Propagation of coupled dark-state polaritons and storage of light in a tripod medium

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We consider slow-light propagation in an atomic medium with a tripod level scheme. We show that the coexistence of two types of dark-state polaritons leads to the propagation dynamics, which is qualitatively different from that in a Λ medium, and allows therefore for very efficient conversion of signal photons into spin excitations. This efficiency is shown to be very close to 1 even for very long signal light pulses, which could not be entirely compressed into a Λ medium at a comparable strength of the control field.

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I. INTRODUCTION

The phenomenon of electromagnetically induced transparency (EIT) based on the creation of a coherent superposition of long-living quantum states in a medium irradiated by a two-laser light field has been known for a long time (see, e.g., the review in Ref. [1]). The EIT has become especially interesting and promising for quantum memory applications [2] since the discovery of the method to "stop the light" by conversion of photons of the weak (signal) into spin excitations of a medium by adiabatically turning off the second (control) field [3,4]. Experimental realizations followed the publication of the idea [3] immediately and employed as the EIT medium ultracold atoms [5], hot atomic vapor in a cell [6], and doped crystal [7].

The Λ scheme containing three quantum levels coupled to a laser radiation is the simplest one that admits coherent population trapping and the light propagation in a medium in the EIT regime. The tripod scheme that contains three low-energy, stable (or metastable) sublevels supports two different quantum superpositions that are decoupled from coherent three-component laser radiation resonant to the optically excited state. Various aspects of the slow-light propagation and storage in a tripod medium have been theoretically studied [8–14]. There are numerous experimental studies of the EIT in media with the tripod-level scheme [15–20], culminating in the demonstration of the storage and retrieval of light pulses at a single-photon level [21].

The dynamical coupling between dark-state polaritons of the two types arising due to the time dependence of the control fields was introduced in Ref. [11] but not fully investigated. Indeed, the Hong-Ou-Mandel interferometer operation in a tripod medium [11] requires the change of the control field parameters during the time interval of no signal photon coming. In this paper we consider the situation of signal photons interacting with a tripod medium where the coupling between the two types of dark-state polaritons is present.

The signal laser pulse can be fully converted into spin excitations in a conventional Λ medium only if the medium is long enough to accommodate the whole slowed-down incoming pulse (spatially compressed in proportion to the ratio of its group velocity in the EIT regime and in the vacuum) [5–7]. In a tripod medium the existence of two coupled dark-state modes allows for a conversion of almost the whole signal light field into spin excitations under less restrictive conditions. Note that in a similar case a conventional Λ-type medium irradiated by a control field of comparable strength and characterized by a comparable slow group velocity would accommodate only a part of the signal pulse.

II. BASIC EQUATIONS

We consider a medium consisting of atoms with the level scheme shown in Fig. 1. The ground-state sublevels $|0\rangle$, $|1\rangle$, and $|2\rangle$ are coupled to an excited state $|e\rangle$ by three coherent fields. The control fields driving the transitions $|1\rangle \leftrightarrow |e\rangle$ and $|2\rangle \leftrightarrow |e\rangle$ are characterized by Rabi frequencies $\Omega_1 \equiv \Omega \cos\beta$ and $\Omega_2 \equiv \Omega \sin\beta$, respectively. These fields are phase locked or obtained from a common source by an acousto-optical modulator in order to provide perfect cross-correlation of their noise and to prevent thus a noise-induced decay of the quantum coherence between the states $|1\rangle$ and $|2\rangle$ [22]. The transition $|0\rangle \leftrightarrow |e\rangle$ is driven by a quantized signal field. We can consider a field propagating freely through the atomic sample [21] or in a nanofiber [23,24]; the propagation direction of the signal field defines the axis z. The quantum field for the signal photons can be expressed as $\hat{\mathcal{E}}(z,t)\exp[-i\omega_{e0}(t-z/c)]$, where $\hat{\mathcal{E}}$ is its slowly varying amplitude subjected to the standard bosonic commutation rules, ω_{e0} is the resonance frequency of the $|0\rangle \leftrightarrow |e\rangle$ atomic transition, and c is the speed of light (in vacuum or in the nanofiber, depending on the type of the setup).

In contrast to Ref. [11], we assume the amplitudes of the two control fields to be constant, and instead we introduce the detuning of the second field $\nu(t)$, which is time dependent in a general case (see Fig. 1). It is convenient to express the atomic collective spin variables through bosonic fields $\hat{f}_\alpha(z,t)$, where $\hat{f}_\alpha(z,t)$ annihilates an atom in the state $|\alpha\rangle$, $\alpha = 0,1,2,e$, at the point z at time t. The set of equations for these bosonic fields and the signal photons is

$$\frac{\partial}{\partial t}\hat{\mathcal{E}} = -c\frac{\partial}{\partial z}\hat{\mathcal{E}} + i\kappa \hat{f}_0^\dagger \hat{f}_e, \tag{1}$$

$$\frac{\partial}{\partial t}\hat{f}_0 = i\kappa \hat{\mathcal{E}}^\dagger \hat{f}_e, \tag{2}$$

$$\frac{\partial}{\partial t}\hat{f}_e = i\kappa \hat{\mathcal{E}} \hat{f}_0 + i\Omega(\cos\beta\, \hat{f}_1 + \sin\beta\, \hat{f}_2), \tag{3}$$

$$\frac{\partial}{\partial t}\hat{f}_1 = i\Omega\cos\beta\, \hat{f}_e, \tag{4}$$

$$\frac{\partial}{\partial t}\hat{f}_2 = i\nu(t)\hat{f}_2 + i\Omega\sin\beta\, \hat{f}_e, \tag{5}$$

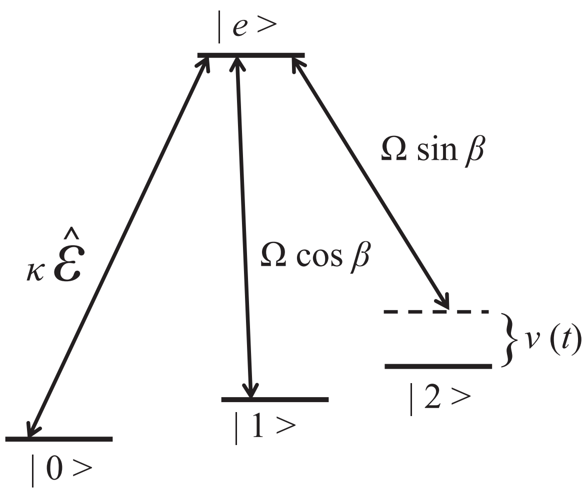

FIG. 1. Tripod scheme of atomic levels.

which is an obvious generalization from the case of EIT in a Λ medium [25] to the case of a tripod medium. The atom-field coupling constant $\kappa = d_{e0}\sqrt{\omega_{e0}/(2\hbar\varepsilon_0 A)}$, where d_{e0} is the electric dipole moment of the transition $|0\rangle \leftrightarrow |e\rangle$ and A is the effective area of the signal beam, can be expressed through the optical density s of the medium for the resonant signal light as $\kappa = \sqrt{\gamma s c/(2N)}$, where N is the number of atoms inside the interaction volume AL and L is the atomic sample length in the propagation direction. The radiative decay rate γ arises due to the coupling of the $|0\rangle \leftrightarrow |e\rangle$ transition to side modes of the electromagnetic field, which is not explicitly written, for the sake of brevity, in Eq. (3). Integrating out the vacuum modes of the electromagnetic field, we would get, instead of Eq. (3),

$$\frac{\partial}{\partial t}\hat{f}_e = i\kappa\hat{\mathcal{E}}\hat{f}_0 + i\Omega(\cos\beta\,\hat{f}_1 + \sin\beta\,\hat{f}_2) - \gamma\hat{f}_e + \hat{\varsigma}_e(z,t), \tag{6}$$

where $\hat{\varsigma}_e(z,t)$ is a δ-correlated Langevin-type operator [3,4] that describes vacuum quantum noise and is needed to preserve bosonic commutation properties of $\hat{f}_e$ after introducing the decay term $-\gamma\hat{f}_e$.

We work in the weak-pulse limit; i.e., we assume that the linear density of dark-state polaritons is always much less than the linear density of atoms, $n_{1D} = N/L$, which are initially all in the state $|0\rangle$ [25,26]. Then we linearize Eqs. (1)–(5) by replacing $\hat{f}_0$ by a number $\sqrt{n_{1D}}$ and find in a standard way [3,4,11], i.e., by adiabatic elimination of excitation modes separated from the dark-state polaritons by large energy gaps, the equations of motion

$$\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial z}\right)\hat{\Psi} = i\tilde{\nu}(t)\sin\tilde{\beta}(\sin\tilde{\beta}\hat{\Psi} + \cos\tilde{\beta}\hat{\Upsilon}), \tag{7}$$

$$\frac{\partial}{\partial t}\hat{\Upsilon} = i\tilde{\nu}(t)\cos\tilde{\beta}(\sin\tilde{\beta}\hat{\Psi} + \cos\tilde{\beta}\hat{\Upsilon}), \tag{8}$$

for two dark-state polariton fields

$$\hat{\Psi} = \cos\theta\hat{\mathcal{E}} - \sin\theta(\cos\beta\,\hat{f}_1 + \sin\beta\,\hat{f}_2), \tag{9}$$

$$\hat{\Upsilon} = \sin\beta\,\hat{f}_1 - \cos\beta\,\hat{f}_2. \tag{10}$$

The mixing angle in Eq. (9) is defined by the usual expression $\tan\theta = \kappa\sqrt{n_{1D}}/\Omega$, and $v_g = c\cos^2\theta$ [3,4]. Also we get $\tilde{\nu}(t) = (\sin^2\theta\sin^2\beta + \cos^2\beta)\nu(t)$ and $\tan\tilde{\beta} = \sin\theta\tan\beta$.

We assume the slow-light regime, $\Omega \ll \kappa\sqrt{n_{1D}}$, and, hence, $\sin\theta \approx 1$, $v_g \ll c$. In this limit, $\tilde{\nu}(t) \approx \nu(t)$ and $\tilde{\beta} \approx \beta$. In what follows, we do not distinguish therefore between the values with or without a tilde and omit this symbol over ν and β.

Before specifying the initial and boundary conditions to Eqs. (7) and (8), we reformulate them for classical complex variables Ψ and Υ. This may be done for coherent states of the dark-state polariton fields as well as for single quantum states. In the latter case, we use the Schrödinger representation and write the wave function of the system as $|\Xi(t)\rangle = \int_0^L dz\,[\Psi(z,t)\hat{\Psi}^\dagger(z,t) + \Upsilon(z,t)\hat{\Upsilon}^\dagger(z,t)]|\text{vac}\rangle + |\Xi_{\text{ph}}(t)\rangle$, where $|\text{vac}\rangle$ is the vacuum state of excitations (all atoms being in their internal state $|0\rangle$), and $|\Xi_{\text{ph}}(t)\rangle$ describes a single photon either before entering the medium (at $z < 0$) or after leaving it (at $z > L$). The evolution of $|\Xi_{\text{ph}}(t)\rangle$ is not interesting for us, and the evolution of the remaining component of $|\Xi(t)\rangle$ is given by Eqs. (7) and (8) with the operators $\hat{\Psi}$ and $\hat{\Upsilon}$ replaced by the complex fields Ψ and Υ, respectively.

It is convenient to introduce new variables, $\tau = t - z/v_g$ and $\zeta = z/v_g$. Then $\nu(t) = \nu(\tau + \zeta)$ and the equations of motion for dark-state polaritons become

$$\frac{\partial}{\partial\zeta}\Psi = i\nu\sin\beta(\sin\beta\Psi + \cos\beta\Upsilon), \tag{11}$$

$$\frac{\partial}{\partial\tau}\Upsilon = i\nu\cos\beta(\sin\beta\Psi + \cos\beta\Upsilon). \tag{12}$$

The boundary and initial conditions are

$$\Psi(0,\tau) = \Psi_0(\tau), \quad \Upsilon(\zeta,0) = 0, \tag{13}$$

where the function $\Psi_0(\tau)$ is determined by the shape of the incoming signal light pulse. We assume that $\Psi_0(\tau) = 0$ for $\tau \leqslant 0$.

III. PROPAGATION DYNAMICS

The main features of the dark-polariton dynamics can be determined from the solution of Eqs. (11)–(13) in the case of constant two-photon detuning, $\nu \equiv \nu_0 = \text{const}$. A simple phase transformation

$$\Psi(\zeta,\tau) = e^{i\chi(\zeta,\tau)}\Psi'(\zeta,\tau), \quad \Upsilon(\zeta,\tau) = e^{i\chi(\zeta,\tau)}\Upsilon'(\zeta,\tau),$$
$$\chi(\zeta,\tau) = \nu_0(\sin^2\beta\,\zeta + \cos^2\beta\,\tau), \tag{14}$$

casts Eqs. (11) and (12) into the form

$$\frac{\partial}{\partial\zeta}\Psi' = i\nu_0\sin\beta\,\cos\beta\Upsilon', \tag{15}$$

$$\frac{\partial}{\partial\tau}\Upsilon' = i\nu_0\sin\beta\,\cos\beta\Psi'. \tag{16}$$

A similar set of equations was derived in Ref. [11] for a different driving protocol of the tripod medium where the coupling between the two dark-state polaritons was induced by changing the angle β in time. We note that the influence of this coupling on the pulse propagation was not studied there.

Equations (15) and (16) can be easily solved by means of Laplace's transform. Also we can note that after elimination of one of the fields the equation for the remaining one is reduced to the relativistic Klein-Gordon equation

$$\frac{\partial}{\partial \zeta}\frac{\partial}{\partial \tau}\Psi' = \frac{1}{4}\left(\frac{\partial^2}{\partial T^2} - \frac{\partial^2}{\partial X^2}\right)\Psi' = -(\nu_0 \sin\beta\, \cos\beta)^2\Psi', \tag{17}$$

where $\frac{\partial}{\partial T} \equiv \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \zeta}$ and $\frac{\partial}{\partial X} \equiv \frac{\partial}{\partial \tau} - \frac{\partial}{\partial \zeta}$. The Green's function for the Klein-Gordon equation is well known [27] and can be used to solve Eqs. (15) and (16).

Finally, we obtain the solutions,

$$\Psi(\zeta,\tau) = e^{i\nu_0 \sin^2\beta\,\zeta}\bigg[\Psi_0(\tau) - \nu_0 \sin\beta\cos\beta \times \int_0^\tau d\tau'\, \Psi_0(\tau-\tau')e^{i\nu_0\cos^2\beta\,\tau'} \times \sqrt{\frac{\zeta}{\tau'}}J_1(2\nu_0\sin\beta\cos\beta\sqrt{\zeta\tau'})\bigg], \tag{18}$$

$$\Upsilon(\zeta,\tau) = i\nu_0\sin\beta\cos\beta\, e^{i\nu_0\sin^2\beta\,\zeta}\int_0^\tau d\tau'\,\Psi_0(\tau-\tau') \times e^{i\nu_0\cos^2\beta\,\tau'}J_0(2\nu_0\sin\beta\cos\beta\sqrt{\zeta\tau'}), \tag{19}$$

where J_0 and J_1 are Bessel functions of the zero and first order, respectively.

First we analyze Eq. (18). The first term in its right-hand side describes, apart from gaining a ζ-dependent phase shift, pulse propagation at the group velocity v_g. However, this regime holds only for small values of ζ, where the integrand in the second term is small because of the small values taken by the Bessel function $J_1(2\nu_0\sin\beta\cos\beta\sqrt{\zeta\tau'})$. This propagation picture is typical for dark-state polaritons in a Λ medium. However, it will be distorted at larger distances, when the second term becomes important. What happens then, one can see from the analysis of the dynamics of the Υ field.

This field corresponds to spin excitations, which possess zero group velocity and are induced by coupling to the Ψ-type polaritons through the finite two-photon detuning ν_0. Equations (15) and (16) together with the initial condition $\Upsilon(\zeta,0) = 0$ yield a simple conservation law

$$\int_0^{\zeta_L} d\zeta\, |\Upsilon(\zeta,\tau)|^2 = \int_0^\tau d\tau'|\Psi(0,\tau')|^2 - \int_0^\tau d\tau'|\Psi(\zeta_L,\tau')|^2, \tag{20}$$

where $\zeta_L = L/v_g$. It relates the total population of the Υ-type mode inside the medium of length L to the loss of the output pulse energy at the exit from the medium compared to the case of propagation in a Λ medium at the group velocity v_g. We define the efficiency of the conversion of signal photons to Υ-type spin excitations as

$$\eta(\tau) = \frac{\int_0^{\zeta_L} d\zeta\, |\Upsilon(\zeta,\tau)|^2}{\int_0^\infty d\tau'|\Psi(0,\tau')|^2}. \tag{21}$$

From Eq. (19) we can evaluate the enumerator of Eq. (21) if we recall the formula for the integral of a product of two Bessel functions [28]:

$$\int_0^{\zeta_L} d\zeta\,|\Upsilon(\zeta,\tau)|^2 = \nu_0\sin\beta\cos\beta\int_0^\tau d\tau_1\int_0^\tau d\tau_2\,\Psi_0(\tau_1)\Psi_0^*(\tau_2)e^{-i\nu_0\cos^2\beta(\tau_1-\tau_2)}\frac{1}{\tau_1-\tau_2}\{\sqrt{\tau-\tau_2} \times J_0[2\nu_0\sin\beta\cos\beta\sqrt{\zeta_L(\tau-\tau_1)}]J_1[2\nu_0\sin\beta\cos\beta\sqrt{\zeta_L(\tau-\tau_2)}] - \sqrt{\tau-\tau_1}J_0[2\nu_0\sin\beta\cos\beta\sqrt{\zeta_L(\tau-\tau_2)}]J_1[2\nu_0\sin\beta\cos\beta\sqrt{\zeta_L(\tau-\tau_1)}]\}. \tag{22}$$

Let τ_p be the typical time scale of the incoming signal light pulse and consider Eq. (22) for $\tau \gtrsim \tau_p$. In this case the time integrals practically converge to their final values on the scale $\tau_{1,2} \lesssim \tau$. Assume that

$$|\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L\tau} \gg 1. \tag{23}$$

In this case we can recall the asymptotic expression [29] for the Bessel function of order n of a large argument $x \to \infty$, $J_n(x) = \sqrt{2/(\pi x)}\cos(x - \frac{n\pi}{2} - \frac{\pi}{4})$, and find an approximation for Eq. (22):

$$\int_0^{\zeta_L} d\zeta\,|\Upsilon(\zeta,\tau)|^2 = \nu_0\sin\beta\cos\beta\int_0^\tau d\tau_1\int_0^\tau d\tau_2\Psi_0(\tau_1) \times \Psi_0^*(\tau_2)e^{-i\nu_0\cos^2\beta(\tau_1-\tau_2)} \times \frac{\sin[|\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L/\tau}\,(\tau_1-\tau_2)]}{\pi(\tau_1-\tau_2)}. \tag{24}$$

If we consider asymptotically long times, such that

$$|\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L/\tau} \ll \frac{1}{\tau_p}, \tag{25}$$

we immediately see that asymptotically

$$\int_0^{\zeta_L} d\zeta\,|\Upsilon(\zeta,\tau)|^2|_{\tau\to+\infty} \approx \frac{\nu_0\sin\beta\cos\beta}{\pi}\sqrt{\frac{\zeta_L}{\tau}} \times \left|\int_0^\infty d\tau'\,\Psi_0(\tau')e^{-i\nu_0\cos^2\beta\,\tau'}\right|^2. \tag{26}$$

This means that the spin excitations in the medium, being coupled to the Ψ-type dark-state polariton mode via the two-photon detuning ν_0, decay in a very slow, nonexponential way, namely, proportionally to $1/\sqrt{\tau}$.

What occurs for τ larger than but close to τ_p requires further analysis. Recall that we are interested in long pulses, which cannot be entirely fit into the medium. Therefore, we assume $\tau_\mathrm{p} \gtrsim \zeta_L = L/v_\mathrm{g}$. Also we need, in order to satisfy condition (23), to have values of β not too close to 0, $\pm\frac{\pi}{2}$, or π. In other words, we assume that $\cos\beta$ and $\sin\beta$ are of the same order. If the incoming signal photons are tuned exactly in resonance with the $|0\rangle \leftrightarrow |e\rangle$ transition, then $|\nu_0 \sin\beta\cos\beta|\sqrt{\zeta_L/\tau} \lesssim |\nu_0|\cos^2\beta$, the convergence of the integrals in Eq. (24) is achieved on the time scale of about $1/(|\nu_0|\cos^2\beta)$, and Eq. (26) remains a satisfactory estimation. The conversion efficiency η thus remains well below 1. However, if the detuning of the signal pulse from the resonance is chosen such that $\Psi_0(\tau) = |\Psi_0(\tau)|\exp(i\nu_0\cos^2\beta\tau)$ then, in order to determine the time scale of convergence of the time integrals in Eq. (24), we have to compare $|\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L/\tau} \sim |\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L/\tau_\mathrm{p}}$ with the spectral width of $|\Psi_0|$, i.e., with $1/\tau_\mathrm{p}$. By taking $|\nu_0|$ large enough, one may attain $|\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L\tau_\mathrm{p}} \gg 1$. In this case, the function $\sin[|\nu_0\sin\beta\cos\beta|\sqrt{\zeta_L/\tau}(\tau_1-\tau_2)]/[\pi(\tau_1-\tau_2)]$ can be approximated by a δ function, $\delta(\tau_1-\tau_2)$. Therefore, the efficiency of conversion of signal photons into spin excitations of the Υ type is

$$\eta(\tau) \approx \frac{\int_0^\tau d\tau'|\Psi(0,\tau')|^2}{\int_0^\infty d\tau'|\Psi(0,\tau')|^2}. \tag{27}$$

For $\tau \gtrsim \tau_\mathrm{p}$ (say, $\tau \approx 3\tau_\mathrm{p}$) this efficiency may get very close to 1. Of course, at very large times $\eta(\tau)$ decays, as we have shown, in proportion to $1/\sqrt{\tau}$. But one can prevent such a long-time decay of the spin excitations by radiating photons out of the medium by suddenly switching off the control fields (or by suddenly changing ν from ν_0 to zero and thus decoupling the Ψ-type and Υ-type polaritons). This means that the use of a tripod medium permits one to trap and convert into spin excitations very long signal light pulses, which would be only partially fit into a medium with a standard Λ scheme of atomic levels. The retrieval of the stored quanta may be implemented using the standard protocol [11] (note also the observed modulation of the retrieved pulse shape [21]). For example, one may retrieve stored signal photons by applying the two control fields with the new amplitudes $\Omega_1^\mathrm{new} = \sin\beta\,\Omega$ and $\Omega_2^\mathrm{new} = -\cos\beta\,\Omega$ and zero detuning, $\nu^\mathrm{new}(t) = 0$. Then the spin excitation stored in the medium turns into a dark-state polariton that moves at the group velocity v_g and leaves the medium without coupling to the dark-state polariton of the other type.

The numerical evaluation of the efficiency of conversion of signal photons into spin excitations based on Eq. (19) is presented in Figs. 2 and 3. The incoming pulse used here is slightly (at the level of 1×10^{-4}) modified in comparison to a Gaussian in order to formally provide its continuity at $\tau = 0$, since $\Psi_0(0) = 0$ by our assumption. We can see from Fig. 2 that for a perfectly resonant signal light the maximum efficiency is always appreciably below 1. On the contrary, if the probe light is detuned by $-\nu_0\cos^2\beta$ from the frequency ω_{e0}, then the values of η very close to 1 can be attained [see Fig. 3(a)]. Note that for $L/(v_\mathrm{g}\tau_\mathrm{p})$ equal to 1.0 and 0.5 the maximum fraction of the Gaussian incident pulse that can be simultaneously contained inside the medium is 0.843 and

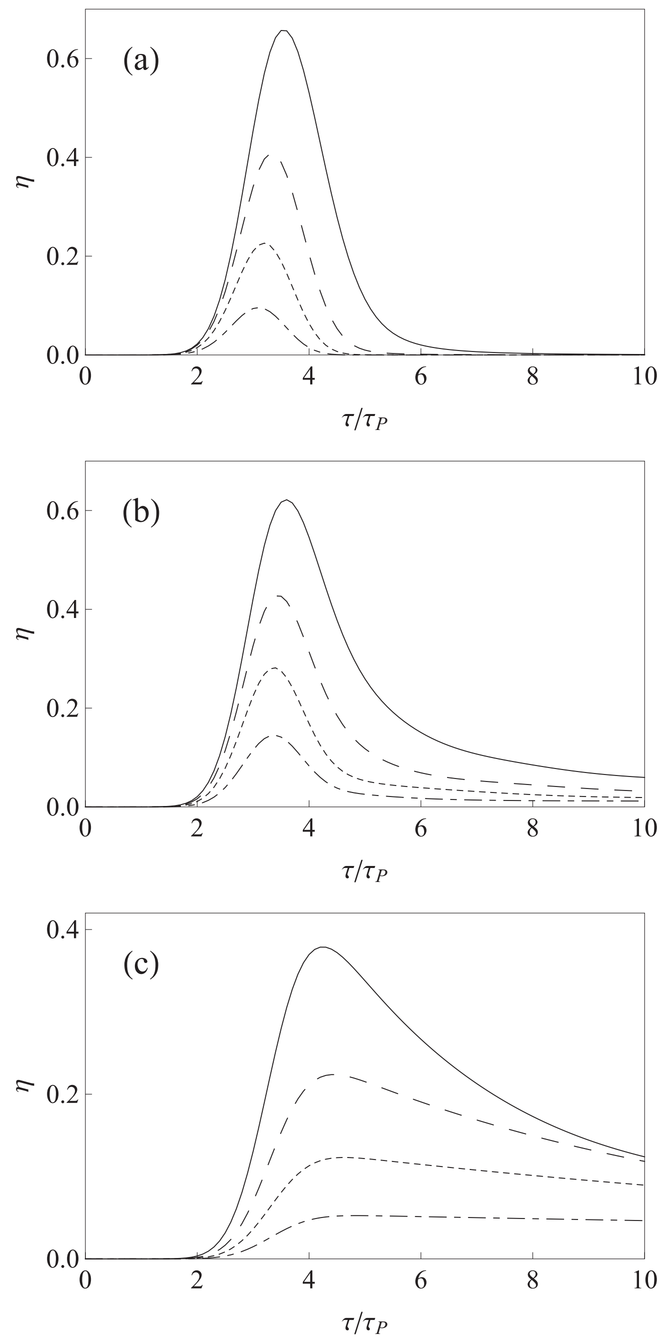

FIG. 2. Efficiency η of the conversion of the signal photons into spin excitations of the Υ type as a function of the retarded time τ for the boundary condition $\Psi_0(\tau) = \Psi_{0\mathrm{A}}\{\exp[-(\tau-3\tau_\mathrm{p})^2/\tau_\mathrm{p}^2] - \exp[-(\tau+3\tau_\mathrm{p})^2/\tau_\mathrm{p}^2]\}$ for $\tau \geqslant 0$ ($|\Psi_{0\mathrm{A}}|^2$ determines the pulse energy). Units on the axes are dimensionless, and the time τ is scaled to the characteristic time τ_p of the pulse duration. $\beta = \pi/4$ for all plots. The detuning ν_0 is (a) $\nu_0 = 1/\tau_p$, (b) $5/\tau_p$, and (c) $10/\tau_p$. On each panel the length L of the medium is $1.0 v_\mathrm{g}\tau_\mathrm{p}$ (solid line), $0.5 v_\mathrm{g}\tau_\mathrm{p}$ (long-dashed line), $0.25 v_\mathrm{g}\tau_\mathrm{p}$ (short-dashed line), and $0.1 v_\mathrm{g}\tau_\mathrm{p}$ (dot-dashed line). The decrease of the dot-dashed line in (b) and (c) is very slow and can bee seen on a time scale $\tau/\tau_\mathrm{p} \sim 10^2$.

0.521, respectively, while the maximum values of η in Fig. 3(a) for the respective lengths are 0.999 (solid line) and 0.990 (long-dashed line).

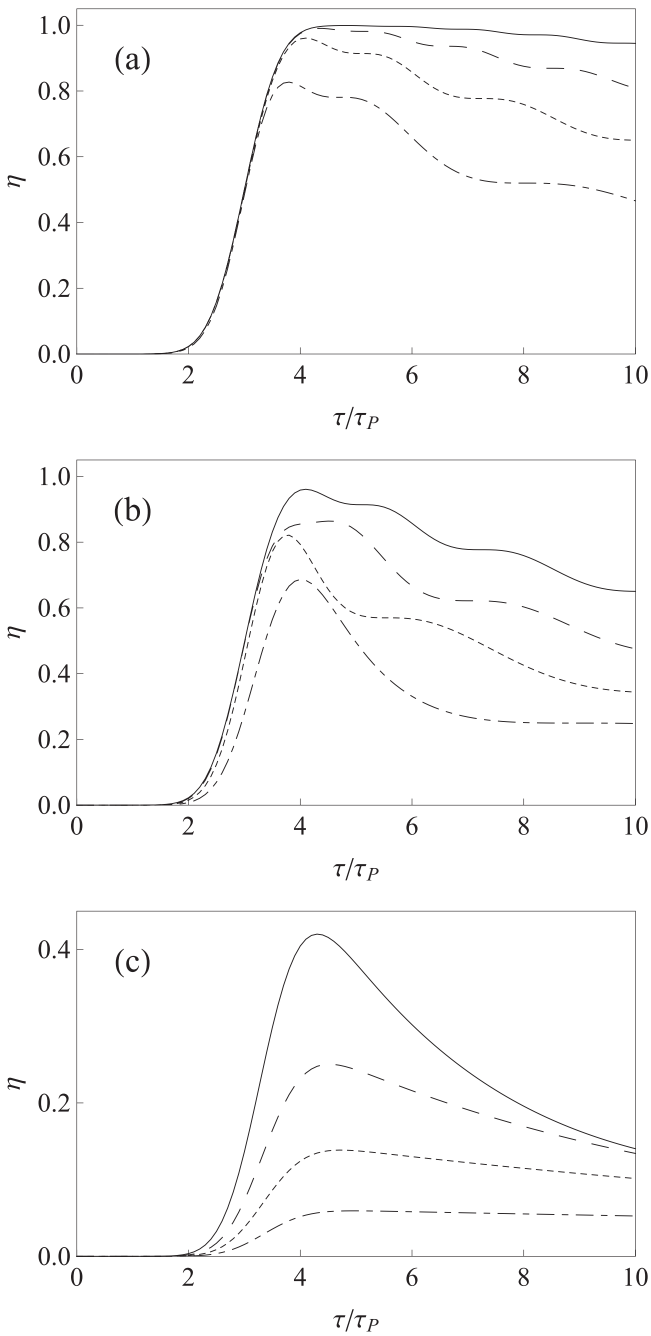

FIG. 3. The same as in Fig. 2, but for $\Psi_0(\tau) = \Psi_{0\,\mathrm{A}}\{\exp[-(\tau - 3\tau_\mathrm{p})^2/\tau_\mathrm{p}^2] - \exp[-(\tau + 3\tau_\mathrm{p})^2/\tau_\mathrm{p}^2]\}\exp(i\nu_0\cos^2\beta\,\tau)$ for $\tau \geqslant 0$. The values of ν_0 assigned to (a)–(c) and of L assigned to the lines of different type as well as β are the same as in the previous plot. Note that for a properly detuned signal pulse $\eta \approx 1$ can be attained.

IV. DISCUSSION

Now we examine the effects of the signal light absorption on the propagation regime considered in the previous section. Equation (23) ensures highly efficient conversion of signal photons into spin excitations on intermediate time scales of about a few incoming pulse duration times τ_p. This high efficiency implies a large optical density of the medium, $s \gg 1$. Also we consider $\beta \approx \frac{\pi}{4}$. Since $v_\mathrm{g} \approx c\Omega^2/(\kappa^2 n_{1\mathrm{D}}) \ll c$ and $\kappa^2 n_{1\mathrm{D}} L = \gamma c s/2$, we can rewrite Eq. (23) as

$$|\nu_0|\sqrt{\frac{\sqrt{s}\,\tau_\mathrm{p}}{\Delta\omega_\mathrm{EIT}}} \gg 1, \tag{28}$$

where

$$\Delta\omega_\mathrm{EIT} = \frac{\Omega^2}{\gamma\sqrt{s}} \tag{29}$$

is the width of the EIT transmission window in an optically dense medium [4,30] (see also the review in Ref. [31]). To minimize the signal pulse absorption, we have to assume its duration to be much longer than $1/\Delta\omega_\mathrm{EIT}$:

$$\tau_\mathrm{p} = \frac{K_\mathrm{p}}{\Delta\omega_\mathrm{EIT}}, \quad K_\mathrm{p} \gg 1. \tag{30}$$

Also the two-photon detuning must be small compared to the width of the EIT window, $|\nu_0| \ll \Delta\omega_\mathrm{EIT}$. This means that for a large optical depth and long enough pulses,

$$\sqrt{s}K_\mathrm{p} \gg \left|\frac{\Delta\omega_\mathrm{EIT}}{\nu_0}\right|^2, \tag{31}$$

condition (23) is satisfied.

To summarize, we investigated theoretically the propagation of weak signal pulses in a medium with a tripod scheme of atomic levels in the slow-light regime. Dark-state polaritons of two kinds exist in such a medium [11]. When they are mutually coupled via nonzero detuning $\nu(t)$ of one of the control fields, the propagation becomes nontrivial. In the case of constant detuning, $\nu(t) \equiv \nu_0$, the propagation bears analogy with the relativistic physics because its Green's function is formally identical to that of the Klein-Gordon equation [27]. The light pulse leaving the medium has a very long "tail" decreasing as $1/\sqrt{\tau}$. Under certain conditions [Eq. (31) together with the detuning of the signal pulse by $-\nu_0\cos^2\beta$ from the single-photon resonance] it is possible to trap temporally almost the entire incoming pulse even if it is so long that it cannot be accommodated in a Λ medium characterized by a comparable reduction of the group velocity, $v_\mathrm{g}/c = \Omega^2/(\kappa^2 n_{1\mathrm{D}}) \ll 1$. Fast switching off of the control fields or setting $\nu(t)$ to zero prevents the spin excitations from decay through the radiation of photons in the forward direction and leads to their storage in the medium. Note that the proposed scheme may be termed "passive," since it, unlike the conventional one [3–7], does not require gradual tuning of the control field during the signal pulse propagation in the medium but implies instead rapid switching off of both the control fields or a fast change of $\nu(t)$ to zero, as soon as the maximum conversion efficiency is achieved.

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