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Nonlocal advantage of quantum coherence

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A bipartite state is said to be steerable if and only if it does not have a single-system description, i.e., the bipartite state cannot be explained by a local hidden state model. Several steering inequalities have been derived using different local uncertainty relations to verify the ability to control the state of one subsystem by the other party. Here, we derive complementarity relations between coherences measured on mutually unbiased bases using various coherence measures such as the l_1-norm, relative entropy, and skew information. Using these relations, we derive conditions under which a nonlocal advantage of quantum coherence can be achieved and the state is steerable. We show that not all steerable states can achieve such an advantage.

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Steering is a kind of nonlocal correlation introduced by Schrödinger [1] to reinterpret the Einstein-Podolsky-Rosen (EPR) paradox [2]. According to Schrödinger, the presence of entanglement between two subsystems in a bipartite state enables one to control the state of one subsystem by its entangled counterpart. Wiseman *et al.* [3] formulated the operational and mathematical definition of quantum steering and showed that steering lies between quantum entanglement and Bell nonlocality on the basis of their strength [4]. The notion of the steerability of quantum states is also intimately connected [5] to the idea of remote state preparation [6,7].

As introduced in Ref. [3], let us consider a hypothetical game to explain the steerability of quantum states. Suppose Alice prepares two quantum systems, say, A and B, in an entangled state ρ_{AB} and sends the system B to Bob. Bob does not trust Alice but agrees with the fact that the system B is quantum. Therefore, Alice's task is to convince Bob that the prepared state is indeed entangled and they share a nonlocal correlation. On the other hand, Bob thinks that Alice may cheat by preparing the system B in a single quantum system, on the basis of possible strategies [8,9]. Bob agrees with Alice that the prepared state is entangled and they share a nonlocal correlation if and only if the state of Bob cannot be written by local hidden state model (LHS) [3]

$$\rho_A^a = \sum_\lambda \mathcal{P}(\lambda)\,\mathcal{P}(a|A,\lambda)\rho_B^Q(\lambda), \tag{1}$$

where $\{\mathcal{P}(\lambda),\rho_B^Q\}$ is an ensemble of LHS prepared by Alice and $\mathcal{P}(a|A,\lambda)$ is Alice's stochastic map to convince Bob. Here, we consider λ to be a hidden variable with the constraint $\sum_\lambda \mathcal{P}(\lambda) = 1$ and $\rho_B^Q(\lambda)$ is a quantum state received by Bob. The joint probability distribution on such states, $P(a_{\mathcal{A}_i},b_{\mathcal{B}_i})$, of obtaining an outcome a for the measurement of observables chosen from the set $\{\mathcal{A}_i\}$ by Alice and an outcome b for the measurement of observables chosen from the set $\{\mathcal{B}_i\}$ by Bob can be written as

$$P\left(a_{\mathcal{A}_i},b_{\mathcal{B}_i}\right) = \sum_\lambda P(\lambda)P\left(a_{\mathcal{A}_i}|\lambda\right)\,P_Q(b_i|\lambda), \tag{2}$$

where $P_Q(b_i|\lambda)$ is the quantum probability of the measurement outcome b_i due to the measurement of $\mathcal{B}_i$.

Several steering conditions have been derived on the basis of Eq. (2) and the existence of a single-system description of a part of the bipartite systems [8–10]. It has also been quantified for two-qubit systems [11]. In the last few years, several experiments have been performed to demonstrate the steering effect with increasing measurement settings [8] and with loophole-free arrangements [12]. For continuous variable systems, the steerability has also been quantified [13].

Recently, quantum coherence has been established as an important notion, especially in the areas of quantum information theory, quantum biology [14–18], and quantum thermodynamics [19–23]. In quantum information theory, it is expected that it can be used as a resource [24–26]. This has been the main motivation for recent studies to quantify and develop a number of measures of quantum coherence [24,25,27,28]. Most importantly, operational interpretations of the resource theory of quantum coherence have also been put forward [29,30]. An intriguing connection between quantum coherence and quantum speed limit (QSL) has been established [31,32]. However, much work needs to be done to really understand how to control and manipulate coherence so as to use it properly as a resource, particularly in multipartite scenarios.

In this Rapid Communication, we study the effects of nonlocality on quantum coherence in a bipartite scenario. We derive a set of inequalities for various quantum coherence measures. The violation of any one of these inequalities by the conditional states of a part of the system implies that it can achieve nonlocal advantage [the advantage, which cannot be achieved by a single system and local operations and classical communication (LOCC)] of quantum coherence. Moreover, these inequalities can also be considered as sufficient steering criteria. Intuitively, for quantum systems, it may seem that all steerable states can achieve a nonlocal advantage on quantum coherence. But here we show that for mixed states, steerability captured by different steering criteria [8–10] based

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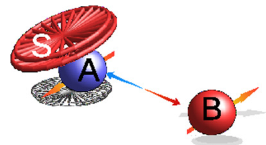

FIG. 1. Coherence of Bob's particle is being steered beyond what could have been achieved by a single system, only by local projective measurements on Alice's particle and classical communications (LOCC).

on uncertainty relations are drastically different from the steerability captured by coherence. In other words, we show that there are steerable states which cannot achieve a nonlocal advantage of quantum coherence.

One should note that we do not aim to derive a stronger steering criteria but aim to establish a connection between the steerability and the quantum coherence (see Fig. 1). This eventually leads us to show the effects of quantum steering on the speed of quantum evolutions (see the Supplemental Material [33]).

To quantify coherence, we consider the l_1-norm and the relative entropy of coherence as a measure of quantum coherence [24]. We also use the skew information [34], which is an observable measure of quantum coherence [25] and is also known as a measure of asymmetry [35–38]. The l_1-norm of the coherence of a state ρ is defined as $C^{l_1}(\rho) = \sum_{\substack{i,j \\ i\neq j}} |\rho_{i,j}|$. Now, if a qubit is prepared in either a spin-up or spin-down state along the z direction, then the qubit is incoherent when we calculate the coherence in the z basis (i.e., $C_z^{l_1} = 0$) and is fully coherent in the x and y bases, i.e., $C_{x(y)}^{l_1} = 1$. The l_1-norm of coherence of a general single qubit $\rho = \frac{1}{2}(I + \vec{n}\cdot\vec{\sigma})$ [where $|\vec{n}| \leqslant 1$ and $\vec{\sigma} \equiv (\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices] in the basis of the Pauli matrix σ_i is given by

$$C_i^{l_1}(\rho) = \sqrt{n_j^2 + n_k^2}, \tag{3}$$

where $k \neq i \neq j$ and $i,j,k \in \{x,y,z\}$.

Therefore, one may ask, what is the upper bound of $\mathcal{C}^{l_1} = C_x^{l_1}(\rho) + C_y^{l_1}(\rho) + C_z^{l_1}(\rho)$ for any general qubit state ρ. Using $C_x^{l_1}C_y^{l_1} + C_x^{l_1}C_z^{l_1} + C_y^{l_1}C_z^{l_1} \leqslant C_x^2 + C_y^2 + C_z^2 \leqslant 2$ (see Ref. [33]), we find that the above quantity is upper bounded by

$$\sum_{i=x,y,z} C_i^{l_1}(\rho) \leqslant \sqrt{6}, \tag{4}$$

where the equality sign holds for a pure state, which is an equal superposition of all the mutually orthonormal states spanning the state space, i.e.,

$$\rho_{\max}^{\mathcal{C}} = \frac{1}{2}\left[I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z)\right], \tag{5}$$

where I is the 2×2 identity matrix. Hence, in the single-system description, the quantity $\mathcal{C}^{l_1}$ cannot be larger than $\sqrt{6}$ and the corresponding inequality (4) can be thought as a coherence complementarity relation.

Another measure of coherence, called the relative entropy of coherence, is defined as [24] $C^E(\rho) = S(\rho_D) - S(\rho)$, where $S(\rho)$ is the von Neumann entropy of the state ρ and ρ_D is the diagonal matrix formed by the diagonal elements of ρ in a fixed basis, i.e., ρ_D is the completely decohered state of ρ. This quantity has also been considered as "wavelike information" in Ref. [39], which satisfies a duality relation. In this case, the sum of coherences of a single-qubit system in the three mutually unbiased bases for qubit systems is bounded by

$$\sum_{i=x,y,z} C_i^E(\rho) = \sum_{i=x,y,z} \mathcal{H}\left(\frac{1+n_i}{2}\right) - 3\mathcal{H}\left(\frac{1+|\vec{n}|}{2}\right),$$
$$\leqslant C_2^m, \tag{6}$$

where $\mathcal{H}(x) = -x\log_2(x) - (1-x)\log_2(1-x)$ and $|\vec{n}| = \sqrt{n_x^2+n_y^2+n_z^2}$. Using the symmetry, one can easily show that the maximum occurs at $n_x = n_y = n_z = 1/\sqrt{3}$ [i.e., for a maximally coherent state given by Eq. (5)] and $C_2^m \approx 2.23$.

Recently, the skew information [34] has also been considered as an observable measure of coherence of a state [25]. The coherence of a state ρ, captured by an observable $\mathcal{B}$, i.e., the coherence of the state in the basis of eigenvectors of the spin observable σ_i, is given by

$$C_i^S = -\frac{1}{2}\mathrm{Tr}[\sqrt{\rho},\sigma_i]^2 = \frac{(n_j^2+n_k^2)(1-\sqrt{1-|\vec{n}|^2})}{|\vec{n}|^2}, \tag{7}$$

which is a measure of the quantum part of the uncertainty for the measurement of the observable σ_i and hence it does not increase under a classical mixing of states [34]. The sum of the coherences measured by skew information in the bases of σ_x, σ_y, and σ_z is upper bounded by

$$\sum_{i=x,y,z} C_i^S(\rho) = 2(1-\sqrt{1-|\vec{n}|^2}) \leqslant 2, \tag{8}$$

where the maximum occurs for the maximally coherent state $\rho_{\max}^{\mathcal{C}}$ given by Eq. (5). The inequalities (4), (6), and (8) are complementarity relations for coherences of a state measured in the mutually unbiased bases.

Let us now describe our steering protocol, which we use to observe the effects of steering of the coherence of a part of a bipartite system. We consider a general two-qubit state of the form of

$$\eta_{AB} = \frac{1}{4}\left(I^A\otimes I^B + \vec{r}\cdot\sigma^A\otimes I^B + I^A\otimes\vec{s}\cdot\vec{\sigma}^B \right.$$
$$\left. + \sum_{i,j=x,y,z} t_{ij}\sigma_i^A\otimes\sigma_j^B\right), \tag{9}$$

where $\vec{r} \equiv (r_x,r_y,r_z)$, $\vec{s} \equiv (s_x,s_y,s_z)$, with $|r| \leqslant 1$, $|s| \leqslant 1$, and (t_{ij}) is the correlation matrix. Alice may, in principle, perform measurements in arbitrarily chosen bases. For simplicity, we derive the coherence steerability criteria for three measurement settings in the eigenbases of $\{\sigma_x,\sigma_y,\sigma_z\}$. When Alice declares that she performs a measurement on the eigenbasis of σ_z and obtains an outcome $a \in \{0,1\}$ with probability $p(\eta_{B|\Pi_z^a}) = \mathrm{Tr}[(\Pi_z^a\otimes I_B)\eta_{AB}]$, Bob measures the coherence randomly with respect to the eigenbasis of (say) the other two of the three Pauli matrices σ_x and σ_y. As Alice's measurement in the σ_k basis affects the coherence of Bob's state, the coherence of the conditional state of $B,\eta_{B|\Pi_k^a}$ in the basis of σ_i becomes

$C_i^{l_1}(\eta_{B|\Pi_k^a}) = \sqrt{\frac{\sum_{j\neq i}\alpha_{jk_a}^2}{\gamma_{k_a}^2}}$, where $\alpha_{ij_a} = s_i + (-1)^a t_{ji}$, $\gamma_{k_a} = 1 + (-1)^a r_k$, and $i,j,k \in \{x,y,z\}$. Note that the violation of any of the inequalities in Eqs. (4), (6), and (8) by the conditional states of Bob implies that a single-system description of the coherence of the system B does not exist. Thus, the criterion for achieving the nonlocal advantage on the quantum coherence of Bob using the l_1-norm comes out to be

$$\frac{1}{2}\sum_{i,j,a} p\left(\eta_{B|\Pi_{j\neq i}^a}\right) C_i^{l_1}\left(\eta_{B|\Pi_{j\neq i}^a}\right) > \sqrt{6}, \tag{10}$$

where $p(\eta_{B|\Pi_j^a}) = \frac{\gamma_{j_a}}{2}$, $i,j \in \{x,y,z\}$, and $a \in \{0,1\}$. This inequality forms a volume in the two-qubit state space.

Let us now derive the same criterion following the relative entropy of coherence measure. We can easily show that the eigenvalues of the conditional state of $B,\eta_{B|\Pi_i^a}$ are given by $\lambda_{i_a}^{\pm} = \frac{1}{2} \pm \frac{\sqrt{\sum_j \alpha_{ji_a}^2}}{2\gamma_{i_a}}$. Therefore, the relative entropy of coherence, when Alice measures in Π_k^a, is given by $C_i^E(\eta_{B|\Pi_k^a}) = \sum_{p=+,-} \lambda_{k_a}^p \log_2 \lambda_{k_a}^p - \beta_{ik_a}^p \log_2 \beta_{ik_a}^p$, where the diagonal element $\beta_{ij_a}^{\pm}$ of the conditional state $\eta_{B|\Pi_j^a}$, when expressed in the σ_i th basis, is given by $\beta_{ij_a}^{\pm} = \frac{1}{2} \pm \frac{\alpha_{ij_a}}{2\gamma_{j_a}}$. Thus, the criterion for achieving the nonlocal advantage of quantum coherence becomes (6)

$$\frac{1}{2}\sum_{i,j,a} p\left(\eta_{B|\Pi_{j\neq i}^a}\right) C_i^{E}\left(\eta_{B|\Pi_{j\neq i}^a}\right) > C_2^m, \tag{11}$$

where $i,j \in \{x,y,z\}$ and $a \in \{0,1\}$. Similarly, we obtain another inequality using the skew information as the observable measure of quantum coherence. The coherence of the conditional state $\eta_{B|\sigma_k^a}$ measured with respect to σ_i in this case is given by $C_i^S(\eta_{B|\Pi_k^a}) = \frac{(\sum_{j\neq i}\alpha_{jk_a}^2)[1-\sqrt{1-(2\lambda_{k_a}^{\pm}-1)^2}]}{\gamma_{k_a}^2(2\lambda_{k_a}^{\pm}-1)^2}$. Thus, from Eq. (8) we get the coherence steering inequality using the skew-information complementarity relation as

$$\frac{1}{2}\sum_{i,j,a} p\left(\eta_{B|\Pi_{j\neq i}^a}\right) C_i^{S}\left(\eta_{B|\Pi_{j\neq i}^a}\right) > 2, \tag{12}$$

where $i,j \in \{x,y,z\}$ and $a \in \{0,1\}$.

It is important to mention here that although the violation of the coherence complementarity relations in Eqs. (4), (6), and (8) implies the steerability of the quantum state and the achievability of the nonlocal advantage of quantum coherence, its violation is highly dependent on the measurement settings [33]. Therefore, the state of Bob (B) can achieve the nonlocal advantage of quantum coherence with the help of Alice if at least one of the inequalities in Eqs. (10)–(12) is satisfied, but it is not necessary. A better choice of projective measurement bases by Alice may reveal steerability of an apparently unsteerable state with respect to the above inequalities. On the other hand, it is also necessary to show that separable states can never violate the coherence complementarity relations using the present protocol.

To show that no separable state can violate the coherence complementarity relations, we use the protocol stated above for an arbitrary number n of measurement settings. We consider a two-qubit separable state ρ_{ab} as

$$\rho_{AB} = \sum_i p_i \rho_A^i \otimes \rho_B^i, \tag{13}$$

with $\sum_i p_i = 1$ and $p_i > 0$ for all i. Suppose Alice performs a projective measurement in an arbitrary basis Π_n^a, where $a \in \{0,1\}$ corresponding to two outcomes of the measurement and $n \in \mathcal{Z}^+$ (set of positive integers), each measurement basis associated with an integer. To compare with the coherence complementarity relations, Alice must choose a $3\mathcal{Z}^+$ number of measurement bases, making n to run up to $3k$ (say), where $k \in \mathcal{Z}+$. This provides Bob a 2^k number of coherence measurement results on a particular Pauli basis. This is due to the fact that for measurements on each basis, Bob can measure coherence randomly only on two of the three mutually unbiased Pauli bases. Bob receives the state $\rho_{B|\Pi_n^a} = \frac{\sum_i p_i \langle n^a|\rho_A^i|n^a\rangle\rho_B^i}{\sum_i p_i\langle n^a|\rho_A^i|n^a\rangle}$ with probability $p(\rho_{B|\Pi_n^a}) = \sum_i p_i \langle n^a|\rho_A^i|n^a\rangle$ due to the projective measurement Π_n^a by Alice. If the proposed protocol is followed, one can show that the above state in Eq. (13) can never violate the coherence complementarity relations. To show that, we start with

$$\begin{aligned}
&\sum_{a=0,n=1,m=0}^{1,3k,1} p\left(\rho_{B|\Pi_n^a}\right) C_{n\oplus m}^q\left(\rho_{B|\Pi_n^a}\right) \\
&\quad\leqslant \sum_{a,n,m,i} p_i C_{n\oplus m}^q\left(\langle n^a|\rho_A^i|n^a\rangle\rho_B^i\right) \\
&\quad\leqslant \sum_{a,n,m,i} p_i\langle n^a|\rho_A^i|n^a\rangle C_{n\oplus m}^q\left(\rho_B^i\right) \\
&\quad= \sum_i \sum_{n=1,m=0}^{3k,1} p_i C_{n\oplus m}^q\left(\rho_B^i\right),
\end{aligned} \tag{14}$$

where we denote $n \oplus m = \text{mod}(n+m,3)+1$ and $q \in \{l_1,E,S\}$, stands for various measures of coherence. In the second and the third inequalities, we used the fact that the coherence and the observable measure of quantum coherence decreases under a classical mixing of states. Here, we use the notation $\{C_1^q,C_2^q,C_3^q\} \equiv \{C_x^q,C_y^q,C_z^q\}$. By taking the summation over n and m, one can show from the last line of Eq. (14) that

$$\begin{aligned}
&\sum_{a,n,m} p\left(\rho_{B|\Pi_n^a}\right) C_{n\oplus m}^q\left(\rho_{B|\Pi_n^a}\right) \\
&\quad\leqslant 2^k \sum_i p_i\left[C_x^q\left(\rho_B^i\right) + C_y^q\left(\rho_B^i\right) + C_z^q\left(\rho_B^i\right)\right] \\
&\quad\leqslant 2^k \sum_i p_i \epsilon^q = 2^k \epsilon^q,
\end{aligned} \tag{15}$$

where $\epsilon^q \in \{\sqrt{6},2.23,2\}$ depending on q. This implies that the coherence complementarity relations can never be violated by any separable state. Mathematically, for any separable state,

$$\frac{1}{2}\sum_{a=0,n=1,m=0}^{1,3,1} p\left(\rho_{B|\Pi_n^a}\right) C_{n\oplus m}^q\left(\rho_{B|\Pi_n^a}\right) \leqslant \epsilon^q, \tag{16}$$

for the three-measurement-settings scenario ($k = 1$). A violation of this inequality implies that the state is steerable and

Bob can achieve a nonlocal advantage of quantum coherence by Alice.

Let us now illustrate the coherence steerability condition with an example, say, a two-qubit Werner state defined by $\rho_w = p|\psi^-_{AB}\rangle\langle\psi^-_{AB}| + \frac{(1-p)}{4} I^A \otimes I^B$, where $|\psi^-_{AB}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ and the mixing parameter p is chosen from the range $0 \leqslant p \leqslant 1$. For this state, $\vec{r} = 0, \vec{s} = 0$, and $t_{xx} = t_{yy} = p, t_{zz} = p$. The state ρ_w is steerable for $p > \frac{1}{2}$, entangled for $p > \frac{1}{3}$, and Bell nonlocal for $p > \frac{1}{\sqrt{2}}$.

Here, the optimal strategy for Alice to maximize the violation of the coherence complementary relation by Bob's conditional state is similar to that stated earlier for the derivation of Eqs. (10)–(12). With the help of these inequalities, it is easy to show that for the Werner state, the coherence of the state of B is steerable for $p > \sqrt{\frac{2}{3}}$ when one uses the l_1-norm as a measure of coherence, $p > 0.914$ when one uses the relative entropy of coherence as a measure, and $p > \frac{2\sqrt{2}}{3}$ for the choice of skew information as a measure of quantum coherence.

Hence, Alice controls the coherence of Bob's system for $p > \sqrt{\frac{2}{3}}$ whereas Alice controls Bob's state for $p > \frac{1}{2}$. This difference occurs due to the presence of the noise part ($\frac{I\otimes I}{4}$) in steering the state, whereas the coherence steerability criteria are never influenced by such classical noise. This raises a natural question: Is it possible to increase the range of p to control the coherence of Bob's system using local filtering operations? It has been shown that filtering operations can improve the steerablity [40]. From Fig. 2, it is clear that the filtering operation on Bob can increase the range of p to some extent for certain values of θ, for which the resulting state can achieve the nonlocal advantage of quantum coherence from Alice to Bob. Moreover, any steerable Werner state can be turned into an unsteerable state by local filtering operations [40] (see Fig. 2).

To summarize, in this Rapid Communication, we use various measures of quantum coherence and derive complementarity relations (4), (6), and (8) between coherences of a single quantum system (qubit) measured in the mutually unbiased bases. Using these complementarity relations, we derive conditions (10)–(12), under which the nonlocal advantage of quantum coherence can be achieved for any general two-qubit bipartite systems. These conditions also provide sufficient criteria for the state to be steerable. We also show that not all steerable states can achieve the nonlocal advantage on quantum coherence.

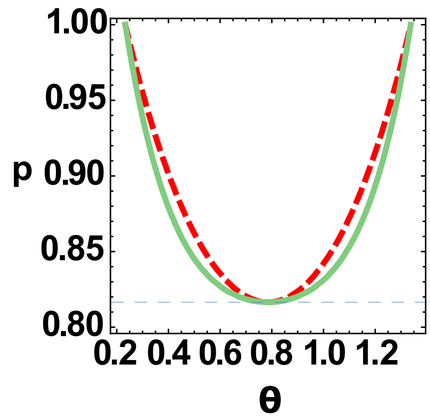

FIG. 2. Filtering operation $F(\theta) = \text{diag}\{1/\cos(\theta), 1/\sin\theta\}$ is applied on the Werner state ρ_w. The red colored dashed line corresponds to the situation when $F(\theta)$ is applied on Alice and the green solid plot is when it is applied on Bob. The nonlocal advantage of quantum coherence is not achievable by the resulting state for the ranges of p under the curves. For example, the resulting state is steerable or the state can achieve the nonlocal advantage of quantum coherence for Bob for $p \geqslant 0.845$, when $F(\theta \approx 0.5)$ is applied on Bob. Here, we assume that Alice is the steering party, and Bob is the party to be steered. The horizontal thin dashed line denotes $p = \sqrt{\frac{2}{3}}$.

Additionally, our results reveal an important connection between quantum nonlocality and the quantum speed limit (see the Supplemental Material [33]). One can show that not all steerable states, or, for that matter, not even all states, for which a nonlocal advantage on quantum coherence is achievable, can, in principle, achieve a nonlocal advantage on the QSL [33]. Only those states which achieve a nonlocal advantage on the observable measure of quantum coherence or asymmetry [35–38] can achieve a nonlocal QSL [33]. One important application of our results has been uncovered in the detection of Unruh effects as well [41].

Note added. Recently, Fan *et al.* presented a study on the quantum coherence of steered states [42]. We consider our works to be complementary. Though examining a similar topic, our approaches are very different [we consider steering from the existence of a local hidden state model rather than from the perspective of the quantum size effect (QSE) formalism].

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[1] E. Schrödinger, Proc. Cambridge Philos. Soc. **31**, 553 (1935); **32**, 446 (1936).

[2] A. Einstein, D. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[3] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007); S. J. Jones, H. M. Wiseman, and A. C. Doherty, Phys. Rev. A **76**, 052116 (2007).

[4] J. S. Bell, Physics **1**, 195 (1964); J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

[5] S. A. Babichev, B. Brezger, and A. I. Lvovsky, Phys. Rev. Lett. **92**, 047903 (2004).

[6] A. K. Pati, Phys. Rev. A **63**, 014302 (2000).

[7] C. H. Bennett, D. P. DiVincenzo, P. W. Shor, J. A. Smolin, B. M. Terhal, and W. K. Wootters, Phys. Rev. Lett. **87**, 077902 (2001); **88**, 099902(E) (2002); C. H. Bennett, P. Hayden, D. W. Leung, P. W. Shor, and A. Winter, IEEE Trans. Inf. Theory **51**, 56 (2005).

[8] D. J. Saunders, S. J. Jones, H. M. Wiseman, and G. J. Pryde, Nat. Phys. **6**, 845 (2010).

[9] T. Pramanik, M. Kaplan, and A. S. Majumdar, Phys. Rev. A **90**, 050305(R) (2014); P. Chowdhury, T. Pramanik, and A. S. Majumdar, *ibid.* **92**, 042317 (2015).
[10] S. P. Walborn, A. Salles, R. M. Gomes, F. Toscano, and P. H. Souto Ribeiro, Phys. Rev. Lett. **106**, 130402 (2011); J. Schneeloch, C. J. Broadbent, S. P. Walborn, E. G. Cavalcanti, and J. C. Howell, Phys. Rev. A **87**, 062103 (2013); Q. Quan, H. Zhu, S.-Y. Liu, S.-M. Fei, H. Fan, and W.-L. Yang, Sci. Rep. **6**, 22025 (2016).
[11] A. C. S. Costa and R. M. Angelo, Phys. Rev. A **93**, 020103(R) (2016).
[12] D. Wittmann *et al.*, New J. Phys. **14**, 053030 (2012); D. H. Smith *et al.*, Nat. Commun. **3**, 625 (2012); A. J. Bennet, D. A. Evans, D. J. Saunders, C. Branciard, E. G. Cavalcanti, H. M. Wiseman, and G. J. Pryde, Phys. Rev. X **2**, 031003 (2012); V. Handchen *et al.*, Nat. Photonics **6**, 598 (2012); S. Steinlechner, J. Bauchrowitz, T. Eberle, and R. Schnabel, Phys. Rev. A **87**, 022104 (2013).
[13] I. Kogias, A. R. Lee, S. Ragy, and G. Adesso, Phys. Rev. Lett. **114**, 060403 (2015).
[14] M. B. Plenio and S. F. Huelga, New J. Phys. **10**, 113019 (2008).
[15] P. Rebentrost, M. Mohseni, and A. Aspuru-Guzik, J. Phys. Chem. B **113**, 9942 (2009).
[16] S. Lloyd, J. Phys.: Conf. Ser. **302**, 012037 (2011).
[17] S. Huelga and M. Plenio, Contemp. Phys. **54**, 181 (2013).
[18] *Quantum Aspects of Life*, edited by D. Abbott, P. Davies, and A. K. Pati (Imperial College Press, London, 2008).
[19] C. A. Rodríguez-Rosario, T. Frauenheim, and A. Aspuru-Guzik, arXiv:1308.1245.
[20] V. Narasimhachar and G. Gour, Nat. Commun. **6**, 7689 (2015).
[21] M. Lostaglio, D. Jennings, and T. Rudolph, Nat. Commun. **6**, 6383 (2015).
[22] M. Lostaglio, K. Korzekwa, D. Jennings, and T. Rudolph, Phys. Rev. X **5**, 021001 (2015).
[23] B. Gardas and S. Deffner, Phys. Rev. E **92**, 042126 (2015).
[24] T. Baumgratz, M. Cramer, and M. B. Plenio, Phys. Rev. Lett. **113**, 140401 (2014).
[25] D. Girolami, Phys. Rev. Lett. **113**, 170401 (2014).
[26] A. Streltsov, U. Singh, H. S. Dhar, M. N. Bera, and G. Adesso, Phys. Rev. Lett. **115**, 020403 (2015).
[27] J. Aberg, arXiv:quant-ph/0612146.
[28] U. Singh, M. N. Bera, H. S. Dhar, and A. K. Pati, Phys. Rev. A **91**, 052115 (2015).
[29] A. Winter and D. Yang, Phys. Rev. Lett. **116**, 120404 (2016).
[30] U. Singh, M. Bera, A. Misra, and A. K. Pati, arXiv:1506.08186.
[31] D. Mondal, C. Datta, and S. Sazim, Phys. Lett. A **380**, 689 (2016).
[32] I. Marvian, R. W. Spekkens, and P. Zanardi, Phys. Rev. A **93**, 052331 (2016).
[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.95.010301 for further proofs, examples and connection with QSL.
[34] S. Luo, Phys. Rev. Lett. **91**, 180403 (2003); Theor. Math. Phys. **143**, 681 (2005).
[35] G. Gour and R. W. Spekkens, New J. Phys. **10**, 033023 (2008).
[36] G. Gour, I. Marvian, and R. W. Spekkens, Phys. Rev. A **80**, 012307 (2009).
[37] M. Ahmadi, D. Jennings, and T. Rudolph, New J. Phys. **15**, 013057 (2013).
[38] I. Marvian and R. W. Spekkens, Phys. Rev. A **90**, 014102 (2014).
[39] R. M. Angelo and A. D. Ribeiro, Found. Phys. **45**, 1407 (2015).
[40] J. Bowles, F. Hirsch, M. T. Quintino, and N. Brunner, Phys. Rev. A **93**, 022121 (2016).
[41] D. Mondal and C. Mukhopadhyay, arXiv:1510.07556.
[42] X. Hu, A. Milne, B. Zhang, and H. Fan, Sci. Rep. **6**, 19365 (2016).