Self-similar localized pulses for the nonlinear Schrödinger equation with distributed cubic-quintic nonlinearity

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We obtain the exact analytical self-similar bright-, dark-, and kink-solitary-wave solutions of the nonlinear Schrödinger equation with localized inhomogeneous cubic-quintic nonlinearity by employing a similarity transformation. This equation could be a model equation of stable pulse propagation beyond ultrashort range in optical fiber communication systems in inhomogeneous media. We have investigated that the self-similar bright- and dark-solitary-wave solitons show interesting compression and amplification features that can be controlled by suitably changing the phase modulation parameter and consequently varying the nonlinear parameter. We unearth a surprising connection between optical self-similar localized waves in graded-index inhomogeneous media and solitons in homogeneous media with the same type of cubic-quintic nonlinearity. Finally, the stability of the solutions is discussed numerically under finite initial perturbations.

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I. INTRODUCTION

For present-day applications in telecommunication and ultrafast signal routing systems and gradual and steady progress in high-repetition-rate, beyond-ultrashort- (autosecond) pulse sources based on fiber technology [1], it is very important to study the management of the solitary-wave solutions of the nonlinear Schrödinger equation (NLSE) family of equations in optical fiber communication systems with distributed coefficients, such as group-velocity dispersion, distributed nonlinearity, and distributed gain and/or loss. In this context, the presence of variable-dependent coefficients of dispersion and higher-degree nonlinear terms in the considered model is more realistic in various situations to handle the optical systems. The reason for this is that constant-coefficient models can only describe the propagation of wave groups in perfect systems. The constant-coefficient nonlinear Schrödinger equation with cubic-quintic nonlinearities and its stationary solitary-wave solutions, such as solitons, have been extensively studied in various fields such as Bose-Einstein condensates [2] and especially nonlinear optical systems [3]. Soliton propagation in nonlinear media [4] has attracted much interest in recent years due to its extensive usage in the field of optical communication and all-optical ultrafast switching devices [5]. It is well known that the cubic-quintic NLSE [5] is a generic model for describing the propagation of femtosecond optical pulses in single-mode fibers [6]. Unlike the ideal uniform fiber material, in a real fiber there is always some nonuniformity in the core medium [7] due to many factors such as variation of lattice parameters and diameter fluctuations. In optical fibers, variation of the nonlinearity can be achieved by varying the type of dopants along the fiber. In such an inhomogeneous medium (graded-index waveguide) for high-intensity optical pulse propagation in Kerr and non-Kerr media we have to consider higher-order nonlinearity, where the refractive index varies as $n(z,t) = n_0 + n_1 f(z)t^2 + n_2\gamma_1(z)I(z,t) - n_4\gamma_2(z)I^2(z,t)$ and the propagation of an optical beam in such a highly nonlinear inhomogeneous waveguide can be described by the cubic-quintic nonlinear Schrödinger equation (CQNLSE) [8]

$$i\Psi_z + \frac{\beta(z)}{2}\Psi_{tt} + g_1\gamma_1(z)|\Psi|^2\Psi + g_2\gamma_2(z)|\Psi|^4\Psi + \frac{f(z)}{2}t^2\Psi - i\Gamma(z)\Psi = 0, \quad (1)$$

where $\Psi(z,t)$ is a slowly varying complex envelope of the electric field in the comoving frame, z is the normalized distance representing the coordinate along the propagation direction, and t is normalized time with the frame of reference moving along the fiber at the group velocity. In addition, $\beta(z)$, $\gamma_1(z)$, and $\gamma_2(z)$ are the group-velocity dispersion, cubic nonlinear, and quintic nonlinear parameters, respectively. The inhomogeneous parameters $f(z)$ and $\Gamma(z)$ are related to the phase modulation and the attenuation (loss) ($\Gamma < 0$) or amplification (gain) ($\Gamma > 0$), respectively. All the above parameters are functions of the propagation distance z. Here n_0 is the linear refractive index coefficient and n_2 and n_4 are the nonlinear refractive index coefficients originating from, respectively, third- and fifth-order susceptibility. The quintic nonlinearity, which arises due to fifth-order susceptibility [9], can be obtained in many optical materials such as semiconductors, semiconductor-doped glasses, polydiacctylene toluene sulfonate, calcogenide glasses, and some transparent organic materials.

To date, most of the theoretical and experimental studies regarding self-similar optical pulse propagations and their interactions in nonuniform graded-index fiber have focused on using the inhomogeneous NLSE up to variable cubic nonlinearity [10]. In this context, self-similarity of a complex nonlinear system points to the presence of an internal order and it is key to extracting physical insight about its evolution [7]. Such formal similarity techniques extend the toolbox available to mathematical physicists and are of particular importance in analyzing nonlinear problems described by nonlinear partial differential equations, which are notoriously difficult to solve

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exactly. With the help of similarity transformations, the original, more complicated problem of solving partial differential equations can be recast into a simplified problem involving well-known differential equations [7,10]. In particular, the self-similar evolution of a nonlinear wave implies that the wave profile remains unchanged and its amplitudes and widths simply scale with the modulation of the system parameters, e.g., time or propagation distance. Recently, many authors have tried to study the cubic-quintic inhomogeneous or nonautonomous NLSE with varying space- or time-dependent coefficients due to its intensive use in nonlinear optics [11] as well as matter wave soliton theory [12]. In particular, the study of topological quasisoliton solutions for the inhomogeneous CQNLSE began with the pioneering work of Serkin *et al.* [13]. By means of the similarity transformations, Dai *et al.* [14] obtained exact self-similar solutions (similaritons), their nonlinear tunneling effects of the generalized CQNLSE, and their higher-dimensional forms with spatially inhomogeneous group-velocity dispersion, cubic-quintic nonlinearity, and amplification or attenuation. It should be noted that the similarity transformations used by many authors [13–15] to study the self-similar properties of the inhomogeneous systems in different fields mainly depend on the inhomogeneous dispersion parameter and the dispersion management technique is relevant for controlling the self-similar properties of the corresponding systems.

II. MODEL EQUATION

In the present work, by making use of a type of similarity transformation that solely depends on the inhomogeneous cubic-quintic nonlinear parameters, we study self-similar optical bright-, dark-, and kink-type-solitary-wave solutions and their scaling profile structures of the inhomogeneous CQNLSE

$$i\Psi_z + \frac{1}{2}\Psi_{tt} + g_1\gamma_1(z)|\Psi|^2\Psi + g_2\gamma_2(z)|\Psi|^4\Psi + \frac{\lambda^2}{2}t^2\Psi = 0. \tag{2}$$

Here λ is a real constant parameter related to phase modulation. When the dispersion parameter $\beta(z)$ and the parameter $f(z)$ related to phase modulation are constant and without an attenuation or amplification term, Eq. (1) becomes Eq. (2). Equation (2) contains only localized inhomogeneity in the nonlinear coefficients. Physically, this equation is very important in modeling the optical propagation through a highly nonlinear inhomogeneous medium because it is a well-established fact from the study of modulational instability that the quintic non-Kerr nonlinear terms are important [15,16] over the cubic Kerr nonlinearity as quintic non-Kerr nonlinearities are responsible for the stability of localized solutions. The importance of the results reported here is twofold. First, the similarity transformation approach leads to an important class of exact self-similar solitary-wave solutions to the NLSE with localized inhomogeneous cubic-quintic nonlinearity in a systematic way. The finding of such self-similar solutions proves the efficiency of the self-similarity technique in searching for exact solutions of an equation having applications in a variety of physical systems and is not integrable by the inverse scattering method. Further, the study of these self-similar solutions has been of great value in understanding widely different nonlinear physical phenomena [7]. The second and more interesting significance of these results lies in their potential application to optical fiber amplifiers, optical fiber compressors, and optical communications links.

Self-similar solutions. To obtain the exact analytic optical solitary-wave solution of Eq. (2) we can employ the transformation

$$\Psi(z,t) = \sqrt{\frac{\gamma_1(z)}{\gamma_2(z)}}u(Z,T)e^{i\phi(z,t)}, \quad Z = G(z),\ T = F(z,t) \tag{3}$$

to reduce Eq. (2) to the standard constant-coefficient CQNLSE of the known form

$$iu_Z + \tfrac{1}{2}u_{TT} + g_1|u|^2u + g_2|u|^4u = 0, \tag{4}$$

only if the following conditions are satisfied:

$$\frac{F_t^2}{\frac{dG}{dz}} = 1, \quad \frac{\gamma_1^2}{\gamma_2\frac{dG}{dz}} = 1, \quad \frac{d\gamma_2}{dz} = 0 \tag{5a}$$

and

$$\lambda^2 = \frac{2}{\gamma_1^2}\left(\frac{d\gamma_1}{dz}\right)^2 - \frac{1}{\gamma_1}\frac{d^2\gamma_1}{d^2z} + \frac{1}{2\gamma_2}\frac{d^2\gamma_2}{d^2z} - \frac{1}{4\gamma_2^2}\left(\frac{d\gamma_2}{dz}\right)^2 - \frac{1}{\gamma_1\gamma_2}\frac{d\gamma_1}{dz}\frac{d\gamma_2}{dz}, \tag{5b}$$

with

$$\phi(z,t) = -\frac{t^2}{2\gamma_1}\frac{d\gamma_1}{dz} + \frac{t^2}{4\gamma_2}\frac{d\gamma_2}{dz}. \tag{5c}$$

Now to determine the real functions $\gamma_1(z)$, $\gamma_2(z)$, $G(z)$, $F(z,t)$, and $\phi(z,t)$ we have to solve Eqs. (5a) and (5b). To satisfy Eqs. (5a) and (5b) we must have

$$\gamma_1(z) = e^{\lambda z}, \quad \gamma_2(z) = \text{const} = c \text{ (say)} \tag{6}$$

and the coordinate transformations are

$$Z = G(z) = \frac{e^{2\lambda z}}{2c\lambda}, \quad T = F(z,t) = \frac{te^{\lambda z}}{\sqrt{c}}. \tag{7}$$

Using Eqs. (5c) and (6), one can solve $\phi(z,t) = -\frac{\lambda t^2}{2}$. The coordinate transformations we have used are different from Galilean or scaling type [17–19]. In this particular context, Gagnon and Winternitz [17] studied the generalized (3+1)-dimensional CQNLSE in the framework of the group-theoretic approach, showed that the equations concerned are invariant under the extended Galilei-similitude group, and found the group invariant solution. More recently, using a technique based on the symmetry reduction method and applying the Galilean coordinate transformation, Schürmann [18] studied the constant-coefficient CQNLSE to find the traveling-wave solutions and Agüero [19] investigated the one, two, and three gray solutions and their interaction features of the CQNLSE, taking recourse in the use of known soliton solutions of the Boussinesq-like equation using scaling transformations.

III. LOCALIZED SOLITARY-WAVE SOLUTIONS

In the following sections we first find bright-, dark-, and kink-solitary-wave localized solutions of the constant-coefficient CQNLSE (4). Then we will employ these three types of localized solutions of the constant-coefficient CQNLSE (4) to construct the self-similar localized solitary-wave solutions of the inhomogeneous CQNLSE (2).

A. A bright solitary wave

We find that Eq. (4) satisfies the bright solitary wave of the form

$$u_b(Z,T) = 2\sqrt{\frac{A_b}{g_1}}\,\frac{\exp\left[iwT - i\left(\rho_b - \frac{w^2}{2}\right)Z\right]}{\sqrt{1 + N_b\cosh[\alpha_b(T - wZ)]}}, \tag{8}$$

where $\alpha_b = 2\sqrt{2A_b}$, $1 - N_b^2 = \frac{16\beta_0 A_b}{3}$, and $A_b = \rho_b - w^2$ with $\beta_0 = -\frac{g_2}{g_1^2}$. Here N_b is a real number and A_b, w, and α_b are related to the amplitude, group velocity, and pulse width of the bright wave profile.

B. A dark solitary wave

The dark-solitary-wave solution of Eq. (4) has the form

$$u_d(Z,T) = \frac{A_d}{\sqrt{-g_1}} \times \frac{\sinh[\alpha_d(T - wZ)]\exp\left[iwT - i\left(\rho_d - \frac{w^2}{2}\right)Z\right]}{\sqrt{1 + N_d\sinh^2[\alpha_d(T - wZ)]}}, \tag{9}$$

where $\alpha_d = \sqrt{\frac{3(N_d - 1)}{2\beta_0(3N_d - 2)^2}}$, $\rho_d - w^2 = \frac{(3N_d - 1)\alpha_d^2}{2}$, and $A_d = \sqrt{(3N_d - 2)\alpha_d^2 N_d}$ with $\beta_0 = \frac{g_2}{g_1^2}$. Here $N_d > 1$ and A_d, w, and α_d are related to the amplitude, group velocity, and pulse width of the dark wave profile.

C. A Kink solitary wave

The kink-solitary-wave solution obtained for Eq. (4) is

$$u_k(Z,T) = \frac{A_k}{\sqrt{g_1}}\sqrt{1 + \tanh[\alpha_k(T - wZ)]} \times \exp\left[iwT - i\left(\rho_k - \frac{w^2}{2}\right)Z\right], \tag{10}$$

where $\alpha_k = A_k$ under the parametric condition $A_k^2 = -\frac{3}{8\beta_0}$ with $A_k = \sqrt{-2(\rho_k - w^2)}$ and $\beta_0 = \frac{g_2}{g_1^2}$. In addition, A_k, w, and α_d are related to the amplitude, group velocity, and pulse width of the kink wave profile.

IV. SELF-SIMILAR LOCALIZED SOLITARY-WAVE SOLUTIONS

Making use of the bright-, dark-, and kink-type solutions given in Eqs. (8), (9), and (10), respectively, of the standard constant-coefficient CQNLSE (4), the transformations in Eq. (3), and Eqs. (5c) and (6), we can construct the self-similar solutions of the inhomogeneous CQNLSE (2). The self-similar bright optical solution of Eq. (2) is given by

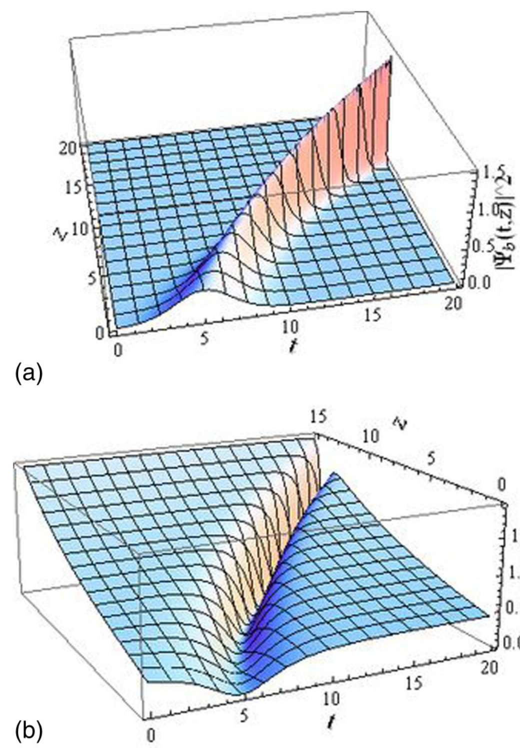

FIG. 1. Evolution of (a) the bright self-similar intensity wave profile $|\Psi_b(t,z)|^2$ as computed from Eq. (11) for the parameter values $\lambda = 0.1, g_1 = 1, g_2 = -1, \beta_0 = -1, N_b = 0.5, w = 1$, and $c = 1$ and (b) the dark self-similar intensity wave profile $|\Psi_d(z,t)|^2$ as computed from Eq. (12) for the parameter values $\lambda = 0.1$, $g_1 = -1$, $g_2 = 1$, $\beta_0 = 1$, $N_d = 3$, $w = 1$, and $c = 1$.

$$\Psi_b(z,t) = \sqrt{\frac{\gamma_1(z)}{\gamma_2(z)}}\,u_b(Z,T)e^{i\phi(z,t)}, \tag{11}$$

the dark one is found to be

$$\Psi_d(z,t) = \sqrt{\frac{\gamma_1(z)}{\gamma_2(z)}}\,u_d(Z,T)e^{i\phi(z,t)}, \tag{12}$$

and the self-similar kink-type solution is of the form

$$\Psi_k(z,t) = \sqrt{\frac{\gamma_1(z)}{\gamma_2(z)}}\,u_k(Z,T)e^{i\phi(z,t)}. \tag{13}$$

Figure 1(a) shows the evolution of the self-similar optical bright-solitary-wave solution calculated with the framework of the inhomogeneous CQNLSE (2) with the parametric values $\lambda = 0.1$, $\beta_0 = -1$, $N_b = 0.5$, $w = 1$, and $c = 1$, while Fig. 1(b) displays the evolution of the self-similar dark soliton. In this case the parametric values used are $\lambda = 0.1$, $\beta_0 = 1$, $N_d = 3$, $w = 1$, and $c = 1$. The self-similar optical bright and dark solutions exist under certain conditions that impose constraints on the coefficients of cubic and quintic nonlinearity. The most interesting features of these self-similar optical bright and dark solutions are their compression and amplitude amplification. In this context, the technique for generating an ultrahigh-repetition-rate soliton train lies in soliton compression. As a means of achieving this, there are

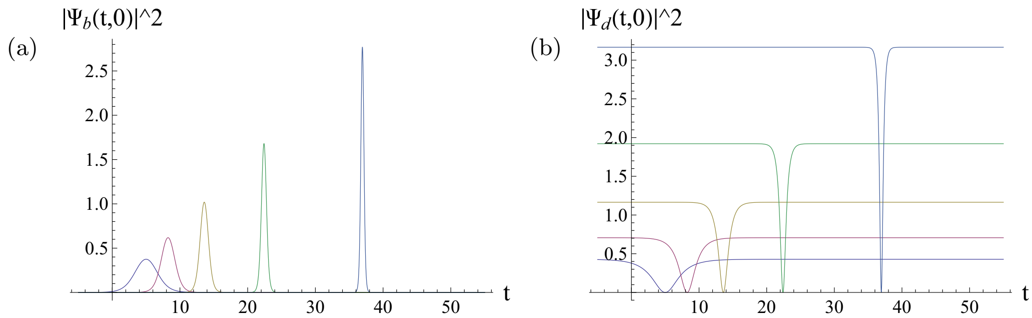

FIG. 2. Self-similar (a) bright and (b) dark pulse compression with the same parametric values as in Figs. 1(a) and 1(b) at different propagation distances.

reports of methods using dispersion-decreasing fiber [20] and compression using nonlinear effects in optical fibers [5]. We can compress the self-similar bright and dark solitons of the inhomogeneous CQNLSE to a desired width and amplitude by suitably changing the constant phase modulation parameter λ and consequently varying the nonlinear parameter $\gamma_1(z)$ given in Eq. (6). We show in Figs. 2(a) and 2(b) the self-similar bright and dark pulse compression and amplitude amplification for different propagation distances z. The parametric values for Figs. 2(a) and 2(b) are similar to those in Figs. 1(a) and 1(b), respectively. For low values of λ we show in Figs. 3(a) and 3(b) the evolution of the bright and dark intensity profiles for the same parametric values as in Figs. 1(a) and 1(b), respectively. Here we use $\lambda = 0.001$. Interestingly, for small values of λ the intensity profiles of the self-similar waves coincide with those solitons supported by homogeneous, passive media with the same type of nonlinearity. Figure 4(a) shows the evolution of self-similar optical kink-solitary-wave solution calculated with the framework of the inhomogeneous CQNLSE (2) with the parametric values $\lambda = 0.1$, $g_1 = 1$, $g_2 = -1$, $w = 1$, and $c = 1$. For low values of λ we show in Fig. 4(b) the evolution of the intensity wave profile of the self-similar kink for the same parametric values as in Fig. 4(a). Here we use $\lambda = 0.001$. Also note that for small values of λ the intensity profiles of the self-similar optical kink waves coincide with the kink soliton supported by homogeneous, passive media with the same type of nonlinearity.

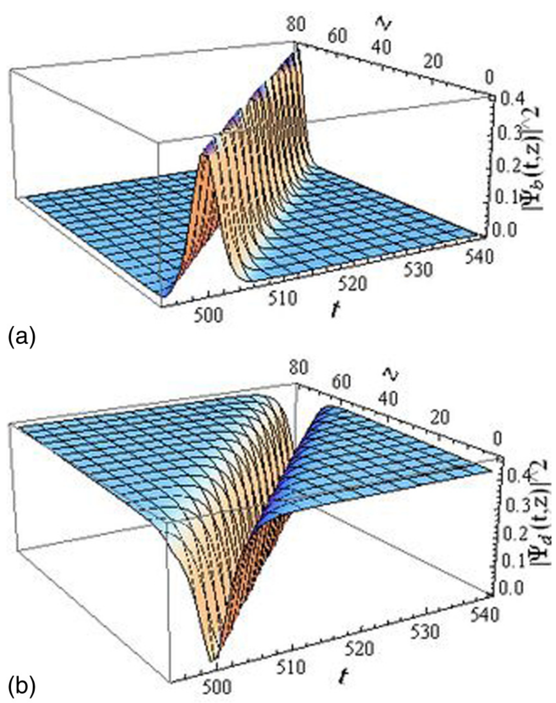

FIG. 3. Evolution of (a) the bright self-similar intensity wave profile $|\Psi_b(t,z)|^2$ as computed from Eq. (11) for the parameter values $\lambda = 0.001$, $g_1 = 1$, $g_2 = -1$, $\beta_0 = -1$, $N_b = 0.5$, $w = 1$, and $c = 1$ and (b) the dark self-similar intensity wave profile $|\Psi_d(z,t)|^2$ as computed from Eq. (12) for the parameter values $\lambda = 0.001$, $g_1 = -1$, $g_2 = 1$, $\beta_0 = 1$, $N_d = 3$, $w = 1$, and $c = 1$.

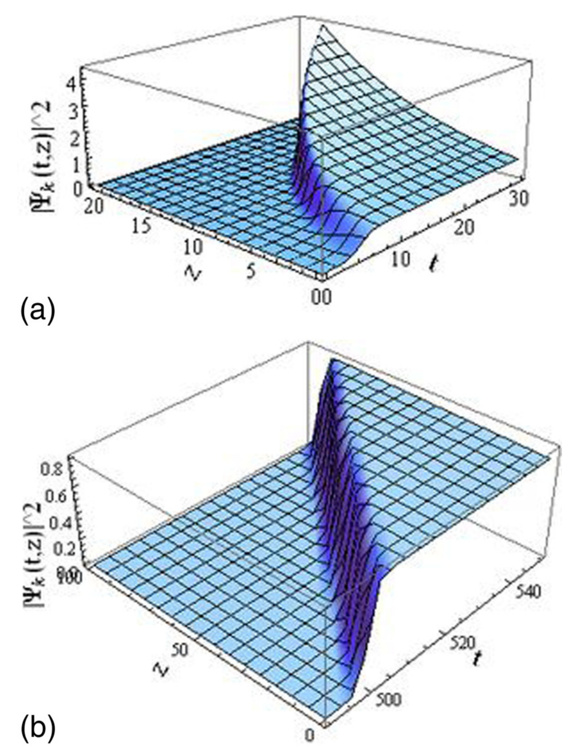

FIG. 4. Evolution of (a) the kink self-similar intensity wave profile $|\Psi_k(t,z)|^2$ as computed from Eq. (13) for the parameter values $\lambda = 0.1$, $g_1 = 1$, $g_2 = -1$, $w = 1$, and $c = 1$ and (b) the intensity wave profile of the self-similar kink $|\Psi_k(z,t)|^2$ as computed from Eq. (13) for the same parametric values as in (a), except that here $\lambda = 0.001$.

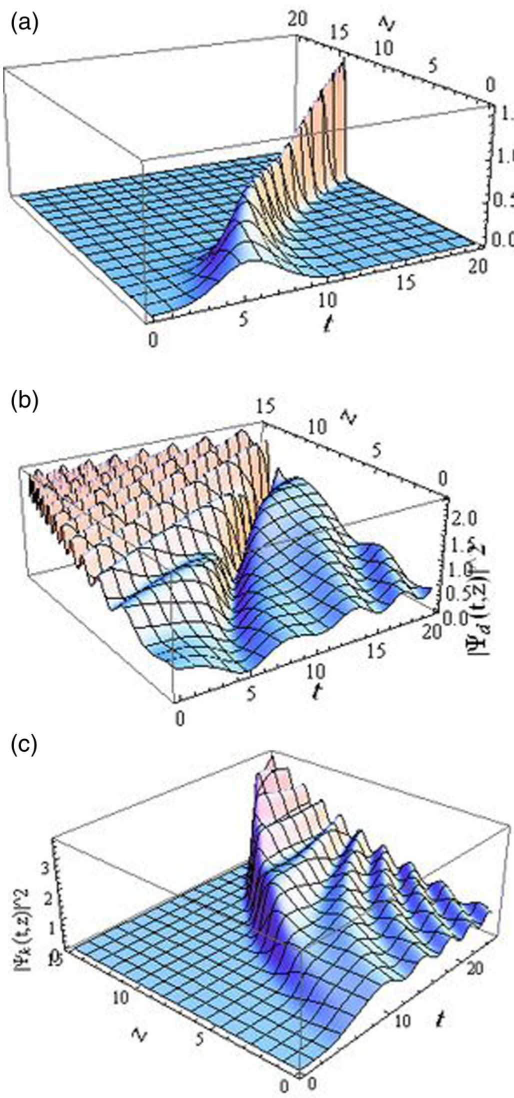

FIG. 5. Evolution of (a) the bright self-similar intensity wave profile $|\Psi_b(t,z)|^2$ as computed from Eq. (11), (b) the dark self-similar intensity wave profile $|\Psi_d(z,t)|^2$ as computed from Eq. (12), and (c) the kink self-similar intensity wave profile $|\Psi_k(t,z)|^2$ as computed from Eq. (13) under the perturbation with 10% initial white noise. The values of the parameters are the same as in Figs. 1(a), 1(b), and 4(a), respectively.

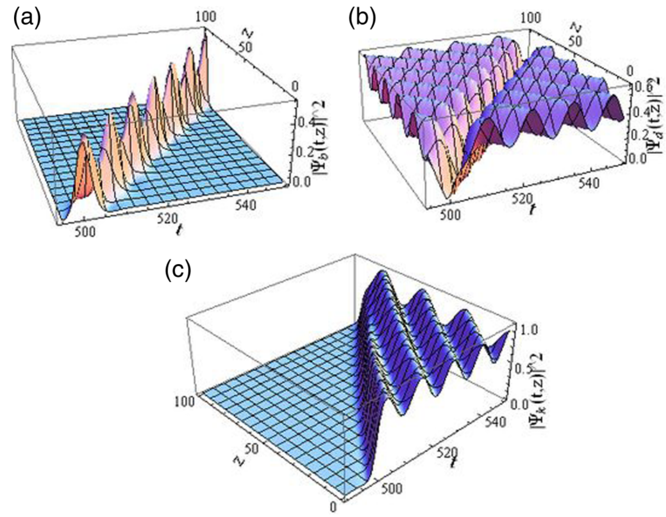

FIG. 6. Evolution of (a) the bright self-similar intensity wave profile $|\Psi_b(t,z)|^2$ as computed from Eq. (11), (b) the dark self-similar intensity wave profile $|\Psi_d(z,t)|^2$ as computed from Eq. (12), and (c) the kink self-similar intensity wave profile $|\Psi_k(t,z)|^2$ as computed from Eq. (13) under the perturbation with 10% initial white noise. The values of the parameters are the same as in Figs. 3(a), 3(b), and 4(b), respectively.

V. STABILITY ANALYSIS

It is a well-known fact that the practical interest of a solitary wave is closely related to its stability and, in particular, its ability to propagate in a perturbed environment over an appreciable distance. Note that only stable (or weakly unstable) solitary waves can be observed experimentally [21]. It is then essential to analyze the stability of exact solutions against finite perturbation. In what follows we demonstrate numerically the stability of the self-similar wave solutions presented above under initial small perturbations. Here we performed direct simulations with initial white noise [22,23] to study the stability of the solutions (11)–(13) compared to Figs. 1(a), 1(b), and 4(a). The evolution plots of self-similar solitary-wave solutions (11)–(13) under the perturbation of 10% white noise are shown in Figs. 5(a)–5(c), respectively. The results reveal that the bright self-similar wave can propagate in a stable way under finite initial perturbations of the additive white noise. Thus we can conclude that the bright solution we obtained is stable. Moreover, from Figs. 5(b) and 5(c) we can see that the shape of dark- and kink-type-solitary-wave solutions change while evolving over distance. This indicates that the white noise has a greater effect upon these solutions and could influence the main character of them. Thus, the dark and kink solutions are unstable. We have also performed numerical simulations to examine the dependence of the stability of the obtained solutions on the phase modulation parameter. Typical results are shown in Figs. 6(a)–6(c) for the case of small values of λ, for example, $\lambda = 0.001$. Compared with Figs. 5(a)–5(c), which correspond to $\lambda = 0.1$, we clearly see that the evolution is the same in both cases. Indeed, in both cases the self-similar bright solution is still very stable after propagating a distance, but dark and kink solutions evolve towards an instable form, indicating that the stability of obtained solutions is independent of the phase modulation parameter.

VI. CONCLUSION

It is an interesting point to note that the bright- and dark-intensity-wave profiles remain unchanged during the propagation, as we saw for the standard constant-coefficient CQNLSE. No pulse compression and amplitude amplification phenomena were observed during the propagation. One can check that for small values of λ, the solutions and the inhomogeneous CQNLSE (2) maps onto the solutions and standard constant-coefficient CQNLSE (4). Our results show an interesting connection between self-similar waves and the existence of solitons in inhomogeneous and homogeneous nonlinear media, respectively.

In this work we have presented the exact analytical self-similar solutions of the nonlinear Schrödinger equation with localized inhomogeneous cubic-quintic nonlinearity using similarity transformations. These self-similar solutions describe the stable optical bright and dark waves, propagating inside a planar, graded-index waveguide. Compression and amplification of soliton pulses propagating in conventional dispersion decreasing optical fibers is a well-established technique [20]. However, the most important feature of our investigated self-similar solutions is its interesting compression and amplification, which can be controlled to get the desired stable high-repetition-rate pulse by changing the phase modulation parameter λ and consequently varying the nonlinear parameter $\gamma_1(z)$. We have shown for small values of λ the intensity profiles of the self-similar waves coincide with those solitons supported by homogeneous passive media with the same type of nonlinearity. We have presented a surprising connection between optical self-similar waves in graded-index inhomogeneous media and solitons in homogeneous media with the same type of cubic-quintic nonlinearity. These stable ultrashort self-similar optical waves are potentially useful for various applications in optical telecommunications, especially in areas such as optical fiber compressors, optical fiber amplifiers, nonlinear optical switches, and optical communications since they can maintain the overall shapes but allow their amplitudes and/or widths to change according to management of the system's parameters. We hope that these solitary-wave solutions can be launched in long-haul telecommunication networks to achieve pulse compression. From our detailed investigation on inhomogeneous systems, using suitable transformation, we strongly believe that inhomogeneous systems can always be transformed into the corresponding homogeneous systems. Also, the stability of the self-similar solitary-wave solutions under the perturbation of white noise whose maximal value is 0.1 has been discussed numerically. The results have shown that the addition of small amounts of random noise could not influence the main character of the bright solitary wave, whereas this perturbation significantly affects the evolution of self-similar dark and kink solitary waves. It would be particularly relevant to extend the application of the used similarity transformation to find self-similar solitary wave solutions of the cubic-quintic-septimal nonlinear Schrödinger equation, which has been recently introduced to describe light propagation in a medium exhibiting nonlinearities up to seventh order [24]. We mention that experimental reports showing the existence of septimal nonlinearity in some materials have been presented recently in Refs. [25–27]. Such studies will be deferred to future work.

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