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# Many-body localization due to random interactions

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The possibility of observing many-body localization of ultracold atoms in a one-dimensional optical lattice is discussed for random interactions. In the noninteracting limit, such a system reduces to single-particle physics in the absence of disorder, i.e., to extended states. In effect, the observed localization is inherently due to interactions and is thus a genuine many-body effect. In the system studied, many-body localization manifests itself in a lack of thermalization visible in temporal propagation of a specially prepared initial state, in transport properties, in the logarithmic growth of entanglement entropy, and in statistical properties of energy levels.

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Almost 60 years ago Anderson [1] showed that disorder has dramatic effects on properties of noninteracting particles. For one-dimensional (1D) systems, even the smallest disorder generically leads to exponential localization of eigenfunctions killing any long-range transport. This is in striking contrast with orderly periodic structures supporting Bloch waves as eigenfunctions. Interestingly, the original idea of Anderson was to consider the effect of disorder on interacting particles. This more difficult problem has not been fully understood up until now. Anderson localization is often described as the result of quantum-mechanical interference of different multiple-scattering paths. This interpretation sheds some light on the interacting-particle problem. In this Rapid Communication, we discuss the physics of ultracold atoms in a 1D optical lattice, in conditions where decoherence is negligible on the time scale of the (numerical) experiment. For weak interactions and small disorder, an effective-mean-field description of the system is possible, leading to the Gross-Pitaevskii equation. The latter, however, is a nonlinear equation, for which no superposition principle works; the concept of interference cannot be easily applied [2]. Even for just two interacting particles it was shown that the localization length rapidly grows with the strength of the interactions [3]. This makes it extremely difficult to observe Anderson localization in the presence of interactions [4–6]. This was the primary reason why the cold-atom observations of Anderson localization were carried out in the noninteracting regime [7,8].

A novel path in the investigation of localization was initiated in the paper of Basko *et al.* [9] where, using a perturbative approach, it was shown that there may exist a transition to localized states for a sufficiently strong disorder. In such a situation, the mean-field approach is not applicable and the full many-body quantum theory has to be used. The latter is linear and the superposition principle holds: The picture of interfering paths is restored. There is, however, another conceptual problem: in what sense one may speak of localization. A possible answer is that one should no longer consider the configuration space but rather think in terms of localization in Hilbert space [10].

Many-body localization (MBL) has recently become a popular topic (a search in arXiv for "many-body localization" in the title or abstract yields more than 150 papers in the past 12 months). Many-body localization is very often connected with a lack of thermalization in the system. While the whole isolated system evolves in a fully coherent way, one may ask whether a small subsystem shows signs of thermalization, i.e., whether the system evolves in such a way that memory about the initial state is eventually lost [11]. Often a lack of thermalization (in the sense of averages of observables) is assumed as a very definition of MBL [10].

Some phenomenological understanding of MBL can be obtained using an effective integrability approach [12,13] or using renormalization-group approaches [14,15]. Most of the treatments, however, are numerical, primarily related to spin-1/2 chains, e.g., the Heisenberg model with a random magnetic field [16] or the $XX$ model (see, e.g., [17]). This is due to the complexity of many-body problems: Spin-1/2 chains may be efficiently treated numerically using time-dependent density-matrix renormalization-group (tDMRG) methods [18]. Much less often one can find simulations for cold-atomic systems (see [19] and references therein).

An attempt to observe MBL in a 1D system has been reported in [20], where a system of interacting fermions in an optical lattice potential (effectively one dimensional) is studied. The initial state is carefully prepared in such a way that a single fermion occupies every second site, other sites being empty. During the temporal evolution in the absence of disorder, the occupations of different sites equalize on average: The system thermalizes. The addition of a sufficiently strong quasirandom disorder (adding a second lattice with a period incommensurate with the primary lattice) allows one to observe a different behavior: A partial asymmetry of occupations of odd and even sites survives for intermediate times, pointing towards a lack of thermalization. This is taken as a signature of MBL. The study is supplemented with simulations using tDMRG that reveal a logarithmic in time increase of the entropy of entanglement during the course of evolution; this is another possible signature of MBL [12,16,17,21]. However, the transition between localization and delocalization takes place at a disorder strength very close to the threshold for single-particle localization in the Aubry-André model [22] and additionally

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only weakly depends on the interaction strength. The MBL observed thus has a predominantly single-particle character and it is a perturbation of the single-particle physics [9].

The aim of this Rapid Communication is to show that ultracold atoms allow us to study models with genuine nonperturbative MBL. One possible way is to consider a system that does not show localization in the absence of interactions. This rules out single-particle localization mechanisms. Consider bosons or fermions in a regular optical lattice in the absence of interactions. This is a perfectly regular system described by Bloch waves as eigenstates. Now we turn on, in a controlled way, interactions that randomly depend on position. Such a situation may be realized close to a Feshbach resonance when the scattering length strongly depends on the magnetic-field value. If the latter fluctuates in space, a system with the desired properties is created. Various phases of this model were studied in [23]. In particular, it was shown that the bosonic Mott insulator entirely disappears for sufficiently large occupations in the system. While the ground state was considered in [23], we analyze here the properties of excited states of the system inspecting their localization properties. We will study eigenvalue statistics for systems of small size (allowing a partial comparison with [19,24,25]) as well as the time propagation (using tDMRG methods) of appropriately prepared initial states (as in [20]).

The Bose-Hubbard Hamiltonian describing a 1D system in an optical lattice within the tight-binding approximation reads, assuming random on-site interactions,

$$\hat{H} = -J\sum_{i}^{L-1}(\hat{a}_{i+1}^{\dagger}\hat{a}_i + \text{H.c.}) + \frac{1}{2}\sum_i U_i\hat{n}_i(\hat{n}_i - 1),$$
$$[\hat{a}_i,\hat{a}_j^{\dagger}] = \delta_{ij}, \quad [\hat{a}_i,\hat{a}_j] = 0, \quad \hat{n}_i = \hat{a}_i^{\dagger}\hat{a}_i, \tag{1}$$

with the first term describing the tunneling while the second term corresponds to interactions. Here, following [23], we assume the interaction strength to depend on the site taking the value $U_i = Ux_i$, with $x_i$ being randomly and uniformly distributed in $[0,1]$. We fix the energy (and time) scale by setting $J = 1$.

Could this system reveal MBL? To check this, let us use an experimental approach [20] and study the temporal dynamics and possible thermalization of some specially prepared state at fixed particle density $N/L = 1.5$. We prepare the system in a density wave (DW) state with odd sites being singly and even sites doubly occupied. We define the magnetization as $M = 3(N_e - N_o)/(N_e + N_o)$, where $N_e$ ($N_o$) corresponds to global populations of even (odd) sites. The thermalization hypothesis suggests that the magnetization, originally equal to unity, would decay to zero in time (after disorder averaging). Yet the numerical results obtained using a homemade implementation of the tDMRG algorithm [18,26–28]) suggest otherwise: In Fig. 1(a), at long times $t$, the magnetization $M$ fluctuates about a nonzero mean value that depends on $U$. Thus the system, despite strong interactions, remembers the initial state, i.e., it does not thermalize. This shows that random interactions partially inhibit transport between neighboring sites.

A characteristic feature predicted for the MBL is the logarithmic in time growth of the entanglement entropy. To be specific, consider $S = -\sum \lambda_i \ln \lambda_i$, where $\lambda_i$ are the Schmidt decomposition coefficients when tracing out a part of the system (since we can arbitrarily split the 1D chain into two parts such different splittings allow for an additional averaging). Indeed, our results display such a logarithmic growth of $S$ as shown in Fig. 1(b).

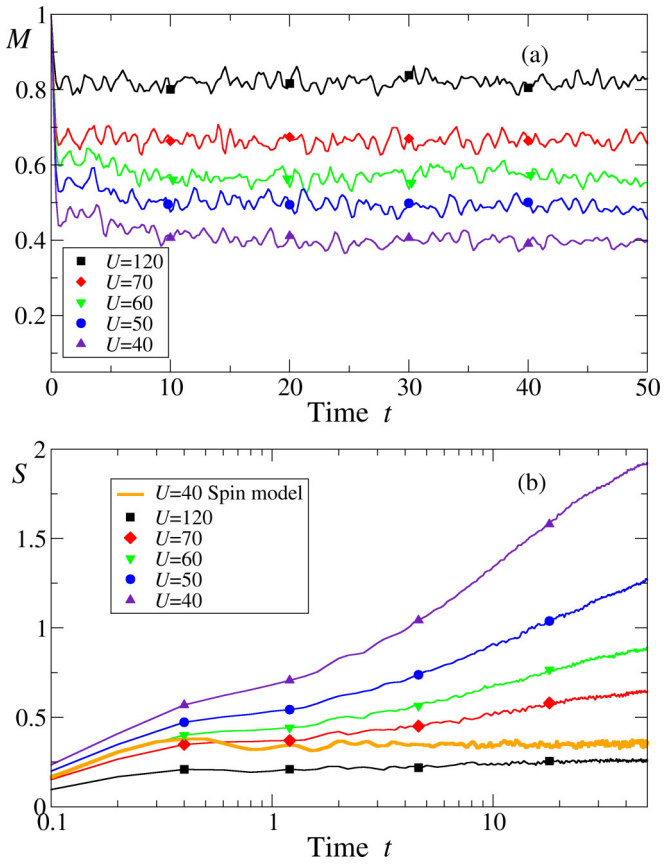


FIG. 1. Many-body localization for a 1D Bose-Hubbard model with random interaction strength. An initial state with a density wave profile is temporally propagated using a tDMRG algorithm. (a) The magnetization $M$ (initially unity) rapidly decays to a nonzero quasistationary value, a clear-cut proof of the absence of thermalization, i.e., of MBL. (b) The corresponding entropy of entanglement $S$ grows logarithmically with time $t$ after the initial transient. The magnetization increases with the disorder strength $U$ correlating with a slower increase of the entanglement entropy. Results are averaged over ten disorder realizations, for system size $L = 60$ and $N = 90$ bosons. The additional curve, labeled "$U = 40$ Spin model," is for the spin model discussed in the text. This simplified model exhibits Anderson localization and gives rise to a magnetization comparable to the one of the full model at large $U$ (see Fig. 2), but the entanglement entropy saturates at long time, emphasizing the difference between single-particle and many-body localization.

Figure 2 shows the quasistationary long-time magnetization $M$ as a function of the disorder strength $U$. The nonzero magnetization, a characteristic feature of the lack of thermalization, depends on the system size $L$ and can be seen to occur already for small $U$. With increasing $L$, the magnetization shifts to the right, converging in the large $L$ limit. Unfortunately, reliable tDMRG calculations for small $U$ cannot be performed for sufficiently long times due to the growth of the entanglement.

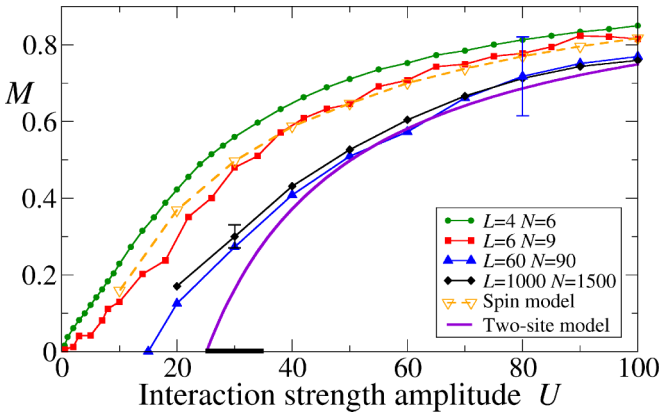


FIG. 2. Quasistationary magnetization $M$ versus $U$ for different system sizes $L$ indicated in the figure. Data are averaged over time for long times $t \in [10,40]$ to remove the oscillatory behavior visible in Fig. 1(a). Data for small system sizes, obtained from exact diagonalization, are averaged over several hundred realizations of disorder. Results for $L = 60$ ($L = 1000$) are obtained using the tDMRG algorithm and are averaged over 20 (4) realizations. For readability, the error bars (one standard deviation) are shown for a single $U$ value, but are very similar for all $U$ values. The quasicoalescence of the $L = 60$ and $L = 1000$ results indicates the absence of finite-size effects. The simplified spin model (dashed curve), discussed in the text, slightly overestimates the magnetization, but catches correctly the asymptotic behavior at large $U$. The two-site model, with its analytic prediction $M \approx 1 - 8\pi/U$ (see the text), reproduces quantitatively the results at large $U$. The black bar in the range $U \in [25,35]$ indicates the region where the transition to MBL occurs; see the discussion in the text.

This indicates a lack of localization for small $U$. On the localized side, the logarithmic in time growth of the entropy is observed for $U$ larger than $\sim$30 only. In the intermediate range $U \in [20,30]$ the numerical results tend to indicate a nonzero magnetization at long time, accompanied by a rapid powerlike growth of the entropy of entanglement, resembling the observations of [29,30] for the $XXZ$ model in the delocalized phase. The lack of reliable numerical results for long times and small $U$ unfortunately prevented us from determining whether the transition is a smooth crossover or a phase transition in the thermodynamic limit; such information would be a very important characteristic property of MBL. The numerical range where the transition occurs is denoted by the black bar in Fig. 2.

While the observed MBL arises solely due to random interactions and seems to have a nonperturbative character, its main properties can be understood from a simple microscopic model. For large $U$, the energy region where the initial state $|1,2,1,2,1,\ldots\rangle$ exists is dominated by states having the same number of single and double occupations in random order (and additionally preserving the total number of atoms). Indeed, moving from a 1,2 configuration to a 2,1 configuration costs no energy on average, while moving to a 0,3 configuration costs about $U$. We thus consider a simplified model, where the occupation numbers of all sites are either 1 or 2 only. This problem maps to a spin model (the $XX$ Heisenberg model for a spin-1/2 chain) in a random magnetic field [see the Hamiltonian (1) in [17] or the Hamiltonian (2) with $\Delta = 0$ in [16]]. A Wigner-Jordan transformation maps the latter system onto noninteracting fermions with random diagonal disorder that exhibit Anderson localization. Restricting occupations accordingly, we obtain in our tDMRG calculations a long-time magnetization comparable to the one of the full model (see Fig. 2) as well as a rapid saturation of the entropy of entanglement [see Fig. 1(b)].

For very large $U$, a simpler two-site model can be built. On-site energies typically differ by much more than $J$, inhibiting transport. It is only when two neighboring sites have an accidentally on-site energy difference of the order of $J$ that a significant transfer can take place. This happens with probability proportional to $J/U$. Consider two states on nearby sites having, respectively, occupations $|2,1\rangle$ and $|1,2\rangle$ and random interaction energies $U_1$ and $U_2$. Averaging over the random disorder as well as over time oscillations, one can compute the magnetization $M \approx 1 - 8\pi J/U$ for large $U$, which reproduces well the numerical observations.

In view of these results, it is also interesting to analyze the statistical properties of the energy spectrum. Indeed, in the delocalized and ergodic phase, one expects the Gaussian orthogonal ensemble (GOE) of random-matrix theory to be relevant, especially with linear repulsion between neighboring levels, corresponding to complete delocalization in the Hilbert space. In contrast, the MBL phase is expected to lead to the absence of level repulsion and Poisson level statistics. A simple indicator is the average ratio $\bar{r}$ between the smallest and the largest adjacent energy gaps: $r_n = \min[\delta_n^E, \delta_{n-1}^E]/\max[\delta_n^E, \delta_{n-1}^E]$, with $\delta_n^E = E_n - E_{n-1}$, and $E_n$ is the ordered list of energy levels [31]. In the ergodic (MBL) phase, one expects $\bar{r}$ to be close to the GOE value $\bar{r}^{\mathrm{GOE}} \approx 0.5307$ ($\bar{r}^{\mathrm{Poisson}} = 2\ln 2 - 1 \approx 0.386$) [32].

The localized or ergodic dynamics depends on energy [33]. For example, we have checked that an initial state $|0,3,0,3,\ldots\rangle$ with the same 1.5 particle density leads similarly to a decay of the magnetization with time, but displays MBL for a significantly smaller $U$ value. The statistical properties of the energy levels are also likely to depend on energy, so it is important to specify the energy range. While [19,34] used arbitrarily the central part of the spectra in their study of $\bar{r}$, we choose the vicinity of the energy of our DW initial state. This is also the region of significant overlaps between eigenstates and the initial state. The results for different system sizes are presented in Fig. 3, providing additional evidence for the transition to MBL for sufficiently large $U$. Crossings of data for numerically accessible system sizes do not allow us to precisely pin down the transition or crossover point, which lies probably around $U \approx 30$, slightly larger than, but compatible with, the magnetization data.

An intriguing possibility [29,30,35] is the possible existence of a delocalized but nonergodic phase below the critical point. Although the numerical simulations are very difficult in this region because of the rapid increase of the entanglement entropy, the data shown in Fig. 2 for large system size tend to show that there is a nonzero magnetization at long time in the $U \in [20,30]$ range, while the statistical properties in Fig. 3 tend to show that this takes place in the delocalized regime. We thus conclude that our results are in favor of the existence of a nonergodic delocalized phase, although we admit that further work is required to confirm this observation.

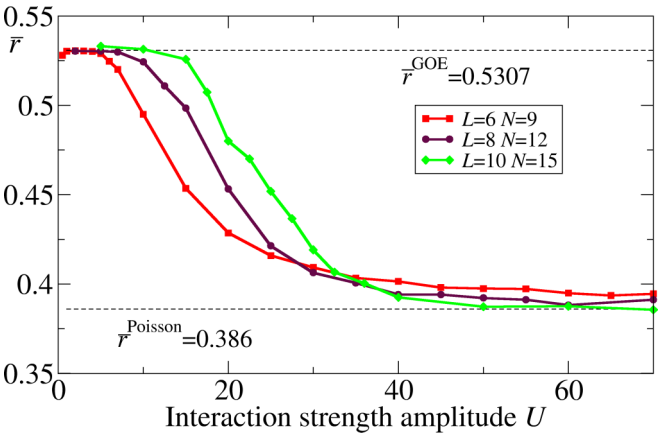


FIG. 3. Average ratio of adjacent energy gaps $\bar{r}$ as a function of the interaction strength amplitude $U$ for different system sizes $L$ and number of bosons $N$ with a fixed density 3/2. Data are averaged over many disorder realizations. Only the energy range where eigenstates significantly overlap with the initial state is taken into account to facilitate a relevant comparison of time and spectral information. The dashed horizontal lines are the GOE and Poisson predictions.

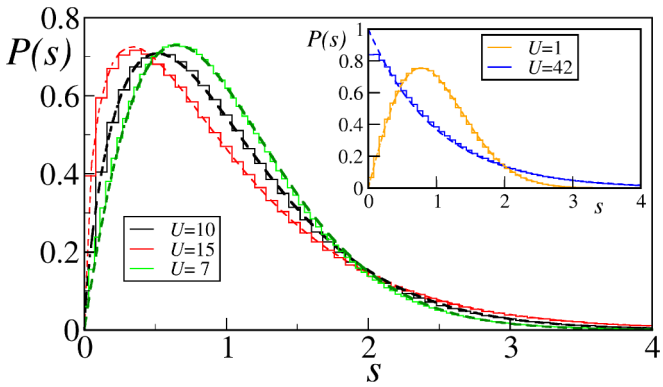


FIG. 4. Level-spacing distributions for $N = 9$ particles on $L = 6$ sites with open boundary conditions after averaging over several realizations of disorder with strength $U$. The $U = 15$ data are accurately described by the semi-Poisson distribution $P(s) \propto s^{\beta} \exp[-(1+\beta)s]$ with $\beta \approx 0.508$, as expected in the critical region. At lower $U$ (7 and 10) there is a transition towards the GOE distribution, where the data are well reproduced by a $P(s) \propto s^{\beta} \exp(-C_2 s^{2-\gamma})$ distribution proposed in [24]. The inset shows two limiting cases: The small (large) $U$ distribution is well reproduced by the GOE (Poisson) prediction.

The analysis of level statistics can be carried further. This requires us to perform the standard unfolding (using a polynomial fit) of the energy spectrum, obtaining level sequences of unit mean spacing (in contrast, the $\bar{r}$ statistics does not require such an unfolding [32]). This allows us to study spacing distributions for different $U$ values and probe the crossover region more carefully [24,25]. For small $U$, very good agreement with the GOE prediction is observed in accord with the $\bar{r}$ value (compare with Fig. 4). With increasing $U$ we observe a transition of spacing distributions towards a typical shape expected for localized distributions. The study of the so-called intermediate statistics has proved useful in the context of single-particle localization [36]. Around the critical point, the data are well described by the semi-Poisson family describing the spacing distribution $P(s) \propto s^{\beta} \exp[-(\beta+1)s]$ (with $\beta$ a real parameter), as shown in Fig. 4 for $U = 15$, and tend to the Poisson distribution (with $\beta = 0$) for very large $U$. On the delocalized side, for $U = 7$ and 10 in Fig. 4, the situation is a bit more complicated, with an intermediate regime well described by the distribution $P(s) \propto s^{\beta} \exp(-Cs^{2-\gamma})$ proposed in [24] in the context of the $XXZ$ spin chain (a similar behavior is observed in [25]). We speculate that it could be related to the transition from a nonergodic delocalized phase to an ergodic one.

In conclusion, we have shown that many-body localization may be observed in one-dimensional systems in optical lattices, under realistic experimental conditions when the disorder is due to random interactions (with no disorder in the chemical potential). The virtue of our model is that it does not show localization without interactions: The MBL effect observed is inherently and solely due to interactions and is not a small perturbation of single-particle physics. We believe that this supports the idea that MBL is robust (as suggested by many-body strongly coupled systems without disorder [37–41]).

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