

# Decoherence by spontaneous emission: A single-atom analog of superradiance

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(Received 7 March 2016; published 16 December 2016)

We show that the decoherence of the atomic center-of-mass induced by spontaneous emission involves interferences corresponding to a single-atom analog of superradiance. We use a decomposition of the stationary decoherence rate as a sum of local and nonlocal contributions obtained to second order in the interaction by the influence functional method. These terms are respectively related to the strength of the coupling between system and environment, and to the quality of the information about the system leaking into the environment. While the local contribution always yields a positive decoherence rate, the nonlocal one may lead to recoherence when only partial information about the system is obtained from the disturbed environment. The nonlocal contribution contains interferences between different quantum amplitudes leading to oscillations of the decoherence rate reminiscent of superradiance. These concepts, illustrated here in the framework of atom interferometry within a trap, may be applied to a variety of quantum systems.

DOI: [10.1103/PhysRevA.94.062114](https://doi.org/10.1103/PhysRevA.94.062114)

## I. INTRODUCTION

Decoherence is a fundamental issue of quantum mechanics that has been the object of innumerable theoretical and experimental studies [1]. The dynamics of open quantum systems is particularly intriguing in the case of non-Markovian environments [2–7]. In this paper we show that an environment of finite memory enables a quantum system with a single atom to mimic collective effects such as superradiance [8], which is still a topic of intense investigation [9–16].

The influence functional [17,18] is an effective theoretical tool to capture the effect of a non-Markovian environment on a quantum system. An important application is to model the interaction between atoms and material surfaces mediated by the quantum electromagnetic field. In this context, nonstationary [19–22] and quantum friction effects [23] can be analyzed with the help of closed-time-path (CTP) integrals describing the density matrix evolution. In the CTP formalism, the finite memory time of the environment induces a coupling between the backward and forward histories of the system within the closed-time-path integrals describing the density matrix evolution. Such coupling leads to an irreducible functional, in the sense that it cannot be split into separate contributions from forward and backward histories. Throughout this paper we assume that the initial system density matrix contains well-separated wave packets about to follow distinct quantum paths while maintaining a negligible overlap between each other. This is typically the case for cold atoms trapped in optical lattices in the Lamb-Dicke regime, or for atom interferometers propagating over a macroscopic scale. After CTP integration, this clear separation between the wave packets enables one to interpret the complex phase arising from this irreducible functional as a genuinely nonlocal phase. The associated phase shifts have been studied in Refs. [20–22] in the context of atom interferometry. When the environment has a finite memory, the corresponding nonlocal contribution to the decoherence rate is essential. In this paper we show that such contribution may be negative (i.e., to be interpreted as recoherence in the sense defined by Ref. [24]), and that it exhibits interference oscillations which constitute an analog of superradiance [8].

The paper is organized as follows. In Sec. II we discuss the coupling between backward and forward histories in the closed-time-path formalism and its relation with nonlocality. An explicit perturbative expansion is developed in Sec. III. Section IV introduces an alternative approach enabling the connection with the quality of which-path information leaking into the environment. In Sec. V we show that the decoherence of a single atom oscillates as a function of the distance between the wave packets, which is reminiscent of superradiance. Concluding remarks are presented in Sec. VI.

## II. COUPLING BETWEEN BACKWARD AND FORWARD HISTORIES

Precisely, we consider a system described by a center-of-mass position  $\mathbf{r}$  interacting with a generic environment. It is well known that the evolution of pure quantum states can be described in terms of forward path integrals. On the other hand, an open quantum system can be suitably described by a density matrix, whose elements contain products of wave functions by their complex conjugates. Therefore, it is possible to write the evolution of a density matrix as a double path integral, with the complex conjugation representing a backward propagation [17,18,25,26]. The reduced density matrix of our system is propagated with the CTP integral

$$\langle \mathbf{r}_f | \rho(\Delta t) | \mathbf{r}'_f \rangle = \int_{\text{CTP}}^{\mathbf{r}_f, \mathbf{r}'_f} \mathcal{D}\mathbf{r} e^{\frac{i}{\hbar}(S_0[\mathbf{r}] - S_0[\mathbf{r}'] + S_{\text{IF}}[\mathbf{r}, \mathbf{r}'])}, \quad (1)$$

where we used the following notation:

$$\int_{\text{CTP}}^{\mathbf{r}_f, \mathbf{r}'_f} \mathcal{D}\mathbf{r} = \int d\mathbf{r}_0 d\mathbf{r}'_0 \int_{\mathbf{r}(0)=\mathbf{r}_0}^{\mathbf{r}(\Delta t)=\mathbf{r}_f} \mathcal{D}\mathbf{r} \times \int_{\mathbf{r}'(0)=\mathbf{r}'_0}^{\mathbf{r}'(\Delta t)=\mathbf{r}'_f} \mathcal{D}\mathbf{r}' \langle \mathbf{r}_0 | \rho(0) | \mathbf{r}'_0 \rangle, \quad (2)$$

with  $\rho(0)$  the initial system density matrix and  $S_0[\mathbf{r}]$  the interaction-free action of the system, i.e., when all couplings to the other degrees of freedom (d.o.f.)—referred to as the environment—are ignored. We have assumed that system and environment are initially uncorrelated. As the actions  $S_0[\mathbf{r}]$  and  $S_0[\mathbf{r}']$  appear with different signs, the paths  $[\mathbf{r}(t)]$

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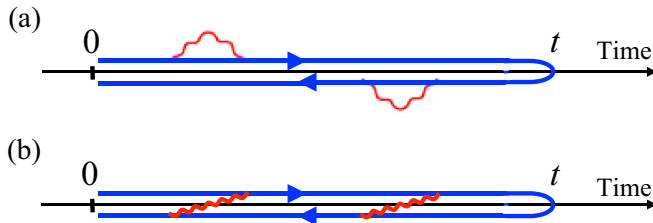


FIG. 1. Closed-time-path diagrams representing (a) local and (b) nonlocal contributions to the influence functional. The nonlocal influence terms couple the backward and forward histories.

and  $[\mathbf{r}'(t)]$  correspond, respectively, to forward and backward histories. The influence of the environment on the system is completely described by the complex influence functional  $S_{\text{IF}}[\mathbf{r}, \mathbf{r}']$ , which can be decomposed into a sum of local and nonlocal terms, namely  $S_{\text{IF}}[\mathbf{r}, \mathbf{r}'] = S_{\text{L}}[\mathbf{r}, \mathbf{r}'] + S_{\text{NL}}[\mathbf{r}, \mathbf{r}']$ . The local influence functional is expressed in terms of the single functional  $S_{\text{single}}$  depending on one path at a time:  $S_{\text{L}}[\mathbf{r}, \mathbf{r}'] = S_{\text{single}}[\mathbf{r}] - S_{\text{single}}^*[\mathbf{r}']$  (explicit expressions are given below). In contrast, the irreducible influence functional  $S_{\text{NL}}[\mathbf{r}, \mathbf{r}']$  cannot be expressed as a combination of single-path functionals. The presence of this nonlocal functional couples the backward and forward histories, as illustrated in Fig. 1. This coupling occurs over a time scale of the order of the memory time of the environment.

The real part of the influence functional describes a relative phase shift between the two paths, while its imaginary part, henceforth denoted by  $S_{\text{L(NL)}}^{\text{Dec}}[\mathbf{r}, \mathbf{r}'] = \text{Im}[S_{\text{L(NL)}}[\mathbf{r}, \mathbf{r}']]$ , provides the exponential attenuation of the interference fringes [27]. Note that the expression of the local influence functional  $S_{\text{L}}[\mathbf{r}, \mathbf{r}']$  as a difference between a functional and its complex conjugate is perfectly consistent with this interpretation. Indeed, this form ensures that its real and imaginary parts are, respectively, antisymmetric and symmetric under the exchange of paths. These features are actually needed to preserve the Hermiticity of the reduced density matrix  $\rho(\mathbf{r}_f, \mathbf{r}'_f; t) = \rho^*(\mathbf{r}'_f, \mathbf{r}_f; t)$  during the propagation. As a consequence, the local decoherence rates associated with  $S_{\text{L}}^{\text{Dec}}[\mathbf{r}, \mathbf{r}']$  sum up.

In order to highlight the role of the irreducible functional  $S_{\text{NL}}[\mathbf{r}, \mathbf{r}']$ , we consider here the decoherence resulting only from the local influence functional  $S_{\text{L}}[\mathbf{r}, \mathbf{r}']$ . We show that it yields results in contradiction with the common interpretation of decoherence as a flow of which-way information from the system to its environment. Let us assume that the system is put in a coherent superposition state of paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$ . For simplicity, we assume that the density matrix corresponds to thin wave packets, so that only the quantum paths lying in a tight bundle around the central paths  $[\mathbf{r}_1(t)], [\mathbf{r}_2(t)]$  significantly contribute to the integrals (1) and (2). We now take the limit where these two paths become arbitrarily close to one another, and make the reasonable assumption that the single-path functional  $S_{\text{single}}[\mathbf{r}]$  is continuous. One gets from  $S_{\text{L}}^{\text{Dec}}[\mathbf{r}_1(t), \mathbf{r}_2(t)] = \text{Im}\{S_{\text{single}}[\mathbf{r}_1(t)] + S_{\text{single}}[\mathbf{r}_2(t)]\}$  a system decoherence rate which is twice the contribution arising from each path. On the other hand, in this limit, the two paths are so close that no matter which way the system goes, it leaves an almost identical record on the environment. Thus, the amount of which-way information flowing to the environment gradually

vanishes, but the local decoherence rate remains finite (similar conclusions hold for wave packets of finite width [28]).

This example shows that the imaginary part of the irreducible influence functional  $S_{\text{NL}}^{\text{Dec}}[\mathbf{r}, \mathbf{r}']$  captures the distinguishability between the two paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$ . This property requires a functional involving simultaneously both paths at a time. In the limit above, when the two paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$  become identical, this contribution should cancel exactly the local contribution from  $S_{\text{L}}[\mathbf{r}, \mathbf{r}']$ , enabling one to retrieve the expected absence of decoherence. Thus, in this limit, the irreducible influence functional should provide complete recoherence to the system. The derivation below confirms this argument: indeed the nonlocal contribution to decoherence has then the same amplitude as the local decoherence, and opposite sign.

The opposite limit is reached when the paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$  are so different that, when followed by the system, their respective perturbations of the environment correspond to orthogonal quantum states. In other words, perfect which-way information flows from the system to the environment: by performing a suitable measurement of the perturbation suffered by the environment, an observer could tell with certainty which path was taken by the system. As established below, the nonlocal decoherence contribution vanishes in this regime, meaning that the decoherence saturates as a function of the distance between the paths. More generally, the total decoherence rate results from a fine tuning between a local decoherence rate of positive sign associated with the probability of a physical process, and a nonlocal decoherence rate of indefinite sign, to be interpreted as a recoherence [24] rate when bearing a negative sign. This nonlocal recoherence measures the lack of which-way information obtained from the system-environment interaction process.

It has been shown previously that the environment may induce, through the influence functional, nonlocal phase shifts in systems such as atom interferometers when considering the dipolar interaction [20,22]. However, in these examples, the nonlocal contribution represents only a tiny (relativistic) correction to other local atomic phases induced by the environment. In contrast, as far as decoherence is concerned, the nonlocal influence contribution [depicted in Fig. 1(b)] can be of the same order of magnitude as the local term. By capturing the quality of the which-way information leaking into the environment, the nonlocal contribution is hence absolutely essential to estimate the decoherence.

### III. LOCAL AND NONLOCAL DECOHERENCE FUNCTIONALS

We address the decoherence effect over a time scale  $\Delta t$  simultaneously much larger than the self-dynamics of the environment and much shorter than the time scale associated with the strength of the coupling with the system. For instance, the natural width of the atomic transition is several orders of magnitude smaller than the transition frequency in usual atomic states, allowing us to consider the intermediate time  $\Delta t$ . The average decoherence rate on this time scale will be referred to as the stationary decoherence rate.

We first use CTP path integrals in order to obtain the local and nonlocal stationary decoherence rates, noted  $\Gamma_{\text{L}}$  and  $\Gamma_{\text{NL}}$ ,

respectively, and defined as

$$\Gamma_{\text{L(NL)}} = \lim_{\Delta t \rightarrow +\infty} \frac{1}{\hbar \Delta t} S_{\text{L(NL)}}^{\text{Dec}}[\mathbf{r}_1(t), \mathbf{r}_2(t)]|_{t \in [0, \Delta t]} \quad (3)$$

for an initial quantum superposition of thin wave packets centered at prescribed paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$  (recoil neglected), defined on the time interval  $[0, \Delta t]$ .

In order to compute the right-hand side of Eq. (3), we assume that the total action has the general form  $S[\mathbf{r}, \mathbf{q}, \mathbf{X}] = S_0[\mathbf{r}] + S_q[\mathbf{q}] + S_X[\mathbf{X}] + S_{\text{int}}[\mathbf{r}, \mathbf{q}, \mathbf{X}]$ , with  $\mathbf{q}$  and  $\mathbf{X}$  representing environment d.o.f. and  $S_q, S_X$  the corresponding free actions. We start by coarse graining over the fastest environment d.o.f., making use of a hierarchy of time scales (see for instance [19]). Here we assume that the continuum of d.o.f.  $\mathbf{X}(x)$  fluctuates on a much faster time scale than the d.o.f.  $\mathbf{q}(t)$ . In the example to be discussed later,  $\mathbf{X}(x)$  represents the electromagnetic (EM) field and  $\mathbf{q}(t)$  the atomic dipole, so that the fluctuations of the former are indeed short lived on an atomic dipole time scale [29]. Thus, we first coarse grain over the environment d.o.f.  $\mathbf{X}(x)$ , yielding an auxiliary influence functional  $S_{\text{IF}}[\mathbf{r}, \mathbf{r}', \mathbf{q}, \mathbf{q}']$ , and then trace out the slower environment d.o.f.  $\mathbf{q}(t)$ . We consider general free actions and only require the interaction to be local, that is,

$$S_{\text{int}}[\mathbf{X}, \mathbf{q}, \mathbf{r}] = \int_0^{\Delta t} dt V(\mathbf{X}(\mathbf{r}, t), \mathbf{q}(t)). \quad (4)$$

For simplicity, we also assume very thin wave packets, since the generalization to more general wave functions is straightforward [22]. We trace out the environment degrees of freedom using perturbation theory and suppose the first order to vanish [that is  $\langle \mathbf{q}(t) \rangle = 0$  or  $\langle \mathbf{X}(\mathbf{r}, t) \rangle = 0$  at any time  $t$ ] [30].

In Appendix A we show that the local and nonlocal influence functionals are then given by

$$S_{\text{L}}[\mathbf{r}_1(t), \mathbf{r}_2(t)] = \frac{i}{\hbar} \int_0^{\Delta t} dt \int_0^{\Delta t} dt' \Theta(t-t') [\langle \tilde{V}(r_1(t)) \tilde{V}(r_1(t')) \rangle + \langle \tilde{V}(r_2(t')) \tilde{V}(r_2(t')) \rangle], \quad (5)$$

$$S_{\text{NL}}[\mathbf{r}_1(t), \mathbf{r}_2(t)] = -\frac{i}{\hbar} \int_0^{\Delta t} dt \int_0^{\Delta t} dt' \langle \tilde{V}(r_2(t)) \tilde{V}(r_1(t')) \rangle, \quad (6)$$

where  $\Theta$  represents the Heaviside step function and  $\tilde{V}(r(t)) = \tilde{V}(\mathbf{r}(t), t)$  denotes the operator  $V$  in the interaction picture obtained from the free evolution of the operators  $\mathbf{X}$  and  $\mathbf{q}$ .  $\langle \dots \rangle$  denotes the expectation value in the initial state of the environment.

A situation of particular interest is the one where the d.o.f.  $\mathbf{q}$  and  $\mathbf{X}$  are linearly coupled:  $V(\mathbf{r}, t) = \mathbf{q}(t) \cdot \mathbf{X}(\mathbf{r}, t)$ . Substituting this equation into Eqs. (5) and (6) and taking the imaginary part we obtain

$$S_{\text{L}}^{\text{Dec}}[\mathbf{r}_1(t), \mathbf{r}_2(t)] = \int_0^{\Delta t} dt \int_0^{\Delta t} dt' [G(r_1(t), r_1(t')) + G(r_2(t'), r_2(t'))], \quad (7)$$

$$S_{\text{NL}}^{\text{Dec}}[\mathbf{r}_1(t), \mathbf{r}_2(t)] = \int_0^{\Delta t} dt \int_0^{\Delta t} dt' G(r_1(t), r_2(t')), \quad (8)$$

where we employed the shorthand notation  $r_i(t) = (t, \mathbf{r}_i(t))$  ( $i = 1, 2$ ) and

$$G(x, x') \equiv \frac{1}{4\hbar} \sum_{i,j} [\Theta(t-t') \langle [q_i(t), q_j(t')] \rangle \langle [X_i(x), X_j(x')] \rangle + \langle \{q_i(t), q_j(t')\} \rangle \langle \{X_i(x), X_j(x')\} \rangle]. \quad (9)$$

In the sum, the indices  $i, j$  span the Cartesian components of the vectors  $\mathbf{q}$  and  $\mathbf{X}(x)$ . Combining these expressions with Eq. (3), one obtains the stationary decoherence rates  $\Gamma_{\text{L}}$  and  $\Gamma_{\text{NL}}$  predicted by the influence functional method.

The nonlocal contribution to decoherence  $S_{\text{NL}}^{\text{Dec}}[\mathbf{r}_1(t), \mathbf{r}_2(t)]$  originates from the mixing of the forward  $[\mathbf{r}_1(t)]$  and backward  $[\mathbf{r}_2(t)]$  histories within the CTP path integral. As suggested by Eqs. (8) and (9), such coupling can only occur in practice if the memory time of the d.o.f.  $\mathbf{q}(t)$ , captured by the decay of the correlation functions  $\langle q_i(t) q_j(t') \rangle$ , is greater than the propagation time of the interaction connecting the two histories. For instance, cold atoms can provide internal d.o.f. with sufficiently long-lived correlations enabling the connection between forward and backward histories at distinct locations. In this sense, cold atomic systems provide an adequate playground for probing non-Markovian environments for open quantum systems.

It is possible to generalize the argument above to multipath atom interferometers [31]. It turns out that the nonlocal contribution to decoherence involves a sum over all possible pairs of paths, similarly to the nonadditive atomic phases induced by electromagnetic vacuum field fluctuations [21].

#### IV. NONLOCALITY AND QUALITY OF INFORMATION

To have a complementary insight on the nature of the nonlocal decoherence, we present below an alternative derivation, based on standard time-dependent perturbation theory. As in the previous derivation, we assume that the system and environment are initially uncorrelated, described by a pure quantum state of the form

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}} (|\psi_1(0)\rangle + |\psi_2(0)\rangle) \otimes |\psi_E(0)\rangle, \quad (10)$$

and that their coupling can be treated perturbatively. The system quantum states  $|\psi_1(0)\rangle$  and  $|\psi_2(0)\rangle$  correspond to propagations along paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$ , respectively, and  $|\psi_E(0)\rangle$  is the initial state of the environment. Although we have chosen a particularly simple initial state for the system, the discussion to follow could be generalized without difficulty to an initial quantum state of the form  $|\psi(0)\rangle = \sum_{k=1}^N \alpha_k |\psi_k(0)\rangle \otimes |\psi_E(0)\rangle$ .

The total Hamiltonian reads  $H = H_0 + H_E + V$ , with  $H_0, H_E$ , and  $V$  representing the system, environment, and interaction terms, respectively. In the limit of thin wave packets, the time evolution of each state  $|\psi_k(0)\rangle \otimes |\psi_E(0)\rangle$  present in the superposition (10) can be obtained by replacing the observable  $\hat{\mathbf{r}}$  by its time-dependent average value  $\mathbf{r}_k(t)$  within the interaction  $V$ . With this replacement, each initial state  $|\psi_k(0)\rangle \otimes |\psi_E(0)\rangle$  evolves into a product state. However, the two state components present in (10) follow different average paths  $\mathbf{r}_k(t)$ . As a consequence, the system and the

environment become entangled at time  $t$  [32]:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(|\psi_1(t)\rangle \otimes |\psi_E^{(1)}(t)\rangle + |\psi_2(t)\rangle \otimes |\psi_E^{(2)}(t)\rangle). \quad (11)$$

The influence of the environment on the system coherence is then captured by the complex amplitude  $\langle \psi_E^{(2)}(t) | \psi_E^{(1)}(t) \rangle$ . As in the previous section, we assume that the leading-order perturbation is of second order in the interaction  $V(\mathbf{r})$ , i.e., the initial state  $|\Psi(0)\rangle$  is such that  $\langle \Psi(0) | V(\mathbf{r}) | \Psi(0) \rangle = 0$  at any position  $\mathbf{r}$ . The resulting influence of the environment can then be compared with the expression for the complex influence functional  $S_{\text{IF}}[\mathbf{r}_1(t), \mathbf{r}_2(t)]$  derived from CTP integrals in the previous section:

$$\begin{aligned} e^{iS_{\text{IF}}[\mathbf{r}_1(t), \mathbf{r}_2(t)]/\hbar} &= \langle \psi_E^{(2)}(t) | \psi_E^{(1)}(t) \rangle \\ &= \langle \psi_E(0) | \tilde{\mathcal{T}} e^{\frac{i}{\hbar} \int_0^{\Delta t} dt \tilde{V}(r_2(t))} \\ &\quad \times \mathcal{T} e^{-\frac{i}{\hbar} \int_0^{\Delta t} dt \tilde{V}(r_1(t))} | \psi_E(0) \rangle, \end{aligned} \quad (12)$$

where  $\mathcal{T}$  and  $\tilde{\mathcal{T}}$  denote, respectively, the time ordering and antitime ordering operators. One retrieves the local influence functional (5) by expanding each evolution operator in (12) separately to second order, whereas the nonlocal functional (6) is obtained by taking the product of first-order terms in (12).

This approach provides a complementary interpretation of the influence functionals  $S_L$  and  $S_{\text{NL}}$  in terms of the environment state vectors appearing in Eqs. (10) and (11). We first develop the local decoherence functional by taking the imaginary part of (5) and using the identity  $\text{Re}\langle AB \rangle = \frac{1}{2}\langle \{A, B\} \rangle$  for generic operators  $A, B$ . We then perform the variable change  $t \leftrightarrow t'$  in the integrals involving antitime ordered terms to find

$$S_L^{\text{Dec}}[\mathbf{r}_1, \mathbf{r}_2] = \frac{1}{2\hbar} \sum_{k=1,2} \int_0^{\Delta t} dt \int_0^{\Delta t} dt' \langle \tilde{V}(r_k(t)) \tilde{V}(r_k(t')) \rangle. \quad (13)$$

We insert this expression into Eq. (3) and use the completeness relation over the Hilbert space associated with the environment to find the local decoherence rate [33]

$$\Gamma_L = \lim_{\Delta t \rightarrow +\infty} \frac{1}{2\Delta t} \sum_{k=1,2} \sum_{|i_E\rangle} \left| \langle i_E | \frac{-i}{\hbar} \int_0^{\Delta t} dt \tilde{V}(r_k(t)) | \psi_E(0) \rangle \right|^2. \quad (14)$$

The sum can be naturally restricted to the environment quantum states  $|i_E\rangle$  coupled to the initial state  $|\psi_E(0)\rangle$  through the interaction potential  $\tilde{V}(\mathbf{r}, t)$ . Physically, the local decoherence rate appears as the average probability per unit time, to leading order in the interaction potential, that the initial state of the environment suffers a transition. Thus, the local decoherence rate is directly connected to the total transition rate from the initial environment state.

The nonlocal decoherence rate is instead related to the quality of the information flowing from the system into the environment. From Eq. (6), the nonlocal influence functional  $S_{\text{NL}}$  can be expressed in terms of the overlap between the environment state perturbations when following each of the two system's paths:

$$S_{\text{NL}}[\mathbf{r}_1, \mathbf{r}_2] = -i\hbar \langle \psi_E^{(1),1}(\Delta t) | \psi_E^{(2),1}(\Delta t) \rangle, \quad (15)$$

with

$$|\psi_E^{(k),1}(\Delta t)\rangle = -\frac{i}{\hbar} \int_0^{\Delta t} dt \tilde{V}(r_k(t)) | \psi_E(0) \rangle \quad (16)$$

representing the first-order perturbation in the interaction picture when taking path  $[\mathbf{r}_k(t)]$ . In particular, the nonlocal decoherence rate vanishes when the propagation of the system along the paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$  leads to orthogonal perturbations of the initial environment state. In this case, the interaction produces a perfect record of the system quantum state in the environment.

More generally, the nonlocal decoherence rate (3) is proportional to the real part of the overlap between the environment state components,

$$\Gamma_{\text{NL}} = - \lim_{\Delta t \rightarrow +\infty} \frac{1}{\Delta t} \text{Re}[\langle \psi_E^{(1),1}(\Delta t) | \psi_E^{(2),1}(\Delta t) \rangle], \quad (17)$$

and thus may be positive or negative in principle. Nevertheless,  $\Gamma_{\text{NL}}$  is certainly negative when the paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$  become arbitrarily close to one another, since the amplitude  $\langle \psi_E^{(1),1}(\Delta t) | \psi_E^{(2),1}(\Delta t) \rangle$  tends by continuity to  $\|\psi_E^{(1),1}(\Delta t)\|^2 = \|\psi_E^{(2),1}(\Delta t)\|^2$  in this case.

We can also discuss this limit by looking at the total decoherence rate  $\Gamma = \Gamma_L + \Gamma_{\text{NL}}$ . From Eqs. (14), (16), and (17), one shows that

$$\Gamma = \lim_{\Delta t \rightarrow +\infty} \frac{1}{\Delta t} \langle \psi_E^{(-)}(\Delta t) | \psi_E^{(-)}(\Delta t) \rangle, \quad (18)$$

where  $|\psi_E^{(-)}(\Delta t)\rangle = \frac{1}{\sqrt{2}}[|\psi_E^{(2),1}(\Delta t)\rangle - |\psi_E^{(1),1}(\Delta t)\rangle]$ . Equation (18) resembles the result of Ref. [34] expressing the decoherence by emission of electromagnetic radiation in terms of a current difference.

When the paths  $[\mathbf{r}_1(t)]$  and  $[\mathbf{r}_2(t)]$  become arbitrarily close to one another, so do the perturbed states  $|\psi_E^{(1),1}(\Delta t)\rangle$  and  $|\psi_E^{(2),1}(\Delta t)\rangle$ , yielding a vanishing total decoherence rate from (18). This results from a fine tuning between a positive local decoherence rate  $\Gamma_L$  and a negative nonlocal decoherence rate  $\Gamma_{\text{NL}}$ . The latter may be interpreted as a recoherence of the system, reflecting that the environment acquires imperfect which-way information about the system. While the local decoherence rate  $\Gamma_L$  guarantees the positivity of the total decoherence rate  $\Gamma$ , only the nonlocal contribution  $\Gamma_{\text{NL}}$  captures the distinguishability between the two possible ‘‘footprints’’ left by the system on the environment when going along path  $[\mathbf{r}_1(t)]$  or path  $[\mathbf{r}_2(t)]$ . This term only survives if the environment has a sufficiently long memory time and in that sense is reminiscent of non-Markovian processes in open quantum systems [3].

## V. DECOHERENCE BY SPONTANEOUS EMISSION AND SUPERRADIANCE-LIKE INTERFERENCE

We now illustrate the general arguments discussed above on a concrete example, which is an idealization of atom interferometry within an optical trap. Inspired by the double well atom interferometry experiments reported in Refs. [35,36], we consider the spontaneous emission of a single excited atom prepared in a coherent superposition of two wave packets located in different wells of an external potential, as illustrated in the inset of Fig. 2. In the spirit of Ref. [32], our formalism

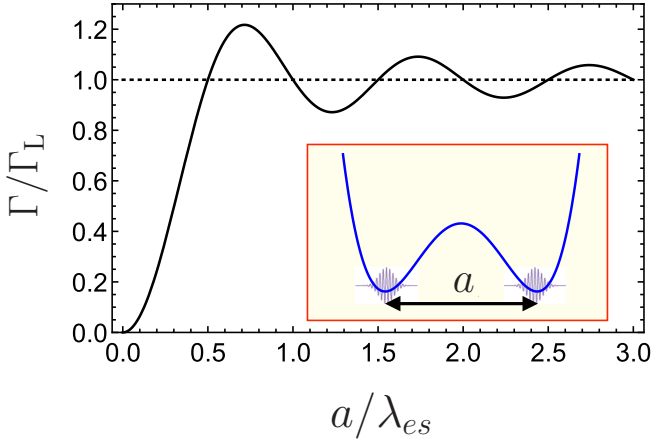


FIG. 2. Decoherence rate variation with the distance  $a$  between the two wells. The total rate  $\Gamma$  is expressed in units of the local contribution  $\Gamma_L$ , which does not depend on  $a$ , whereas the distance is expressed in units of the transition wavelength  $\lambda_{es}$ . For simplicity, we have considered a two-level atom in the excited state. The quantum state of the atom center-of-mass is initially a coherent superposition of localized wave packets in a double-well potential, as illustrated by the inset.

allows us to interpret the decoherence either as resulting from a random phase, associated with the influence functional developed in Secs. II and III, or in terms of entanglement with the dipole and electromagnetic field, discussed in Sec. IV. We show that the decoherence rate oscillates as a function of the distance  $a$  between the wave packets. These oscillations, resulting from quantum interferences in the transition amplitudes, enable us to build an analogy with superradiance. This is remarkable since our system contains only a single atom, while superradiance [8] is in essence a collective phenomenon.

A closely related experiment [37]—an atom interferometer undergoing photon scattering—reported oscillations of the interference fringe contrast as a function of the distance between the arms. Other studies focused on the decoherence of the atomic internal degree of freedom [38,39] or on the influence of a largely spread center-of-mass wave function [40].

We consider an atom prepared in a coherent superposition state of the form  $|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle + e^{i\varphi}|\psi_-\rangle)$ , where  $|\psi_{\pm}\rangle$  correspond to the trapped wave packets in the two wells (tunneling neglected) located at  $\mathbf{r} = \pm \frac{1}{2}\mathbf{a}$  (see inset of Fig. 2) assumed distant enough so that  $\langle\psi_-|\psi_+\rangle = 0$ , i.e., the atoms are trapped in the optical lattice in the Lamb-Dicke regime. We also assume that such superposition may be produced by an atomic beam splitter leaving unaffected the atomic internal state—taken as an excited eigenstate  $|e\rangle$ —and that the two wave functions are recombined coherently at a later time.

We apply Eqs. (5) and (6) to the case of a dipolar interaction Hamiltonian (in the interaction picture)  $\tilde{V}(\mathbf{r}, t) = -\mathbf{d}(t) \cdot \mathbf{E}(\mathbf{r}, t)$ . The decoherence rates then involve bilinear correlations in both dipole and electric field operators. Our first step is then to analyze such correlations. The dipole correlation reads  $\langle e|d_i(t)d_j(t')|e\rangle = \frac{1}{3}\delta_{ij} \sum_s |\mathbf{d}_s|^2 e^{i\omega_{es}\tau}$ , the sum being performed over all possible internal atomic states  $s$  with the Bohr frequencies

$\omega_{es} = \frac{1}{\hbar}(E_e - E_s)$  and  $\mathbf{d}_s = \langle e|\mathbf{d}(0)|s\rangle$ . The vacuum electric field correlation is well known [41]:  $\langle 0|E_i(\mathbf{r}, t)E_j(\mathbf{r}', t')|0\rangle = \int d^3\mathbf{k} \sum_{\lambda} \left(\frac{\hbar\omega}{2\pi^2\hbar}\right) e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')-i\omega(t-t')} (\boldsymbol{\epsilon}_{\mathbf{k}\lambda})_i (\boldsymbol{\epsilon}_{\mathbf{k}\lambda})_j$ , with  $\omega = k/c$  and  $\boldsymbol{\epsilon}_{\mathbf{k}\lambda}$  representing a transverse unit vector associated with polarization  $\lambda$ . The decoherence rates read ( $\alpha = L, NL$ )

$$\Gamma_{\alpha} = -\frac{1}{3} \sum_s |\mathbf{d}_s|^2 \int d^3\mathbf{k} \left(\frac{\omega}{2\pi^2\hbar}\right) \int_{-\infty}^{+\infty} d\tau e^{i(\omega_{es}-\omega)\tau} \xi_{\alpha}, \quad (19)$$

where  $\xi_L = 1$  and  $\xi_{NL} = -e^{i\mathbf{k}\cdot\mathbf{a}}$ .

As one may expect intuitively, the decoherence rates are closely related to the spontaneous emission rate  $\gamma = \frac{1}{\Delta t} \sum_s \int d^3\mathbf{k} |(\mathbf{1}_{\mathbf{k}} \otimes \langle s| - \frac{i}{\hbar} \int_0^{\Delta t} dt V(\mathbf{r}, t)|e\rangle \otimes |0\rangle|^2$ . From Eq. (19), one derives that the local decoherence rate is indeed the half-sum of the spontaneous emission rates in each well. Since those are identical and independent of the center-of-mass position, one finds  $\Gamma_L = \gamma$  for the local decoherence rate. The nonlocal decoherence rate is obtained from Eq. (19) as

$$\Gamma_{NL} = -\sum_s \Gamma_{es} \operatorname{sinc}\left(2\pi \frac{a}{\lambda_{es}}\right), \quad (20)$$

where the  $\operatorname{sinc}(x) = \sin(x)/x$  and  $\Gamma_{es} = \omega_{es}^3 |\mathbf{d}_s|^2 / (6\pi\hbar c^2)$  represents the spontaneous transition rate from level  $e$  to level  $s$ , associated with the transition wavelength  $\lambda_{es} = 2\pi c/\omega_{es}$ . In Fig. 2 we plot the total decoherence rate  $\Gamma = \Gamma_L + \Gamma_{NL}$  as a function of the distance  $a$ , for the case of a two-level atom (in this case  $\gamma = \Gamma_{es}$ ).

The local and nonlocal decoherence rates only differ through the presence of the interference factor  $\xi_{NL} = -e^{i\mathbf{k}\cdot\mathbf{a}}$  within the integral (19) for the latter, which originates from the two separate positions  $\mathbf{r}_{\pm} = \pm \frac{1}{2}\mathbf{a}$  taken on the right-hand side of Eq. (6) [see also Fig. 1(b)]. If the well separation is such that  $a \ll \min_s\{\lambda_{es}\}$ , the interference factor is then  $\xi_{NL} \approx -\xi_L = -1$ , and the nonlocal recoherence effect cancels the local decoherence, as illustrated by Fig. 2. Physically, the wavelength of the photons emitted by the excited atom is too large for the two wells to be resolved within the diffraction limit. Therefore, an observer detecting a spontaneously emitted photon could not tell from which well the radiation was emitted, and this lack of information is at the origin of the nonlocal recoherence.

In the opposite limit  $a \gg \max_s\{\lambda_{es}\}$ , the nonlocal rate  $\Gamma_{NL}$  becomes negligible since it is given by the integration of a fast oscillating complex exponential. Thus, the total decoherence rate saturates in the long-distance limit to the value fixed by the spontaneous emission rate, which is represented by a horizontal dashed line in Fig. 2. In this regime, increasing the distance  $a$  between the wells will not improve the which-path information contained in each emitted photon, which already reveals almost certainly which well is occupied by the atom (see also Ref. [42] for a related discussion). Alternatively, this can also be interpreted as follows: in the regime  $a \gg \max_s\{\lambda_{es}\}$ , we have  $a/c \gg 1/\omega_{\max}$  ( $\omega_{\max}$  representing the largest Bohr frequency), so the time-of-flight between wells is much longer than the characteristic time of the atomic dipole correlation function, and then the coupling between forward and backward histories illustrated by Fig. 1(b) vanishes. In other words, the nonlocal decoherence rate contributes

significantly only if the two atomic wave packets are closer than a characteristic length scale given by the memory time of the environment times the propagation velocity of the interaction. In Appendix B we study another example where the quality of which-path information saturates, for the case where this information is encoded in the frequency of the emitted photon.

The most interesting regime corresponds to intermediate values for the distance between the wave packets, for which the decoherence rate displays oscillations with a period set by the transition wavelength  $\lambda_{es}$  as shown in Fig. 2. Those oscillations are in contradiction with the naive expectation that the decoherence rate should be a monotonic increasing function of the separation. They may be interpreted in terms of interference effects from photons coproduced by each atomic wave packet in the superposition. For certain distances, decoherence can be faster than in the case of infinite separation, where the which-way information would in principle be the most accurate. This is in close analogy with the behavior of the photon emission rate in the case of superradiance [8] with two atoms separated by a distance smaller or of the order of the transition wavelength. As in the problem of superradiance, the oscillation amplitude decays as the inverse of the distance between the wave packets. Intuitively, this power law results from the electromagnetic field Green's function connecting the two separated wave packets (nonlocal contribution), as illustrated in Fig. 1(b) and indicated in Eqs. (8) and (9). Hence, the decoherence rate saturates to the value defined by the local contribution  $\Gamma_L = \Gamma_{es}$  in the long distance limit.

Differently from the case of superradiance, in our single-atom effect we have interference of quantum amplitudes associated with information on the atomic position, but not in the photon emission rate. This is consistent with previous works [43–45] showing that the shape of the external wave function of a single atom does not influence its total spontaneous emission rate, even though it may change the emission spectrum. The oscillation of the decoherence rate, combined with a fixed spontaneous emission rate, is indeed to be interpreted as an oscillation of the quantity of which-way information contained in each emitted photon. A possible definition of this quantity would be the ratio  $\Gamma/\Gamma_L$  between the total decoherence and the spontaneous emission rates shown in Fig. 2. Photons emitted by wave packets located far apart carry a which-way information equal to unity, while those emitted by less distant wave packets may have a which-way information enhanced or suppressed by interference effects.

## VI. CONCLUSION

In conclusion, we have shown that quantum interferences may enhance or reduce the decoherence of the center-of-mass position of a single excited atom. As a consequence, this decoherence can be fastest at a finite separation. The resulting oscillations suggest an analogy with superradiance, with the photon emission rate replaced by the decoherence rate. Instead of multiple atoms, here we have multiple wave packets giving rise to different quantum paths. This enhancement of decoherence does not result from an increased spontaneous emission rate, but rather from interferences involving the quantum states of the environment associated with distinct paths.

The quantum interferences observed in the decoherence rate are entirely related to the contribution of the nonlocal functional, which results from the coupling between forward and backward histories. This contribution, often neglected in the dynamics of open quantum systems, captures the quality of the which-way information flowing from the system into the environment. It is actually essential to estimate the decoherence of a system interacting with a non-Markovian environment.

## ACKNOWLEDGMENTS

The authors are grateful to Robin Kaiser, Ryan Behunin, and Claudio Ccapa-Ttira for enlightening suggestions and discussions. This work was supported by the Brazilian agencies CNPq and FAPERJ.

## APPENDIX A: EXPLICIT DERIVATION OF THE INFLUENCE FUNCTIONAL

In this Appendix we present the CTP derivation of Eqs. (5) and (6) for the local and nonlocal influence functionals, respectively. For the sake of clarity, from now on we identify the operators with a hat. To simplify the notation, we start with a simple system composed only of two d.o.f.  $X$  and  $q$ . For convenience, we recall the formal expression of the influence functional, obtained by integrating over the d.o.f.  $X$  [17,18,25]:

$$e^{\frac{i}{\hbar}S_{\text{IF}}[q,q']} = \int dX_f \int_{\text{CTP}}^{X_f, X_f} \mathcal{D}X e^{\frac{i}{\hbar}(S_X[X] - S_X[X'])} \times e^{\frac{i}{\hbar}(S_{\text{int}}[X,q] - S_{\text{int}}[X',q'])}, \quad (\text{A1})$$

where  $S_X[X]$  stands for the free action involving the d.o.f.  $X$  only and  $S_{\text{int}}[X,q]$  is the interaction term, which couples  $X$  and  $q$ . We require the interaction to be local:  $S_{\text{int}}[X,q] = \int_0^T dt V(X(t), q(t))$ , where  $V$  is a generic potential. This action is identical to (4) except for the position dependence of the potential, which we omit for the time being to render the presentation simpler.

We also remind the definition of the CTP integration (2)

$$\int_{\text{CTP}}^{X_f, X_f} \mathcal{D}X = \int dX_0 dX'_0 \int_{X(0)=X_0}^{X(\Delta t)=X_f} \mathcal{D}X \times \int_{X'(0)=X'_0}^{X'(\Delta t)=X'_f} \mathcal{D}X' \langle X_0 | \rho(0) | X'_0 \rangle. \quad (\text{A2})$$

We assume that the coupling can be treated perturbatively, and that the leading order term is of second order. The influence functionals are then written as

$$S_{\text{L}}^{\text{IF}}[q,q'] = \frac{1}{4\hbar} \int dX_f \int_{\text{CTP}}^{X_f, X_f} \mathcal{D}X e^{\frac{i}{\hbar}(S_X[X] - S_X[X'])} \times (S_{\text{int}}^2[X,q] + S_{\text{int}}^2[X',q']), \quad (\text{A3})$$

$$S_{\text{NL}}^{\text{IF}}[q,q'] = -\frac{1}{2\hbar} \int dX_f \int_{\text{CTP}}^{X_f, X_f} \mathcal{D}X e^{\frac{i}{\hbar}(S_X[X] - S_X[X'])} \times (S_{\text{int}}[X,q]S_{\text{int}}[X',q']). \quad (\text{A4})$$

Let us develop in detail the expression of the nonlocal influence functional (the local one can be obtained following similar steps). A key point is that the integrals (A3) can be performed separately on the paths  $X$  and  $X'$ . For example, we perform the integral over the path  $X$ :

$$\mathcal{I}[q] = \int_{X_0}^{X_f} \mathcal{D}X e^{\frac{i}{\hbar} S_X[X]} S_{\text{int}}[X, q]. \quad (\text{A5})$$

We may write (A5) as

$$\begin{aligned} \mathcal{I}[q] &= \int_0^{\Delta t} dt \int dX(t) \left( \int_{X_0}^{X(t)} \mathcal{D}X e^{\frac{i}{\hbar} S_X[X]} \right) V(X(t), q(t)) \\ &\times \left( \int_{X(t)}^{X_f} \mathcal{D}X e^{\frac{i}{\hbar} S_X[X]} \right). \end{aligned} \quad (\text{A6})$$

We now proceed to evaluate this quantity with the help of the propagator

$$\langle X_2 | \hat{U}(t, t') | X_1 \rangle = \int_{X(t')=X_1}^{X(t)=X_2} \mathcal{D}X e^{\frac{i}{\hbar} S_X[X]}, \quad (\text{A7})$$

where  $\hat{U}(t, t')$  is the evolution operator. As in Ref. [18], we note that the potential  $V(X(t), q(t))$  is an eigenvalue associated with the vector  $|X(t)\rangle$ . We write

$$\begin{aligned} \langle X_f | \hat{U}(\Delta t, t) | X(t) \rangle V(X(t), q(t)) \\ = \langle X_f | \hat{U}(\Delta t, t) V(\hat{X}, q(t)) | X(t) \rangle, \end{aligned} \quad (\text{A8})$$

so that one obtains

$$\mathcal{I}[q] = \int_0^{\Delta t} dt \langle X_f | \hat{U}(\Delta t, t) V(\hat{X}, q(t)) \hat{U}(t, 0) | X_0 \rangle. \quad (\text{A9})$$

The path integral over  $X'$ ,

$$\mathcal{I}'[q'] = \int_{X'_0}^{X'_f} \mathcal{D}X' e^{-\frac{i}{\hbar} S_{X'}[X']} S_{\text{int}}[X', q'] \quad (\text{A10})$$

can be expressed similarly as

$$\mathcal{I}'[q'] = \int_0^{\Delta t} dt' \langle X'_0 | \hat{U}(0, t') V(\hat{X}', q(t')) \hat{U}(t', \Delta t) | X'_f \rangle. \quad (\text{A11})$$

Note the backward evolution, which results from taking the action with the  $-1$  prefactor in (A10). Using the expressions (A9) and (A11) for  $\mathcal{I}[q]$  and  $\mathcal{I}'[q']$  as well as the completeness relation, one obtains

$$\begin{aligned} S_{\text{NL}} &= -\frac{i}{\hbar} \int_0^{\Delta t} dt \int_0^{\Delta t} dt' \int dX_f \\ &\times \langle X_f | V(\hat{X}(t'), q(t')) V(\hat{X}(t), q(t)) \hat{\rho}(0) | X_f \rangle \\ &= -\frac{i}{\hbar} \int_0^{\Delta t} dt \int_0^{\Delta t} dt' \langle V(\hat{X}(t'), t') V(\hat{X}(t), t) \rangle. \end{aligned}$$

We have introduced the interaction picture operators  $\hat{X}(t) = U(0, t) \hat{X} U(t, 0)$  and used the cyclic property of the trace as well as the group property of the evolution operator  $\hat{U}(t_3, t_2) \hat{U}(t_2, t_1) = \hat{U}(t_3, t_1)$ .

Finally, we now include the possibility of  $X$  to be a field so that the interaction of  $X$  with  $q$  becomes dependent of the center-of-mass position of the system and given by (4) in Sec. III. We assume the wave packet width to be negligible

(for a detailed account of the width see Ref. [22]). In this case, the derivation provided above remains essentially the same, except for the replacement of  $\hat{X}(t)$  and  $\hat{X}'(t')$  by  $\hat{X}(\mathbf{r}_1(t), t)$  and  $\hat{X}'(\mathbf{r}_2(t'), t')$ , respectively. With such replacements, one obtains a functional depending on two paths, i.e., Eq. (A11) becomes

$$S_{\text{NL}}[\mathbf{r}_1(t), \mathbf{r}_2(t)] = -\frac{i}{\hbar} \int_0^{\Delta t} \int_0^{\Delta t} dt dt' \langle V(\hat{X}(\mathbf{r}_2(t'))) V(\hat{X}(\mathbf{r}_1(t))) \rangle$$

with the four-vector notation  $r_k(t) = (\mathbf{r}_k(t), t)$ . This form is equivalent to Eq. (5). The local influence functional can be derived by following the same steps.

## APPENDIX B: QUALITY OF INFORMATION IN THE TIME DOMAIN

In this Appendix we discuss another example showing that the nonlocal decoherence contribution represents the quality of which-way information. We consider the evolution of the system during a finite time  $\Delta t$  much greater than the decay time of the dipole and field correlations. One expects that only nearly resonant processes may contribute significantly to the decoherence rate over the considered time scale  $\Delta t$ . This corresponds to the emission of photons with frequencies in the vicinity of the atomic Bohr frequencies  $\omega_{es}$ . Below we show that a small mismatch of the Bohr frequencies  $\omega_{es}$  in the two wells affects the local and nonlocal decoherence rates in very different ways.

From now on, we assume that the atomic levels are light shifted differently in each well, yielding the distinct Bohr frequencies  $\omega_{es}^{\pm}$ . To simplify the discussion, we assume that the values of the atomic transition matrix elements  $|\mathbf{d}_s|^2$  are not significantly modified. To obtain the local and nonlocal decoherence rates, one uses again Eqs. (14), (16), and (17).

The local decoherence rate involves two successive transitions through intermediate states of identical energies. One obtains  $\Gamma_L = \frac{1}{2}(\Gamma_L^+ + \Gamma_L^-)$ , where the rates  $\Gamma_L^+$  and  $\Gamma_L^-$  correspond to the local rate  $\Gamma_L$  as given by Eq. (19) up to a change of the Bohr frequencies from  $\omega_{es}$  to  $\omega_{es}^+$  and  $\omega_{es}^-$ , respectively. Typically, the change of the local decoherence rate is extremely small. In contrast, the nonlocal decoherence rate involves transitions through atomic states dressed differently by the light field. Precisely, the nonlocal decoherence rate involves the integration of oscillating functions

$$\begin{aligned} \Gamma_{\text{NL}} &= -\frac{1}{3\Delta t} \sum_s |\mathbf{d}_s|^2 \int d^3\mathbf{k} \left( \frac{\omega}{2\pi^2\hbar} \right) e^{i\mathbf{k}\cdot\mathbf{a}} \\ &\times \int_0^{\Delta t} dt_m e^{-i(\omega_{es}^+ - \omega_{es}^-)t_m} \int_{2t_m - 2\Delta t}^{2t_m} d\tau e^{-\frac{i}{2}(\omega_{es}^+ + \omega_{es}^-)\tau}. \end{aligned} \quad (\text{B1})$$

where we introduced the time variables  $t_m = \frac{1}{2}(t + t')$  and  $\tau = t - t'$ . The integral over  $\tau$  can be extended to infinity and yields, together with the integral over  $\mathbf{k}$ , a finite result. The integral over the time  $t_m$  fails to provide a term proportional to  $\Delta t$  because of the oscillatory complex exponential arising from the light shift mismatch. One then finds that the nonlocal decoherence rate is significant only for short monitoring times or small frequency mismatches,  $|\omega_{es}^+ - \omega_{es}^-| \lesssim 1/\Delta t$ .

Again, we may interpret this result in terms of quality of information. We assume here that the two wells are close  $a/\lambda_{es} \ll 1$  for all levels  $s$ , so that which-path information can only be stored in the frequency value  $\omega_{es}^+$  or  $\omega_{es}^-$ . By monitoring the system during  $\Delta t$ , the frequency resolution is limited by  $\delta\omega \simeq 1/\Delta t$ . If  $|\omega_{es}^+ - \omega_{es}^-| \lesssim 1/\Delta t$ , an observer detecting a photon emitted from one of the wells would not be able to resolve the frequency with the accuracy needed to determine the location of the emitting atom. Thus, the emission process carries which-way information of low quality in this case. Consistently with our previous interpretation,

the nonlocal decoherence term is significant and of negative sign, thus providing recoherence to the system. On the other hand, if  $|\omega_{es}^+ - \omega_{es}^-| \gg 1/\Delta t$ , the nonlocal decoherence rate associated with the level  $s$  vanishes. In this limit, the detection of a photon emitted from a transition  $e \rightarrow s$  provides perfect information on the atomic position given the frequency resolution achievable during the monitoring time  $\Delta t$ . This is another example showing that the presence of a nonlocal stationary recoherence rate is tied to imperfections in the which-way information gained through the interaction process.

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- [1] W. H. Zurek, *Rev. Mod. Phys.* **75**, 715 (2003).
- [2] H.-P. Breuer, E.-M. Laine, and J. Piilo, *Phys. Rev. Lett.* **103**, 210401 (2009).
- [3] H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, *Rev. Mod. Phys.* **88**, 021002 (2016).
- [4] F. C. Lombardo and P. I. Villar, *Phys. Rev. A* **91**, 042111 (2015).
- [5] S. Groeblacher, A. Trubarov, N. Prigge, G. D. Cole, M. Aspelmeyer, and J. Eisert, *Nat. Commun.* **6**, 7606 (2015).
- [6] M. Genkin, D. W. Schönleber, S. Wüster, and A. Eisfeld, *J. Phys. B* **49**, 134001 (2016).
- [7] V. Semin and F. Petruccione, *Europhys. Lett.* **113**, 20004 (2016).
- [8] R. H. Dicke, *Phys. Rev.* **93**, 99 (1954).
- [9] R. Bachelard, H. Bender, P. W. Courteille, N. Piovella, C. Stehle, C. Zimmermann, and S. Slama, *Phys. Rev. A* **86**, 043605 (2012).
- [10] L. Ostermann, H. Zoubi, and H. Ritsch, *Opt. Express* **20**, 29634 (2012).
- [11] R. A. de Oliveira, M. S. Mendes, W. S. Martins, P. L. Saldanha, J. W. R. Tabosa, and D. Felinto, *Phys. Rev. A* **90**, 023848 (2014).
- [12] D.-W. Wang, R.-B. Liu, S.-Y. Zhu, and M. O. Scully, *Phys. Rev. Lett.* **114**, 043602 (2015).
- [13] M. O. Araújo, I. Krešić, R. Kaiser, and W. Guerin, *Phys. Rev. Lett.* **117**, 073002 (2016).
- [14] P. Tighineanu, R. S. Daveau, T. B. Lehmann, H. E. Beere, D. A. Ritchie, P. Lodahl, and S. Stobbe, *Phys. Rev. Lett.* **116**, 163604 (2016).
- [15] D. Bhatti, S. Oettel, R. Wiegner, G. S. Agarwal, and J. von Zanthier, *Phys. Rev. A* **94**, 013810 (2016).
- [16] F. Damanet, D. Braun, and J. Martin, *Phys. Rev. A* **93**, 022124 (2016).
- [17] R. P. Feynman and F. L. Vernon, *Ann. Phys. (N.Y.)* **24**, 118 (1963).
- [18] E. A. Calzetta and B.-L. Hu, *Nonequilibrium Quantum Field Theory* (Cambridge University Press, Cambridge, 2008).
- [19] R. O. Behunin and B.-L. Hu, *Phys. Rev. A* **84**, 012902 (2011).
- [20] F. Impens, R. O. Behunin, C. C. Ttira, and P. M. Neto, *Europhys. Lett.* **101**, 60006 (2013).
- [21] F. Impens, C. C. Ttira, and P. A. M. Neto, *J. Phys. B: At. Mol. Opt. Phys.* **46**, 245503 (2013).
- [22] F. Impens, C. C. Ttira, R. O. Behunin, and P. A. M. Neto, *Phys. Rev. A* **89**, 022516 (2014).
- [23] M. B. Farfás and F. C. Lombardo, *Phys. Rev. D* **93**, 065035 (2016).
- [24] F. D. Mazzitelli, J. P. Paz, and A. Villanueva, *Phys. Rev. A* **68**, 062106 (2003).
- [25] K. A. Milton, *Schwinger's Quantum Action Principle* (Springer, Berlin, 2015).
- [26] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981).
- [27] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2007).
- [28] Note that, strictly speaking, in this limit the phase associated with the irreducible functional loses its nonlocal character as the wave packets merge when the two paths  $[\mathbf{r}_1(t), \mathbf{r}_2(t)]$  become identical.
- [29] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions: Basic Processes and Applications* (Wiley, New York, 1998).
- [30] Such a condition is met, for instance, if one takes for the potential  $V(\mathbf{r})$  the dipolar interaction, and if the atoms are initially in an unpolarized quantum state or in the presence of the vacuum field.
- [31] A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, *Rev. Mod. Phys.* **81**, 1051 (2009).
- [32] A. Stern, Y. Aharonov, and Y. Imry, *Phys. Rev. A* **41**, 3436 (1990).
- [33] The prefactor  $\frac{1}{2}$  in (14) results from taking identical amplitudes for the two system states  $|\psi_1(0)\rangle$  and  $|\psi_2(0)\rangle$  in the initial product state (10).
- [34] H.-P. Breuer and F. Petruccione, *Phys. Rev. A* **63**, 032102 (2001).
- [35] Y. Shin, M. Saba, T. A. Pasquini, W. Ketterle, D. E. Pritchard, and A. E. Leanhardt, *Phys. Rev. Lett.* **92**, 050405 (2004).
- [36] T. Schumm, S. Hofferberth, L. M. Andersson, S. Wildermuth, S. Groth, I. Bar-Joseph, J. Schmiedmayer, and P. Krüger, *Nat. Phys.* **1**, 57 (2005).
- [37] M. S. Chapman, T. D. Hammond, A. Lenef, J. Schmiedmayer, R. A. Rubenstein, E. Smith, and D. E. Pritchard, *Phys. Rev. Lett.* **75**, 3783 (1995).
- [38] T. Yu and J. H. Eberly, *Phys. Rev. Lett.* **93**, 140404 (2004).
- [39] Z. Ficek and R. Tanaś, *Phys. Rev. A* **77**, 054301 (2008).
- [40] O. Steuernagel and H. Paul, *Phys. Rev. A* **52**, R905(R) (1995).
- [41] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, Oxford, 1954).
- [42] J. R. Anglin, J. P. Paz, and W. H. Zurek, *Phys. Rev. A* **55**, 4041 (1997).
- [43] K. Rzazewski and W. Zakowicz, *J. Phys. B* **25**, L319 (1992).
- [44] M. V. Fedorov, M. A. Efremov, A. E. Kazakov, K. W. Chan, C. K. Law, and J. H. Eberly, *Phys. Rev. A* **72**, 032110 (2005).
- [45] J. Robert, C. Miniatura, O. Gorceix, S. L. Boiteux, V. Lorent, J. Reinhardt, and J. Baudon, *J. Phys. II France* **2**, 601 (1992).