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Suppressing gate errors in frequency-domain quantum computation through extra physical systems coupled to a cavity

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We propose a scheme for frequency-domain quantum computation (FDQC) in which the errors due to crosstalk are suppressed using extra physical systems coupled to a cavity. FDQC is a promising method to realize large-scale quantum computation, but crosstalk is a major problem. When physical systems employed as qubits satisfy specific resonance conditions, gate errors due to crosstalk increase. In our scheme, the errors are suppressed by controlling the resonance conditions using extra physical systems.

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I. INTRODUCTION

Implementation of a large number of qubits is a major challenge to realize universal quantum computation. Practical quantum computation requires fault tolerance because the qubits, which are quantum superpositions, are fragile [1]. In fault-tolerant quantum computation, a logical qubit must be composed of many physical qubits. In an estimation of Ref. [2], 10^8 physical qubits are required to solve factorization of 1024 bits by Shor's algorithm using 10^4 logical qubits. An implementation to employ this large number of physical qubits is required.

One strategy to implement a large number of physical qubits is to employ solid-state systems containing a large number of candidate qubits which have a long coherence time, such as rare-earth-ion-doped crystals [3–6], nitrogen-vacancy centers in a diamond [7,8], and donors in isotopically purified silicon [9,10]. If the candidates in these systems are addressable and controllable, these systems may provide a large number of qubits without having to artificially manufacture each qubit to have a long coherence time.

Frequency-domain quantum computation (FDQC), in which qubits are identified by their frequencies, has been proposed as a method of addressing the qubits [3]. When qubits are identified by not only their positions but also their frequencies, a large number of qubits will be employed in quantum computing systems.

FDQC has gate errors due to crosstalk because the positions of qubits are not identified and driving fields are applied to the whole system including unintended qubits. These gate errors can be suppressed by avoiding a specific resonance condition between eigenenergies of the whole system and frequency differences among qubits [11]. However, when a whole system for FDQC has a large number of qubits, the system has a large number of resonance conditions, and it is difficult to find physical systems for avoiding the resonance conditions. This results from uncontrollable parameters in the conditions, because the parameters are given by actual physical systems such as ions in a crystal. This paper proposes an implementation of FDQC to employ a large number of qubits by controlling the resonance conditions.

We propose a method of controlling the resonance conditions using extra physical systems coupled to a cavity mode. The resonance conditions derived in Ref. [11] depend on the number of physical systems coupled to a cavity (N_c) and coupling constant g. Because the coupling constant g should be large for a high-fidelity gate, g is not suitable as a control parameter. On the other hand N_c can be controlled using optical pumping to transfer atoms to or from shelving states. Therefore, we propose an implementation of FDQC in which N_c is employed as a control parameter of the resonance conditions by introducing extra physical systems coupled to a cavity mode.

Our model of FDQC with extra physical systems is described in Sec. II, the resonance conditions in the model are described in Sec. III, the effect of the extra physical systems is shown in Sec. IV, and some implementation of the extra physical systems are proposed in Sec. V.

II. MODEL OF FDQC WITH EXTRA PHYSICAL SYSTEMS

The model used to investigate gate errors due to crosstalk in FDQC including extra physical systems is based on the model given in Ref. [11]. We consider only a cavity mediated adiabatic passage (CMAP) operation, which is part of a two-qubit gate affected by crosstalk nontrivially [12–16].

We investigate the fidelity of this CMAP which is performed in $N + N'$ four-level systems including extra physical systems (Fig. 1).

In $N + N'$ four-level systems X_i ($i = 1,2,3,\dots,N + N'$), systems of $i = 1,\dots,N$ are employed as qubits, and systems of $i = N + 1,\dots,N + N'$ are employed as extra systems. In the systems for qubits, $|0\rangle_i$ and $|1\rangle_i$ are states for qubits, and $|2\rangle_i$ are ancilla states. $|2\rangle_i - |e\rangle_i$ transitions of all the systems couple to a common cavity mode with a coupling constant g. We assume that the energy relaxation rate of physical systems and the cavity are zero for the strong coupling limit. Δ_j denotes frequency differences between $|1\rangle_1 - |e\rangle_1$ transition and $|1\rangle_j - |e\rangle_j$ transitions ($j = 2,3,\dots,N$).

We now consider that inhomogeneous broadening of lower states $|0\rangle_i$, $|1\rangle_i$, and $|2\rangle_i$ in $N + N'$ four-level systems is larger than the homogeneous broadening of transitions between these lower states and $|e\rangle$. Therefore, although all $|2\rangle_i - |e\rangle_i$ transitions have the same transition frequency, $|1\rangle_i - |e\rangle_i$ transitions can have various transition frequencies. The subscripts of

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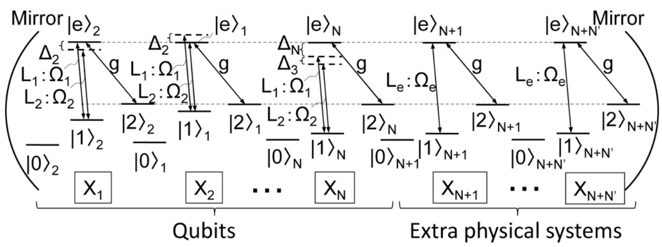

FIG. 1. A model of $N + N'$ four-level systems including extra physical systems.

the systems are in ascending order of transition frequencies of $|1\rangle_i - |e\rangle_i$. We assume that the transition frequencies of $|1\rangle_i - |e\rangle_i$ of extra systems ($i = N+1, N+2, \ldots, N+N'$) are substantially different from those of $|1\rangle_i - |e\rangle_i$ for qubits ($i = 1,2,\ldots,N$) sufficient to ignore the effects of driving fields to $|1\rangle_i - |e\rangle_i$ ($i = N+1, N+2, \ldots, N+N'$).

We consider a CMAP using systems of $i = 1,2$ which is operated by driving fields L_1 and L_2. L_1 resonates with $|1\rangle_1 - |e\rangle_1$ transition, and L_2 resonates with $|1\rangle_2 - |e\rangle_2$ transition. Furthermore, additional driving field L_e is irradiated to $|1\rangle_i - |e\rangle_i$ ($i = N+1, N+2, \ldots, N+N'$) resonantly. For simplicity, we assume that Rabi frequencies due to L_1 are a common Ω_1, Rabi frequencies due to L_2 are a common Ω_2, and Rabi frequencies due to L_e are a common Ω_e. Additional driving field L_e works to keep the state of extra physical systems in $|2\rangle_i$, and make the dark state in the system including extra physical systems as described below.

Equation (1) shows the Hamiltonian of the whole system for CMAP including extra physical systems in Fig. 1.

$$\begin{aligned} H(t) &= H_1(t) + V(t)H_1(t)/\hbar \\ &= \sum_{i=1}^{N+N'} g a \sigma_{e2}^{(i)} + \Omega_1(t)\sigma_{e1}^{(1)} + \Omega_2(t)\sigma_{e1}^{(2)} \\ &\quad + \sum_{i=N+1}^{N+N'} \Omega_e(t)\sigma_{e1}^{(i)} + \text{H.c.}, \\ V(t)/\hbar &= \Omega_1(t)\left\{ e^{-i\Delta_2 t}\sigma_{e1}^{(2)} + \sum_{j=3}^{N} e^{i\Delta_j t}\sigma_{e1}^{(j)} \right\} \\ &\quad + \Omega_2(t)\left\{ e^{i\Delta_2 t}\sigma_{e1}^{(1)} + \sum_{j=3}^{N} e^{i(\Delta_2+\Delta_j)t}\sigma_{e1}^{(j)} \right\} + \text{H.c.}, \end{aligned} \tag{1}$$

where, $\sigma_{jk}^{(i)}$ is an operator to transfer a state from $|k\rangle_i$ to $|j\rangle_i$. $a^\dagger$ and a are creation and annihilation operators of the cavity mode, respectively. CMAP operation is performed by Gaussian pulses $\Omega_i = \Omega_0 \exp[-(t-\tau_i)^2/2\sigma^2]$ for L_i ($i = 1,2$). When a state of system X_i is state $|\psi_i\rangle_i$ ($\psi_i = 0,1,2,e$) and the photon number of the cavity mode is n_c, the state of the whole system is denoted by $|\psi_1\rangle_1|\psi_2\rangle_2\ldots|\psi_N\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|n_c\rangle_c$. The Hamiltonian [Eq. (1)] has a dark state [Eq. (2)], which is an eigenstate without excited states of physical systems.

$$\begin{aligned} |\psi_0\rangle &= g\Omega_2\Omega_e|1\rangle_1|2\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|0\rangle_c \\ &\quad + g\Omega_1\Omega_e|2\rangle_1|1\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|0\rangle_c \\ &\quad + g\Omega_1\Omega_2|2\rangle_1|2\rangle_2|1\rangle_3\ldots|1\rangle_N \\ &\quad \times[(|1\rangle_{N+1}|2\rangle_{N+2}\ldots|2\rangle_{N+N'}) \\ &\quad + (|2\rangle_{N+1}|1\rangle_{N+2}|2\rangle_{N+3}\ldots|2\rangle_{N+N'}) \\ &\quad + (|2\rangle_{N+1}\ldots|2\rangle_{N+N'-1}|1\rangle_{N+N'})]|0\rangle_c \\ &\quad - \Omega_1\Omega_2\Omega_e|2\rangle_1|2\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|1\rangle_c. \end{aligned} \tag{2}$$

This dark state is available by the driving field Le for extra physical systems [17]. If extra physical systems are introduced without L_e, eigenstates of the whole system are states including excited states of ions. When Rabi frequencies satisfy the condition of $\Omega_{1,2} \ll \Omega_{e,g}$, cavity excited states and states including $|1\rangle_i$ of extra physical systems are suppressed, and the dark state is described as

$$\begin{aligned} |\psi_0\rangle &\sim g\Omega_2\Omega_e|1\rangle_1|2\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|0\rangle_c \\ &\quad + g\Omega_1\Omega_e|2\rangle_1|1\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|0\rangle_c. \end{aligned} \tag{3}$$

When Ω_1 and Ω_2 are Gaussian pulse, Ω_e is constant value, and $\Omega_0 \ll g$, the condition of $\Omega_{1,2} \ll \Omega_{e,g}$ are satisfied at the tail of the Gaussian pulses, and eigenstates of the system are approximately represented by Eq. (3). Therefore, CMAP operation using dark state can be performed by Gaussian pulses under $\tau_2 < \tau_1$, and a state is transferred from an initial state $|1\rangle_1|2\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|0\rangle_c$ to a final state $|2\rangle_1|1\rangle_2|1\rangle_3\ldots|1\rangle_N|2\rangle_{N+1}\ldots|2\rangle_{N+N'}|0\rangle_c$ by the operation.

III. RESONANCE CONDITIONS IN FDQC WITH EXTRA PHYSICAL SYSTEMS

Resonance conditions in the FDQC with extra physical systems can be derived by an analysis of gate errors due to the crosstalk. In the one of a system for FDQC with extra physical systems also, gate errors due to crosstalk are described the same way as Ref. [11] under the assumption that the differences in transition frequencies between $|1\rangle_i - |e\rangle_i$ ($i = 1,2,\ldots,N$) of qubits and $|1\rangle_i - |e\rangle_i$ ($i = N+1, N+2, \ldots, N+N'$) of extra physical systems are large. Concretely, gate errors occur only from interactions between L_1 and transitions $|1\rangle_i - |e\rangle_i$ ($i = 2,3,\ldots,N$), and interactions between L_2 and transitions $|1\rangle_i - |e\rangle_i$ ($1,3,\ldots,N$). The effects depend on detunings between transition frequencies of qubits and driving fields. The detunings correspond with the differences among transition frequencies of qubits, because driving fields are resonant with some transitions of the qubits. Detuning parameters between the $|1\rangle_1 - |e\rangle_1$ transition and $|1\rangle_j - |e\rangle_j$ transitions ($j = 2,3,\ldots,N$) are denoted by Δ_j. Detuning parameters Δ_j are parameters given by actual physical systems such as ions in a crystal.

The gate errors due to the crosstalk are evaluated by the same perturbation theory as Ref. [11]. When the initial state is the dark state $|\psi_0(0)\rangle$ at $t = 0$, a state $|\psi(t)\rangle$ at the time of t is described as

$$|\psi(t)\rangle = |\psi_0\rangle + \sum_n C_n^{(1)}(t)|\psi_n\rangle + \sum_n C_n^{(2)}(t)|\psi_n\rangle + O(V^3), \tag{4}$$

where $|\psi_n\rangle$ $(n \neq 0)$ is an eigenstate of the whole system. The coefficients $C_n^{(1,2)}$, which determine error probabilities, are described by

$$C_n^{(1)}(t) = \frac{1}{i\hbar}\int_0^t dt' e^{-(E_n-E_0)t'/i\hbar}\langle\psi_n|V(t')|\psi_0\rangle,$$

$$C_n^{(2)}(t) = \left(\frac{1}{i\hbar}\right)^2 \int_0^t dt' \int_0^{t'} dt'' e^{-(E_n-E_k)t'/i\hbar} e^{-(E_k-E_0)t''/i\hbar} \times \langle\psi_n|V(t')|\psi_k\rangle\langle\psi_k|V(t'')|\psi_0\rangle, \tag{5}$$

where E_n are eigenvalues corresponding to eigenstates $|\psi_n\rangle$, and E_0 is an eigenvalue corresponding to the dark state. When detuning parameters and eigenenergies accidentally satisfy "resonance conditions," $C_n^{(1,2)}$ diverge, and the gate errors increase. The resonance conditions are evaluated as

$$(E_n - E_0)/\hbar = \pm\Delta_2, \pm 2\Delta_2, \pm\Delta_j, \pm 2\Delta_j, \pm(\Delta_2+\Delta_j), \pm(\Delta_2 \pm 2\Delta_j), \pm(2\Delta_2 \pm \Delta_j). \tag{6}$$

The driving fields resonantly interact with unintended transitions on these conditions. The eigenvalues in the resonance conditions are one of ensemble two-level systems coupled to a cavity mode. The eigenvalues can be analytically evaluated as

$$E_n/\hbar = 0, \pm\sqrt{N+N'-1}g, \pm\sqrt{4(N+N')-2}g, \pm\sqrt{N+N'-2}g. \tag{7}$$

The eigenvalue of dark state E_0 is zero. The resonance conditions which are obtained by assigning Eq. (7) to Eq. (6) are represented as relations among detuning parameters Δ_i, cavity coupling constant g, and number of two-level systems coupled to the cavity mode N'.

If a system has a frequency distribution which avoids the resonance conditions, the gate errors due to the crosstalk are suppressed, and CMAP can be performed with high fidelity. When a large number of qubits are implemented, there are a large number of combinations of eigenenergies and transitions. Therefore, it is difficult to find systems which can avoid the resonance conditions, because the conditions are uncontrollable in the conventional FDQC ($N' = 0$). In FDQC with extra physical systems, we can create systems which can avoid the resonance condition by controlling the number of extra physical systems N'.

IV. EFFECT OF EXTRA PHYSICAL SYSTEMS

We now investigate the effects of extra physical systems by numerical calculation. We calculate the time evolution of CMAP operation from initial states $|1\rangle_1, |2\rangle_2, |1\rangle_i\ (i = 3,4,\ldots,N),\ |2\rangle_j\ (j = N+1, N+2, \ldots, N+N')$, and $|0\rangle_c$ by the time-dependent Schrödinger equation with the Hamiltonian in Eq. (1). Fidelity is obtained by a population of the state $|2\rangle_1,\ |1\rangle_2, |1\rangle_i\ (i = 3,4,\ldots,N),\ |2\rangle_j\ (j = N+1, N+2, \ldots, N+N')$, and $|0\rangle_c$ in the final states.

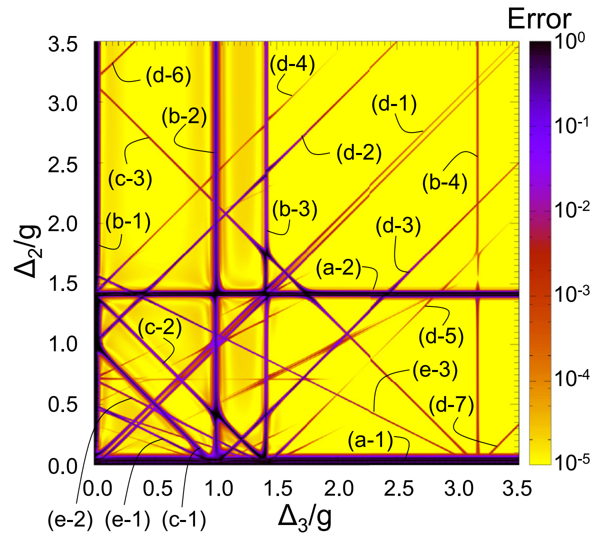

FIG. 2. Contour diagram of gate errors in a three-qubit system [11].

Figure 2 shows contour diagrams of gate errors (= 1−fidelity) in the Δ_2–Δ_3 plane in the case of a three-qubit system ($N = 3$, $N' = 0$).

This figure is the same graph as in Ref. [11], and the gate errors are calculated using driving fields with $\Omega_0 = 0.05g$, $\tau_1 = 2404/g$, $\tau_2 = 1596/g$, and $\sigma = 400/g$. Each symbol of large error regions in Fig. 2 corresponds to the equation in Table I, which represents resonance conditions. The gate errors notably are large in some regions in which resonance conditions are accidentally satisfied, and the gate errors are suppressed in off-resonant regions.

Figure 3 shows contour diagrams of gate errors in the Δ_2–Δ_3 plane in the case of the three-qubit system with three extra physical systems ($N = 3$, $N' = 3$).

The errors are calculated using driving fields with $\Omega_0 = \Omega_e = 0.05g$, $\tau_1 = 2404/g$, $\tau_2 = 1596/g$, and $\sigma = 400/g$. The optimal value of Ω_e is a similar value of Ω_0. Because cases of $\Omega_e \ll \Omega_0$ and $\Omega_e \gg \Omega_0$ have some problems for high fidelity. Population of $|1\rangle_i$ in extra physical systems during the gate is too much in the case of $\Omega_e \ll \Omega_0$, and Ω_e become near level with g in the case of $\Omega_e \gg \Omega_0$. Each symbol of large-error regions in Fig. 3 corresponds to the equations in Table II, which represent resonance conditions.

The large-error regions of Fig. 3 are shifted from those of Fig. 2 owing to the introduction of extra physical systems. Therefore, introducing extra physical systems can be employed to control resonance conditions. For example, when detuning parameters are given as $\Delta_2 = 0.7g$ and $\Delta_3 = g$, although the high-fidelity gate is unavailable in the three-qubit systems

TABLE I. Equations corresponding to the labels in Fig. 2.

(a − 1,2)	$\Delta_2 = 0, \sqrt{2}g$
(b − 1,2,3,4)	$\Delta_3 = 0, g, \sqrt{2}g, \sqrt{10}g$
(c − 1,2,3)	$\Delta_2 + \Delta_3 = g, \sqrt{2}g, \sqrt{10}g$
(d − 1,2,3,4,5,6,7)	$\Delta_2 - \Delta_3 = 0, \pm g, \pm\sqrt{2}g, \pm\sqrt{10}g$
(e − 1,2,3)	$2\Delta_2 + \Delta_3 = g, \sqrt{2}g, \sqrt{10}g$

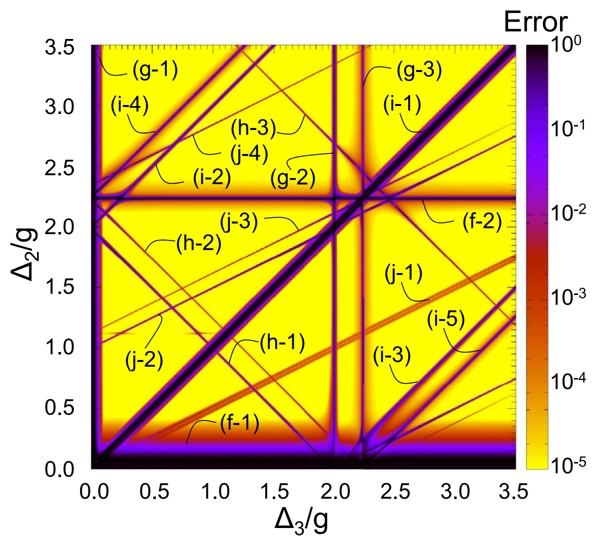

FIG. 3. Contour diagram of gate errors in a three-qubit system with three extra physical systems.

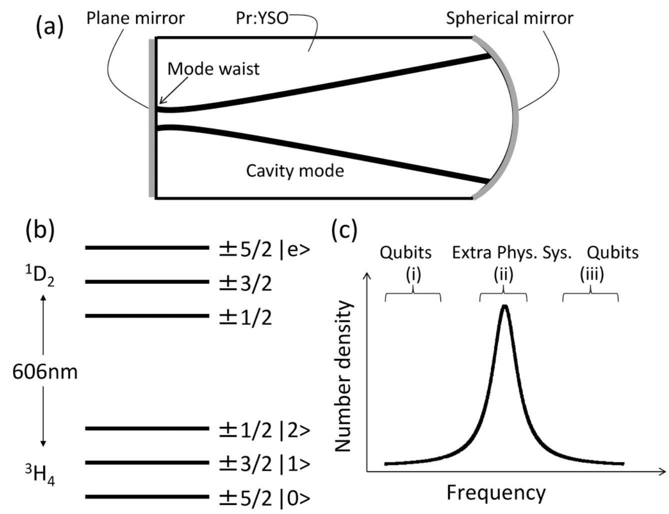

FIG. 4. Implementation of proposed FDQC. (a) A device to implement our scheme. (b) State assignment. (c) Assignment of qubits and extra physical systems in the frequency domain.

without extra physical systems, high-fidelity gates are available by introducing three extra physical systems. Finally, if extra physical systems can be introduced to the whole system, resonance conditions are controllable, and the gate errors due to crosstalk can be suppressed.

V. IMPLEMENTATION OF EXTRA PHYSICAL SYSTEMS

We discuss an implementation of FDQC with extra physical systems in an actual system. This implementation requires some conditions for the system: containing many candidate qubits which have a long coherence time, large inhomogeneous broadening compared with homogeneous broadening, and strong coupling between qubits and cavity mode. Some solid-state systems which have a long coherence time, such as rare-earth-ion-doped crystals, donors in isotopically purified silicon, and nitrogen-vacancy centers in diamond, will satisfy all the conditions by improving some of the properties. For instance, consider the case of Pr^{3+}:Y_2SiO_5 (Pr:YSO), which is a rare-earth-ion-doped crystal.

Pr:YSO has a long coherence time, and is a promising material. We consider a device which is a monolithic Fabry-Perot cavity made of the crystal [Fig. 4(a)]. Figure 4(b) shows the state assignment in ions.

Ions which have $|2\rangle_i - |e\rangle_i$ transition resonant with a cavity mode are employed as qubits and extra physical systems. Ions strongly coupling to the cavity mode are employed as qubits. Ions coupling to the cavity mode and ions unemployed as qubits can be employed as extra physical systems. Ions

TABLE II. Equations corresponding to the labels in Fig. 3.

(f − 1,2)	$\Delta_2 = 0, \sqrt{5}g$
(g − 1,2,3)	$\Delta_3 = 0, 2g, \sqrt{5}g$
(h − 1,2,3)	$\Delta_2 + \Delta_3 = 2g, \sqrt{5}g, \sqrt{22}g$
(i − 1,2,3,4,5)	$\Delta_2 - \Delta_3 = 0, \pm 2g, \pm\sqrt{5}g$
(j − 1,2,3,4)	$2\Delta_2 + \Delta_3 = 0, 2g, \sqrt{5}g, \sqrt{22}g$

for extra physical systems are prepared at state $|2\rangle_i$ in the initial state, and additional driving fields are irradiated to $|1\rangle_i - |e\rangle_i$ transitions of the ions during the gate operation. Ions unemployed as qubits and unemployed as extra physical systems are pumped out to suppress any effects by the ions.

We now consider a cavity for strong coupling between ions and the cavity mode. For the strong coupling, we assume that the cavity has a mode waist of wavelength size, and we employ ions only around antinodes of electric field within Rayleigh length region as qubits. When the cavity length is 3 mm, wavelength is 606 nm, and size of mode waist is 1 μm, the Rayleigh length of the mode is about 10 μm. When we can employ 50% (around antinode) of ions in the 44-μm^3 space region, it is estimated that the number of ions is 10^8 in the case of 10^{-3} at % (3×10^6 ions/μm^3) of the density of ions in the crystal.

If these ions are addressable, the ions can be employed for FDQC. The number of available ions depends on homogeneous broadening of optical transitions, inhomogeneous broadening of lower states, and inhomogeneous broadening of optical transitions, which are 1 kHz, 70 kHz, and 10 GHz in Pr:YSO, respectively [18,19]. The ions in the frequency region which is about the coupling-constant value are selected from inhomogeneous broadening of optical transition. Therefore, in the case of a coupling constant of 1 MHz for strong coupling, it is estimated that the number of ions strongly coupled to the cavity mode is about 10^4 ($= 10^8 \times 1$ MHz/10 GHz).

Frequency distribution of optical transitions is a power law with long tail [20]. When frequency distribution of lower states has also long tail [21,22], we can employ many ions whose transition frequencies are outside [Fig. 4(c), (i) and (iii)] of their so-called inhomogeneous width (70 kHz). Transition frequency differences between one of three lower states and one of three excited states [Fig. 4(b)] are larger than 1 MHz. Therefore, it is estimated that ions in about the 1-MHz range which is free from the influence of driving fields to transitions of another combination of a lower and an excited state can be employed as qubits. Frequency differences of lower states of ions for qubits must be larger than homogeneous broadening of optical transition (1 kHz) to identify each ion in frequency region. Finally, it is estimated that the number of ions which can be employed as qubits is 10^3 (= 1 MHz/1 kHz).

The number of ions which cannot be distinguished in the frequency domain [Fig. 4(c), (ii)] is 10^3–10^4. These ions can be employed as extra physical systems. Therefore, even if ions for extra physical systems are introduced, the number of qubits is not compromised. The initial states of the ions employed as extra physical systems are prepared at state $|2\rangle_i$. The initial states of the ions employed neither as qubits nor as extra physical systems are prepared at state $|0\rangle_i$. $|1\rangle_i$–$|e\rangle_i$ transitions of ions employed as extra physical systems are irradiated by the resonant driving field L_e during gate operations to keep the states at $|2\rangle_i$. Additional crosstalk due to L_e is negligible in either of the following cases. First, frequency differences between the region of Fig. 4(c), (ii), and the region of Fig. 4(c), (i) and (iii), are large enough to suppress the influence of L_e to qubits. Second, qubit ions and extra ions are from a different location. For example, qubit ions are from near the mode waist, and extra ions are from near the spherical mirror. Extra ions do not strongly couple to the cavity mode near the spherical mirror. However, the ions can be employed to control resonance conditions, because the number of ions is very large.

Finally, it is concluded that extra physical systems are implementable in the cavity system using rare-earth-ion-doped crystal.

VI. SUMMARY

In summary, we have proposed an implementation of FDQC with extra physical systems to suppress crosstalk. We have also shown the effects of the extra physical systems by numerical calculations. We can expect that FDQC implements 10^3 times as many qubits as that using only distinction in the space domain. FDQC will be implemented in some actual systems including many candidate qubits which have a long coherence time. Properties required to realize the FDQC other than long coherence time are inhomogeneous broadening of lower states which is larger than homogeneous broadening of optical transitions, and strong coupling between ions and a cavity mode. When these requirements are fulfilled, our proposed implementation is expected to be useful to realize universal quantum computation using a large number of physical qubits.

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