# Reverse engineering of a nonlossy adiabatic Hamiltonian for non-Hermitian systems

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We generalize the quantum adiabatic theorem to the non-Hermitian system and build a strict adiabaticity condition to make the adiabatic evolution nonlossy when taking into account the effect of the adiabatic phase. According to the strict adiabaticity condition, the nonadiabatic couplings and the effect of the imaginary part of adiabatic phase should be eliminated as much as possible. Also, the non-Hermitian Hamiltonian reverse-engineering method is proposed for adiabatically driving an artificial quantum state. A concrete two-level system is adopted to show the usefulness of the reverse-engineering method. We obtain the desired target state by adjusting extra rotating magnetic fields at a predefined time. Furthermore, the numerical simulation shows that certain noise and dissipation in the systems are no longer undesirable but play a positive role in the scheme. Therefore, the scheme is quite useful for quantum information processing in some dissipative systems.

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## I. INTRODUCTION

As an essential task in many areas of quantum information science ranging from quantum information processing [1,2] and coherent manipulation of quantum systems [3] to high-precision measurements [4,5], quantum-state engineering (QSE) [6–12] has attracted much attention, which promotes the development of experimental techniques and theoretical schemes. The quantum adiabatic theorem (QAT), an important way of realizing QSE, has been widely studied, and the basic properties of the QAT are being scrutinized both theoretically and experimentally [13–19]. The basic idea of QAT can be summarized as follows: if the control parameters in the time-dependent Hamiltonian change slowly, the system will follow closely along an eigenstate trajectory up to an adiabatic phase factor when it is initially in one of the eigenstates. Therefore, the adiabatic phase is a complicated factor which can be divide into a dynamical phase and a geometrical phase [13]. Interestingly, in the Hermitian adiabatic Hamiltonians scenario, one can focus on the dynamics of the eigenstate and neglect the complicated phase factor since it can be discarded as the common pure phase. However, in practice the quantum system inevitably interacts with the surrounding environment, e.g., the non-Hermitian (NH) systems [20–32]. In this case, the complicated adiabatic phase factor cannot be simply discarded as the common pure phase any more since it generally is not a pure (real) phase factor. Then, the ideal robustness and the intended dynamics may be spoiled by the accumulation of the imaginary part of the adiabatic phase due to noise and undesired interactions. Thus, it is very worthwhile to look for the novel methods which are robustness, and figure out the strict adiabaticity condition for NH Hamiltonians when taking into account the effect of adiabatic phase.

In fact, several authors have paid attention to the study of adiabaticity in NH systems [27–31]. For example, Miniatura *et al.* set a rough estimate of an adiabaticity condition by analogy with the Hermitian counterpart and recognized the importance of the nonadiabatic transition [27]. Subsequently, Sun studied the generalization of the high-order adiabatic approximation method for the NH quantum systems by using perturbation theory and integration by parts and obtained an adiabaticity condition similar to the Hermitian one with the damping factor and the oscillating factor [28]. Recently, Dridi *et al.* established a generalization of the Davis-Dykhne-Pechukas formula by the complex time method and showed a general adiabatic approximation for lossy two-state models [29,30]. More recently, Ibáñez and Muga generalized the concept of population for NH systems to characterize adiabaticity and worked out an approximate adiabaticity criterion [31]. Indeed, the adiabaticity of a given NH system has been discussed well using those excellent methods [27–31]. However, in principle the above methods did not give a clear quantitative analysis of the dynamics of the bare state in the eigenstate and the adiabatic phase. In some cases, we may observe a false adiabaticity due to the problematic or obscure population concept [24–26]. In addition, they will also be limited severely by the presence of the strong dissipation effects in some applications [25,31]. The above problems make designing the perfect scheme to reach the intended dynamics for the NH systems very challenging.

In this paper, we will introduce a method to solve the problems shown above. Different from the previous schemes [28,30,31] which were proposed to explore the adiabatic approximation condition for a given NH system, we are dedicated to setting a strict adiabaticity condition to make the adiabatic evolution nonlossy when taking into account the effect of the adiabatic phase and exploring the Hamiltonian which will exactly satisfy the strict condition via the reverse-engineering method. The scheme has the following advantages: (1) We take the effect of the adiabatic phase into consideration and make the adiabatic process be a strict one without loss in the NH system. (2) By using the reverse-engineering method, we can design the Hamiltonian to realize the intended dynamics according to the demand. (3) The noise or certain dissipation in the systems can play a positive role in the scheme. We can obtain the desired target state by adjusting extra rotating magnetic fields at a predefined time even in the dissipative system. Therefore, the scheme makes it possible to realize the QSE for some dissipative systems.

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The rest of this paper is arranged as follows. In Sec. II, we briefly review some important properties of the NH Hamiltonians and build a strict adiabaticity condition which contains two parts: an auxiliary adiabaticity condition with respect to the adiabatic phase and a general adiabatic condition given via the Feshbach $P$-$Q$ partitioning technique [33–35]. In Sec. III, we explicitly discuss how to engineer a NH Hamiltonian which could exactly satisfy the strict adiabaticity condition. Then, we consider a concrete two-level-system example to show the usefulness of our reverse-engineering method. Both experimental feasibility and population engineering are discussed step by step in Sec. IV. Finally, we give a summary in Sec. V.

## II. BASIC THEORIES

### A. NH Hamiltonians: Basic formulas

For NH systems, the usual approximations and criteria are not necessarily valid, so the results which are applicable for Hermitian systems have to be reconsidered and modified. We first briefly recall some important properties of the NH Hamiltonians [20,21]. Consider an arbitrary time-dependent NH Hamiltonian $H(t)$ with $N$ nondegenerate instantaneous eigenstates $\{|\phi_n(t)\rangle\}, n = 1,2,\ldots,N$. It satisfies the following eigenvalue equation:

$$H(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle. \tag{1}$$

As the adjoint operator of $H(t)$, $H(t)^\dagger$ will also satisfy the following eigenvalue equation:

$$H(t)^\dagger|\widehat{\phi}_n(t)\rangle = E_n^*(t)|\widehat{\phi}_n(t)\rangle, \tag{2}$$

where $\{|\widehat{\phi}_n(t)\rangle\}$ are the instantaneous eigenstates of $H(t)^\dagger$ and also the biorthogonal partners of $\{|\phi_n(t)\rangle\}$ and the asterisk indicates complex conjugate. The biorthogonal partners are normalized to satisfy the biorthogonality relation

$$\langle\widehat{\phi}_n(t)|\phi_m(t)\rangle = \delta_{nm}, \tag{3}$$

and the closure relation

$$\sum_n |\widehat{\phi}_n(t)\rangle\langle\phi_n| = \sum_n |\phi_n(t)\rangle\langle\widehat{\phi}_n(t)| = 1. \tag{4}$$

With the above properties, the Hamiltonian and its adjoint can be rewritten as

$$H(t) = \sum_n |\phi_n(t)\rangle E_n(t)\langle\widehat{\phi}_n(t)|,$$
$$H(t)^\dagger = \sum_n |\widehat{\phi}_n(t)\rangle E_n^*(t)\langle\phi_n(t)|. \tag{5}$$

### B. The auxiliary adiabaticity condition for the NH systems with respect to the adiabatic phase

According to the adiabatic theorem, a state with the initial condition $|\phi(0)\rangle = |\phi_n(0)\rangle$ will evolve adiabatically if its dynamics is well approximated by $|\phi(t)\rangle = e^{i\beta_n(t)}|\phi_n(t)\rangle$. Furthermore, if $|\phi_n(t)\rangle$ is the instantaneous state of the system Hamiltonian $H(t)$ and $|\phi(t)\rangle$ satisfies the Schrödinger equation ($\hbar = 1$)

$$i|\dot{\phi}(t)\rangle = H(t)|\phi(t)\rangle, \tag{6}$$

we can obtain the adiabatic phase

$$\beta_n(t) = \int_0^t [-E_n(t') + i\langle\widehat{\phi}_n(t')|\dot{\phi}_n(t')\rangle]dt'. \tag{7}$$

However, this ansatz of the adiabaticity for the NH system is not strict. The imaginary part of the adiabatic phase will induce the decay of system and cause confusion about the validity of the adiabaticity. Consequently, it is necessary to forcibly eliminate $\mathrm{Im}[\beta_n(t)]$ to keep the adiabatic scheme working well; that is, we should ensure

$$-\mathrm{Im}[E_n(t)] + \mathrm{Re}[\langle\widehat{\phi}_n(t)|\dot{\phi}_n(t)\rangle] = 0. \tag{8}$$

Then, the adiabatic phase can be safely discarded as a common pure phase when we investigate the dynamics of the target state, even in the NH systems. Notice that Eq. (8) is the auxiliary adiabaticity condition which allows one to make the adiabatic evolution nonlossy with respect to adiabatic phase, which is the primary result to be used in following work.

### C. The general adiabatic condition for the NH systems

In general, a state at time $t$ can be expressed as

$$|\Psi(t)\rangle = \sum_n \Psi_n(t)e^{i\beta_n(t)}|\phi_n(t)\rangle, \tag{9}$$

where the phase factor $\beta_n(t)$ satisfies Eq. (7) for arbitrary $n$ and $\Psi_n(t)$ is considered a complex function. It is obvious that $\Psi_n(t)$ is the key coefficient associated with the dynamics of $|\phi_n(t)\rangle$. Therefore, an exact dynamical equation for $\Psi_n(t)$ is highly desirable. Assuming $|\Psi(t)\rangle$ satisfies the Schrödinger equation, we can obtain the following equations:

$$i\dot{\Psi}_n(t) = -i\sum_{m\neq n}\langle\widehat{\phi}_n(t)|\dot{\phi}_m(t)\rangle e^{i[\beta_m(t)-\beta_n(t)]}\Psi_m(t)$$
$$= \sum_{m\neq n} H'_{mn}\frac{\Psi_m(t)}{\Psi_n(t)}\Psi_n(t), \tag{10}$$

$$i|\dot{\Psi}'_n(t)\rangle = \sum_{m\neq n} H'_{mn}|\Psi_m(t)\rangle\langle\Psi_n(t)|\Psi'_n(t)\rangle = H'(t)|\Psi'_n(t)\rangle, \tag{11}$$

where $H'_{mn} \equiv -i\langle\widehat{\phi}_n(t)|\dot{\phi}_m(t)\rangle e^{i[\beta_m(t)-\beta_n(t)]}$. According to the adiabatic theorem, $|\phi_n(t)\rangle$ will evolve adiabatically if the term on the left-hand side of Eq. (10) approaches zero. Moreover, it is interesting to find that the form of Eq. (10) is similar to the form of the artificial Schrödinger equation (11) for the vector $|\Psi'_n(t)\rangle = [\Psi_1(t),\Psi_2(t),\Psi_3(t),\ldots,\Psi_n(t)]^T$ (the superscript $T$ denotes the transpose operator) with the rotating representation Hamiltonian $H'(t)$. So we can deal with $\Psi_n(t)$ with the help of Eq. (11). In fact, $H'(t)$ describes the coupling transitions between the instantaneous eigenstates $\{|\phi_n(t)\rangle\}$, the so-called nonadiabatic couplings.

We should stress that in this paper we do not intend to research fully adiabatic dynamics (for all modes). The problem we address is the adiabatic dynamics of one target component (for one mode). Without loss of generality, the target component can be denoted as $\Psi_1(t)$, corresponding to the target eigenstate $|\phi_1(t)\rangle$ of $H(t)$. In order to obtain a better understanding of the adiabatic dynamics of $\Psi_1(t)$, the

Feshbach $P$-$Q$ partitioning technique [34,35] is introduced. According to the $P$-$Q$ partitioning technique, the state $|\Psi_n'(t)\rangle$ and the rotating representation Hamiltonian $H'(t)$ in the Schodinger equation (11) can always be partitioned into the following form:

$$|\Psi_n'(t)\rangle = \begin{bmatrix} P \\ \overline{Q} \end{bmatrix}, \quad H'(t) = \begin{pmatrix} 0 & R \\ W & D \end{pmatrix}, \tag{12}$$

where $P$, associated with the target state, is equal to $\Psi_1(t)$, while $Q$, associated with the $(N-1)$-dimensional vector, denotes the rest of the state spaces. The vector $R \equiv [R_2, R_3, \ldots, R_n]$, with $R_m = -i\langle\widehat{\phi_1}(t)|\dot{\phi}_m(t)\rangle e^{i[\beta_m(t)-\beta_1(t)]}$ $(m \geqslant 2)$, while the vector $W \equiv [W_2, W_3, \ldots, W_n]^T$, with $W_m = -i\langle\widehat{\phi_m}(t)|\dot{\phi}_1(t)\rangle e^{i[\beta_1(t)-\beta_m(t)]}$. The $(N-1)\times(N-1)$ matrix $D \equiv \sum_{m\neq n} D_{mn}|\Psi_m(t)\rangle\langle\Psi_n(t)|$, where $D_{mn} = -i\langle\widehat{\phi_n}(t)|\dot{\phi}_m(t)\rangle e^{i[\beta_m(t)-\beta_n(t)]}(m,n \geqslant 2)$.

Substituting Eq. (12) into Eq. (11), we obtain the following equations:

$$i\dot{P} = RQ, \quad i\dot{Q} = WP + DQ. \tag{13}$$

The formal solution of Eq. (13) can be written as

$$i\dot{P} = -iR(t)\int_0^t G(t,s)W(s)P(s)ds + R(t)G(t,0)Q(0), \tag{14}$$

where $G(t,s) = \mathcal{T}_{\leftarrow}\{\exp[-i\int_s^t D(s')ds']\}$ is the time-ordered evolution operator. Under the condition $P(0) = 1$ and $Q(0) = 0$, we have the exact dynamical equation for the $P$ part,

$$\dot{P} = -R(t)\int_0^t G(t,s)W(s)P(s)ds = -\int_0^t g(t,s)P(s)ds, \tag{15}$$

where $g(t,s) = R(t)G(t,s)W(s)$ is an effective propagator which plays a very important role in the analysis of adiabaticity. Notice that the general adiabatic approximation condition is $\int_0^t g(t,s)P(s)ds = 0$; that is, the propagator $g(t,s) = 0$ or $g(t,s)$ is factored by a rapid oscillating function [35,36], which is also the primary result to be used in following work.

For an effective two-level system, the associated rotating representation Hamiltonian $H'(t)$ reads

$$H'(t) = -i\begin{pmatrix} 0 & \langle\widehat{\phi_1}(t)|\dot{\phi}_2(t)\rangle e^{i\Delta\beta(t)} \\ \langle\widehat{\phi_2}(t)|\dot{\phi}_1(t)\rangle e^{-i\Delta\beta(t)} & 0 \end{pmatrix}, \tag{16}$$

where $\Delta\beta(t) = \beta_2(t) - \beta_1(t)$. When the effective two-level system is initially in the eigenstate $|\phi_1(0)\rangle$, the propagator $g(t,s)$ reads

$$g(t,s) = -\langle\widehat{\phi_1}(t)|\dot{\phi}_2(t)\rangle\langle\widehat{\phi_2}(s)|\dot{\phi}_1(s)\rangle e^{i\int_s^t(\dot{\beta}_2(s')-\dot{\beta}_1(s'))ds'}. \tag{17}$$

Notice that Eqs. (16) and (17) are also the primary results to be used in following work.

## III. THE NH HAMILTONIAN REVERSE-ENGINEERING METHOD AND APPLICATIONS

### A. The NH Hamiltonian reverse-engineering method

In this section, we will start with an engineering method about how to engineer the yet unknown NH Hamiltonian which could exactly satisfy the strict adiabaticity condition. From the special properties of the NH Hamiltonian [see Eq. (5)], one can conclude that the design process can be divided into two steps: designing the eigenvectors and modifying the eigenvalues. Here we should make some remarks on the eigenvector designs. (1) The goal of our scheme is driving the eigenvectors of an initial Hamiltonian into those of a final Hamiltonian, so the designed eigenvectors must connect the initial state with the target state. (2) Our scheme is, working in the NH Hamiltonian scenario, the eigenvectors must satisfy the biorthogonality relation and the closure relation. (3) The eigenvectors must evolve adiabatically; that is, they should satisfy the general adiabatic condition which has been discussed in Sec. II C. Once the eigenvector designs are completed, we can reconsider and modify the eigenvalues resorting to normalization ambiguities in the eigenvectors of NH Hamiltonians. More specifically, we should consider the auxiliary adiabaticity condition with respect to the adiabatic phase for the new eigenvector in this step.

Before elaborating on manipulating a two-level system to the target state, we will give a simple restriction on eigenvectors to satisfy the biorthogonality relation and the closure relation from the view of mathematics. Without loss of generality, for an $n$-dimensional system, we assume the eigenstates $\{|\phi_n(t)\rangle\}$ of $H(t)$ read

$$\begin{aligned} |\phi_1(t)\rangle &= A_{11}(t)|1\rangle + A_{21}(t)|2\rangle + \cdots + A_{n1}(t)|n\rangle, \\ |\phi_2(t)\rangle &= A_{12}(t)|1\rangle + A_{22}(t)|2\rangle + \cdots + A_{n2}(t)|n\rangle, \\ &\vdots \\ |\phi_n(t)\rangle &= A_{1n}(t)|1\rangle + A_{2n}(t)|2\rangle + \cdots + A_{nn}(t)|n\rangle, \end{aligned} \tag{18}$$

where $|l\rangle$ $(l = 1,2,3,\ldots,n)$ is the bare state for the system and $A_{jk}(t)$ $(j,k = 1,2,3,\ldots,n)$ is a devisable function associated with the bare state $|j\rangle$ in $|\phi_k(t)\rangle$. In a similar manner, the biorthogonal states of $\{|\phi_n(t)\rangle\}$ are expressed as

$$\begin{aligned} \langle\widehat{\phi_1}(t)| &= A'_{11}(t)\langle 1| + A'_{12}(t)\langle 2| + \cdots + A'_{1n}(t)\langle n|, \\ \langle\widehat{\phi_2}(t)| &= A'_{21}(t)\langle 1| + A'_{22}(t)\langle 2| + \cdots + A'_{2n}(t)\langle n|, \\ &\vdots \\ \langle\widehat{\phi_n}(t)| &= A'_{n1}(t)\langle 1| + A'_{n2}(t)\langle 2| + \cdots + A'_{nn}(t)\langle n|, \end{aligned} \tag{19}$$

where $\langle l|$ $(l = 1,2,3,\ldots,n)$ also is the bare state for the system and $A'_{jk}(t)$ $(j,k = 1,2,3,\ldots,n)$ is a devisable function associated with the state $\langle k|$ in $\langle\widehat{\phi_j}(t)|$. Let's introduce two matrices constructed by $A_{jk}(t)$ and $A'_{jk}(t)$, respectively,

$$A^T(t) = \begin{pmatrix} A_{ll}(t) & A_{21}(t) & \cdots & A_{n1}(t) \\ A_{12}(t) & A_{22}(t) & \cdots & A_{n2}(t) \\ \vdots & \vdots & \cdots & \vdots \\ A_{1n}(t) & A_{2n}(t) & \cdots & A_{nn}(t) \end{pmatrix},$$

$$A'(t) = \begin{pmatrix} A'_{11}(t) & A'_{12}(t) & \cdots & A'_{1n}(t) \\ A'_{21}(t) & A'_{22}(t) & \cdots & A'_{2n}(t) \\ \vdots & \vdots & \cdots & \vdots \\ A'_{n1}(t) & A'_{n2}(t) & \cdots & A'_{nn}(t) \end{pmatrix}, \tag{20}$$

where the superscript $T$ denotes the transpose operator.

In order to satisfy the biorthogonality relation and the closure relation as shown in Eqs. (3) and (4), $A(t)$ and $A'(t)$

should satisfy the following relation:

$$A'(t)\cdot A(t)=A^{T}(t)\cdot A'^{T}(t)=[A'(t)\cdot A(t)]^{T}=\mathbf{1}_{n}, \quad (21)$$

where $\mathbf{1}_n$ is the $n$-dimensional unit matrix. We can easily verify that Eq. (21) will be satisfied if $A(t)$ is the reverse matrix of $A'(t)$. That is, we just need to make sure $A'(t)$ or $A(t)$ is the invertible matrix. Therefore, mathematically, the determinants of $A'(t)$ and $A(t)$ should never be zero for the reverse-engineered biorthogonal partners.

### B. Engineering quantum states using the reverse-engineering method

As an example, we now demonstrate how to engineer a quantum state of a single qubit by means of the reverse-engineering method. For the sake of simplicity, we assume the eigenstates $\{|\phi_n(t)\rangle\}$ of $H(t)$ read

$$\begin{aligned}|\phi_1(t)\rangle &= A_{11}(t)|1\rangle + A_{21}(t)|2\rangle,\\ |\phi_2(t)\rangle &= A_{12}(t)|1\rangle + A_{22}(t)|2\rangle.\end{aligned} \quad (22)$$

The choice of coefficients $A_{11}(t)$ and $A_{21}(t)$ is variable, and we can choose the state in which we are interested as the target state $|\phi_1(t)\rangle$. Without loss of generality, by setting $A_{11}(t)=-\lambda(t)\sin\alpha(t)$, $A_{21}(t)=\cos\alpha(t)$, $A_{12}(t)=\lambda(t)\cos\alpha(t)$, and $A_{22}(t)=\sin\alpha(t)$, we can obtain

$$A(t)=\begin{pmatrix}-\lambda(t)\sin\alpha(t) & \lambda(t)\cos\alpha(t)\\ \cos\alpha(t) & \sin\alpha(t)\end{pmatrix}, \quad (23)$$

where $\lambda(t)$ and $\alpha(t)$ are time-dependent complex functions. Obviously, $A(t)$ will be an invertible matrix if $\lambda(t)\neq 0$ is established all the time. Then, we can obtain the accurate solution of $A'(t)$,

$$A'(t)=\begin{pmatrix}\frac{-1}{\lambda(t)}\sin\alpha(t) & \cos\alpha(t)\\ \frac{1}{\lambda(t)}\cos\alpha(t) & \sin\alpha(t)\end{pmatrix}. \quad (24)$$

Now, we start to consider the general adiabatic condition for the designed system and calculate the following matrix elements:

$$\begin{aligned}\langle\widehat{\phi_1}(t)|\dot{\phi}_1(t)\rangle &= \frac{\dot{\lambda}(t)}{\lambda(t)}\sin^2\alpha(t),\\ \langle\widehat{\phi_2}(t)|\dot{\phi}_2(t)\rangle &= \frac{\dot{\lambda}(t)}{\lambda(t)}\cos^2\alpha(t),\\ \langle\widehat{\phi_2}(s)|\dot{\phi}_1(s)\rangle &= -\dot{\alpha}-\frac{\dot{\lambda}(s)}{\lambda(s)}\sin\alpha(s)\cos\alpha(s),\\ \langle\widehat{\phi_1}(t)|\dot{\phi}_2(t)\rangle &= \dot{\alpha}-\frac{\dot{\lambda}(t)}{\lambda(t)}\sin\alpha(t)\cos\alpha(t).\end{aligned} \quad (25)$$

Then, $|\phi_1(t)\rangle$ will adiabatically evolve if the propagator $g(t,s)=0$ or $g(t,s)$ is factored by a rapid oscillating function. Mathematically, the simplest choice is setting $\langle\widehat{\phi_1}(t)|\dot{\phi}_2(t)\rangle=0$ (we also can set $\langle\widehat{\phi_2}(s)|\dot{\phi}_1(s)\rangle=0$), and $\lambda(t)$ can be solved as

$$\lambda(t)=\tan\alpha(t), \quad (26)$$

where $\alpha(t)\neq\eta\pi/2$, $\eta\in Z$. Here, we should note that $g(t,s)$ will also be factored by a rapid oscillating function if $\lambda(t)$ is a constant and $\dot{\alpha}\approx 0$. In fact, this kind of setting was examined in detail in Ref. [31] by Ibáñez and Muga. However, the weakness of this kind of setting is quite obvious; the target state $|\phi_1(t)\rangle$ could not be engineered to reach an arbitrary target state in a short time as $\dot{\alpha}\approx 0$. For the sake of generality and giving more choices for the realization of QSE, $\lambda(t)$ will be chosen as Eq. (26) in this paper. Up to now, we have successfully completed the eigenvector designs and obtained the following unnormalized eigenvectors:

$$\begin{aligned}|\phi_1(t)\rangle &= -\frac{\sin^2\alpha(t)}{\cos\alpha(t)}|1\rangle+\cos\alpha(t)|2\rangle,\\ |\phi_2(t)\rangle &= \sin\alpha(t)|1\rangle+\sin\alpha(t)|2\rangle.\end{aligned} \quad (27)$$

According to Eq. (5), the system Hamiltonian takes the form

$$H(t)=\begin{pmatrix}E_1(t)+\Delta_E(t)\cos^2\alpha(t) & \Delta_E(t)\sin^2\alpha(t)\\ \Delta_E(t)\cos^2\alpha(t) & E_1(t)+\Delta_E(t)\sin^2\alpha(t)\end{pmatrix}, \quad (28)$$

where $\Delta_E(t)\equiv E_2(t)-E_1(t)$ is the eigenvalue difference of the system and it cannot equal zero due to the nondegeneracy.

At this point, the eigenvalues of NH Hamiltonians are still undetermined, although the eigenvector designs have been completed. We should reconsider and modify the eigenvalues, resorting to normalization ambiguities in the eigenvectors of NH Hamiltonians. One can find that the following states are also the eigenvectors of Eq. (28) with the same eigenvalues:

$$\begin{aligned}|\phi_1'(t)\rangle &= f_1(t)|\phi_1(t)\rangle,\\ |\phi_2'(t)\rangle &= f_2(t)|\phi_2(t)\rangle,\end{aligned} \quad (29)$$

where $f_1(t)$ and $f_2(t)$ can be arbitrary nonzero functions. Then, the biorthogonal partners of $\{|\phi_1'(t)\rangle,|\phi_2'(t)\rangle\}$ read

$$\begin{aligned}\langle\widehat{\phi_1'}(t)| &= \frac{1}{f_1^*(t)}\langle\widehat{\phi_1}(t)|,\\ \langle\widehat{\phi_2'}(t)| &= \frac{1}{f_2^*(t)}\langle\widehat{\phi_2}(t)|.\end{aligned} \quad (30)$$

By calculating, we can find the propagator $g(t,s)$ is also factored by a rapid oscillating function for the new eigenvector $|\phi_1'(t)\rangle$. That is, $|\phi_1'(t)\rangle$ will continue to evolve adiabatically in the current system without additional Hamiltonians, even though $f_1(t)$ is an arbitrary nonzero function. Substituting $|\phi_1'(t)\rangle$ into Eq. (6), we obtain

$$|\phi(t)\rangle=e^{i\beta_1'(t)}|\phi_1'(t)\rangle=e^{i\beta_1(t)}f_1(0)|\phi_1(t)\rangle, \quad (31)$$

where the adiabatic phase for the new eigenvector reads

$$\begin{aligned}\beta_1'(t) &= \int_0^t[-E_1(t')+i\langle\widehat{\phi_1'}(t')|\dot{\phi}_1'(t')\rangle]dt'\\ &= \int_0^t[-E_1(t')+i\langle\widehat{\phi_1}(t')|\dot{\phi}_1(t')\rangle+id\ln f_1(t)]dt'.\end{aligned} \quad (32)$$

As a consequence, the normalization ambiguities in the eigenvectors generate only a constant multiplication factor $f_1(0)$, and the target state $|\phi_1(t)\rangle$ always evolves adiabatically in the current system. Furthermore, when the auxiliary adiabaticity condition with respect to the adiabatic phase [see Eq. (8)] is taken into account,

$$\mathrm{Im}[E_1(t)]=\mathrm{Re}[\langle\widehat{\phi_1}(t)|\dot{\phi}_1(t)\rangle]=\mathrm{Re}[\sin\alpha(t)\cos\alpha(t)], \quad (33)$$

the target state will not suffer strong exponential variations, which is remarkable for quantum information processing.

We can find that Eq. (28) can be expressed in terms of the Pauli matrices as

$$H(t)=\frac{\Delta_E(t)}{2}\sigma_x-i\frac{\delta(t)}{2}\sigma_y+\frac{\delta(t)}{2}\sigma_z+E_0'(t)\mathbf{1}, \quad (34)$$

where $\delta(t)=\Delta_E(t)\cos[2\alpha(t)]$ and $E_0'(t)=E_1(t)+\Delta_E(t)/2$ are the time-dependent variables and $\mathbf{1}$ denotes the unit matrix. In fact, the real part of $E_0'(t)$ can be ignored by applying the appropriate energy shift, which does not play a negative role in the investigation of population of system. The system can be mapped onto the Hamiltonian

$$H(t)=\tfrac{1}{2}[\Delta_E(t)\sigma_x-i\delta(t)\sigma_y+\delta(t)\sigma_z]+i\mathrm{Im}[E_0'(t)]\mathbf{1}. \quad (35)$$

It can be easily found that there are only two variables, $\Delta_E(t)$ and $\alpha(t)$, in Eq. (35). Thus, the crucial NH Hamiltonian engineering can be cast into the $\Delta_E(t)$ design and the $\alpha(t)$ design. Theoretically speaking, besides the consistency condition [$\alpha(t)\neq 2\eta/\pi$, $\Delta_E(t)\neq 0$ and $\sin\alpha(0)\approx 1$; it should be noted that the initial state could make a connection with the target state $|\phi_1(t)\rangle$ by setting $\sin\alpha(0)\approx 1$ according to Eq. (27)], there is almost no limit on the choices of $\Delta_E(t)$ and $\alpha(t)$ for engineering the system to reach an arbitrary target state at a predefined time. However, the choices of $\alpha(t)$ and $\Delta_E(t)$ will affect the evolution speed of the target state and the feasibility in the practical realization. Especially, when the term $\mathrm{Im}[E_0'(t)]$ in Eq. (35) does not equal zero, the practical realization of this Hamiltonian is significantly challenged in experiments. We shall explore in the following section an appropriate physical model that can incorporate the resulting Hamiltonian.

## IV. EXPERIMENTAL FEASIBILITY AND NUMERICAL EXAMPLES

For the purpose of convenience, we consider a simple case of Eq. (35),

$$\mathrm{Im}[E_0'(t)]=\mathrm{Im}[E_1(t)]+\mathrm{Im}\left[\frac{\Delta_E(t)}{2}\right]\approx 0. \quad (36)$$

The Hamiltonian of the system reduces to

$$H(t)=\tfrac{1}{2}[\Delta_E(t)\sigma_x-i\delta(t)\sigma_y+\delta(t)\sigma_z]. \quad (37)$$

In general, there is no simple "real" field interaction leading to Eq. (37) since the off-diagonal terms of the resulting Hamiltonian are different. For example, we assume a semiclassical description of the interaction between a "real" magnetic field $\boldsymbol{B}(t)$ and a rotating spin qubit, where $\boldsymbol{B}(t)=[B_x(t)e_x+B_y(t)e_y+B_z(t)e_z]/2M_b$, $e_r$ $(r=x,y,z)$ is the unit vector along the $r$ axis, $M_b=\hbar e/(2m)$ is the Bohr magneton, and $B_r(t)$ is a real variable. Then, the Hamiltonian of this system reads

$$H(t)=\begin{pmatrix}B_z(t) & B_x-iB_y(t)\\ B_x(t)+iB_y(t) & -B_z(t)\end{pmatrix}, \quad (38)$$

and we can find that the off-diagonal terms are the complex conjugate of each other, which does meet the requirements. However, we may obtain the resulting Hamiltonian if the magnetic field $\boldsymbol{B}(t)$ is the complex signal field rather than the real signal field; for example,

$$\begin{aligned}B_x(t)\to A_x(t)e^{I\Theta_x(t)}&=\mathrm{Re}[\Delta E(t)]+i\mathrm{Im}[\Delta E(t)],\\ B_y(t)\to A_y(t)e^{I\Theta_y(t)}&=\mathrm{Im}[\delta(t)]-i\mathrm{Re}[\delta(t)],\\ B_z(t)\to A_z(t)e^{I\Theta_z(t)}&=\mathrm{Re}[\delta(t)]+i\mathrm{Im}[\delta(t)],\end{aligned} \quad (39)$$

where $A_r$ is the amplitude and $\Theta_r$ is the phase. In fact, a similar complex signal field has been discussed in detail in Refs. [37,38] (and references therein). Additionally, the phase $\Theta_r$ can also be considered the dissipation factor which is introduced by the noise (e.g., the dephasing effects due to the collisions or phase fluctuations of the magnetic fields or when the rotating-wave approximation fails for the strong magnetic fields [38]). Therefore, the resulting Hamiltonian (37) is accessible experimentally with the complex signal field or the real signal field under some dissipation effects.

Now, let's focus on how to design $\Delta_E(t)$ and $\alpha(t)$ from an experimental view point. First, we can write $\alpha(t)$ in polar form,

$$\alpha(t)=\rho(t)\exp[i\theta(t)], \quad (40)$$

where $\rho(t)$ and $\theta(t)$ are time-dependent real variables. It is useful to rewrite $\Delta_E(t)$, taking into account Eqs. (33) and (36), as

$$\begin{aligned}\Delta_E(t)=&\,\mathrm{Re}[\Delta_E(t)]\\ &-i\sin[2\rho(t)\cos\theta(t)]\cosh[2\rho(t)\sin\theta(t)],\end{aligned} \quad (41)$$

where the real part of $\Delta_E(t)$ is an undetermined parameter and the selection of $\mathrm{Re}[\Delta_E(t)]$ seems quite arbitrary mathematically. However, $\Delta_E(t)$ is physically associated with the eigenvalue difference of the system [see Eq. (28)]. Thus, we should guarantee the modulus of $\mathrm{Re}[\Delta_E(t)]$ is relatively large; otherwise, the system will undergo transitions between $|\phi_1(t)\rangle$ and $|\phi_2(t)\rangle$ constantly. Furthermore, $\Delta_E(t)$ is also associated with the magnetic field; we should consider the experimental technology for the magnetic-field engineering. Once $\mathrm{Re}[\Delta_E(t)]$, $\rho(t)$, and $\theta(t)$ are fixed, the magnetic field $B$ is fixed. However, it should be emphasized that an arbitrary choice of $\rho(t)$ and $\theta(t)$ will typically lead to singularities on the magnetic field. We will discuss this problem in detail with the following physical model.

In the above derivation, we have considered a simple case of Eq. (35), that is, $\mathrm{Im}[E_0'(t)]\approx 0$. Now, we will discuss the experimental feasibility of the physical model when $\mathrm{Im}[E_0'(t)]\gg 0$. For convenient discussion, we assume $\mathrm{Im}[\delta(t)]=\Gamma(t)$, where $\Gamma(t)$ is a time-dependent real coefficient. In this case, Eq. (35) can be written as

$$\begin{aligned}H(t)=&\,\tfrac{1}{2}\{\Delta_E(t)\sigma_x-i\delta(t)\sigma_y+\mathrm{Re}[\delta(t)]\sigma_z\}\\ &+i\begin{pmatrix}\mathrm{Im}[E_0'(t)]+\frac{\Gamma(t)}{2} & 0\\ 0 & \mathrm{Im}[E_0'(t)]-\frac{\Gamma(t)}{2}\end{pmatrix}.\end{aligned} \quad (42)$$

Note that the difference in the order of magnitude between $\mathrm{Im}[E_0'(t)]$ and $\Gamma(t)$ is small; otherwise, the problem seems to be equivalent to the above simple example. More specifically, setting

$$\mathrm{Im}[E_0'(t)]+\Gamma(t)/2=0, \quad (43)$$

we will find the resulting Hamiltonian in Eq. (42) can be accessible in the following physically setting: a spin

qubit or atom passes through a region of rapidly varying magnetic field $\boldsymbol{B} = \{\Delta_E(t)e_x - i\delta(t)e_y + \mathrm{Re}[\delta(t)]e_z\}/2M_b$, and the spin qubit or atom suffers a radiation process with the dissipation rate [32,39,40] $\Gamma(t)$ [e.g., the spontaneous decay; in some cases, $\Gamma(t)$ can be controlled as an effective decay rate by further interactions; see, e.g., Ref. [40]]. This is remarkable since the noise and certain dissipation in the systems are no longer undesirable but play an integral part in our scheme.

From an experimental view point, we should consider the $\Delta_E(t)$ design and the $\alpha(t)$ design for the current physical model. Similar to the above derivation, $\alpha(t)$ is still in polar form. Substituting Eqs. (33) and (40) into Eq. (43), we will find $\Delta_E(t)$ satisfies the following equation:

$$-\mathrm{Re}[\Delta_E(t)]\Omega_1 = \mathrm{Im}[\Delta_E(t)](1+\Omega_2) + \Omega_3, \qquad (44)$$

where

$$\begin{aligned}\Omega_1 &= \sin[2\rho(t)\cos\theta(t)]\sinh[-2\rho(t)\sin\theta(t)],\\ \Omega_2 &= \cos[2\rho(t)\cos\theta(t)]\cosh[2\rho(t)\sin\theta(t)],\\ \Omega_3 &= \sin[2\rho(t)\cos\theta(t)]\cosh[2\rho(t)\sin\theta(t)].\end{aligned} \qquad (45)$$

Furthermore, $\Gamma(t)$ can be simplified as $\Gamma(t) = -\mathrm{Im}[\Delta_E(t)] - \Omega_3$. Apparently, once $\mathrm{Im}[\Delta_E(t)]$ is specified, the magnetic field $B$ and $\Gamma(t)$ are straightforwardly calculated with Eqs. (44) and (45). On the other hand, the form of $\mathrm{Im}[\Delta_E(t)]$ can be derived with the inversion strategy if the form of dissipation rate $\Gamma(t)$ is fixed. This is remarkable since we can choose appropriate extra magnetic fields to adiabatically drive an artificial quantum state for a certain dissipative quantum system. Up to now, we have, in principle, constructed the magnetic fields according to the $\alpha(t)$ design and specified dissipation rate $\Gamma(t)$. However, the $\alpha(t)$ designs are problematic, as an arbitrary choice of $\rho(t)$ and $\theta(t)$ will typically lead to singularities on the right-hand side of Eq. (45) [for instance, $\Omega_i$ $(i = 1,2,3)$ will jump abruptly when $2\rho(t)\cos\theta(t) = \eta\pi$ or $2\rho(t)\sin\theta(t) \approx 0$]. In general, $\Omega_i$ will also introduce singularities in magnetic fields; then, we cannot construct the finite and smooth magnetic fields. Thus, we should design $\rho(t)$ and $\theta(t)$ to avoid the singularities. It is advisable to fix $\rho(t)$ or $\theta(t)$ first, then design the other one to avoid the singularities. A simple example is

$$\rho(t) = \frac{\pi}{2} - o - \xi\sin\mu t, \quad \theta(t) = \zeta + \sin\nu t, \qquad (46)$$

where $\mu$ and $\nu$ are constant frequencies related to the concrete phase engineering and $o$ is an extremely small constant to keep the consistency condition. By choosing appropriate parameters (such as $\xi = 0.4\pi$, $\zeta = 0.08\pi$, and $\mu = \nu = 0.5\Omega$), we can construct the finite and smooth magnetic fields. For an intuitive grasp of the change of magnetic fields with different parameters in the dissipation system, we plot the time evolution of magnetic fields in Fig. 1. As shown in Fig. 1(a), when the

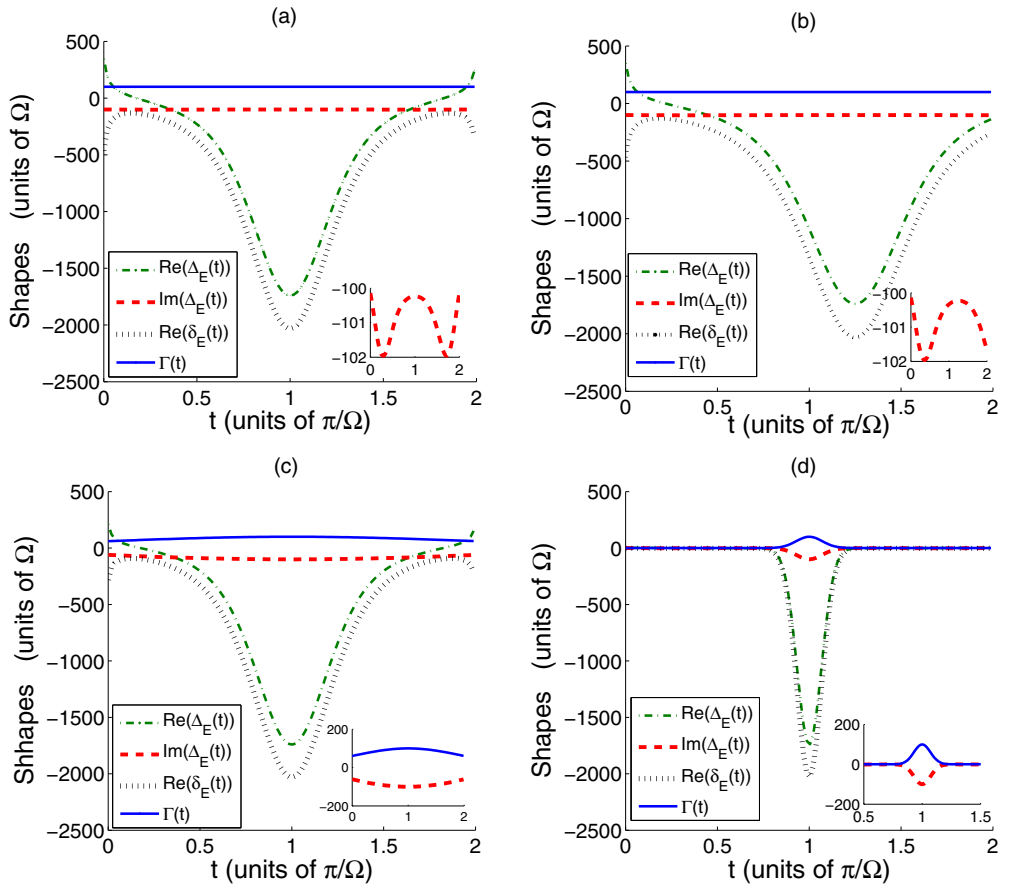


FIG. 1. Time evolution of the magnetic fields with different parameters. For (a) and (b) the dissipation rate $\Gamma(t) = 100\Omega$: (a) $\mu = \nu = 0.5\Omega$ and (b) $\mu = \nu = 0.4\Omega$ (the other parameters are $\xi = 0.4\pi$ and $\zeta = 0.08\pi$). For (c) and (d) the dissipation rate $\Gamma(t)$ is based on Eq. (47): (c) $T = \sqrt{2/\Omega}$ and (d) $T = \sqrt{0.01/\Omega}$ (the other parameters are $\Omega' = 100\Omega$, $t_0 = \pi/\Omega$, $\mu = \nu = 0.5\Omega$, $\xi = 0.4\pi$, and $\zeta = 0.08\pi$).

dissipation rate $\Gamma(t)$ is a constant [$\Gamma(t) = 100\Omega$], the shape of the magnetic fields is not very complex, and the maximum value of the magnetic fields $\Omega_{\max}$ is about $2000\Omega$. From an experimental view point, if $\Omega = 2\pi \times 10$ KHz, $\Omega_{\max}$ is about $2\pi \times 20$ MHz, which is feasible with present experimental techniques [41–46]. Thus, the magnetic fields in our scheme are not hard to realize in practice. Figures 1(a) and 1(b) share the same dissipation rate $\Gamma(t)$, while the parameters $\rho(t)$ and $\theta(t)$ are different. In fact, we are also interested in the time evolution of magnetic fields for a dissipation system with a time-dependent dissipation rate $\Gamma(t)$. Without loss of generality, we take a Gaussian dissipation rate $\Gamma(t)$ as an example,

$$\Gamma(t) = \Omega' e^{\left[-\left(\frac{t-t_0}{T}\right)^2\right]}, \tag{47}$$

where $\Omega'$ is a constant frequency, while $T$ and $t_0$ are time constants. We should emphasize that $T$ is related to the time scale of $\Gamma(t)$ physically; it should be chosen appropriately to keep the validity of the noise or certain dissipation. Figures 1(c) and 1(d) display the time evolution of magnetic fields with different $T$, while the other parameters are identical. Apparently, the magnetic fields in Figs. 1(b) and 1(c) are similar to the magnetic fields in Fig. 1(a); all of them are feasible in practice. However, the magnetic fields in Fig. 1(d) are quite different from the others. We can clearly see that magnetic fields can be neglected most of the time; specifically, $\Delta_E(t)$ is equal to zero for a long time, which means the consistency condition is invalid [$\Delta_E(t) \neq 0$ since $\Delta_E(t)$ is also associated with the difference between the eigenvalues of the system physically]. Therefore, the choice of the parameters in Fig. 1(d) is problematic or false.

Now we start to study the population engineering of the bare state in the target state. However, as shown in Eqs. (27) and (29), the target state does not seem to be a natural normalization. For an intuitive grasp of the change in the population engineering of the bare states, we will use the relative population $P_i^r$ ($i = 1,2$) to study the effects of different magnetic fields on the population engineering, where the relative population is defined as $P_i^r = P_i/(P_1 + P_2)$ and $P_i$ is the population of the bare state $|i\rangle$. We consider a realistic case of an extremely small population in the bare state $|2\rangle$ for the initial state

$$|\phi(0)\rangle \simeq |\phi_1'(0)\rangle = \sqrt{1-o^2}|1\rangle + o|2\rangle, \tag{48}$$

where $o$ is an extremely small constant. In Fig. 2, we plot the time evolution of the relative populations of bare states $|1\rangle$ and $|2\rangle$ with the same parameters as shown in the caption of Fig. 1. We find that the relative populations $P_1^r$ and

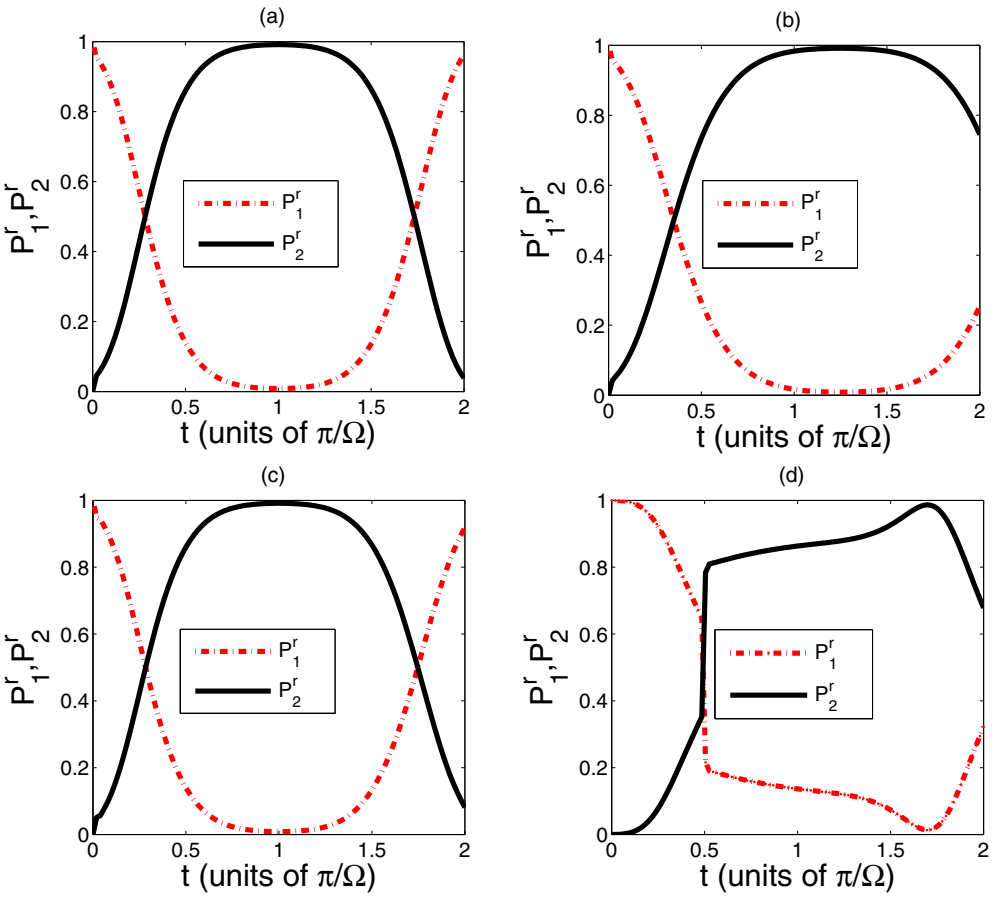


FIG. 2. Time evolution of the relative populations for states $|1\rangle$ and $|2\rangle$ with different magnetic-field parameters. The parameters are the same as shown in the caption of Fig. 1: (a) $\Gamma(t) = 100\Omega$, $\mu = \nu = 0.5\Omega$; (b) $\Gamma(t) = 100\Omega$, $\mu = \nu = 0.4\Omega$ ($\xi = 0.4\pi$ and $\zeta = 0.08\pi$); (c) $\Gamma(t) = 100\Omega \exp[-(t-t_0)^2/T^2]$, $T = \sqrt{2/\Omega}$; and (d) $\Gamma(t) = 100\Omega \exp[-(t-t_0)^2/T)^2]$, $T = \sqrt{0.01/\Omega}$ ($t_0 = \pi/\Omega$, $\mu = \nu = 0.5\Omega$, $\xi = 0.4\pi$, and $\zeta = 0.08\pi$).

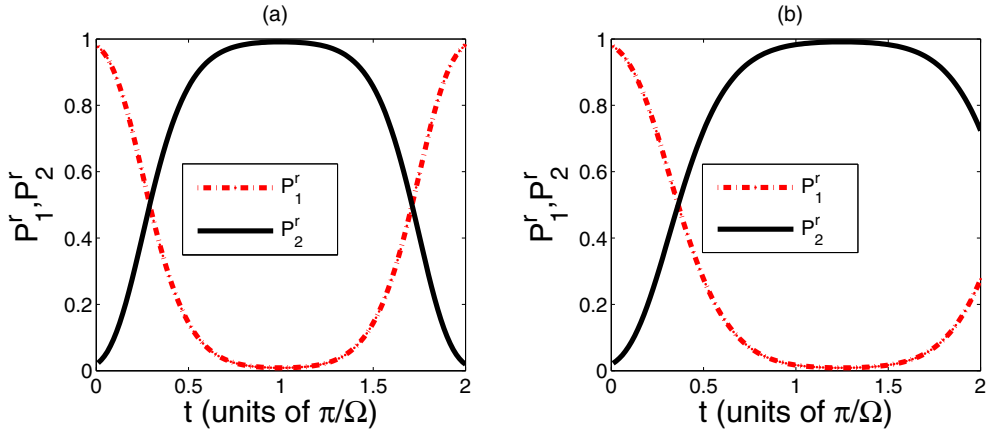


FIG. 3. The ideal population engineering with different $\alpha(t)$. $\alpha(t)$ is based on Eqs. (40) and (46): (a) $\mu = \nu = 0.5\Omega$, $\xi = 0.4\pi$, and $\zeta = 0.08\pi$; (b) $\mu = \nu = 0.4\Omega$, $\xi = 0.4\pi$, and $\zeta = 0.08\pi$.

$P_2^r$ almost have the same evolving tendency in Figs. 2(a) and 2(c), and a perfect full relative population inversion takes place when $\Omega t = \pi$. It should be noted that the time for a full relative population inversion is about 50 ns, which is short, if $\Omega = 2\pi \times 10$ kHz. Figure 2(b) also clearly shows a full relative population inversion when $\Omega t \approx 1.3\pi$. However, the time evolution of the relative populations in Fig. 2(d) is complicated and is quite different from the others. The reason for this result is that the choice of the parameters in Fig. 2(d) is problematic or false; particularly, $T$ is too short, and the consistency condition is invalid in this case.

To judge the validity of our scheme for adiabatic driving, we should compare the real population engineering with the ideal population engineering [see Eq. (27)]. The ideal population engineering with different $\alpha(t)$ is given in Fig. 3. As shown in Eq. (27), the ideal population engineering depends on only the $\alpha(t)$ design. In other words, the population engineering will be identical for the same $\alpha(t)$ design independent of other parameters. Thus, if our scheme is valid, Figs. 2(a), 2(c), and 3(a) [Figs. 2(b) and 3(b)] should be identical since their $\alpha(t)$ designs are identical. Obviously, the results are consistent with our deduction; hence, our scheme can work well even under noise if the parameters are chosen appropriately. In addition, we can get more target states of interest with different $\alpha(t)$ designs.

## V. DISCUSSION AND CONCLUSION

We have generalized the quantum adiabatic theorem to the NH system and provided a strict adiabaticity condition to make the adiabatic evolution nonlossy. The strict adiabaticity condition can be regarded as a nontrivial generalization of adiabaticity conditions for the Hermitian Hamiltonians presented by Jing *et al.* [35]. According to the strict adiabaticity condition, one should eliminate the nonadiabatic couplings and the effect of the imaginary part of the adiabatic phase as much as possible. The NH Hamiltonian reverse-engineering method has been proposed to adiabatically drive an artificial quantum state. A concrete two-level system example was discussed to show the usefulness of the reverse-engineering method in the paper, and numerical simulation showed that our scheme can work well even under noise if the parameters are chosen appropriately. Furthermore, we can obtain the desired target state by adjusting extra rotating magnetic fields at a predefined time. Specifically, the noise and certain dissipation in the systems are no longer undesirable but play a positive role in our scheme. Therefore, our scheme is powerful and reliable for quantum information processing.

The present work has some elements in common with the quantum control in open quantum systems, including the idea of using dissipation as a resource [e.g., dissipative quantum dynamics (DQD) [47–50] and the NH shortcuts to adiabaticity schemes [32,38]], so it is worth stressing the similarities and differences. In fact, the basic idea of DQD can be summarized as follows: the interaction between the system and the environment is modulated to make the target state become the stationary state of the system. Therefore, some specific dissipative factors are no longer undesirable but can be regarded as important resources. For the NH shortcuts to adiabaticity schemes, the dissipative factors are also introduced to the system to cancel somehow the nonadiabatic losses. In this way, one can improve dramatically the fidelity of the adiabatic passage. However, a common problem which one may encounter via DQD or the NH shortcuts to adiabaticity is how to use the specific dissipative factors or employ the appropriate interaction between the system and the environment. Furthermore, those methods may also be limited severely for some applications (the nonadiabatic dynamics processes) since their starting point generally is to improve a given (adiabatic) dynamics process.

Among the differences with recent works [30–32,38,47–50], the most prominent point is as follows: using the reverse-engineering method, we can easily obtain the Hamiltonian to realize the intended dynamics without loss, which allows one to design the Hamiltonian according to the demand. The main task we should consider is how to physically realize the resulting NH Hamiltonian. Sometimes, the resulting NH Hamiltonian may be hard to realize (a common potential problem of the NH shortcuts to adiabaticity). However, we should note that the difficulty to realize the NH Hamiltonian may be solved by enlarging the system with the aid of Naimark extensions [51]. Furthermore, in a sense, all the resulting NH Hamiltonians (even the problematic NH Hamiltonian) may help us with a deeper understanding of the problem: which

dissipative factors are the specific dissipative factors that can be used as a resource to realize QSE and promote the development of quantum information science in NH system frames.

Furthermore, for any quantum system whose Hamiltonian can be simplified into the form in Eq. (35) (the basic vectors for the simplified Hamiltonian can be arbitrary dressed states as long as the dressed states satisfy the biorthogonality relation and closure relation), the scheme can be implemented straightforwardly. This might lead to a useful step toward realizing fast and noise-resistant quantum information processing for multiqubit systems with current technology. The applications or extensions of this work may be in fields, such as $n$-dimensional systems [52,53] (for instance, the three-dimensional systems for the stimulated Raman adiabatic passage), superadiabatic treatments [54,55], and nonadiabatic evolution of NH quantum systems [32].

## ACKNOWLEDGMENTS

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