# Imbalance of group velocities for amplitude and phase pulses propagating in a resonant atomic medium

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The dynamics of light pulses with amplitude and phase modulations is investigated for a medium of resonant two-level atoms. It is shown that the pulse-like variations of the phase can be also described in terms of group velocity. It is found that in the nonlinear regime of atom-field interaction, the group velocities of amplitude and phase pulses can have a large imbalance. Namely, amplitude pulses travel at a velocity less than $c$, whereas the group velocity of phase pulses is greater than the velocity of light in free space or it is even negative. The predicted imbalance of the group velocities can be important in the case of chirped pulses propagating in a resonant medium.

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## I. INTRODUCTION

Investigation of the propagation of light pulses in the atomic medium is a fundamental problem in optics. The medium has the greatest effect on the dynamics of pulses under the conditions of resonant interaction. In particular, the resonant response of the atomic medium is accompanied by strong frequency (temporal) dispersion, which leads to a significant modification of the group velocity of pulses [1–8]. For example, large slowing of light pulses can take place [9–15] or, on the contrary, their group velocity can exceed the light velocity in free space (or even become negative) [16–23].

At present, chirped pulses [i.e., pulses with simultaneous amplitude and phase (frequency) modulation] find wide application [24], including the following: the optical frequency-domain reflectometry [25,26], Fourier-transform microwave spectroscopy [27–29], coherent control of population transfer [30–32], and so on. It is well known [4,8] that for a medium of resonant two-level atoms in the linear regime, the group velocities of amplitude and phase modulations are equal (negative or greater than the light speed in free space $c$); therefore, a chirped pulse propagates as a whole. However, in the nonlinear regime, the problem of the dynamics of chirped pulses has been insufficiently studied. On the one hand, the mathematical description becomes more complicated in comparison with the linear regime and often it requires the development of nonstandard approaches [33–35]. On the other hand, the nonlinear dynamics is more interesting from the viewpoint of the appearance of new effects, for example, self-phase modulation, four-wave mixing, self-induced transparency, soliton regime, and so on [2,36,37].

In this paper, the propagation of amplitude and phase pulses is investigated in a medium of resonant two-level atoms in the nonlinear regime. For clarity, we assume that the amplitude and phase pulses correspond to variations of the field amplitude or phase in the pulse shape. We have found that in the nonlinear regime, the group velocities of amplitude and phase pulses can be significantly different. Namely, the amplitude pulses propagate with slowing down, whereas the phase pulses move with acceleration. Such a difference in the group velocities results in a shift of the amplitude modulation with respect to the phase modulation. This imbalance of the group velocities is predicted by analyzing the coefficients of a reduced Maxwell equation derived in the adiabatic approximation and is confirmed by numerically solving the Maxwell-Bloch equations.

## II. BASIC EQUATIONS

Consider the propagation of a running electromagnetic wave

$$E = \widetilde{E}(t,z)\, e^{-i(\omega t - kz)} + \text{c.c.} \tag{1}$$

through a gas of two-level atoms (see Fig. 1), where $\widetilde{E}(t,z)$ is the slowly varying complex amplitude of the field, $\omega$ is the carrier frequency, and $k = \omega/c$ is the wave number. For simplicity, we use a model of motionless atoms, which is valid, for instance, in the case of atomic cells with a buffer gas. The single-atom density matrix $\hat{\rho}$ satisfies the following operator equation:

$$\frac{\partial}{\partial t}\hat{\rho} + \hat{\Gamma}\{\hat{\rho}\} = -\frac{i}{\hbar}[\hat{H}_0,\hat{\rho}] - \frac{i}{\hbar}[\hat{V},\hat{\rho}], \tag{2}$$

where $\hat{H}_0$ is the Hamiltonian of an unperturbed atom, the operator $\hat{V} = -(\hat{d}\, E)$ describes the interaction of the atom with the optical field in the electric-dipole approximation ($\hat{d}$ is the operator of the electric dipole moment), and the operator $\hat{\Gamma}\{\hat{\rho}\}$ describes the relaxation processes. In the rotating-wave approximation, $\hat{V}$ is given by

$$\hat{V} = -d_{21}\widetilde{E}\, e^{-i(\omega t - kz)}|2\rangle\langle 1| + \text{H.c.}, \tag{3}$$

where $d_{21} = \langle 2|\hat{d}|1\rangle$ is a dipole matrix element. The fast oscillating dependencies in Eq. (2) can be eliminated by

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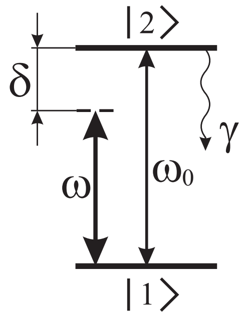


FIG. 1. Two-level atomic resonance. Field of frequency $\omega$ excites the transition $|1\rangle \to |2\rangle$ with resonance frequency $\omega_0$, $\gamma$ is the decay rate of the excited state.

following transformation:

$$\rho_{21} = \tilde{\rho}_{21}\, e^{-i(\omega t - kz)}, \quad \rho_{12} = \tilde{\rho}_{12}\, e^{i(\omega t - kz)}. \tag{4}$$

Then the time evolution of the density matrix is described by the optical Bloch equations

$$\frac{\partial \rho_{11}}{\partial t} = \gamma \rho_{22} - i\widetilde{\Omega}\tilde{\rho}_{12} + i\widetilde{\Omega}^{*}\tilde{\rho}_{21}, \tag{5}$$

$$\frac{\partial \tilde{\rho}_{12}}{\partial t} = -(\Gamma + i\delta)\tilde{\rho}_{12} - i(\rho_{11} - \rho_{22})\widetilde{\Omega}^{*}, \tag{6}$$

$$\frac{\partial \tilde{\rho}_{21}}{\partial t} = -(\Gamma - i\delta)\tilde{\rho}_{21} + i(\rho_{11} - \rho_{22})\widetilde{\Omega}, \tag{7}$$

$$\frac{\partial \rho_{22}}{\partial t} = -\gamma \rho_{22} + i\widetilde{\Omega}\tilde{\rho}_{12} - i\widetilde{\Omega}^{*}\tilde{\rho}_{21}, \tag{8}$$

where $\gamma$ is the decay rate of the excited state $|2\rangle$, $\Gamma$ is the coherence decay rate, $\delta = (\omega - \omega_0)$ is the detuning of the field frequency from the atomic resonance frequency, and $\widetilde{\Omega}$ is the complex Rabi frequency

$$\widetilde{\Omega}(t,z) = \frac{d_{21}\widetilde{E}(t,z)}{\hbar}. \tag{9}$$

Equations (5)–(7) correspond to the closed atomic transition $|1\rangle \to |2\rangle$, and the normalization condition for the populations is satisfied:

$$\mathrm{Tr}\{\hat{\rho}\} = \rho_{11} + \rho_{22} = 1. \tag{10}$$

To solve Eqs. (5)–(8) we will use the adiabatic approach, which has been developed in Ref. [38]. The essence of this approach is the following: the solution for the density matrix $\hat{\rho}$ can be constructed in the form of a series in the time derivatives $\partial/\partial t$:

$$\hat{\rho} = \hat{\rho}^{(0)} + \hat{\rho}^{(1)} + \hat{\rho}^{(2)} + \cdots, \tag{11}$$

where $\hat{\rho}^{(k)}$ has $k$th order of $\partial/\partial t$ (i.e., $\partial^k/\partial t^k$). Indeed, let us write Eq. (2) in symbolic form:

$$\frac{\partial \hat{\rho}}{\partial t} = \hat{L}\{\hat{\rho}\}, \tag{12}$$

where $\hat{L}\{\hat{\rho}\}$ is the linear operator functional. Then the terms of series (11) can be found from the recurrent equations

$$\hat{L}\{\hat{\rho}^{(0)}\} = 0, \tag{13}$$

$$\hat{L}\{\hat{\rho}^{(k)}\} = \frac{\partial}{\partial t}\hat{\rho}^{(k-1)} \quad (k \geqslant 1). \tag{14}$$

The normalization condition for $\hat{\rho}^{(k)}$ has the following form:

$$\mathrm{Tr}\{\hat{\rho}^{(0)}\} = 1, \tag{15}$$

$$\mathrm{Tr}\{\hat{\rho}^{(k)}\} = 0 \quad (k \geqslant 1). \tag{16}$$

For calculations we will employ the described algorithm for Eqs. (5)–(8), which can be rewritten in vector form

$$\frac{\partial \vec{\rho}}{\partial t} = \hat{\mathcal{L}}\vec{\rho}, \tag{17}$$

where the column vector $\vec{\rho}$ we construct from elements of density matrix $\hat{\rho}$ as follows:

$$\vec{\rho} = \begin{pmatrix} \rho_{11} \\ \tilde{\rho}_{12} \\ \tilde{\rho}_{21} \\ \rho_{22} \end{pmatrix}, \tag{18}$$

and matrix $\hat{\mathcal{L}}$ has the form

$$\hat{\mathcal{L}} = \begin{pmatrix} 0 & -i\widetilde{\Omega} & i\widetilde{\Omega}^{*} & \gamma \\ -i\widetilde{\Omega}^{*} & -\Gamma - i\delta & 0 & i\widetilde{\Omega}^{*} \\ i\widetilde{\Omega} & 0 & -\Gamma + i\delta & -i\widetilde{\Omega} \\ 0 & i\widetilde{\Omega} & -i\widetilde{\Omega}^{*} & -\gamma \end{pmatrix}. \tag{19}$$

In accordance with Eq. (11), the expansion for vector $\vec{\rho}$ by powers of derivatives $\partial/\partial t$ has the following form:

$$\vec{\rho} = \vec{\rho}^{\,(0)} + \vec{\rho}^{\,(1)} + \vec{\rho}^{\,(2)} + \cdots. \tag{20}$$

The expression for atomic polarization $P(t,z)$ can be obtained from the definition as the average dipole moment per unit volume:

$$P(t,z) = N_a\,\mathrm{Tr}\{\hat{d}\,\hat{\rho}\} = \tilde{P}(t,z)\,e^{-i(\omega t - kz)} + \text{c.c.}, \tag{21}$$

where $N_a$ is the atomic density, and $\tilde{P}(t,z)$ is defined as

$$\tilde{P}(t,z) = N_a d_{12}\tilde{\rho}_{21}. \tag{22}$$

Using expressions (11) and (22), we obtain the corresponding expansion for $\tilde{P}(t,z)$:

$$\tilde{P}(t,z) = \sum_{k=0}^{\infty} \tilde{P}^{(k)}, \quad \tilde{P}^{(k)} = N_a d_{12}\tilde{\rho}_{21}^{(k)}. \tag{23}$$

We assume that the complex field amplitude $\widetilde{E}(t,z)$ varies rather slowly, and for $\tilde{P}^{(k)}$ $(k \geqslant 1)$ the adiabatic conditions

$$\begin{aligned} \mathrm{Re}\{\tilde{P}^{(k+1)}\} &\ll \mathrm{Re}\{\tilde{P}^{(k)}\}, \\ \mathrm{Im}\{\tilde{P}^{(k+1)}\} &\ll \mathrm{Im}\{\tilde{P}^{(k)}\}, \end{aligned} \tag{24}$$

hold. In this case, it is quite enough to take into account only the first two terms of the expansion (20), which can be found from equations

$$\hat{\mathcal{L}}\vec{\rho}^{\,(0)} = 0, \tag{25}$$

$$\hat{\mathcal{L}}\vec{\rho}^{\,(1)} = \frac{\partial}{\partial t}\vec{\rho}^{\,(0)}, \tag{26}$$

using the following normalization conditions:

$$\rho_{11}^{(0)} + \rho_{22}^{(0)} = 1, \tag{27}$$

$$\rho_{11}^{(1)} + \rho_{22}^{(1)} = 0. \tag{28}$$

## III. PROPAGATION OF AMPLITUDE AND PHASE PULSES: ADIABATIC APPROACH

In the slowly-varying-envelope approximation (i.e., when the temporal and spatial scale of the envelope variation is much greater than the period and the wavelength of the carrier) [39], the propagation of light pulses is described by the reduced Maxwell equation

$$\frac{\partial \tilde{E}}{\partial z} + \frac{1}{c}\frac{\partial \tilde{E}}{\partial t} = 2i\pi k \tilde{P}. \tag{29}$$

Using formulas (9) and (22), the last equation can be rewritten in the following form:

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\widetilde{\Omega}(t,z) = i\frac{2\pi N_a \omega |d_{21}|^2}{\hbar c}\tilde{\rho}_{21}. \tag{30}$$

Let us express the complex Rabi frequency $\widetilde{\Omega}$ in terms of the real amplitude $\Omega$ and phase $\varphi$:

$$\widetilde{\Omega}(t,z) = \Omega(t,z)e^{i\varphi(t,z)}. \tag{31}$$

Substituting the solution of Eqs. (25) and (26) for the density matrix into Eq. (30) for the complex Rabi frequency and equating the real and imaginary parts to zero, we obtain

$$\frac{\partial \Omega^2}{\partial z} + \frac{1}{c}(1+\beta a_\Omega)\frac{\partial \Omega^2}{\partial t} + \frac{\beta}{c}a_0\Omega^2 + \frac{\beta}{c}a_\varphi\frac{\partial \varphi}{\partial t} = 0, \tag{32}$$

$$\frac{\partial \varphi}{\partial z} + \frac{1}{c}(1+\beta b_\varphi)\frac{\partial \varphi}{\partial t} + \frac{\beta}{c}b_0 + \frac{\beta}{c}b_\Omega\frac{\partial \Omega^2}{\partial t} = 0, \tag{33}$$

where the following notation is used:

$$\beta = \frac{2\pi N_a}{\hbar}\omega|d_{21}|^2. \tag{34}$$

For convenience, in Eqs. (32) and (33) the squared amplitude $\Omega^2$ (which is proportional to the light intensity) is used instead of $\Omega$. The coefficients of these equations are given by

$$a_\Omega = \frac{\gamma}{[\gamma(\Gamma^2+\delta^2)+4\Gamma\Omega^2]^3}\{-\gamma^2(\Gamma^4-\delta^4) + 4\Gamma[2\Gamma(\Gamma^2+\delta^2)+\gamma(\Gamma^2-\delta^2)]\Omega^2\}, \tag{35}$$

$$a_0 = \frac{2\gamma\Gamma}{\gamma(\Gamma^2+\delta^2)+4\Gamma\Omega^2}, \tag{36}$$

$$a_\varphi = \frac{4\delta\gamma^2\Gamma\Omega^2}{[\gamma(\Gamma^2+\delta^2)+4\Gamma\Omega^2]^2}, \tag{37}$$

$$b_\varphi = -\frac{\gamma[\gamma(\Gamma^2-\delta^2)+4\Gamma\Omega^2]}{[\gamma(\Gamma^2+\delta^2)+4\Gamma\Omega^2]^2}, \tag{38}$$

$$b_0 = \frac{\delta\gamma}{\gamma(\Gamma^2+\delta^2)+4\Gamma\Omega^2}, \tag{39}$$

$$b_\Omega = \frac{1}{\Omega^2}\frac{\delta\gamma}{[\gamma(\Gamma^2+\delta^2)+4\Gamma\Omega^2]^3}\{8\Gamma\Omega^4 + 2[2\Gamma(\Gamma^2+\delta^2)+\gamma(\Gamma^2-\delta^2)]\Omega^2 - \Gamma\gamma^2(\Gamma^2+\delta^2)\}. \tag{40}$$

It can be seen from Eqs. (32) and (33) that the last terms, $a_\varphi$ and $b_\Omega$, lead to amplitude-phase cross modulation. This complicates the dynamics of propagation of both the amplitude and phase pulses. However, the situation is considerably simplified when the detuning is zero ($\delta = 0$). In this case, $b_0 = 0$ and the "engagement" coefficients $a_\varphi$ and $b_\Omega$ vanish as well. Therefore, the equations for the squared amplitude $\Omega^2$ and phase $\varphi$ become separated:

$$\frac{\partial \Omega^2}{\partial z} + \frac{1}{v_\Omega}\frac{\partial \Omega^2}{\partial t} + \frac{\beta}{c}\frac{2\gamma\Omega^2}{\gamma\Gamma+4\Omega^2} = 0, \tag{41}$$

$$\frac{\partial \varphi}{\partial z} + \frac{1}{v_\varphi}\frac{\partial \varphi}{\partial t} = 0. \tag{42}$$

Equation (42) for $\varphi$ has a typical form of the reduced Maxwell equation for well-known amplitude pulses, i.e., pulse-like variations of the phase can be also described in terms of group velocity with possibilities of slow or fast pulses, in the general case. We previously considered the phase pulses in the context of propagation of polarized pulses in the medium of degenerated two-level atoms [38] and propagation of bichromatic pulses in the medium of three-level $\Lambda$-atoms [40]. Thus, the coefficients $v_\Omega$ and $v_\varphi$ in Eqs. (41) and (42) determine the instantaneous local group velocities (i.e., at the instant of time $t$ and the coordinate $z$) of the amplitude and phase pulses, respectively:

$$v_\Omega = c\left[1 + \beta\frac{4\gamma(2\Gamma+\gamma)\Omega^2 - \gamma^3\Gamma}{(\gamma\Gamma+4\Omega^2)^3}\right]^{-1}, \tag{43}$$

$$v_\varphi = c\left[1 - \beta\frac{\gamma}{\Gamma(\gamma\Gamma+4\Omega^2)}\right]^{-1}. \tag{44}$$

Note that in the linear approximation [under the condition $\Omega^2(t,z) \ll \gamma^2/8$] we obtain the well-known expression for the group velocity [4,8]

$$v_\Omega \approx v_\varphi \approx v_g = c\left[1 - \frac{\beta}{\Gamma^2}\right]^{-1}, \tag{45}$$

i.e., the group velocities of amplitude and phase pulses become equal. Due to this the system of Eqs. (41) and (42) can be reduced to one equation for complex amplitude:

$$\frac{\partial \widetilde{\Omega}}{\partial z} + \frac{1}{v_g}\frac{\partial \widetilde{\Omega}}{\partial t} + \frac{\beta}{c}\frac{1}{\Gamma}\widetilde{\Omega} = 0. \tag{46}$$

Thus, the pulse with simultaneous amplitude and phase modulations propagates in the linear medium as a whole. The group velocity can have two types of values: those exceeding the velocity of light in free space, $v_g > c$ (for $\beta \leqslant \Gamma^2$) or those that are negative, $v_g < 0$ (for $\beta > \Gamma^2$).

However, in the nonlinear regime [under the condition $\Omega^2(t,z) \geqslant \gamma^2/8$] the situation changes radically. In this case, the group velocities of amplitude [Eq. (43)] and phase [Eq. (44)] pulses differ in magnitude. Moreover, two diametrically opposite effects take place: slowing-down of the amplitude pulses ($v_\Omega < c$) and speeding-up of the phase pulses ($v_\varphi > c$ or $v_\varphi < 0$). Thus, if at the boundary the amplitude and phase of the field are simultaneously modulated, then during propagation these pulses become shifted with respect to each other in time and space.

Equations (41) and (42) were obtained within the framework of the adiabatic approximation for the density matrix (i.e., $|\mathrm{Re}[\rho_{21}^{(2)}]| \ll |\mathrm{Re}[\rho_{21}^{(1)}]|$, $|\mathrm{Im}[\rho_{21}^{(2)}]| \ll |\mathrm{Im}[\rho_{21}^{(1)}]|$) and they adequately describe the dynamics of light pulses under the following restrictions on the rate of change of the field parameters:

$$\left|\frac{\partial^2\varphi}{\partial t^2}\right| \ll \Gamma\left|\frac{\partial\varphi}{\partial t}\right|, \tag{47a}$$

$$\left|\frac{\partial^2\Omega}{\partial t^2}\right| \ll 2\Gamma\left|\frac{\partial\Omega}{\partial t}\right| \quad \text{for} \quad \Omega^2 \gg \Gamma^2, \tag{47b}$$

$$\left|\frac{\partial^2\Omega}{\partial t^2}\right| \ll \frac{4\Omega^2}{\Gamma}\left|\frac{\partial\Omega}{\partial t}\right| \quad \text{for} \quad \gamma\Gamma \ll \Omega^2 \ll \Gamma^2, \tag{47c}$$

$$\left|\frac{\partial^2\Omega}{\partial t^2}\right| \ll \gamma\left|\frac{\partial\Omega}{\partial t}\right| \quad \text{for} \quad \Omega^2 \ll \gamma\Gamma. \tag{47d}$$

From Eqs. (47a)–(47d) we can obtain an estimate of the minimum duration of the amplitude and phase pulses:

$$\tau_\varphi \gg \frac{1}{\Gamma}, \tag{48a}$$

$$\tau_\Omega \gg \frac{1}{2\Gamma} \quad \text{for} \quad \Omega^2 \gg \Gamma^2, \tag{48b}$$

$$\tau_\Omega \gg \frac{\Gamma}{4\Omega^2} \quad \text{for} \quad \gamma\Gamma \ll \Omega^2 \ll \Gamma^2, \tag{48c}$$

$$\tau_\Omega \gg \frac{1}{\gamma} \quad \text{for} \quad \Omega^2 \ll \gamma\Gamma. \tag{48d}$$

Let us demonstrate the imbalance of the group velocities for amplitude and phase pulses. Usually the temporal variation of the field amplitude $\Omega^2(t)$ from the zero level is investigated [see Fig. 2(a)]. However, in the general case, the field amplitude

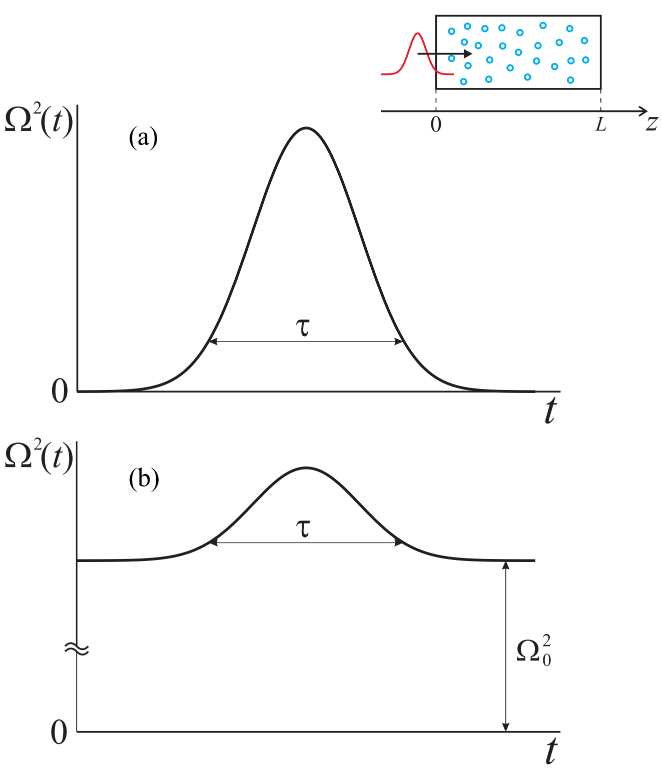


FIG. 2. Schematic illustration of two types of pulses: (a) field amplitude $\Omega^2(t)$ varies from zero, (b) field amplitude $\Omega^2(t)$ varies from the background $\Omega_0^2$. $\tau$ is the pulse duration.

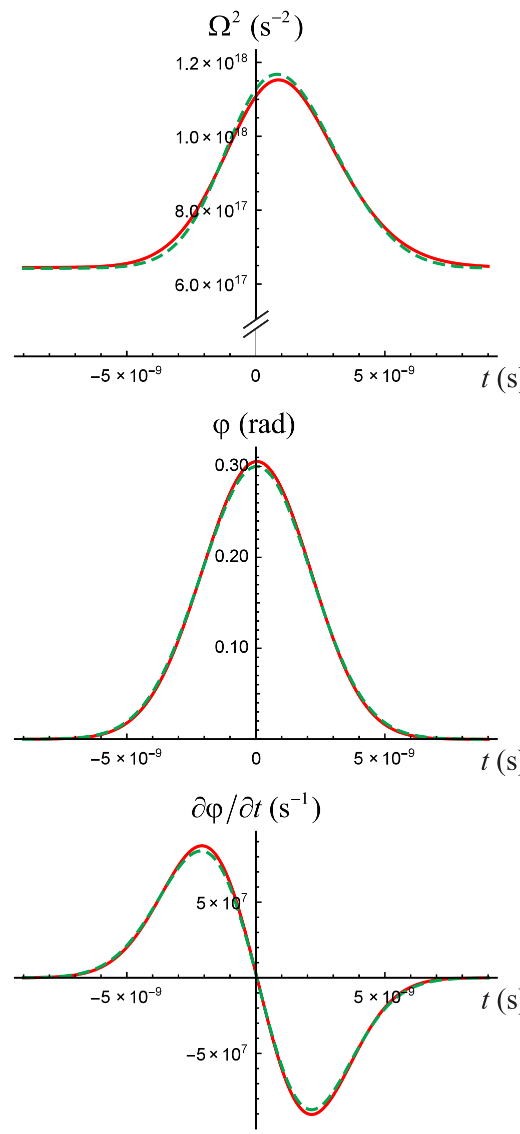


FIG. 3. Intensity $\Omega_0^2$, phase $\varphi$, and frequency $\partial\varphi/\partial t$ vs time $t$ after propagation through an atomic medium. The solution of the Maxwell-Bloch equations (5)–(7), and (30) is shown by a solid line and the solution of Eqs. (41) and (42), derived in the adiabatic approximation, is shown by a dashed line. The calculations were performed with the following parameters: atom density $N_a = 10^{12}\,\mathrm{cm}^{-3}$, spontaneous relaxation rate $\gamma = 6$ MHz, coherence decay rate $\Gamma = 500$ MHz, length of atomic medium $L = 9$ cm, and wavelength of resonant transition $\lambda = 795$ nm. The boundary conditions are specified by the formulas (49) and (50), where $\Omega_0 = 370$ MHz, $\Delta_\Omega = 0.1 \times \Omega_0^2$, $\Delta_\varphi = 0.3$, and $\tau = 3$ ns.

can be varied starting from a constant (in time) background [see Fig. 2(b)]. A strong constant component $\Omega_0^2$ provides a nonlinear response throughout the atomic medium. Consider the propagation of light pulses with a nonzero background of the amplitude through an atomic medium under the following boundary conditions

$$\Omega^2(t,z=0) = \Omega_0^2 + \Delta_\Omega\, e^{-t^2/\tau^2}, \tag{49}$$

$$\varphi(t,z=0) = \Delta_\varphi\, e^{-t^2/\tau^2}, \tag{50}$$

where $\Delta_\Omega$ and $\Delta_\varphi$ are the variations of intensity (in terms of the Rabi frequency) and phase, respectively; $\Omega_0^2$ is the stationary amplitude (background) of the input field, and $\tau$ is the duration of the pulses. The background $\Omega_0^2$ is chosen so as to provide the nonlinear regime in the atomic medium. To verify the validity of our conclusions based on the expressions for instantaneous group velocities in Eqs. (43) and (44), we compare the solution of approximate Eqs. (41) and (42) with the numerical calculations of Maxwell-Bloch equations (5)–(7) and (30). The results of the computations are shown in Fig. 3. In the plots one can see an appreciable shift of the amplitude pulse with respect to the phase pulse. Moreover, it can be seen that the adiabatic approximation is in rather good agreement with the exact solution. Some differences in the shapes of pulses are associated with the influence of higher orders of dispersion, which are not taken into account in Eqs. (41) and (42). In the considered example, the average group velocity (determined as the ratio between the length of the medium and the delay time of the pulse) can be estimated as $c/2.95$ for the amplitude pulse and $6.8c$ for the phase pulse. Thus, due to the imbalance of the group velocities, the center of the frequency chirp is displaced with respect to the center of the amplitude pulse.

## IV. CONCLUSIONS

The propagation of light pulses with amplitude and phase modulations through a two-level atomic medium under resonance conditions has been theoretically investigated. It has been shown that in the nonlinear regime of atom-field interaction, the group velocity of amplitude pulses is less than the velocity of light in free space $c$, whereas the group velocity of phase pulses exceeds $c$ or is even negative. This results in a shift (in time and space) of the amplitude modulation with respect to the phase (frequency) modulation for chirped pulses. This effect of group velocity imbalance has been described in analytical form within the adiabatic approximation and confirmed by numerical calculations of Maxwell-Bloch equations.

Note that the main advantage of the adiabatic approach for the density matrix is that analytical expressions can be obtained for the coefficients of the reduced Maxwell equations. Analyzing these coefficients (which nonlinearly depend on the field in the general case), one can predict various effects of light pulse propagation without solving the equations themselves. Thus, the adiabatic approach can be successfully used to find and estimate new effects and to interpret the results obtained in experiments and numerical simulations.

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