix of the reservoir is $\rho^{(B)} = \prod_{\mathbf{k}}\rho_{\mathbf{k}}^{(B)} \equiv \prod_{\mathbf{k}}(1-e^{\varepsilon_{\mathbf{k}}/(k_B T)})e^{-\varepsilon_{\mathbf{k}}b_{\mathbf{k}}^\dagger b_{\mathbf{k}}}$, with k_B being the Boltzmann constant. The dephasing factor of the qubit is then [31,50]

$$\gamma(t) = \int_0^\infty d\omega G(\omega,T)[1-\cos(\omega t)], \tag{6}$$

where

$$G(\omega,T) = \frac{J(\omega)}{\omega^2}\coth\left(\frac{\hbar\omega}{2k_B T}\right), \tag{7}$$

with $J(\omega) = \sum_{\mathbf{k}\neq 0}|g_{\mathbf{k}}|^2\delta(\omega - \varepsilon_{\mathbf{k}}/\hbar)$ being the *reservoir spectral density*. In particular, in the continuum limit, $S^{-1}\sum_{\mathbf{k}} \to (2\pi)^{-2}\int d\mathbf{k}$, we have

$$J(\omega) = Q\hbar\omega_z^3\ell_B^4\int_0^\infty dk\frac{k^3 e^{-\ell_A^2 k^2/2}}{\varepsilon(k)}\delta\left(\omega - \frac{\varepsilon(k)}{\hbar}\right), \tag{8}$$

where $Q = na_{AB}^2\ell_B^2(m_A+m_B)^2/[m_A^2(\ell_A^2+\ell_B^2)]$ is a dimensionless parameter measuring the qubit-reservoir coupling and in the continuum limit $\varepsilon_{\mathbf{k}}$ is denoted as $\varepsilon(k)$, i.e., $\varepsilon(k)\equiv\varepsilon_{\mathbf{k}}$.

To measure the degree of non-Markovianity, we note that, for purely dephasing dynamics described by Eq. (5), information flows from the system to the environment when $\gamma'(t) > 0$ and it flows reversely if $\gamma'(t) < 0$. Therefore, based on Breuer *et al.* [3], the measure of non-Markovianity is

$$\mathcal{N} = \int_{\gamma'(t)<0} de^{-\gamma(s)}, \tag{9}$$

where the integration is over all intervals in which $\gamma'(t) < 0$.

III. RESULTS

To present our results, we consider a single ^{87}Rb atom immersed in a BEC of Dy atoms [39] which possess a magnetic dipole moment of $10\mu_B$. In this work, the relative DDI strength χ is treated as a control parameter that is tunable through the Feshbach resonance. To fix the values of P and Q, we assume a typical trap frequency $\omega_z = 2\pi\times 10^3$ Hz;

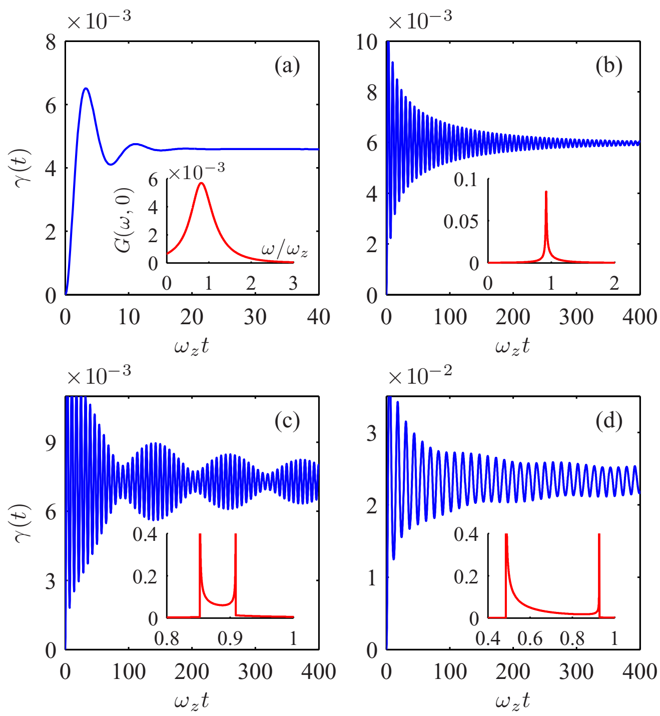

FIG. 2. Time dependence of the dephasing factor for (a) $\chi = 2$, (b) 4.1, (c) 4.6, and (d) 5.4. The insets show the corresponding $G(\omega,0)$ (in units of ω_z^{-1}).

the corresponding harmonic oscillator width is $\ell_B \simeq 2.5\times10^{-5}$ cm. Next, we consider a typical condensate peak density of $10^{14}\,\text{cm}^{-3}$; the area density is then $n = 4.4\times10^{9}\,\text{cm}^{-2}$. Consequently, we have $P \sim 1.4$ if a_B is taken as 5.9 nm [51]. To find Q, we assume that the s-wave scattering length between Rb and Dy atoms is $a_{AB} \sim 5$ nm and the width of the impurity trap ℓ_A equals to ℓ_B; we therefore have $Q \sim 4.6\times10^{-3}$. Without loss of generality, we shall take $P = 2$ and $Q = 4\times10^{-3}$ in the results presented below. It has been verified that changing the values of P and Q will not change our main results qualitatively. In fact, the dephasing factor γ is simply proportional to Q, whose value can be efficiently tuned through the density n, the interspecies scattering length a_{AB}, and the mass of the qubit atom m_A. Finally, for simplicity, we shall consider only the zero-temperature case in this work.

Since the non-Markovianity depends only on γ, let us examine the time dependence of the dephasing factor. Figure 2 plots the typical behaviors of $\gamma(t)$ for $\chi \neq 0$. In the absence of the DDI, $\gamma(t)$ increases monotonically from zero to a steady value in a short period of time, indicating that the system is Markovian. In fact, it was shown that, independent of the contact interaction strength P, the dynamics of the impurity qubit is always Markovian for a 2D environment with $\chi = 0$ [30]. When DDI is switched on, $\gamma(t)$ exhibits very distinct behaviors for different χ's. For an intermediate DDI strength, $\chi = 2$, $\gamma(t)$ starts to show nonmonotonic behavior, but it quickly converges to the asymptotic value after a few oscillations, demonstrating a weak non-Markovianity in the qubit. Remarkably, when χ is close to the critical value χ^*, $\gamma(t)$ becomes a damped oscillating function that oscillates for a very long period of time. More interestingly, for $\chi > \chi^*$, $\gamma(t)$ appears as a damped beat-frequency oscillation, which suggests that it originates from the interference of two frequencies. Finally, under an even stronger DDI, the beat-frequency oscillation is gradually washed out, implying that one of the frequencies starts to dominate.

To understand the behavior of $\gamma(t)$, we plot the corresponding $G(\omega,0)$ in the insets of Fig. 2. As χ increases, the position of the peak of $G(\omega,0)$ moves from $\omega_p = 0$ for $\chi = 0$ to $\omega_p \neq 0$ for $\chi \neq 0$. In particular, $G(\omega,0)$ becomes a sharply peaked function of ω as χ approaches χ^*. Intuitively, a peak of $G(\omega,0)$ located at ω_p indicates that the spectral density is particularly high at ω_p, which picks up the frequency ω_p for $\gamma(t)$ through Eq. (6) and results in an oscillating $\gamma(t)$. To see this more clearly, we approximate $G(\omega,0)$ as a δ function, i.e., $G(\omega,0) \propto \delta(\omega - \omega_p)$, which immediately leads to $\gamma(t) \propto 1 - \cos(\omega_p t)$, an undamped oscillating function of t. The same procedure can also be used to explain the beat-frequency oscillations of $\gamma(t)$ when $\chi > \chi^*$. In fact, the two peaks on $G(\omega,0)$ give rise to two frequency components for $\gamma(t)$ through Eq. (6), which naturally leads to beat. However, in order to explain the damping in $\gamma(t)$, we need to be more careful about how to choose the approximation of $G(\omega,0)$.

Let $k_i(\omega)$ be the roots of the equation $\varepsilon(k) = \hbar\omega$; the reservoir spectral density, Eq. (8), can be explicitly expressed as

$$J(\omega) = Q\hbar\omega_z^3\ell_B^4 \sum_i \frac{f(k_i(\omega))}{\omega}\left|\frac{d\varepsilon(k)}{dk}\right|^{-1}_{k=k_i(\omega)}, \tag{10}$$

where $f(k) \equiv k^3 e^{-\ell_A^2 k^2/2}$. When $\chi > \chi^*$, the excitation spectrum has a local maximum at k_M and a local minimum at k_m; the corresponding excitation energies are $\hbar\omega_M = \varepsilon(k_M)$ and $\hbar\omega_m = \varepsilon(k_m)$, respectively. Based on Eq. (10), $J(\omega)$ diverges at ω_M and ω_m. To accurately take into account the contributions from these singularities to $G(\omega,0)$, let us focus on $\varepsilon(k)$ in the vicinities of k_M where the excitation energy can be approximated as $\varepsilon(k) \approx \hbar\omega_M + \varepsilon''(k_M)(k - k_M)^2/2$. Using Eq. (10), it can then be shown that, in the vicinity of ω_M, we have $G(\omega,0) \approx g_M(\omega_M - \omega)^{-1/2}$ for $\omega < \omega_M$, where $g_M = (2\hbar)^{1/2} Q\omega_z^3\ell_B^4 f(k_M)|\varepsilon''(k_M)|^{-1/2}\omega_M^{-3}$. Similarly, in the vicinity of the local minimum, we have $G(\omega,0) \approx g_m(\omega - \omega_m)^{-1/2}$ for $\omega > \omega_m$, where $g_m = (2\hbar)^{1/2} Q\omega_z^3\ell_B^4 f(k_m)[\varepsilon''(k_m)]^{-1/2}\omega_m^{-3}$. Now, by assuming that these two singularities give rise to the largest contribution to $G(\omega,0)$, we define the function

$$\tilde{G}(\omega,0) = g_M\frac{H(\omega_M - \omega)}{\sqrt{\omega_M - \omega}} + g_m\frac{H(\omega - \omega_m)}{\sqrt{\omega - \omega_m}} \tag{11}$$

as the approximate of $G(\omega,0)$, where $H(x)$ is the Heaviside step function.

To evaluate $\gamma(t)$, we note, from Eq. (6), that it contains two parts: the time-independent part $\gamma_0 = \int_0^\infty d\omega G(\omega,0)$ and the time-dependent part $\gamma_1(t) = -\int_0^\infty d\omega G(\omega,0)\cos(\omega t)$. Since γ_0 diverges with the approximation equation (11), we shall compare only the time-dependent parts $\gamma_1(t)$ and $\tilde{\gamma}_1(t)$, which are evaluated using $G(\omega,0)$ and $\tilde{G}(\omega,0)$, respectively. After some straightforward calculations, we find analytically that

$$\tilde{\gamma}_1(t) \simeq -\frac{\Gamma\left(\frac{1}{2}\right)}{t^{1/2}}\left[g_M\cos\left(\omega_M t - \frac{\pi}{4}\right) + g_m\cos\left(\omega_m t + \frac{\pi}{4}\right)\right], \tag{12}$$

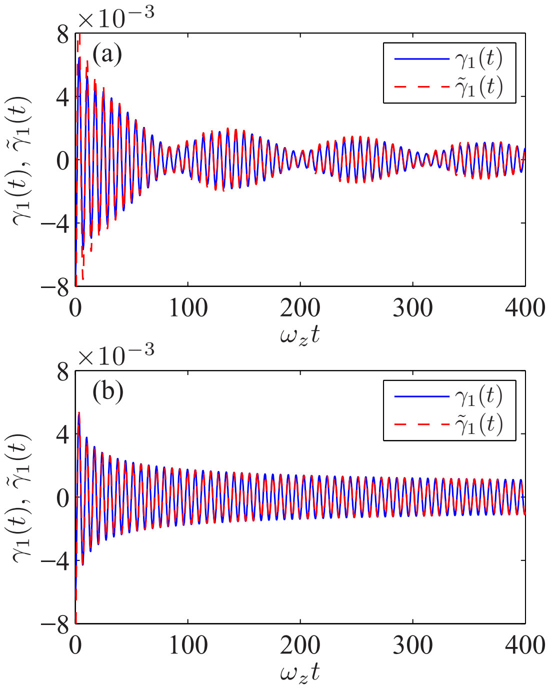

FIG. 3. Comparison of $\gamma_1(t)$ and $\tilde{\gamma}_1(t)$ for (a) $\chi = 4.6$ and (b) $\chi = \chi^*$. Parameters used in (a) are $\omega_z^{1/2} g_M = 4.57\times10^{-3}$, $\omega_M/\omega_z = 0.909$, $\omega_z^{1/2} g_m = 8.84\times10^{-3}$, and $\omega_m/\omega_z = 0.853$ and in (b) are $\omega_z^{1/3} g_I = 4.65\times10^{-3}$ and $\omega_I/\omega_z = 0.912$.

where $\Gamma(\cdot)$ denotes the Gamma function. In Fig. 3(a), we compare $\gamma_1(t)$ for $\chi = 4.6$ with the corresponding $\tilde{\gamma}_1(t)$. The parameters $g_{m,M}$ and $\omega_{m,M}$ used in $\tilde{\gamma}_1(t)$ are all obtained with the given χ using the excitation spectrum equation (2). As can be seen, the agreement is remarkable. In fact, we numerically find that such agreement still holds for $\omega_z t$ over 2000. As the DDI strength is increased, for instance, to $\chi = 5.4$, ω_M (ω_m) is increased (decreased) such that g_m becomes much larger than g_M. As a result, the frequency component ω_m dominates in Eq. (12), and as shown in Fig. 2(d), the beat-frequency oscillation is washed out.

For $\chi < \chi^*$, there does not exist a simple model that can capture the peak of $G(\omega,0)$. However, at $\chi = \chi^*$, $G(\omega,0)$ is singular at the inflection point k_I where $\varepsilon'(k_I) = \varepsilon''(k_I) = 0$ and $\varepsilon'''(k_I) \neq 0$. Let $\hbar\omega_I = \varepsilon(k_I)$; the excitation energy in the vicinity of the inflection point is $\varepsilon(k) \approx \hbar\omega_I + \varepsilon'''(k_I)(k - k_I)^3/6$. It can then be shown that the approximation of $G(\omega,0)$ is

$$\widetilde{G}(\omega,0) = \frac{g_I}{(\omega - \omega_I)^{2/3}}, \tag{13}$$

where $g_I = 3^{-2/3}(2\hbar)^{1/3} Q \omega_z^3 \ell_B^4 f(k_I)[\varepsilon'''(k_I)]^{-1/3}\omega_I^{-3}$. Now it can be easily shown that

$$\tilde{\gamma}_1(t) \simeq -\frac{\Gamma\left(\frac{1}{3}\right) g_I}{t^{1/3}}\left[\cos\left(\omega_I t - \frac{\pi}{6}\right) + \cos\left(\omega_I t + \frac{\pi}{6}\right)\right]. \tag{14}$$

Figure 3(b) compares $\gamma_1(t)$ and $\tilde{\gamma}_1(t)$ for $\chi = \chi^*$, which again exhibits remarkable agreement.

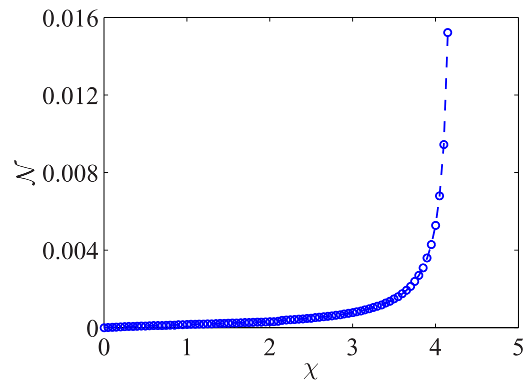

FIG. 4. Non-Markovianity $\mathcal{N}$ versus χ.

It is now clear that the strong oscillation of $\gamma(t)$ is associated with the singularities of $G(\omega,0)$. To gain more insight into its physical origin, we consider the density of states $\rho(\omega) \propto \int_0^\infty dk k\delta[\omega - \varepsilon(k)/\hbar]$, which can be rewritten as

$$\rho(\omega) \propto \sum_i k_i(\omega)\left|\frac{d\varepsilon(k)}{dk}\right|^{-1}_{k=k_i(\omega)}. \tag{15}$$

Apparently, each singularity on $G(\omega,0)$ has a one-to-one correspondence to that on $\rho(\omega)$, which is, in fact, the so-called *Van Hove singularity* [52]. Now, if $\rho(\omega)$ is singular at certain frequencies, the modes corresponding to those frequencies then dominate in the reservoir. As a result, the condensate can be regarded as a highly structured monochromatic or bichromatic reservoir which can lead to the oscillating $\gamma(t)$.

As to the non-Markovianity of the impurity qubit, because $\gamma_1(t)$ decays slower than t^{-1} for $\chi \geqslant \chi^*$, $\mathcal{N}$ must diverge if χ is in the roton spectrum regime. Figure 4 shows the χ dependence of the non-Markovianity at zero temperature for χ up to 4.15 that is less than χ^*. For a slightly larger χ value, we find that it is very difficult to obtain a converged $\mathcal{N}$. As expected, $\mathcal{N}$ increases very quickly when χ approaches χ^*, signaling that it is close to the divergence.

IV. CONCLUSION

In conclusion, we have studied the quantum non-Markovian dynamics of an atomic impurity qubit coupled to a quasi-2D dipolar BEC. It has been shown that, by increasing DDI, the dephasing dynamics of the qubit undergoes a transition from being Markovian to highly non-Markovian such that the non-Markovianity can even diverge. We have also proved that this transition is due to the roton mode softening of the collective excitations, which leads to the Von Hove singularities in the excitation density of states. Our study therefore reveals a wide range of non-Markovian BEC reservoirs that can be tuned efficiently via their rotonlike excitation spectra.

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