Collapse of the wave field in a one-dimensional system of weakly coupled light guides

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The analytical and numerical study of the radiation self-action in a system of coupled light guides is fulfilled on the basis of the discrete nonlinear Schrödinger equation (DNSE). We develop a variational method for qualitative study of DNSE and classify self-action modes. We show that the diffraction of narrow (in grating scale) wave beams weakens in discrete media and, consequently, the "collapse" of the one-dimensional wave field with power exceeding the critical value occurs. This results in the ability to self-channel radiation in the central fiber. Qualitative analytical results were confirmed by numerical simulation of DNSE, which also shows the stability of the collapse mode.

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I. INTRODUCTION

The advance in studies of nonlinear wave processes in spatially periodic media [1–3] led to active development of this field of nonlinear science, specifically, research into specific features of supercontinuum generation [4,5], control over the wave-field structure and formation of light bullets [6–8], localization of laser radiation in a certain light guide [9–11], shortening of pulse duration [11], the possibility of generation of intense laser fields in an active system of light guides [12,13], etc. Due to the complexity of the problems considered, the main results were obtained on the basis of numerical simulation.

The basic model for studying self-action of wave fields in a continuous medium is the nonlinear Schrödinger equation (NSE). Expansion of this model for the description of nonlinear processes in spatially periodic media was also successful. For example, the dynamics of self-action in a system of coupled light guides under the conditions of weak overlapping of guiding modes is determined by the nonlinear discrete Schrödinger equation (DNSE) [14,16]. A similar approach in solid-state physics and molecular systems [2] leads to the same equation for the description of nonlinear excitations of the polariton type.

In contrast to continuum media, no analytical methods have been developed for qualitative studies of specific features of self-action of wave fields in spatially periodic media. Only the soliton-type solutions, which are localized on the scale of the order of magnitude of the grating period [1,2], have been considered, and their stability has been investigated. In the case of initial field distributions, which are smooth on the characteristic scale of the spatially periodic system, several qualitative conclusions about system behavior (the possibility of wave beam self-compression [4,11], wave flow control [11], etc.) can be made on the basis of the continuum model. Numerical studies show that the wave dynamics turns out to be richer within the framework of the discrete model than in the continuum one [1,2]. Therefore, it is important to develop methods for approximate description of the wave-field dynamics which would allow for specific features of a microstructured medium.

Variational methods are widely used to obtain analytical results in physics. In the annex to the discrete NSE, stationary structures of the soliton type [17–21] were found based on the variational method. The variational approach with the Gaussian ansatz (which is similar to the one used in our paper) was performed in [22]. This paper considers self-action dynamics in a Bose-Einstein condensate (BEC) in a periodic trap, which corresponds to *defocusing* nonlinearity. We study the self-action in media with *focusing* Kerr nonlinearity, which have essential specific features.

In this work, we perform a detailed analysis of the specific features of self-focusing of wave beams in the framework of the simplest discrete model of a microstructured medium [14,16] in the one-dimensional case. We show on the basis of numerical simulation that there exists a regime of self-action of wave fields which is typical of the discrete problem. The conditions are determined on the basis of the variational method, in which a wide field distribution (on the scale of the medium microstructure period) is collected in one light guide. We use the term "collapse" to denote the situation where most of a pulse power due to self-focusing will be located in the single light guide. This is a discrete analog of collapses in the continuum limit. The method of an approximate analytical study of this regime of radiation collapse is developed. In many aspects, this method is similar to the aberration-free approximation in the NSE theory, where it is assumed that wave beams have a Gaussian profile and a parabolic phase front. As a result, the problem is reduced to studying a simpler system of ordinary equations for the width of the wave beam and curvature of the phase front.

This paper is structured as follows. In Sec. II, a DNSE system is formulated, which describes self-action of radiation in the one-dimensional system of coupled light guides. Based on numerical simulation of these equations, self-action regimes are categorized. Section III generalizes the aberration-free description of the spatial evolution of the system to the discrete NSE. As in the continuum limit, we derive the DNSE Lagrangian for parameters of Gaussian wave beams using the variational approach. It gives us equations for the wave beam width and phase-front curvature which allow for discreteness of the initial problem. Analysis of these equations shows that medium discreteness leads to weakening of the wave beam diffraction. This results in the possibility of a collapse of the one-dimensional distribution of the wave

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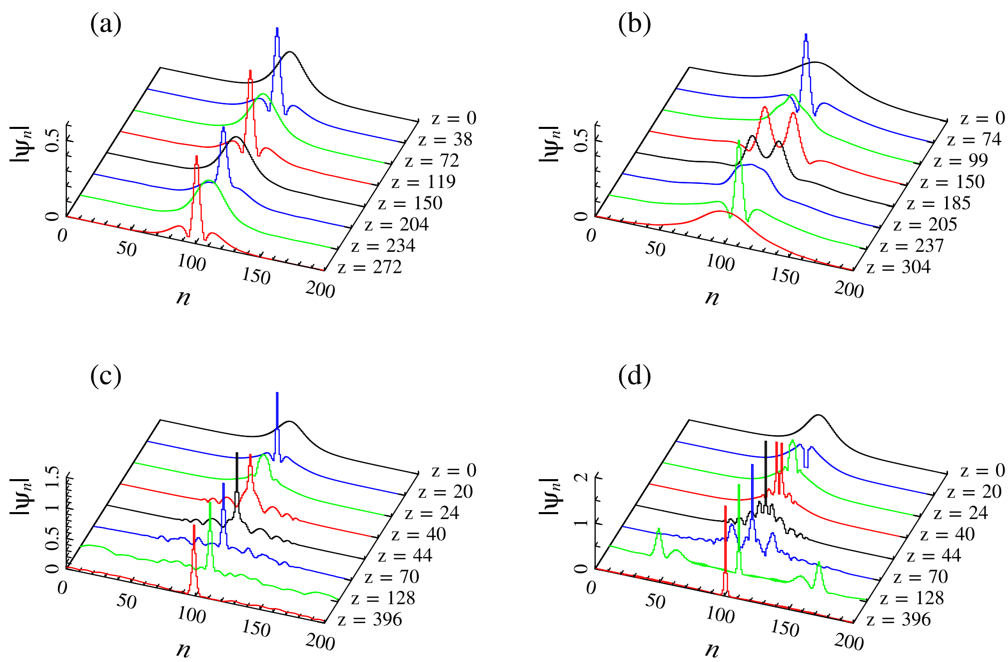

FIG. 1. Dynamics of the beam amplitude $|\Psi(z,n)|$ for different values of the parameters δ and $\mathcal{N}$: (a) $\delta = 0.1, \mathcal{N} = 2$ ($\mathcal{P} = 1.6 < \mathcal{P}_{cr}$); (b) $\delta = 0.05, \mathcal{N} = 3$ ($\mathcal{P} = 1.8 < \mathcal{P}_{cr}$); (c) $\delta = 0.1, \mathcal{N} = 3$ ($\mathcal{P} = 3.6 > \mathcal{P}_{cr}$); and (d) $\delta = 0.1, \mathcal{N} = 5$ ($\mathcal{P} = 10 > \mathcal{P}_{cr}$).

field with the total power exceeding a certain critical value. Section V presents the results of the numerical simulation of the approximate aberration-free equations. The comparison of the results with the data from numerical studies of the initial DNSE from Sec. II shows that the presented aberration-free approximation describes rather well the characteristic features of the self-action in a discrete medium. In the same section, the stability of the collapse regime is studied on the basis of the initial DNSE equations.

II. FORMULATION OF THE PROBLEM: NUMERICAL SIMULATION OF THE FIELD EVOLUTION

Consider the simplest discrete model [1,2] in the one-dimensional case. Assume that the evolution of a slowly varying field envelope in the nth light guide in the process of propagating along the z axis is determined by the following factors: the third-order nonlinearity of the medium and interaction with only the nearest light guides. For a system of equidistantly located lossless light guides, we use the discrete NSE, which has the following form [14,16]:

$$i\frac{\partial \Psi_n}{\partial z} + \Psi_{n+1} + \Psi_{n-1} + |\Psi_n|^2\Psi_n = 0. \tag{1}$$

This equation has a Hamiltonian structure, like the continuum NSE. Additionally, Eq. (1) retains the wave-field power

$$\mathcal{P} = \sum_{n=-\infty}^{+\infty} |\Psi_n|^2. \tag{2}$$

Equation (1) is well known as the discrete self-channeling equation [1,14]. It also has many other physical applications [1,2].

We will consider mainly the spatial evolution of wave beams with the characteristic initial size a_0 being much longer than the distance between the light guides ($a_0 \gg 1$). In the continuum limit, the system dynamics is determined by the power $\mathcal{P}$. A typical value is the power $\mathcal{P} = \mathcal{P}_0$, at which a uniform light guide channel (spatial soliton) is formed. At $\mathcal{P}$ slightly different from $\mathcal{P}_0$, the wave beam width changes periodically along the propagation path. At $\mathcal{P} \gg P_0$, the initial field distribution disintegrates into a set of solitons. In the discrete case, the situation becomes essentially different.

Equation (1) was solved numerically for the initial distributions

$$|\Psi_n| = \frac{\sqrt{2}\mathcal{N}\delta}{\cosh[(n-n_0)\delta]} \tag{3}$$

with a plane phase front. Here, $1/\delta$ determines the characteristic quantity of light guides in which the field is concentrated (or the characteristic size of the area occupied by the field $a_0 = 1/\delta$). For the power, we have $\mathcal{P} = 4\delta\mathcal{N}^2$.

In the continuum problem distribution (3) describes the spatial soliton at $\mathcal{N} = 1$. The parameter $\mathcal{N}$ determines the number of solitons contained in the initial distribution.

The results of studying Eq. (1) numerically are shown in Fig. 1. Figures 1(a) and 1(b) show that the evolution of smooth wave-field distributions (on the scale of a cell) is similar to that in the continuum problem. However, as the power increases [see Figs. 1(c) and 1(d)], the self-action regime becomes essentially different: irreversible collapse of the wave beam occurs, and it is localized on the scale approximately equal to the medium microstructure period. Numerical calculations show that the critical self-focusing power is nearly independent of the width of the initial field distribution a_0. Also it does not depend on the types of boundary conditions since wave fields become localized far from boundaries. The question of the value of the critical self-focusing power stays undetermined to a certain degree. The matter is that the decrease in the wave field and, correspondingly, the decrease in the power in the narrow central part of the beam are observed in the process of the collapse [see Fig. 1(c)]. For example, self-focusing of the wave beam in Fig. 1(c) occurs as the power decreases from $\mathcal{P}_{\rm in} = 3.6$ to $\mathcal{P}_{\rm out} = 2.7$. This self-action regime, which is typical for the discrete problem, will be considered in what follows in more detail.

III. ANALYTICAL STUDY OF SPATIAL EVOLUTION OF WAVE BEAMS

The ability of wave beam collapse can be understood from the following simple considerations. References [14,15] show that highly localized wave structures with the size of a few cells can exist in an array of coupled light guides if the wave power exceeds a certain critical value $\mathcal{P}^\star$. It is well known from the self-focusing theory that significant wave beam self-focusing occurs at the initial stage in continuous nonlinear media if the input solitonlike wave field has an amplitude much higher than the soliton amplitude. Obviously, the beam size may become comparable to the cell size at beam self-focusing even in a one-dimensional discrete system if the beam power exceeds the critical value. As a result, part of the radiation will be captured in a localized structure and channeled in one waveguide.

Next, consider the evolution of the wide wave beams. As in the continuum medium, it is convenient to use the variational method for an approximate description of the wave-field evolution. The initial parameter is the Lagrangian of the system (1),

$$\mathcal{L} = \sum_{n=-\infty}^{+\infty} \mathcal{L}_n = \sum \frac{i}{2}\left(\Psi_n \frac{\partial \Psi_n^*}{\partial z} - \Psi_n^* \frac{\partial \Psi_n}{\partial z}\right) - \Psi_{n+1}\Psi_n^* - \Psi_{n+1}^*\Psi_n - \frac{1}{2}|\Psi_n|^4. \quad (4)$$

Using the Poisson summation formula for continuous-argument function $F(x)$,

$$\sum_{n=-\infty}^{\infty} F(n) = \int_{-\infty}^{\infty} F(x) \sum_{n=-\infty}^{\infty} \exp(2\pi i n x) dx, \quad (5)$$

we transform Lagrangian (4) into a form which is more convenient for our approach:

$$\mathcal{L} = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{\infty} \left[\frac{i}{2}\left(\Psi \frac{\partial \Psi^*}{\partial z} - \Psi^* \frac{\partial \Psi}{\partial z}\right) - \frac{1}{2}|\Psi(x)|^4 - \Psi(x+1)\Psi^*(x) - \Psi^*(x+1)\Psi(x)\right] e^{2\pi i n x} dx. \quad (6)$$

As in the continuum problem, we will consider Gaussian wave packets with a parabolic phase front,

$$\Psi(x) = \sqrt{\frac{\mathcal{P}}{a\sqrt{\pi}}} \exp\left(-\frac{x^2}{2a^2} + i\beta x^2\right), \quad (7)$$

where $\mathcal{P}$ is the integral of the one-dimensional problem (2) and parameters $a(z)$ and $\beta(z)$ characterize the wave-packet width and phase-front curvature.

Integrating Eq. (6) with respect to the continuous variable x, we obtain

$$\mathcal{L} = \frac{\sqrt{\pi}}{2}\frac{d\beta}{dz}\mathcal{P}a^2 \sum_{n=-\infty}^{\infty}(1 - 2\pi^2 a^2 n^2)e^{-\pi^2 a^2 n^2} - 2\mathcal{P}\sqrt{\pi} e^{-\beta^2 a^2 - 1/4a^2} \sum_{n=-\infty}^{\infty} \cos(\pi n) e^{-\pi^2 a^2 n^2 - 2\pi n \beta a^2} - \frac{\sqrt{\pi}}{\sqrt{2}}\frac{\mathcal{P}^2}{2a} \sum_{n=-\infty}^{\infty} e^{-\pi^2 a^2 n^2/2}. \quad (8)$$

One can see that the terms of this series decrease fast as n increases. For wave beams with a width $a \gg 1/\pi$, we can confine ourselves to only one term with $n = 0$ in order to describe the processes. As a result, we obtain a reduced Lagrangian of the system,

$$\mathcal{L}_c = \frac{d\beta}{dz}\frac{\mathcal{P}a^2}{2} - 2\mathcal{P}\exp\left(-\frac{1}{4a^2} - \beta^2 a^2\right) - \frac{\mathcal{P}^2}{2a}\sqrt{\frac{1}{2\pi}}. \quad (9)$$

We turn our attention to the following fact. Since in this case the evolution of wave packets is determined by only one term in Eq. (8) with $n = 0$, it follows from Eq. (6) that the system dynamics is described by the Lagrangian

$$\mathcal{L}_0 = \frac{i}{2}\left(\Psi \frac{\partial \Psi^\star}{\partial z} - \text{c.c.}\right) - \frac{1}{2}|\Psi(x)|^4 - \Psi(x+1)\Psi^\star(x) - \Psi^\star(x+1)\Psi(x). \quad (10)$$

This means that the evolution of field distributions with the characteristic scale $L_\perp \gg \frac{1}{\pi}$ is described by the following equation in finite differences:

$$i\frac{\partial \Psi}{\partial z} + \Psi(x+1) + \Psi(x-1) + |\Psi|^2\Psi = 0. \quad (11)$$

One can see from the comparison of the reduced Lagrangian (9) with the corresponding Lagrangian in the continuum limit that the strongest modification in the discrete medium was undergone by the middle term in Eq. (9). It describes the wave-field diffraction. One can see that medium discreteness leads to exponential weakening of the diffraction as the wave beam width decreases. That is why the wave-field collapse occurs in the one-dimensional case under consideration [see Figs. 1(c) and 1(d)].

Let us study thoroughly the consequences of diffraction weakening on the basis of the equations for the wave beam parameters. Using the Euler equation

$$\frac{d}{dz}\frac{\partial \mathcal{L}_c}{\partial q_z} - \frac{\partial \mathcal{L}_c}{\partial q} = 0, \quad (12)$$

where q is the parameter of the (a,β) system, we find

$$\frac{d\beta}{dz} = -\frac{\mathcal{P}}{2\sqrt{2\pi}a^3} + \frac{1 - 4\beta^2 a^4}{2a^2}\frac{2}{a^2}e^{-\beta^2 a^2 - 1/4a^2}, \quad (13a)$$

$$\frac{da}{dz} = 4\beta a e^{-\beta^2 a^2 - 1/4a^2}. \quad (13b)$$

To study the role of discreteness in the nonlinear system dynamics, let us turn first to Eq. (13a). It is the eikonal equation for the case of wave beams with a parabolic phase front. The first term in Eq. (13a) describes the nonlinear aberration of the phase front, and the second term describes the diffraction one. One can see that in the discrete system, the diffraction term decreases exponentially with a decrease in the beam width, and therefore, the role of nonlinearity increases.

Consider now stationary homogeneous wave structures (spatial solitons). In the case of a collimated wave beam, setting in Eqs. (13) $\beta = 0$ and $d\beta/dz = 0$, we find the relationship

$$\mathcal{P} = \frac{2\sqrt{2\pi}}{a}\exp\left(-\frac{1}{4a^2}\right), \tag{14}$$

which connects the characteristic field distribution width a and the power $\mathcal{P}$. Analysis of Eq. (14) shows that the maximum transmitted power of the homogeneous light guide system with width $a = 1/\sqrt{2}$ is $\mathcal{P}_m = 4\sqrt{\pi/e}$. At $\mathcal{P} \leqslant \mathcal{P}_m$, two solitons with a width of the order of the magnitude of the cell size exist: narrow ($a < 1/\sqrt{2}$) and wide ($a > 1/\sqrt{2}$). They have been studied sufficiently thoroughly in the discrete problem. Within the approach considered here, these solitons are described by the continuous Gaussian function [Eq. (6)]. For narrow solitons, this is valid, until $a > 1/\pi$.

To study specific features of spatial evolution of the wave beams, which are sufficiently wide initially, let us turn to the system of equations (13). It has the integral

$$\exp\left(-\frac{1}{4a^2} - \beta^2 a^2\right) = \mathcal{C} - \frac{\mathcal{P}}{4\sqrt{2\pi}a}, \tag{15}$$

where $\mathcal{C}$ is the integration constant. For the Hamiltonian system, $\mathcal{C}$ is proportional to the Hamiltonian. Excluding β in Eq. (13b) using integral (15), we obtain the following equation:

$$\frac{da}{dz} = \pm 4\left(\mathcal{C} - \frac{\mathcal{P}}{4\sqrt{2\pi}a}\right)\sqrt{-\ln\left(\mathcal{C} - \frac{\mathcal{P}}{4\sqrt{2\pi}a}\right) - \frac{1}{4a^2}}. \tag{16}$$

The right-hand part of Eq. (16) contains two cofactors. The first reflects specific features of the discrete problem. The stationary point ($da/dz = 0$)

$$a_1 = \frac{\mathcal{P}}{4\sqrt{2\pi}\mathcal{C}}, \tag{17}$$

which is determined by this cofactor, corresponds to the self-action regime, which is different from homogeneous light guide channels [Eq. (14)]. It is seen from Eq. (15) that it is realized at $\beta \to \infty$, i.e., for wave beams with the nonplane phase front ($\beta \neq 0$).

Consider specific features of the system evolution, which are determined by the second cofactor in Eq. (16). In the case of an initially wide ($a = a_0 \gg 1$) collimated ($\beta = 0$) wave beam, the integration constant, as it follows from Eq. (15), is $\mathcal{C} \geqslant 1$. At the initial stage ($a \gg a_1$), the beam evolution is described by the equation which is written conveniently as

$$\frac{da}{dz} \approx \pm\frac{2\mathcal{C}}{a}\sqrt{\ln\mathcal{C}}\sqrt{(a_0 - a)(a - a_2)}, \tag{18}$$

where $a_2 = \sqrt{2\pi}\mathcal{C}/\mathcal{P} - \mathcal{P}/8\sqrt{2\pi}\mathcal{C}$. When obtaining this equation from Eq. (16), we expanded the logarithmic function in a series at $\mathcal{C} \gg \mathcal{P}/4a\sqrt{2\pi}$ and represented the quadratic polynomial in the root expansion. It describes the periodic oscillation of the width between the initial value a_0 and the minimal value a_2, as in the continuum problem. With increasing $\mathcal{P}$, the value of a_2 decreases and becomes comparable to the value of the beam width (17), which is determined by the first cofactor in Eq. (16). This results in bifurcation from the oscillation regime to the monotonic one for

$$\mathcal{P} > \mathcal{P}_{cr} = \frac{4\sqrt{\pi}}{\sqrt{3}}\mathcal{C} \approx 4.1\mathcal{C}.$$

A more accurate estimate can be obtained by keeping higher-order terms in the series expansion of the transcendent right-hand part of Eq. (16). In particular, keeping the cubic term in the logarithm expansion, we obtain

$$\mathcal{P}_{cr} = \sqrt{\frac{48\pi}{11}}\mathcal{C} \approx 3.7\mathcal{C}. \tag{19}$$

Unfortunately, the formulas become too complicated if we keep higher terms of the logarithm expansion. Direct numerical simulation of Eqs. (13) gives just a slightly smaller value $\mathcal{P}_{cr} \approx 3.3\,\mathcal{C}$. So the estimate (19) for critical power has appropriate accuracy.

Expression (19) for local critical power depends on the beam parameter through the coefficient $\mathcal{C}$, similar to ones in continuum media. However, the discrete case has a key difference: the beam width has to be larger than one cell, i.e., $a \geqslant 1$. This provides the finite maximal value of $\mathcal{P}_{cr}$. Using Eq. (15) for $\mathcal{C}$, we obtain the following estimate for this maximal value:

$$\mathcal{P}_{\text{cr}}^{\max} \approx 4.9, \tag{20}$$

which does not depend on beam parameters and is the "true" critical power in the sense that a beam with arbitrary parameters will collapse if its power exceeds $\mathcal{P}_{\text{cr}}^{\max}$.

The solution of Eq. (18) at $\mathcal{P} > \mathcal{P}_{cr}$ at the stage of the wave beam collapse ($a \gg a_2$) has the form

$$\sqrt{a(a_0 - a)} + a_0\left[\arcsin\left(\frac{a_0 - 2a}{a_0}\right) + \frac{\pi}{2}\right] = 2\mathcal{C}\sqrt{\ln\mathcal{C}}z. \tag{21}$$

Here, beam width a decreases monotonically from a_0 to a_1. It follows from this formula that the beam width in this region decreases obeying the law, which is close to the parabolic one, and becomes zero at

$$z_0 = \frac{\pi}{2}\frac{a_0}{\mathcal{C}\sqrt{\ln\mathcal{C}}}. \tag{22}$$

Determining $\mathcal{C}$ from Eq. (15), we find that for wide beams $\mathcal{C} = 1 + \frac{\mathcal{P}}{4\sqrt{2\pi}a_0}$. As a result, we obtain

$$z_0 = \frac{\pi\sqrt[4]{2}a_0^{3/2}\sqrt[4]{\pi}}{\sqrt{\mathcal{P}}}; \tag{23}$$

that is, z_0 decreases obeying the law $\propto\frac{1}{\sqrt{\mathcal{P}}}$.

In the process of self-focusing, the system passes to the state in which the first cofactor in Eq. (16) tends to zero. The evolution of the bandwidth is determined at this stage by Eq. (16), in which one can neglect the term $\frac{1}{4a^2}$ under the square-root sign. The solution of this equation describes the exponential decrease in the beam width to the minimal size a_1 obeying the law

$$a = a_1 + \exp(-4\mathcal{C}^2 z^2). \tag{24}$$

One can see here that the typical length of the approaching a_1 coincides with Eq. (22) almost completely. This means that the transition from self-focusing to the asymptotic regime occurs at $z \simeq z_0$.

It should be noted that in the asymptotic limit the self-action regime, which we considered, contains a homogeneous wave structure which differs strongly from Eq. (14). The phase of this structure is not planar. Moreover, it is seen from Eq. (13a) that β increases along the wave beam propagation path obeying the law

$$\beta = -\frac{\mathcal{P}}{2a_1^3\sqrt{2\pi}}z. \tag{25}$$

This is similar to the collapse of axisymmetric beams, which is well known in the self-focusing theory. However, the collapse occurs not to a point but to a distribution with the characteristic scale being of the order of magnitude of the medium stratification period. Note that the growing parabolic phase for high-power beams will break the well-known periodical regime of a NLS soliton in the continuum media (see Fig. 2).

To achieve deeper understanding of the self-action pattern in a discrete system, we studied the wave beam evolution numerically on the basis of the equations of the aberration-free approximation (13). Two self-action regimes are discerned within the continuum one-dimensional problem: one (spatial soliton) with the wave beam width being greater and the other being smaller than the characteristic size of a homogeneous wave channel. In the discrete system, an analytical study can be performed only in the first case, which is related to the transcendence of the right-hand part of Eq. (16).

Figure 2 shows the evolution of the wave beam width along the propagation path at different values of the power $\mathcal{P}$. At $\mathcal{P} < \mathcal{P}_{cr}$, this behavior is identical to that in the continuum medium: wide wave beams become narrower at the initial stage of the periodic process [see Fig. 2(a)], and the narrow beams become wider [see Fig. 2(b)]. At $\mathcal{P} > \mathcal{P}_{cr}$, the self-action regime changes qualitatively: the collapse of the radiation to a single wave channel takes place.

This self-channeling process differs from the usual one. The phase front of the wave field inside the channel is parabolic, rather than planar. The curvature β of the phase front increases obeying the linear law [Eq. (25)] along the propagation path. Therefore, this process is a collapse in our continuous description of the discrete system. For wide wave beams, $\mathcal{P}_{cr} \simeq 3.4$, while for narrow beams, $\mathcal{P}_{cr} \simeq 3.9$, which is rather close to estimate (20).

Upon the whole, one can say that the results of studying the evolution of wave beams in discrete systems on the basis of approximation of the discrete field distribution with a smooth distribution (in accordance with Poisson's formula) yield a pattern which largely coincides with the data for numerical simulation of initial equation (1) in Sec. III. It should be noted that the law of the self-focusing length $z_0 \propto 1/\sqrt{\mathcal{P}}$ decreasing with increasing $\mathcal{P}$ is well confirmed in our calculations.

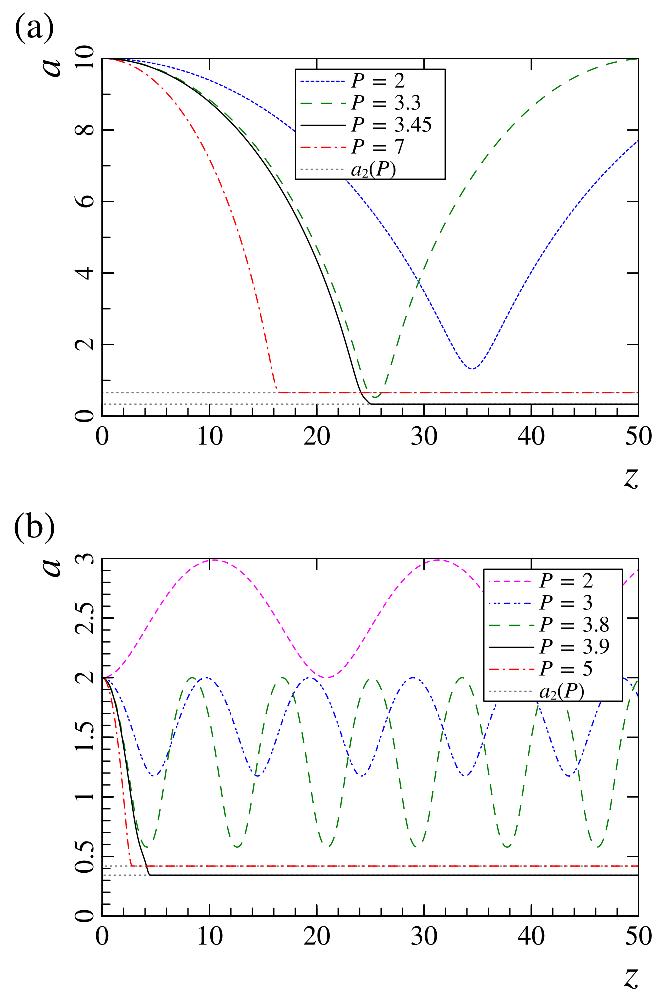

FIG. 2. Dependence of the transverse size a of the wave packet on the evolution variable z for two different initial beam size values: (a) $a_0 = 10$, (b) $a_0 = 2$, and for different power values $\mathcal{P}$.

Probably, the most important thing here is the description of the collapse regime. It takes place at a power level exceeding critical power (19). This agrees well with the data for the numerical calculations in Sec. II.

For the width of the field localization area in the collapse process, the analytical value [Eqs. (17) and (24)] proves to be somewhat overstated. For a more accurate value, one should allow for the release of the wave field during self-focusing. For example, at the input power $\mathcal{P}_{\text{in}} = 3.6$, the power in the central part of the beam (with respect to the level of the intensity decrease by $1/e^2$) decreases to $\mathcal{P}_{\text{out}} = 2.7$, and in the case of $\mathcal{P}_{\text{in}} = 10$, the value $\mathcal{P}_{\text{out}}$ is almost two times smaller. Thus, allowing for the wave field release yields the width of the beam localization area, which is comparable to the distance between the light guides.

It should also be noted that the power $\mathcal{P}$ in the compressed distribution of the field with allowance for the radiation processes stays greater than the critical values ($\mathcal{P} > \mathcal{P}_{cr}$). This means that the structure formed in the process of radiation self-focusing differs from homogeneous wave channels (spatial solitons) of the type of Eq. (14).

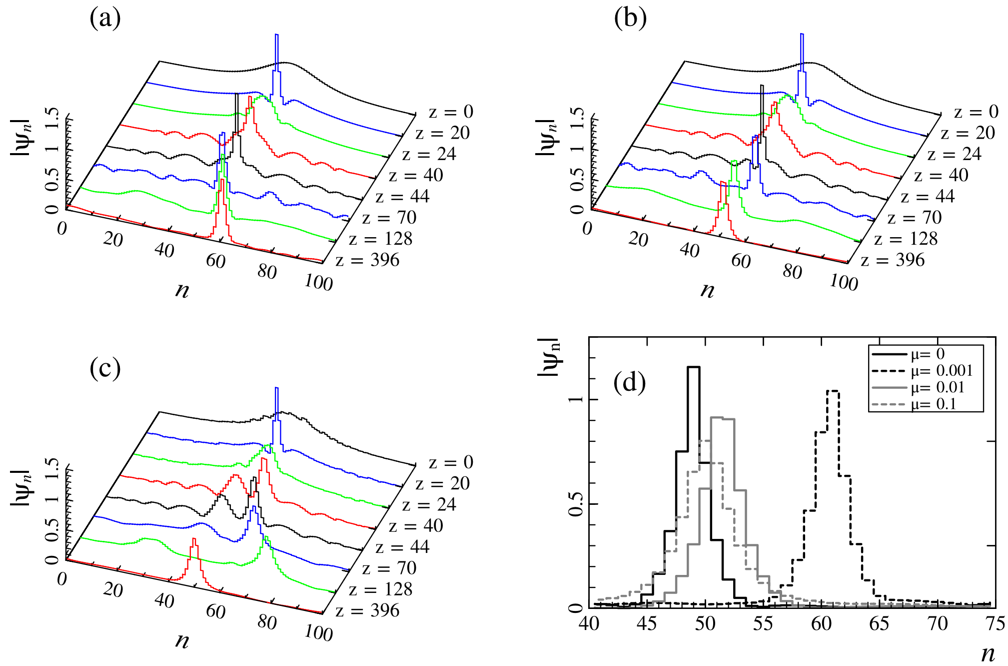

FIG. 3. Dynamics of the beam amplitude $|\Psi(z,n)|$ in an array of coupled light guides for the case when the initial distribution (26) is specified at the medium input at different noise levels: (a) $\mu = 0.001$, (b) $\mu = 0.01$, and (c) $\mu = 0.1$. (d) The amplitude distribution at $z = 396$ for different values of μ. Here, $\delta = 0.1$ and $\mathcal{Q}$ is the noise signal.

IV. CONTINUATION OF NUMERICAL SIMULATION OF RADIATION SELF-ACTION REGIMES

In what follows, we consider the question of the stability of the wave-field self-focusing regime in a one-dimensional system of weakly coupled light guides in the presence of initial perturbations of the noise type. Figure 3 shows the evolution of the wave field specified at the input of the nonlinear medium in the form

$$\Psi_n = \frac{3\sqrt{2}\delta[1 + \mu\mathcal{Q}(n)]}{\cosh[\delta(n - n_0)]} \tag{26}$$

at $\delta = 0.1$ and different values of μ: $\mu = 0.001$ [see Fig. 3(a)], $\mu = 0.01$ [see Fig. 3(b)], and $\mu = 0.1$ [see Fig. 3(c)]. Here, $\mathcal{Q}(n)$ is the noise signal.

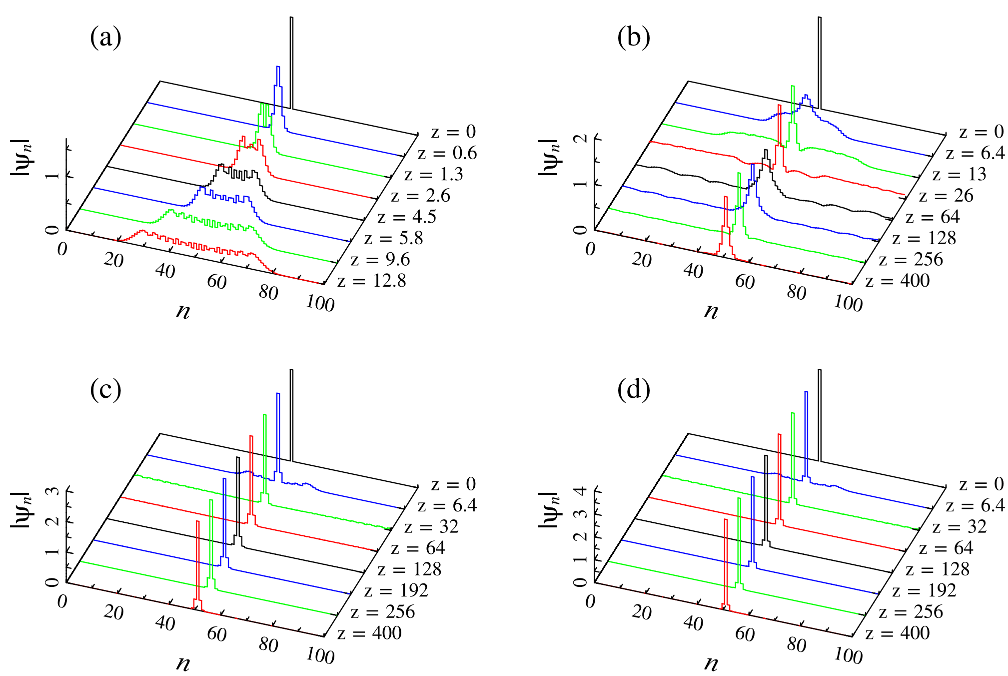

FIG. 4. Dynamics of the beam amplitude $|\Psi(z,n)|^2$ in an array of coupled light guides for the case when the initial distribution $\Psi(z = 0,n) = \mathcal{A}_0\,\delta_{n,n_0}$ is specified at the medium input at different values of $\mathcal{A}_0$: (a) $\mathcal{A}_0 = 1.7$, (b) $\mathcal{A}_0 = 2$, (c) $\mathcal{A}_0 = 3$, and (d) $\mathcal{A}_0 = 4$.

For the sake of comparison, Fig. 1(c) presents the results of numerical simulation of the evolution of the corresponding distribution [Eq. (3)] at $\mathcal{N} = 3$ in the absence of noise ($\mu = 0$). It follows from Figs. 3(a), 3(b), and 3(c) that the process of wave-field self-focusing in an array of light guides is accompanied by irregular and asymmetric radiation release into neighboring channels, in contrast to the case of $\mu = 0$. One can see that in the process of self-focusing, purification of the wave packet of the noise component and further self-channeling of the field occur, as in the case of $\mu = 0$ [see Fig. 1(c)]. In this case, the share of the energy in the central part of the beam decreases as the noise level μ increases. Figure 3(d) shows amplitude distributions of the self-channeled wave structure at $z = 396$ for different μ. One can see that an increase in the noise amplitude in the initial distribution leads to an increase in the beam size compared with the case of $\mu = 0$. It is also seen from Fig. 3(d) that the initial noise will lead to an irregular position of the center of the self-channeled structure. This requires additional study which will be done in further works.

To conclude this section, we will turn to the question of the existence of wave structures which are strongly localized near one light guide. This was discussed in [14]. Figure 4 shows the dynamics of the field amplitude $|\Psi(z,n)|$ in an array of coupled light guides in the case when only the central channel is powered at the input:

$$\Psi(z = 0,n) = \mathcal{A}_0\, \delta_{n,n_0}, \tag{27}$$

where n_0 is the location of the central channel. It is seen from Fig. 4 that in the case where $\mathcal{A}_0 \lesssim 1.9$ [see Fig. 4(a)], the wave beam spreads rather fast due to the linear coupling between the light guides. However, as one can see from Fig. 4(b), at $\mathcal{A} \simeq 2$ the wave beam spreads and becomes several times wider ($z \sim 6.4$), but then the beam focuses to approximately the initial value ($z \sim 256$). As follows from Fig. 4(b), the field amplitude in the central channel decreased by only two times compared with the initial value. At a further increase in the initial amplitude [see Figs. 4(c) and 4(d)], the wave field does not undergo diffraction and is localized entirely in the central channel. Note that the power $\mathcal{P} = \mathcal{A}_0^2$, at which the wave field does not diffract, is comparable with the critical self-focusing power in discrete system (19). Thus, the nonlinear mode (27) can be regarded as the final stage of self-focusing of wide wave beams at $\mathcal{P} > \mathcal{P}_{cr}$.

V. CONCLUSION

In this paper, we generalize a method for approximate description of self-action of wave fields in a one-dimensional system of weakly coupled light guides. This method is the variational generalization of the aberration-free approximation for Gaussian wave beams with a parabolic phase front, which is used successfully in continuous media. As a result, we could analyze analytically the specific features of the evolution of the wave field with the characteristic scale, which is comparable to the period of a microstructured medium. It is shown that these features are related to weakening of the diffraction in a periodically microstructured medium. The main result of the work is the conclusion about the possibility of a collapse in a one-dimensional discrete system of an initially wide (on the scale of the medium inhomogeneity) wave beam. It is realized at the power level, which exceeds the critical value, and leads to self-channeling of the wave beam in a region with the size approximately equal to the grating size.

The conclusions made herein about the regimes of self-action of wave beams (periodic changes in the beam width in the propagation process, collapse and stationary soliton-type structures with the characteristic scale being of the order of magnitude of the grating size) and the critical power for field self-channeling are confirmed by the results of numerical simulation of both the aberration-free approximation equations and the basic initial equation (1). The question of the stability of the collapse regime was considered especially in the framework of the one-dimensional DNSE equation in the presence of initial noise-type perturbations. It was shown that in the self-focusing process, the wave beam is purified of noise and the wave field is self-localized, as in the case of smooth beams. At a high noise level (~10%), the self-channeled structure is retained, but its intensity center undergoes a minor irregular displacement.

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