Nonlocal correlations in a macroscopic measurement scenario

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Nonlocality is one of the main characteristic features of quantum systems involving more than one spatially separated subsystem. It is manifested theoretically as well as experimentally through violation of some *local realistic* inequality. On the other hand, classical behavior of all physical phenomena in the macroscopic limit gives a general intuition that any *physical* theory for describing microscopic phenomena should resemble classical physics in the macroscopic regime, the so-called macrorealism. In the 2-2-2 scenario (two parties, with each performing two measurements and each measurement having two outcomes), contemplating all the no-signaling correlations, we characterize which of them would exhibit *classical* (*local realistic*) behavior in the macroscopic limit. Interestingly, we find correlations which at the single-copy level violate the Bell-Clauser-Horne-Shimony-Holt inequality by an amount less than the optimal quantum violation (i.e., Cirel'son bound $2\sqrt{2}$), but in the macroscopic limit gives rise to a value which is higher than $2\sqrt{2}$. Such correlations are therefore not considered physical. Our study thus provides a sufficient criterion to identify some of unphysical correlations.

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I. INTRODUCTION

In our everyday experience almost all physical phenomena satisfy the laws of classical physics. However, at the microscopic scale the physical world follows the rules of quantum physics. The description of quantum physics is different from its classical counterpart both conceptually and mathematically [1]. This raises the question of quantum to classical transition, i.e., when and how do the systems stop behaving quantum mechanically and begin to behave classically? Several novel ideas, like collapse models [2], the concept of decoherence [3], etc., were introduced long ago to address these questions. More recently, in a conceptually different approach, it has been shown that under coarse-grained measurements, the classical world arises out of quantum physics [4]. All these studies result in a general dictum that at the macroscopic level, the nonclassical behaviors of quantum theory or any physical theory (possibly postquantum) should subside, and consequently, classicality should emerge. The aim of this paper is to study the emergence of such classical behavior in terms of the strength of correlations for generalized no-signaling theories and identify some of the generalized no-signaling correlations as *unphysical*.

One of the most fundamental contradictions of quantum mechanics (QM) with classical physics is its nonlocal behavior as established by Bell in his 1964 seminal work [5] (see also [6]). Whereas all correlations in the classical world are *local realistic*, correlations obtained from multipartite entangled quantum systems may violate the empirically testable local realistic inequality (called Bell-type inequalities in general) which establishes that such quantum correlations do not allow a *local realistic* explanation. Quantum nonlocality does not contradict the relativistic causality principle or, more generally, the no-signaling principle. Moreover, QM is not the only possible theory that exhibits nonlocality along with satisfying the no-signaling principle; there can be nonquantum no-signaling correlations exhibiting nonlocality. One extreme example of such a correlation (more nonlocal than QM) was constructed by Popescu and Rohrlich (PR) [7]. Whereas the PR correlation violates the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) [8] inequality by algebraic maximum, the optimal Bell-CHSH violation in quantum theory is restricted by Tsirelson's bound [9]. This raises another important question: Which nonlocal correlations are physical? This question is also important from a practical perspective since nonlocality has been proved to be an important resource in numerous applications [10–19]. An endeavor to answer this question was initiated by van Dam, who showed that the existence of superstrong nonlocal correlations (e.g., PR correlation) would trivialize the problem of communication complexity [20]. It may be noted that principles like information causality (IC) [21] and macroscopic locality (ML) [22] do help us towards understanding the physicality of some of the postquantum correlations. Apart from these, other conceptually different proposals have been introduced to single out Tsirelson's bound [23–27]. But, to date, identifying the boundary between quantum correlations and postquantum ones has not been done completely, and it remains an active area of research (see [28]). Here we aim to approach this problem using a macroscopic measurement scheme different from the one used in ML.

In order to study macroscopic properties of a correlation, one must create a measurement scheme using many copies of the correlation where the identities of individual particles involved in the correlation have not been revealed [29]. A practically relevant scheme for studying such macroscopicity of correlations is to consider a case when the identities of the individual particles in the correlations get lost during the distribution of the correlated state. One can, of course, interact microscopically with particles in the correlation, but in general, it is difficult to address them individually [30]. So whatever microscopic interaction one intends to use will affect, in general, all the particles of the beam at the same time. In this context, Bancal *et al.* studied the violation of Bell inequalities of entangled states considering a general multipair scenario [31]. They showed that the nonlocality of the quantum entangled state decreases in this multipair scenario with the increase in the number of independent entangled pairs; that is, in the macroscopic limit of having infinitely many copies of entangled pairs, one cannot get nonlocal correlation. This observation is compatible with the general dictum that classicality emerges at the macroscopic level.

Here, in the simplest scenario, i.e., two parties, with each performing one of the two possible measurements and each measurement having two possible outcomes (i.e., the 2-2-2 scenario), we consider the same approach as that of Bancal *et al.* [31]. But instead of considering only correlations in entangled quantum states, we contemplate general correlations that may be stronger than quantum ones in exhibiting nonlocal behavior yet weak enough to prohibit instantaneous signaling. We characterize *all* such correlations which, in the macroscopic limit, display classicality that is considered, in our context, to be the *local realistic* behavior of the correlations. It is worth mentioning that such classical behavior of any correlation at the macroscopic level is not sufficient to certify the correlation to be perceived in some physical theory; it is rather a necessary criterion. We find examples of such correlations that in the macroscopic level behave classically but do not fulfill other necessary criteria, like *nonlocality distillation* [32–35] or IC [21], and hence cease to be considered physical correlations. Interestingly, on the other hand, we find examples of correlations that indeed satisfy the necessary criteria of IC but at the macroscopic scale exhibit strong nonlocal behavior, going against our general dictum, and hence fail to be considered physical correlations.

At this point it is important to note that the ML principle [22] can also identify unphysical correlations. However, our approach is different from that of ML. In ML, one also considers beams of correlated particles (M in number, with a value much greater than 1, i.e., in the thermodynamic limit), and to make a relationship with classical physics these beams are assumed to be continuous fields. In other words, Alice's and Bob's detectors can perform only coarse-graining measurements; that is, these detectors cannot resolve the beams to their constituent particles (also, this implies that they cannot perform different measurements on different particles; they are able to perform the same interactions on the whole beam). Hence, the resolution of their detectors should not be perfect, and such a poor resolution can provide information about only the mean value. The detectors work with such precision that one could observe the deviations of the intensity fluctuations from the mean value of the order $\sqrt{M}$ because in that case the resultant distributions will be described by classical physics.

On the other hand, very recently, Rohrlich showed that, at the macroscopic scale, PR box correlations violate relativistic causality and hence has no realization in the classical world [36]. Moreover, this result has been generalized to all stronger-than-quantum bipartite correlations, constituting a derivation of Tsirelson's bound without assuming quantum mechanics [36,37]. In Refs. [36,37] the authors showed unphysicality of stronger-than-quantum correlation by showing signaling of those correlations in the macroscopic limit; we do the same but via distillation of nonlocality in the macroscopic limit, and therefore our approach is completely different from that adopted in [36,37]. Furthermore, in our approach some of the correlations having weaker nonlocality than the optimal quantum nonlocality (in the sense that Bell-CHSH violation is strictly less than Tsirelson's bound) turn out to be unphysical.

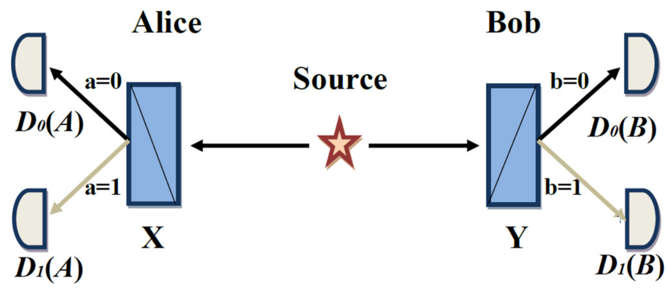

FIG. 1. Single-pair setup: $X,Y \in \{0,1\}$ are Alice's and Bob's measurements, respectively. After the measurement interaction, going through the paths $a=0$ and $a=1$, particles are collected at Alice's detectors $D_0(A)$ and $D_1(A)$, respectively, and similarly on Bob's end at the detectors $D_0(B)$ and $D_1(B)$.

The organization of this paper is as follows: in Sec. II we discuss the setup to study a general bipartite correlation in single-pair and multipair scenarios; in Sec. III we present our results. Section IV contains a comparative discussion of our procedure and other methods. Last, we present our conclusion in Sec. V.

II. SETTING UP THE SCENARIO

A. Single-pair setting

Consider the following bipartite scenario: a particle pair is produced by some source, and two spatially separated experimentalists (say, Alice and Bob) receive one particle each. Alice (Bob) can have one of two interactions, denoted by $X=0,1$ ($Y=0,1$), with her (his) particle. Each interaction results in Alice's (Bob's) particle following one of two possible paths, called outcomes; let us denote these paths by a (b), with $a \in \{0,1\}$ ($b \in \{0,1\}$). Eventually, the particle will impinge on one of Alice's (Bob's) two detectors $D_a(A)$ [$D_b(B)$; see Fig. 1]. Repeating this experiment many times, they can estimate the relative frequencies $P(ab|XY)$, i.e., the probability that Alice's and Bob's outcomes are a and b, respectively, when they apply the interactions X,Y. The joint probabilities $\{P(ab|XY)\}$ form an entire correlation vector. The positivity, normalization, and nonsignaling constraints lead to this correlation vector being a point of an eight-dimensional polytope [38], called the no-signaling polytope $\mathcal{P}_{\rm NS}$. Local correlations are of the form $P(ab|XY) = \int d\lambda \rho(\lambda) P(a|X,\lambda) P(b|Y,\lambda)$, where $P(a|X,\lambda)$ is the probability of getting the outcome a when Alice performs the measurement X given the knowledge of (local hidden) variable λ, $P(b|Y,\lambda)$ is similar for Bob, and $\rho(\lambda)$ is a probability distribution over the variable λ. The collection of all such local correlations forms another polytope $\mathcal{L}$ strictly residing in $\mathcal{P}_{\rm NS}$ with trivial facets determined by positivity constraints and nontrivial facets determined by Bell-CHSH inequalities, which up to relabeling of inputs and outputs read

$$I_{\rm CHSH} := |\langle 00\rangle + \langle 01\rangle + \langle 10\rangle - \langle 11\rangle| \leqslant 2, \qquad (1)$$

where $\langle XY\rangle := \sum_{a,b}(-1)^{a\oplus b} P(ab|XY)$ and $\oplus$ denotes modulo-2 sum. Correlations with the form $P(ab|XY) = \mathrm{Tr}[\rho_{AB}(\Pi_X^a \otimes \Pi_Y^b)]$ are called quantum, where ρ_{AB} is some density operator on some composite Hilbert space and $\{\Pi_X^a\}$ ($\{\Pi_X^a\}$) is some positive operator-valued measure on Alice's

(Bob's) side. The set of quantum correlations $\mathcal{Q}$ forms a convex set (with a continuous boundary) lying strictly between $\mathcal{P}_{\rm NS}$ and $\mathcal{L}$, i.e., $\mathcal{L} \subset \mathcal{Q} \subset \mathcal{P}_{\rm NS}$. There are 24 vertices of the polytope $\mathcal{P}_{\rm NS}$, 16 of which are the extreme points of the polytope $\mathcal{L}$, called local or deterministic vertices, and the remaining 8 are called nonlocal vertices. Since $\sum_{a,b} P(ab|XY) = 1$ (due to normalization), $I_{\rm CHSH}$ can be written as

$$I_{\rm CHSH} = |2 + 2(A_{11} - A_{00} - A_{01} - A_{10})|, \tag{2}$$

with $A_{XY} := P(01|XY) + P(10|XY)$. The deterministic vertices (i.e., the correlations giving deterministic outcomes for all measurements) that saturate inequality (1) are readily seen to be the following ones [39]:

$$\mathcal{D}_1^r = \{P(ab|XY) : a(X) = r, b(Y) = r\}, \tag{3a}$$

$$\mathcal{D}_2^r = \{P(ab|XY) : a(X) = X \oplus r, b(Y) = r\}, \tag{3b}$$

$$\mathcal{D}_3^r = \{P(ab|XY) : a(X) = r, b(Y) = Y \oplus r\}, \tag{3c}$$

$$\mathcal{D}_4^r = \{P(ab|XY) : a(X) = X \oplus r, b(Y) = Y \oplus r \oplus 1\}, \tag{3d}$$

with $r,X,Y \in \{0,1\}$. Any no-signaling correlation can be expressed as a convex mixture of local correlations and a single extremal nonlocal point on top of each CHSH facet, with the representative defined as

$$\mathcal{C}_{\rm PR} \equiv P(ab|XY) := \begin{cases} \frac{1}{2} & \text{if } a \oplus b = XY, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

This is called PR correlation (PR box), as introduced by Popescu and Rohrlich [7]. Any no-signaling correlation $P_{\rm NS} \equiv \{P(ab|XY)\}$ can be written as [39]

$$P_{\rm NS} = C_1\mathcal{D}_1^0 + C_2\mathcal{D}_1^1 + C_3\mathcal{D}_2^0 + C_4\mathcal{D}_2^1 + C_5\mathcal{D}_3^0 + C_6\mathcal{D}_3^1 + C_7\mathcal{D}_4^0 + C_8\mathcal{D}_4^1 + C_9\mathcal{C}_{\rm PR}, \tag{5}$$

with $0 \leqslant C_i \leqslant 1\, \forall\, i$ and $\sum_{i=1}^9 C_i = 1$. Such a correlation $P_{\rm NS}$ is nonlocal iff $P_{\rm NS} \in \mathcal{P}_{\rm NS}$ but $P_{\rm NS} \notin \mathcal{L}$.

In the following section we will consider different special forms of no-signaling (NS) correlations (5) and discuss the violation of the Bell-CHSH inequality (1) in the multipair setting of these correlations.

B. Multipair setting

Consider that a source produces M independent identical pairs, or, equivalently, M independent sources, each producing one and the same pair. Alice and Bob each receive a beam of M particles (we are assuming that there is no particle loss). Although the pairs are created independently, in experiment, it is very hard to address them individually (as already discussed earlier). Alice and Bob perform a measurement on the beam of particles they received; that is, they interact with *all* the particles in same manner as earlier (Alice performs measurement $X \in \{0,1\}$ on *all* the particles she receives, and similarly, Bob performs $Y \in \{0,1\}$). However, during the interaction the classical information about the identity of the individual pair is lost; that is, it is not possible to identify a correlated pair from the beam of particles (see Fig. 2). Let the correlation of each pair be $P_{\rm NS} = \{P(ab|XY)\}$, and let us denote the global correlation for M pairs as $P_M \equiv P_{\rm NS}^M$. The number of particles collected in two detectors (one each on

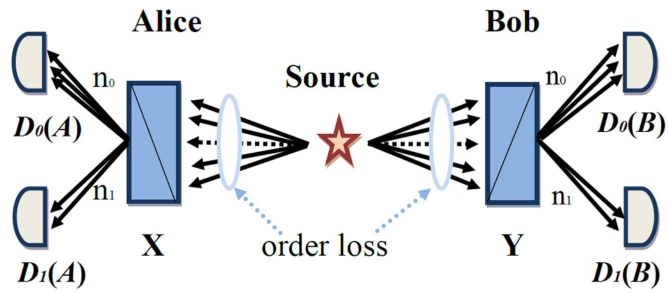

FIG. 2. Multipair setup: Source produces M independent pairs of particles. Since information about ordering between Alice's and Bob's particles is lost during their transmission, they address the beams of particles as a whole. A particle gets in the detector $D_s(\kappa)$ if $s = a \in \{0,1\}$ ($s = b \in \{0,1\}$) is the outcome in the measurement $X \in \{0,1\}$ ($Y \in \{0,1\}$) on $\kappa = A$, i.e., Alice's particle ($\kappa = B$, i.e., Bob's particle). A few particles (n_0 in number) are collected at the detector $D_0(\kappa)$, and the rest are collected at the detector $D_1(\kappa)$, where $\kappa = A,B$.

Alice's side and Bob's side) are counted, and n_0 and n_1 are the number of particles counted in the two detectors. For perfectly efficient detectors ($\eta = 1$), one has $M = n_0 + n_1$. Our aim is to study the nonlocal strength (particularly, the amount of Bell-CHSH inequality violation) of the global correlation P_M. For this purpose, Alice and Bob must transform their data into a binary input-output correlation which we denote in bold letters, i.e., $\{P(\mathbf{ab}|\mathbf{XY})\}$, where $\mathbf{a},\mathbf{b},\mathbf{X},\mathbf{Y} \in \{\mathbf{0},\mathbf{1}\}$. Here Alice's and Bob's interactions are denoted by bold letters, $\mathbf{X}$ and $\mathbf{Y}$, respectively, which imply that they apply the same interaction X or Y on each particle of the incoming beam of particles. One can get binary outputs by invoking any of the following voting procedures: (a) majority voting, (b) unanimous voting, or (c) any intermediate possibility. According to majority voting, if the number of particles collected in detector $D_0(\kappa)$, i.e., n_0, is greater than or equal to the number of particles collected in detector $D_1(\kappa)$, then the outcome will be denoted as $\mathbf{0}$; otherwise, the outcome will be denoted as $\mathbf{1}$:

$$\text{Majority voting} \Rightarrow \begin{cases} n_0 \geqslant n_1 \;\rightarrow \mathbf{0}, \\ \text{otherwise} \rightarrow \mathbf{1}. \end{cases} \tag{6}$$

Thus from M independent identical pairs of correlations majority voting gives a binary input-output probability distribution $\{P(\mathbf{ab}|\mathbf{XY})\}$. Instead of majority voting one can also follow voting procedure (b) or (c). However, here our aim is to follow a voting procedure which may exhibit nonlocal behavior of the given binary input-output correlation even in the macroscopic limit. It may happen that a correlation becomes local in the macroscopic limit under a particular voting protocol, whereas the same correlation exhibits nonlocal behavior under another voting protocol. It has been shown in [31] (see [40,41] for experiments that consider majority voting) that majority voting yields the largest violation, and we also checked that the PR correlation retains its nonlocal behavior in the macroscopic limit under majority voting, while it becomes local under other voting protocols. For this reason, we consider here the majority voting for our study. In fact, if a NS correlation turns out to be nonlocal in the macroscopic limit under any one of the above-mentioned voting procedures (or even by using some other counting method in the macroscopic scenario), it will be enough, according to the notion of macrorealism, to discard such a correlation as a physical one.

III. CORRELATION IN THE MULTIPAIR SETTING

First, we will consider the PR correlation and then arbitrary no-signaling correlations.

A. PR correlation

Before considering the general case of M independent pairs, let us first assume that a source emits two independent pairs of particles, each being correlated according to the PR correlation of Eq. (4). Alice (Bob) performs the *same* measurement, either $X=0$ or $X=1$ ($Y=0$ or $Y=1$), on both particles she (he) receives. After the measurement they count the number of particles detected on their detectors $D_0(\kappa)$ and $D_1(\kappa)$, $\kappa=A,B$. Then, according to the majority-voting condition, they declare their output to be either $\mathbf{0}$ or $\mathbf{1}$ and thus prepare the new binary input-output probability distribution $P(\mathbf{ab}|\mathbf{XY})$. For example, let us consider that both Alice and Bob perform measurement $X=Y=0$ on each particle of their respective beams. The particles can be collected in the detectors in one of the following three ways (see Fig. 2 for reference):

(i) On Alice's side both particles are detected in the $D_0(A)$ detector. Due to strict correlation of the PR box [see Eq. (4)] both particles on Bob's side will also be detected in the detector $D_0(B)$. According to the majority vote, both Alice and Bob declare their output to be $\mathbf{0}$, i.e., $\mathbf{a}=\mathbf{b}=\mathbf{0}$. The probability of occurrence of this case is $P(\mathbf{a}=\mathbf{0},\mathbf{b}=\mathbf{0}|\mathbf{X}=\mathbf{0},\mathbf{Y}=\mathbf{0}) = 2!P^2(00|00)/2!$. For PR correlation, $P(00|00)=1/2$.

(ii) On Alice's side both particles are detected in detector $D_1(A)$. Due to a similar argument both Alice and Bob declare their output to be $\mathbf{1}$, i.e., $\mathbf{a}=\mathbf{b}=\mathbf{1}$, and the probability $P(\mathbf{a}=\mathbf{1},\mathbf{b}=\mathbf{1}|\mathbf{X}=\mathbf{0},\mathbf{Y}=\mathbf{0})$ of this case occurring is $2!P^2(11|00)/2!$, where $P(11|00)=1/2$ for the PR correlation.

(iii) On Alice's side one particle is detected in detector $D_0(A)$, and the other is detected in detector $D_1(A)$. Due to strict correlation [see Eq. (4)], the same is true on Bob's side. The majority-voting condition allows them to declare their output to be $\mathbf{0}$. The probability of this case occurring is $P(\mathbf{a}=\mathbf{0},\mathbf{b}=\mathbf{0}|\mathbf{X}=\mathbf{0},\mathbf{Y}=\mathbf{0}) = 2!P(00|00)P(11|00)/(1!)^2$.

Thus the new probability distribution for the measurement setting $\mathbf{XY}=\mathbf{00}$ (i.e., $X=0$ for both Alice's particles and $Y=0$ for both Bob's particles) reads

$$P(\mathbf{00}|\mathbf{00}) = 2!\left[\frac{P^2(00|00)}{2!} + \frac{P(00|00)P(11|00)}{(1!)^2}\right],$$

$$P(\mathbf{01}|\mathbf{00}) = P(\mathbf{10}|\mathbf{00}) = 0, \quad P(\mathbf{11}|\mathbf{00}) = 2!\left[\frac{P^2(11|00)}{2!}\right].$$

For the measurement settings $XY=01$, the corresponding *new* probability distribution has the form

$$P(\mathbf{00}|\mathbf{01}) = 2!\left[\frac{P^2(00|01)}{2!} + \frac{P(00|01)P(11|01)}{(1!)^2}\right],$$

$$P(\mathbf{01}|\mathbf{01}) = P(\mathbf{10}|\mathbf{01}) = 0, \quad P(\mathbf{11}|\mathbf{01}) = 2!\left[\frac{P^2(11|01)}{2!}\right],$$

and the case is similar for $XY=10$. But for $XY=11$ we have

$$P(\mathbf{00}|\mathbf{11}) = 2!\left[\frac{P(01|11)P(10|11)}{(1!)^2}\right], \quad P(\mathbf{11}|\mathbf{11}) = 0,$$

$$P(\mathbf{01}|\mathbf{11}) = 2!\left[\frac{P^2(01|11)}{2!}\right], \quad P(\mathbf{10}|\mathbf{11}) = 2!\left[\frac{P^2(10|11)}{2!}\right].$$

To obtain the CHSH value of this new probability distribution we calculate $A^{(2)}_{\mathbf{XY}} = P(\mathbf{01}|\mathbf{XY}) + P(\mathbf{10}|\mathbf{XY})$, which in this case become

$$A^{(2)}_{\mathbf{00}} = A^{(2)}_{\mathbf{01}} = A^{(2)}_{\mathbf{10}} = 0,$$
$$A^{(2)}_{\mathbf{11}} = 2!\left[\frac{P^2(01|11)}{2!} + \frac{P^2(10|11)}{2!}\right].$$

Here the superscript denotes the number of independent pairs used in the experiment. Hence, according to Eq. (2), we have

$$\mathbf{I}^{(2)}_{\text{CHSH}} = 2 + 2\big(A^{(2)}_{\mathbf{11}} - A^{(2)}_{\mathbf{00}} - A^{(2)}_{\mathbf{01}} - A^{(2)}_{\mathbf{10}}\big) \tag{7a}$$
$$= 2 + 2A^{(2)}_{\mathbf{11}}. \tag{7b}$$

For a source emitting M (let M be even) independent pairs of particles, with each paired in the PR correlation, a similar analysis gives

$$A^{(M)}_{\mathbf{11}} = M!\sum_{j=0}^{(\frac{M}{2}-1)} \frac{1}{(M-j)!j!}[\beta^{(M-j)}\delta^j + \beta^j\delta^{(M-j)}]$$
$$= (\beta+\delta)^M - \frac{M!}{(M/2)!}(\beta\delta)^{M/2},$$
$$A^{(M)}_{\mathbf{00}} = A^{(M)}_{\mathbf{01}} = A^{(M)}_{\mathbf{10}} = 0, \tag{8}$$

where $\beta := P(01|11) = 1/2 = P(10|11) =: \delta$. Figure 3 (with $\beta=0.5$) shows that at large M the value of $A^{(M)}_{\mathbf{11}}$ gets close to unity [42], which further implies that $\mathbf{I}^{(M)}_{\text{CHSH}} = 2 + 2A^{(M)}_{\mathbf{11}} \cong 4$ for large M; that is, it reaches the maximum algebraic value of the CHSH inequality. Therefore in the macroscopic limit, under majority voting, the PR correlation does not exhibit classical (more precisely, local) behavior and hence fails to be considered a physical correlation.

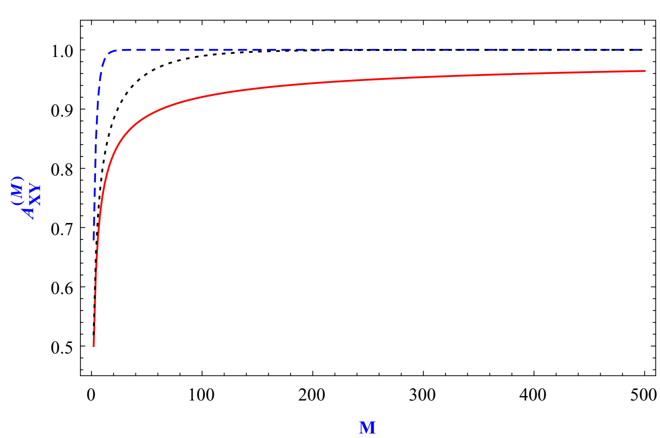

FIG. 3. $A^{(M)}_{\mathbf{XY}}$ for the probability distribution $\mathcal{P} := \{P(00|XY), P(01|XY), P(10|XY), P(11|XY)\} = (0,\beta,\delta,0)$. Solid red curve: $\beta=0.5$; dotted black curve: $\beta=0.4$; dashed blue curve: $\beta=0.8$.

B. Noisy correlation

Before going into explicit examples of NS correlations of Eq. (5), we first consider different representative cases and study how $A_{\mathbf{XY}}^{(M)}$ get modified in the macroscopic limit. Here, in all the special cases discussed below, the joint probabilities $P(ab|XY)$ are prescribed for all $XY \in \{00,01,10,11\}$.

Case 1. For some particular measurement setting $XY \in \{00,01,10,11\}$, let the probability distribution be

$$P(00|XY)=\alpha,\quad P(01|XY)=0,$$
$$P(10|XY)=0,\quad P(11|XY)=\gamma, \tag{9}$$

with $0 \leqslant \alpha,\gamma \leqslant 1, \alpha+\gamma=1$. From the above discussion (this case is analogous to the case $XY=00$ of the PR box) it is evident that with majority voting, $A_{\mathbf{XY}}^{(M)}=0$ for an arbitrary number of pairs M.

Case 2. For the measurement setting XY the probability distribution reads

$$P(00|XY)=0,\quad P(01|XY)=\beta,$$
$$P(10|XY)=\delta,\quad P(11|XY)=0, \tag{10}$$

with $0 \leqslant \beta,\delta \leqslant 1, \beta+\delta=1$. An analysis similar to the PR scenario indicates that $A_{\mathbf{XY}}^{(M)}$ look identical to Eq. (8). For different values of β, the variation of $A_{\mathbf{XY}}^{(M)}$ with increasing M under majority voting is shown in Fig. 3 (with $\beta=0.8$ and $\beta=0.4$), where it is evident that $A_{\mathbf{XY}}^{(M)}$ approaches unity at the large-M limit.

Case 3. Let the probability distribution read

$$P(00|XY)=\alpha,\quad P(01|XY)=\beta,$$
$$P(10|XY)=\delta,\quad P(11|XY)=0, \tag{11}$$

with $0 \leqslant \alpha,\beta,\delta \leqslant 1, \alpha+\beta+\delta=1$. In this case we have

$$A_{\mathbf{XY}}^{(M)} = M! \sum_{k=0}^{(\frac{M}{2}-1)} \sum_{j=0}^{(\frac{M}{2}-k-1)} \frac{\alpha^k}{k!\,j!\,(M-k-j)!} \times [\beta^{(M-k-j)}\delta^j + \beta^j\delta^{(M-k-j)}]. \tag{12}$$

For different choices of β,δ, variations of $A_{\mathbf{XY}}^{(M)}$ with M are plotted in Fig. 4, where it is evident that $A_{\mathbf{XY}}^{(M)}$ approaches zero for large M.

Case 4. Here we have

$$P(00|XY)=0,\quad P(01|XY)=\beta,$$
$$P(10|XY)=\delta,\quad P(11|XY)=\gamma, \tag{13}$$

with $0 \leqslant \beta,\delta,\gamma \leqslant 1, \beta+\delta+\gamma=1$. In this case we get

$$A_{\mathbf{XY}}^{(M)} = M! \sum_{k=0}^{\frac{M}{2}} \sum_{j=0}^{(\frac{M}{2}-k-1)} \frac{\gamma^k}{k!\,j!\,(M-k-j)!} \times [\beta^{(M-k-j)}\delta^j + \beta^j\delta^{(M-k-j)}], \tag{14}$$

which is plotted in Fig. 5 and also resembles the behavior in case 3.

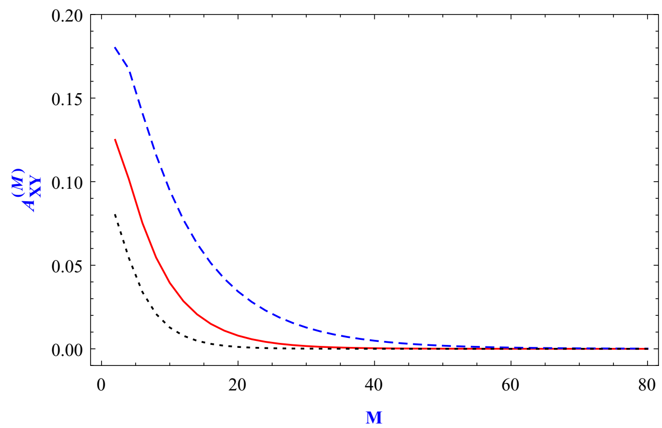

FIG. 4. $A_{\mathbf{XY}}^{(M)}$ for the probability distribution $\mathcal{P}=(\alpha,\beta,\delta,0)$. $\beta=\delta$ and $\alpha=0.4$ (dotted black curve), $\alpha=0.5$ (solid red curve), and $\alpha=0.6$ (dashed blue curve).

Case 5. The probability distribution is given by

$$P(00|XY)=\alpha,\quad P(01|XY)=0,$$
$$P(10|XY)=\delta,\quad P(11|XY)=\gamma, \tag{15}$$

with $0 \leqslant \alpha,\delta,\gamma \leqslant 1, \alpha+\delta+\gamma=1$. Here we have

$$A_{\mathbf{XY}}^{(M)} = M! \sum_{k=0}^{(\frac{M}{2}-1)} \sum_{j=0}^{(\frac{M}{2}-k)} \left[\frac{\alpha^k\delta^{(M-k-j)}\gamma^j}{k!\,j!\,(M-k-j)!} + \$ \sum_{n=j+1}^{\frac{M}{2}} \frac{\alpha^k\delta^{(M-k-n)}\gamma^n}{k!\,n!\,(M-k-n)!} \right], \tag{16}$$

where $\$=1$ when $k+j=\frac{M}{2}$ and otherwise $\$=0$. $A_{\mathbf{XY}}^{(M)}$ is plotted in Fig. 6, where $A_{\mathbf{XY}}^{(M)}$ approaches 1 for large M.

Case 6. The probability distribution reads

$$P(00|XY)=\alpha,\quad P(01|XY)=\beta,$$
$$P(10|XY)=0,\quad P(11|XY)=\gamma, \tag{17}$$

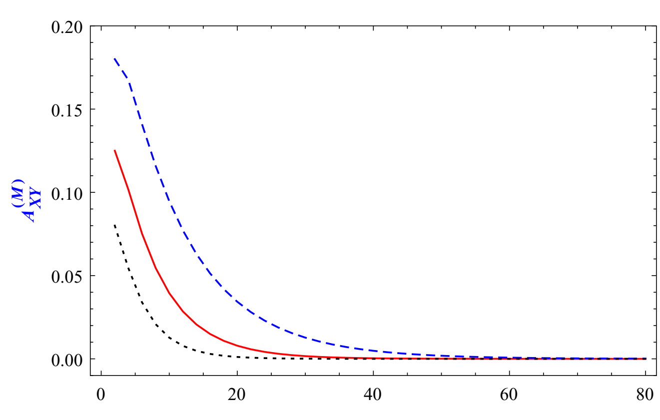

FIG. 5. $A_{\mathbf{XY}}^{(M)}$ for the probability distribution $\mathcal{P}=(0,\beta,\delta,\gamma)$. $\beta=\delta$ and $\gamma=0.6$ (dotted black curve), $\gamma=0.5$ (solid red curve), and $\gamma=0.4$ (dashed blue curve).

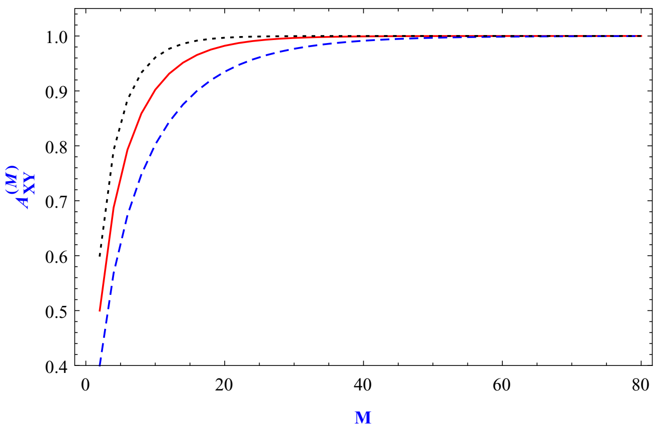

FIG. 6. $A^{(M)}_{\mathbf{XY}}$ for the probability distribution $\mathcal{P} = (\alpha,0,\delta,\gamma)$. $\alpha = \gamma$ and $\alpha = 0.2$ (dotted black curve), $\alpha = 0.25$ (solid red curve), and $\alpha = 0.3$ (dashed blue curve).

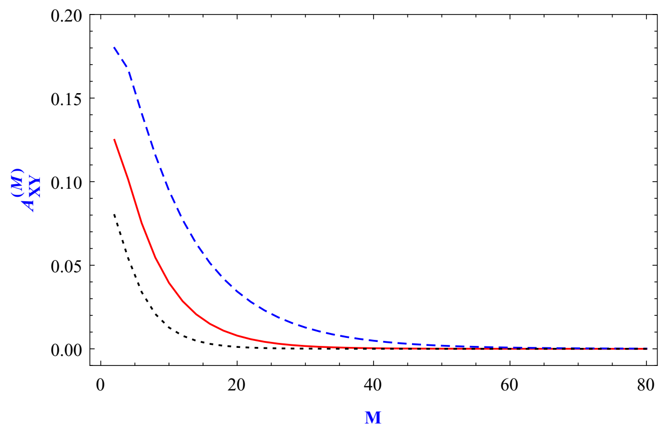

FIG. 7. $A^{(M)}_{\mathbf{XY}}$ for the probability distribution $\mathcal{P} = (\alpha,\beta,\delta,\gamma)$. $\alpha = \gamma, \beta = \delta$ and $\alpha = 0.2$ (dashed blue curve), $\alpha = 0.25$ (solid red curve), and $\alpha = 0.3$ (dotted black curve).

with $0 \leqslant \alpha,\beta,\gamma \leqslant 1, \alpha+\beta+\gamma = 1$. In this case we get

$$A^{(M)}_{\mathbf{XY}} = M! \sum_{k=0}^{(\frac{M}{2}-1)} \sum_{j=0}^{(\frac{M}{2}-k)} \left[\frac{\alpha^k \beta^{(M-k-j)} \gamma^j}{k!j!(M-k-j)!} + \$ \sum_{n=j+1}^{\frac{M}{2}} \frac{\alpha^k \beta^{(M-k-n)} \gamma^n}{k!n!(M-k-n)!} \right], \tag{18}$$

where $\$ = 1$ when $k + j = \frac{M}{2}$ and otherwise $\$ = 0$. In this case $A^{(M)}_{\mathbf{XY}}$ looks similar to case 5, but δ is replaced by β.

Case 7. The probability distribution is given by

$$P(00|XY) = \alpha, \quad P(01|XY) = \beta,$$
$$P(10|XY) = \delta, \quad P(11|XY) = \gamma, \tag{19}$$

with $0 \leqslant \alpha,\beta,\delta,\gamma \leqslant 1, \alpha+\beta+\delta+\gamma = 1$. In this case we have

$$A^{(M)}_{\mathbf{XY}} = \sum_{k_1=0}^{(\frac{M}{2}-1)} \sum_{k_2=0}^{\frac{M}{2}} \sum_{j=0}^{(\frac{M}{2}-k_1-k_2-1)} \frac{M!\alpha^{k_1}\gamma^{k_2}}{k_1!k_1!j!(M-k_1-k_2-j)!} \times [\beta^{(M-k_1-k_2-j)}\delta^j + \beta^j\delta^{(M-k_1-k_2-j)}], \tag{20}$$

which is plotted in Fig. 7, from which it is evident that $A^{(M)}_{\mathbf{XY}}$ approaches zero for large M and, consequently, $\mathbf{I}^{(M)}_{\text{CHSH}}$ becomes 2.

We are now in a position to consider some particular nonlocal correlations and thereby test their CHSH values in the macroscopic measurement scenario.

C. Different classes of no-signaling correlations

In this section we will study the nonlocal strengths of different representative classes of no-signaling correlations in the macroscopic measurement setting.

Class I. Let the no-signaling probability distribution [see Eq. (5)] be given by

$$P_{\text{NS}} = C_9\mathcal{C}_{\text{PR}} + C_1\mathcal{D}^0_1 := p\mathcal{C}_{\text{PR}} + (1-p)\mathcal{D}^0_1, \tag{21}$$

with $0 < C_9(:=p) < 1$. The CHSH value of this correlation is $I_{\text{CHSH}} = 2 + 2p$. Let the source emit M independent pairs of this nonlocal correlation. The joint outcome distributions for the measurement settings $XY = 00,01,10$ are of the form $P(00|XY) = 1 - p/2, P(01|XY) = P(10|XY) = 0$ and $P(11|XY) = p/2$, which is similar to case 1 discussed in Sec. III B. So, according to majority voting, in the macroscopic measurement scenario $A^{(M)}_{\mathbf{XY}} = 0$ for the large-M limit, with $\mathbf{XY} \in \{\mathbf{00},\mathbf{01},\mathbf{10}\}$. For the measurement setting $XY = 11$, the probability distribution will be of the form $P(00|11) = 1 - p$, $P(01|11) = P(10|11) = p/2, P(11|11) = 0$, similar to case 3 in Sec. III B, and hence $A^{(M)}_{\mathbf{11}} = 0$ for large M. The CHSH value of the microscopic correlation thus becomes $\mathbf{I}^{(M)}_{\text{CHSH}} = 2$. Hence the original microscopic nonlocal correlation becomes local in the macroscopic limit. The same is true for the correlation $P_{\text{NS}} = p\mathcal{C}_{\text{PR}} + (1-p)\mathcal{D}^1_1$.

Class II. Let the no-signaling probability distribution be of the form

$$P_{\text{NS}} = p\mathcal{C}_{\text{PR}} + (1-p)\mathcal{D}^0_2. \tag{22}$$

Here also the CHSH value is $I_{\text{CHSH}} = 2 + 2p$. The outcome probability distribution for the measurement settings $XY = 00,01$ will be of the form $P(00|XY) = 1 - p/2$, $P(01|XY) = P(10|XY) = 0, P(11|XY) = p/2$, similar to case 1 in Sec. III B, which implies $A^{(M)}_{\mathbf{00}} = A^{(M)}_{\mathbf{01}} = 0$ for large M. For the measurement setting 10, the probability distribution is $P(00|01) = P(11|01) = p/2, P(01|10) = 0$, $P(10|01) = (1-p)$, which is identical to case 5 and hence implies $A^{(M)}_{\mathbf{10}} = 1$. For the measurement setting 11, the probability distribution $P(00|11) = 0, P(01|11) = p/2, P(10|XY) = 1 - p/2, P(11|11) = 0$ is similar to case 2, and hence $A^{(M)}_{\mathbf{11}} = 1$. Thus for large M, the CHSH value turns out to be

$$\mathbf{I}^{(M)}_{\text{CHSH}} = 2 + 2\left(A^{(M)}_{\mathbf{11}} - A^{(M)}_{\mathbf{10}} - A^{(M)}_{\mathbf{01}} - A^{(M)}_{\mathbf{00}}\right) = 2. \tag{23}$$

A similar conclusion holds well for the correlations of the forms $P_{\text{NS}} = p\mathcal{C}_{\text{PR}} + (1-p)\mathcal{D}^1_2$ and $P_{\text{NS}} = p\mathcal{C}_{\text{PR}} + (1-p)\mathcal{D}^r_s$ for $s = 3,4$ and $r = 0,1$.

Class III. Let the probability distribution be given by

$$P_N S = p_1\mathcal{C}_{\text{PR}} + p_2\mathcal{D}^0_1 + p_3\mathcal{D}^1_1, \tag{24}$$

with $0 < p_i < 1$, $\sum p_i = 1$, and the CHSH value is $I_{\text{CHSH}} = 2 + 2p_1$. The outcome probability distribution for the measurement settings $XY = 00,01,10$ is of the form of case 1, and for $XY = 11$, it is of the form of case 7 in Sec. III B, implying $A^{(M)}_{\mathbf{XY}} = 0$ for all $\mathbf{XY}$. This further implies that $\mathbf{I}^{(M)}_{\text{CHSH}} = 2$ for large M.

Class IV. Let the probability distribution be of the form

$$P_{\text{NS}} = p_1\mathcal{C}_{\text{PR}} + p_2\mathcal{D}^0_2 + p_3\mathcal{D}^1_2, \tag{25}$$

with $0 < p_i < 1$, $\sum p_i = 1$, and the CHSH value is therefore $I_{\text{CHSH}} = 2 + 2p_1$. For the measurement settings $XY = 00,01$ the outcome distribution will have a form similar to case I, while for $XY = 10$ it will resemble case 7 in Sec. III B, and thus this implies $A^{(M)}_{\mathbf{XY}} = 0$ for measurement settings $\mathbf{XY} \in \{\mathbf{00},\mathbf{01},\mathbf{10}\}$ for large M. On the other hand, for the measurement setting $XY = 11$, the outcome distribution will be of the form of case 2 in Sec. III B, implying $A^{(M)}_{\mathbf{11}} = 1$. This further indicates that at large M we have $\mathbf{I}^{(M)}_{\text{CHSH}} = 4$. A similar conclusion holds well for the correlations belonging to the classes $P_{\text{NS}} = p_1\mathcal{C}_{\text{PR}} + p_2\mathcal{D}^0_s + p_3\mathcal{D}^1_s$ with $s = 3,4$. Therefore, for these classes of correlations, the original *weak* microscopic nonlocality becomes maximally nonlocal in the macroscopic limit under the majority-voting condition.

Class V. Let the probability distribution be given by

$$P_{\text{NS}} = p_1\mathcal{C}_{\text{PR}} + p_2\mathcal{D}^0_1 + p_3\mathcal{D}^0_2 + p_4\mathcal{D}^0_3 + p_5\mathcal{D}^0_4, \tag{26}$$

with $0 < p_i < 1$, $\sum p_i = 1$, and $I_{\text{CHSH}} = 2 + 2p_1$.The outcome probability distribution for the measurement settings $XY = 00,01$ is similar to case 6, while for the measurement settings $XY = 10$ and $XY = 11$, they are similar to case 5 and case 3 in Sec. III B, respectively. So for large M we have

$$\mathbf{I}^{(M)}_{\text{CHSH}} = 2 + 2\big(A^{(M)}_{\mathbf{XY}} - A^{(M)}_{\mathbf{XY}} - A^{(M)}_{\mathbf{XY}} - A^{(M)}_{\mathbf{XY}}\big) = -4. \tag{27}$$

Thus, in this case also the original *weak* microscopic nonlocal correlations become maximally nonlocal (i.e., CHSH value of 4) in the macroscopic limit according to the majority-voting condition. A similar result holds for the other correlations of the forms $P_{\text{NS}} = p_1\mathcal{C}_{\text{PR}} + p_2\mathcal{D}^r_1 + p_3\mathcal{D}^t_2 + p_4\mathcal{D}^u_3 + p_5\mathcal{D}^v_4$, with $r,t,u,v \in \{0.1\}$.

Along the lines of the aforementioned analysis one can consider *any* 2-2-2 NS correlation of Eq. (5) and can find its nonlocal strength in the macroscopic limit.

IV. UNPHYSICAL CORRELATIONS: NONLOCALITY DISTILLATION AND INFORMATION CAUSALITY

If one *believes* that nature does not allow one to perform all distributed computations with a trivial amount of communication or one *believes* in the principle that the amount of information that an observer (say, Bob) can gain about a data set belonging to another observer (say, Alice) using all of his local resources (which may be correlated with her resources) and using classical communication obtained from Alice is bounded by the information volume of the communication, then, under the these beliefs, not all no-signaling correlations can be considered physical. In this context, the *nonlocality-distillation* and *information-causality* principles are two well-known tests to determine whether a given no-signaling correlation is unphysical.

Nonlocality distillation. This idea was proposed by Forster *et al.* [32]. Starting with several copies of a nonlocal box with a given CHSH value (greater than 2), it is possible via wiring (classical circuitry to produce a new binary-input–binary-output box, or, in other words, postprocessing of the data without any communication) to obtain a final box which has a larger CHSH value. Using this idea, in Ref. [34], the authors identified a specific class of postquantum nonlocal boxes that make communication complexity trivial, and therefore such correlations are very unlikely to exist in nature. In our analysis, we find that correlations belonging to classes I and II in Sec. III C are local in the macroscopic measurement scenario under majority voting. However, as shown in [33], these correlations can be distilled arbitrarily close to the maximally nonlocal correlation, implying *trivial communication complexity*; hence such correlations are considered to be unphysical (according to the aforesaid belief).

Information Causality. Pawlowski *et al.* proposed the principle of information causality (IC) as a generalization of the no-signaling principle. It can be formulated quantitatively through an information-processing game played between two parties [21]. If Alice communicates m bits to Bob, the total information obtainable by Bob, using all his local resources (which may be correlated with Alice's resources) and the classical communications from her, cannot be greater than m. For $m = 0$, IC reduces to the standard no-signaling principle. Both classical and quantum correlations have been proved to satisfy the IC principle. Furthermore, it has been shown that if Alice and Bob share arbitrary two-input and two-output no-signaling correlations, then by applying the protocol of van Dam [20] and Wolf *et al* [43], one can derive a necessary condition for respecting the IC principle, which can be expressed as

$$E_1^2 + E_2^2 \leqslant 1, \tag{28}$$

where $E_i = 2Q_i - 1$ for $i = 1,2$ and $Q_1 = \frac{1}{2}[P(a = b|00) + P(a = b|10)]$, $Q_2 = \frac{1}{2}[P(a = b|01) + P(a \neq b|11)]$.

For the probability distributions belonging to class V in Sec. III C, we have $E_1 = 1 - (p_3 + p_5)$ and $E_2 = 1 - (p_2 + p_4)$. The necessary condition of IC thus implies

$$p_1^2 - 2(p_3 + p_5)(p_2 + p_4) \leqslant 0; \tag{29}$$

that is, the probability distributions belonging to class V in Sec. III C will satisfy the necessary condition of IC as long as the function $F := p_1^2 - 2(p_3 + p_5)(p_2 + p_4)$ is not positive. Since the Bell-CHSH expression for the probability distributions belonging to class V is $2 + 2p_1$, they violate Tsirelson's bound if $p_1 > \sqrt{2} - 1$ and hence are not quantum. So we are interested in the range $0 \leqslant p_1 \leqslant \sqrt{2} - 1$. Now letting $y = p_3 + p_5$ (clearly, $0 \leqslant y \leqslant 1$) and using the probability normalization condition, i.e., $p_1 + p_2 + p_3 + p_4 + p_5 = 1$, we get

$$F = p_1^2 - 2y + 2p_1 y + 2y^2. \tag{30}$$

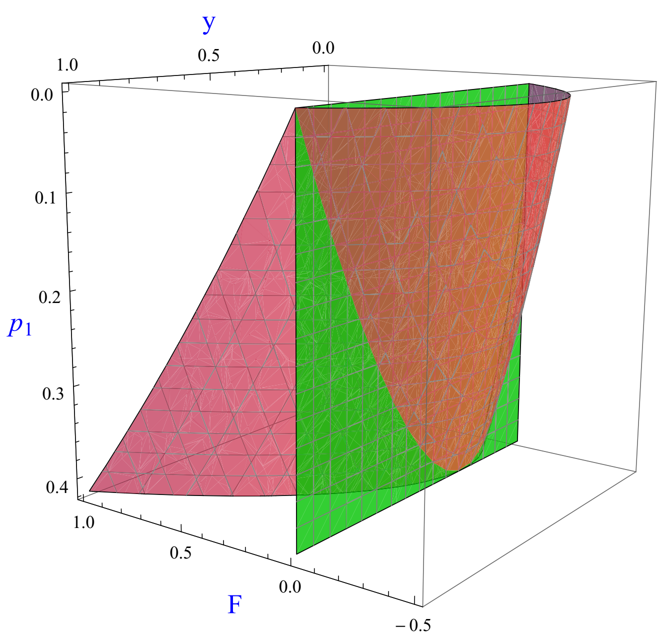

FIG. 8. The red surface represents the function $F(p_1,y)$ in Eq. (30). The green surface represents an $F=0$ surface. The points (p_1,y) on the right side of the green surface satisfy the necessary condition of IC.

We plot the function $F(p_1,y)$ in Fig. 8, which shows that, in the ranges of parameter p_1 (i.e., $0\leqslant p_1\leqslant\sqrt{2}-1$) that interest us, there exist correlations which satisfy the necessary condition of IC. Therefore the necessary condition of IC fails to identify those correlations as unphysical. However, our earlier analysis points them out as unphysical ones since these correlations show extreme nonlocal behavior (i.e., Bell-CHSH value of 4) and hence fail to exhibit the expected classical feature (i.e., the local behavior of the correlation) in the macroscopic limit even though at the single-copy level they do not violate Tsirelson's bound.

V. CONCLUDING REMARKS

Identifying the set of all quantum correlations is a very important problem in the research area of quantum foundation. This also has practical relevance since nonlocal correlations are resources for various device-independent tasks. In the last few years, different approaches, based on information-theoretic or physical principles, have been proposed to identify the quantum correlations [21,22]. Whereas in [21] the authors introduced an information-theoretic principle, namely, IC, in [22] the authors introduced a physical principle, namely, ML. In this paper we took a different approach which is closer to the second one. Whereas, according to ML, the coarse-grained extensive observations of macroscopic sources of M independent particle pairs should admit a local hidden-variable model in the limit $M\to\infty$, we considered the majority-voting approach (like [31]) to get a new probability distribution from M independent particle pairs and demanded that in the limit $M\to\infty$ this new correlation should behave locally. For the simplest scenario (the 2-2-2 case) we showed how one can characterize which correlations become local and which do not. Correlations exhibiting nonlocal behavior in the large-M limit are sure to be unphysical. We also found that for some sets of correlations, our method is better than the necessary criterion of the IC principle in identifying them as unphysical ones. Moreover, our approach identifies some no-signaling correlations that do satisfy Tsirelson's bound *nonmaximally* but give rise to maximum nonlocality in the macroscopic measurement setup. For future work it will be interesting to extend this study to more general input-output correlations rather than just the 2-2-2 scenario.

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