Hybrid quantum magnetic-field sensor with an electron spin and a nuclear spin in diamond

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Recently, magnetic-field sensors based on an electron spin of a nitrogen vacancy center in diamond have been studied both from an experimental and theoretical point of view. This system provides a nanoscale magnetometer, and it is possible to detect a precession of a single spin. In this paper, we propose a sensor consisting of an electron spin and a nuclear spin in diamond. Although the electron spin has a reasonable interaction strength with magnetic field, the coherence time of the spin is relatively short. On the other hand, the nuclear spin has a longer lifetime while the spin has a negligible interaction with magnetic fields. We show that, through the combination of such two different spins via the hyperfine interaction, it is possible to construct a magnetic-field sensor with the sensitivity far beyond that of previous sensors using just a single electron spin.

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I. INTRODUCTION

Measurement of the weak magnetic field with high spatial resolution is an important objective in the field of metrology. Many sensitive magnetic-field sensors such as superconducting quantum interference devices [1], Hall sensors in semiconductors [2], and force sensors [3] have been developed. Also, a magnetic-field sensor using entanglement has been also studied both from an experimental and theoretical point of view [4–6]. A spin amplification is another way to improve the sensitivity where ancillary spins are utilized to enhance the target signal [7,8], and there are some practical and scalable proposals about the spin amplification with specific configurations of the ancillary qubits [9–13]. One of the goals in this field is to measure a nuclear spin, because of a wide variety of potential applications in many fields such as material science and biomedical science.

Especially, much effort is being devoted to use nitrogen vacancy (NV) centers for the realization of the field sensor to detect a single spin [6,14–17]. NV defects in diamond consist of a nitrogen atom and a vacancy in the adjacent site, which substitute for carbon.

Single qubit gates and readout of the spins in NV centers have already been demonstrated [18–21]. There is an optical transition between its electron-spin triplet ground state and a first excited spin triplet state [18] in an NV center, and the quantum state of the electron spin can be measured via the fluorescence emission which has a dependency on the electron-spin state [19,20]. Also, Rabi oscillations of single electron spins in NV centers have been observed by using the optical detection [21]. All these properties are prerequisite in the construction of a sensitive and high-resolution sensor [14–16].

It is possible to use both electron spin and nuclear spin in diamond for quantum information processing. Here, each spin has its own distinct advantages. An electron spin offers strong interactions with other systems, and therefore can efficiently mediate the information to the other system. A nuclear spin presents excellent isolation from the environment, and this spin works as a quantum memory to store the information. Interestingly, NV centers provide both an electron spin and a nuclear spin (^{13}C, ^{14}N, or ^{15}N), and these spins are coupled via a hyperfine coupling.

This hybrid system of the electron spin and nuclear spin in diamond has been investigated to realize a quantum computer [22–24]. The electron spin in the NV center can be coupled with an optical photon [25]. So we can entangle distant NV centers by an interference of the photons while the nuclear spin can be used to store the information [26]. Also, the nuclear spin can be used as an ancillary qubit to implement a distillation protocol to increase the fidelity of the entanglement between the electron spins [27–29]. These techniques provide us with a scalable way to implement quantum computation [30,31].

In this paper, we propose a scheme to improve the sensitivity of magnetic-field sensors by using a hybrid system of an electron spin and nuclear spin in diamond. The electron spin has a strong coupling with the magnetic fields, and so we use this to accumulate the phase from the fields. On the other hand, since the nuclear spin has a longer coherence time than the electron spin, we can store the phase information in the nuclear spin. We show that, by using this hybrid system, it is possible to detect static magnetic fields with the sensitivity far beyond that of previous sensors using just a single electron spin.

An NV center in diamond has a spin 1 with the three levels $|0\rangle_e$ and $|\pm 1\rangle_e$, and it is possible to use just two of them to construct a two level system, namely, a qubit. The NV center has a zero-field splitting to be along the axis between the nitrogen and the vacancy. We apply an external magnetic field parallel to this axis, and we set Zeeman splitting between states $|1\rangle_e$ and $|-1\rangle_e$. This detuning allows us to use an effective two level system. Throughout this paper, we assume to use $|0\rangle_e$ and $|1\rangle_e$ as a two level system to construct a magnetic-field sensor.

II. MAGNETIC FIELD SENSING

Let us summarize the conventional strategy to use just an electron spin of the NV center for the detection of the magnetic field [4]. In the present description, we make the assumption of no decoherence for simplicity. First, we prepare a superposition of the spin $|+\rangle_e = \frac{1}{\sqrt{2}}|0\rangle_e + \frac{1}{\sqrt{2}}|1\rangle_e$. Second,

let this spin expose a magnetic field for a time t, and we obtain $|\psi(t)\rangle_e = \frac{1}{\sqrt{2}}|0\rangle_e + \frac{1}{\sqrt{2}}e^{-i\omega t}|1\rangle_e$ where ω denotes the detuning due to the Zeeman splitting induced by the target magnetic field. Finally, we measure the state $|\psi\rangle_e$ in the basis of $\hat{\sigma}_y = |1_y\rangle_e\langle 1_y| - |0_y\rangle_e\langle 0_y|$ where $|1_y\rangle_e = \frac{1}{\sqrt{2}}|0\rangle_e + \frac{i}{\sqrt{2}}|1\rangle_e$ and $|0_y\rangle_e = \frac{1}{\sqrt{2}}|0\rangle_e - \frac{i}{\sqrt{2}}|1\rangle_e$. Note that we can construct a projection about $\hat{\sigma}_y$ by rotating the spin with microwaves before the optical fluorescence measurement. By repeating the above three processes M times, we can obtain a probability to project the state $|\psi(t)\rangle_e$ into $|1_y\rangle_e$ as $P = \frac{1}{2} - \frac{1}{2}\sin\omega t$. The uncertainty of the estimated value is then given by

$$|\delta\omega| = \frac{1}{\left|\frac{dP}{d\omega}\right|}\sqrt{\frac{P(1-P)}{M}} = \frac{1}{\sqrt{M}t}. \quad (1)$$

Therefore, for a longer exposure time of the sensor to the field, the uncertainty of the estimated value becomes smaller.

However, in the actual circumstance, the noise from the environment induces decoherence, and the nondiagonal term of the quantum state disappears in a finite time. Typically, a relaxation time of the electron spin in the NV center is much longer than the dephasing time [32], and so we consider only dephasing through this paper. Since it is necessary to measure the state within the lifetime of the electron spin, we set the exposure time as $t = \alpha T_{2e}^*$ where T_{2e}^* denotes a dephasing time of the electron spin and α denotes a small constant. So the uncertainty is approximately calculated as $|\delta\omega| \simeq \frac{1}{\alpha\sqrt{MT_{2e}^*}}$. This shows that the sensitivity of the field sensing is limited by the lifetime of the electron spin.

Unfortunately, there is a tradeoff relationship between the sensitivity and the spatial resolution of the field sensor. In order to achieve a spatial resolution, one needs to use a smaller nanocrystal. However, the miniaturization of the crystal typically leads to the degradation of the coherence time of the electron spin in the diamond [14,33,34].

III. HYBRID MAGNETIC FIELD SENSING

We introduce our scheme to overcome the short lifetime of the electron spin by using a nuclear spin in the diamond as described in the Fig. 1. A nuclear spin is well isolated from the environment, and so we can use this as a quantum memory to store the information. Actually, the coherence time of the nuclear spin in the NV center exceeds 1 s at room temperature by using the spin echo [35]. Instead, the coupling of the nuclear spin with the target magnetic field is three orders of magnitude smaller than that of the electron spin. Fortunately, since the nuclear spin is coupled with an electron spin via a hyperfine coupling, it is possible to transfer the information attained by the electron spin to the nuclear spin for the storage. Actually, a controlled-NOT (C-NOT) gate between the electron spin and the nuclear spin has been already demonstrated [36]. Thus, we can construct an efficient hybrid magnetic-field sensor to combine the preferable properties of these two different systems.

The Hamiltonian of the NV center with an electron spin and a nuclear spin is described as follows. The Hamiltonian is

$$H = D\hat{S}_{z,e}^2 + g^{(e)}\mu_B(B_{ex}+B)\hat{S}_{z,e} + A\hat{S}_{z,e}\hat{\sigma}_{z,n} + \frac{A'}{2}(\hat{S}_{+,e}\hat{\sigma}_{-,n} + \hat{S}_{-,e}\hat{\sigma}_{+,n}) + g^{(n)}\mu_B(B_{ex}+B)\hat{\sigma}_{z,n}, \quad (2)$$

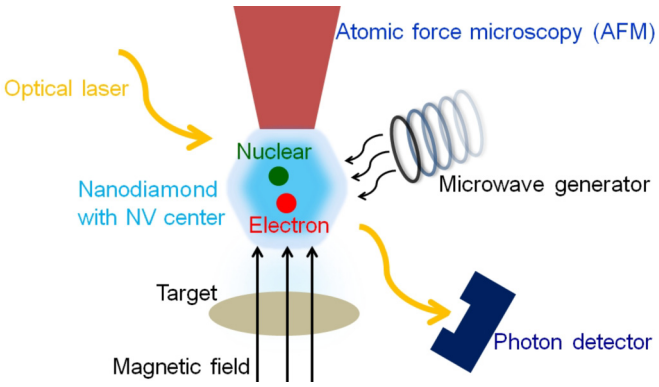

FIG. 1. The structure of our hybrid NV center sensor: a diamond containing an NV center with an electron spin and a nuclear spin is attached to an AFM tip. Single qubit rotations and a C-NOT gate can be performed by directing a microwave into the diamond. The electron-spin state can be measured by the optical laser and photodetectors. The electron spin has a reasonable interaction with the target magnetic field, and the nuclear spin works as a quantum memory to store the information from the magnetic field. By combining these two systems, it is possible to improve the sensitivity of the field sensor.

where $\omega = g\mu_B B$, $g^{(e)}$ ($g^{(n)}$), μ_B, and B (B_{ex}) denote the Zeeman splitting, an electron (nuclear) spin g factor, Bohr magneton, and target (external) magnetic field, respectively. Since the Zeeman splitting of the nuclear spin due to the target magnetic field is much weaker than the other values, we ignore this effect. Also, flip flops between the electron spin and the nuclear spin can be neglected because of the energy difference between them. In addition, the Zeeman splitting induced by the external magnetic field allows us to detune $|-1\rangle_e$ and to isolate a two level subsystem spanned by $|0\rangle_e$ and $|1\rangle_e$. So we obtain the following effective Hamiltonian:

$$H \simeq (D + g^{(e)}\mu_B(B_{ex}+B))|1\rangle_e\langle 1| + A|1\rangle_e\langle 1|\otimes\hat{\sigma}_{z,n} + g^{(n)}\mu_B B_{ex}\hat{\sigma}_{z,n}. \quad (3)$$

We make a unitary transformation

$$U = e^{i(D+g^{(e)}\mu_B B_{ex})|1\rangle_e\langle 1| + g^{(n)}\mu_B B_{ex}\hat{\sigma}_{z,n}t}, \quad (4)$$

into a rotating frame, and this yields the following Hamiltonian in the frame:

$$H' \simeq g^{(e)}\mu_B B|1\rangle_e\langle 1| + A|1\rangle_e\langle 1|\otimes\hat{\sigma}_{z,n}. \quad (5)$$

We describe the prescription to detect the target magnetic field by our field sensor (see Fig. 2). For simplicity, we assume no decoherence for both the electron spin and the nuclear spin. First, we prepare $|0\rangle_e \otimes (\frac{1}{\sqrt{2}}|0\rangle_n + \frac{1}{\sqrt{2}}|1\rangle_n)$. Second, we perform a C-NOT gate between them where the electron is the target and the nuclear spin is the control, and we obtain $\frac{1}{\sqrt{2}}|0\rangle_e|0\rangle_n + \frac{1}{\sqrt{2}}|1\rangle_e|1\rangle_n$. Third, let this state evolve under the effect of the target magnetic field for a time $t = kT_{2e}^*$ and we obtain $\frac{1}{\sqrt{2}}|0\rangle_e|0\rangle_n + \frac{1}{\sqrt{2}}e^{-i\omega t}|1\rangle_e|1\rangle_n$ where k and T_{2e}^* denote a constant number and the dephasing time of the electron spin, respectively. Also, we define $\omega = g^{(e)}\mu_B B + A$ as a resonant frequency of the state $|11\rangle_{en}$. Fourth, we perform the C-NOT gate again to obtain a separable state $|0\rangle_e \otimes (\frac{1}{\sqrt{2}}|0\rangle_n + \frac{1}{\sqrt{2}}e^{-i\omega t}|1\rangle_n)$ where

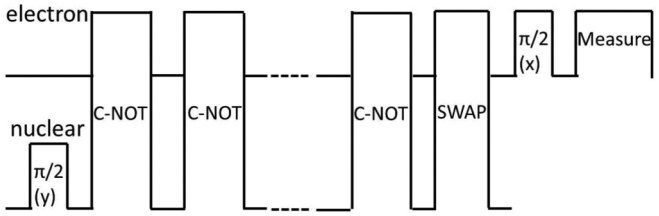

FIG. 2. A pulse sequence to detect magnetic fields with an electron spin and a nuclear spin in the NV center. In this sequence, we perform a single qubit rotation around the y axis, C-NOT gates, a SWAP gate, a single qubit rotation around the x axis, and optical detections. We acquire the phase information from the coupling between the electron spin and magnetic field, and transfer the information to the nuclear-spin state. We perform this transfer N times by implementing C-NOT gates $2N$ times, and finally readout the accumulated phase information.

we transfer the phase information acquired by the electron spin to the nuclear spin. By repeating the above four processes N times, we obtain $|0\rangle_e \otimes (\frac{1}{\sqrt{2}}|0\rangle_n + \frac{1}{\sqrt{2}}e^{-iN\omega t}|1\rangle_n) = |0\rangle_e \otimes (\frac{1}{\sqrt{2}}|0\rangle_n + \frac{1}{\sqrt{2}}e^{-ikN\omega T_{2e}^*}|1\rangle_n)$. We measure the nuclear spin in $\hat{\sigma}_y$ basis which can be constructed by a SWAP gate between the electron spin and nuclear spin, a rotation of the electron spin, and the optical detection. Thus, we obtain the information of the phase $\theta = kN\omega T_{2e}^*$ stored in the superposition. Since the lifetime of the nuclear spin is much longer than that of the electron spin, it is possible to transfer the phase information several times from the electron spin to the nuclear spin before the nondiagonal term of the nuclear spins disappears. By repeating such transfer, we can increase the amount of the phase accumulated from the target magnetic field, which enhances the sensitivity of the field sensor.

However, if we consider the effect of decoherence, there is of course a difficulty with this simple picture, namely, a propagation of the error from the electron spin to the nuclear spin. Due to the dephasing effect of the electron spin, the nondiagonal term of the entangled state $\frac{1}{\sqrt{2}}|0\rangle_e|0\rangle_n + \frac{1}{\sqrt{2}}|1\rangle_e|1\rangle_n$ decreases as quickly as that of the electron spin does. This dephasing error might be accumulated in the nuclear spins, which could destroy the phase information obtained from the target magnetic field.

Especially, if the dephasing noise is Markovian, the sensitivity of the hybrid field sensor is as small as that of the conventional one, due to the error propagation. In the Markovian dephasing model, the nondiagonal term of the density matrix decays exponentially, and so the error probability to have a phase flip during the free evolution is calculated as $\epsilon = \frac{1-e^{-\frac{t}{T_{2e}^*}}}{2} \simeq \frac{k}{2}$ for $k \ll 1$. When we implement the phase-information transfer from the electron spin to the nuclear spin N times, the total probability to have a phase flip on the nuclear spin is calculated as $N\epsilon = \frac{Nk}{2}$. In order to suppress the dephasing effect, we need a condition as $Nk < 1$. However, the acquired phase information from the target magnetic field in this case is $\theta = kN\omega T_{2e}^* < \omega T_{2e}^*$ which is comparable as that of the conventional field sensor. So we cannot obtain any improvement of the hybrid sensor in this case.

Fortunately, since the relevant dephasing in the NV center is induced by low-frequency noise [37,38] which is not Markovian, we can suppress the error accumulation as follows. Under the effect of low-frequency noise, the nondiagonal term of the density matrix decays quadratically. Due to this property, the initial decay of the non-Markovian noise is slower than that of the Markovian noise. The error probability to have a phase flip during the free evolution is calculated as $\epsilon = \frac{1-e^{-(\frac{t}{T_{2e}^*})^2}}{2} \simeq \frac{k^2}{2}$ for $k \ll 1$. We need a condition $N\frac{k^2}{2} < 1$ to suppress the dephasing effect after the N times transfer, and so the scaling of k should be $k \propto \frac{1}{\sqrt{N}}$. Thus, we have the acquired phase information from the target magnetic field as $\theta = kN\omega T_{2e}^* \propto \sqrt{N}\omega T_{2e}^*$, which can be larger as we increase the number of the transfer. Therefore, we can improve the sensitivity of the hybrid magnetic field.

We perform more rigorous calculation to show the efficiency of our magnetic-field sensor. The relevant noise in this scheme is the dephasing on the electron spin during the free evolution with the target magnetic field, and so we only consider this error. The density matrix after the N times transfer can be described as follows:

$$\rho = \frac{1}{2}|00\rangle_{en}\langle 00| + \frac{e^{Ni\omega t - N(\frac{t}{T_{2e}^*})^2}}{2}|00\rangle_{en}\langle 11| + \frac{e^{-iN\omega t - N(\frac{t}{T_{2e}^*})^2}}{2}|11\rangle_{en}\langle 00| + \frac{1}{2}|11\rangle_{en}\langle 11|. \quad (6)$$

Here, since the coherence time of the nuclear spin is much longer than that of the electron spin, we ignore the decoherence on the nuclear spin [35]. We set $t = \frac{\alpha}{\sqrt{N}}T_{2e}^*$ where α denotes a constant number. We can calculate the uncertainty of the estimated value as

$$|\delta\omega| = \frac{e^{\alpha^2}}{\alpha}\frac{1}{\sqrt{M}}\frac{\sqrt{1 - e^{-2\alpha^2}\sin^2(\alpha\sqrt{N}\omega T_2^*)}}{\sqrt{N}T_2^*|\cos(\alpha\sqrt{N}\omega T_2^*)|}, \quad (7)$$

where M denotes the number of the repetition of the experiment. Since we try to detect a weak magnetic field, it is valid to assume $\alpha\sqrt{N}\omega T_{2e}^* \ll 1$, and so we obtain $|\delta\omega| \simeq \frac{e^{\alpha^2}}{\alpha}\frac{1}{\sqrt{M}}\frac{1}{\sqrt{N}T_2^*}$. Thus, the minimum uncertainty is attained for $\alpha = \frac{1}{\sqrt{2}}$ as $|\delta\omega|_{opt} = \sqrt{2}e^{\frac{1}{2}} \cdot \frac{1}{\sqrt{M}}\frac{1}{\sqrt{N}T_{2e}^*}$. On the other hand, the minimum uncertainty in the conventional scheme is calculated as $|\delta\omega_{conv}|_{opt} = \sqrt{2}e^{\frac{1}{2}}\frac{1}{\sqrt{M}T_{2e}^*}$ for $t = \frac{1}{\sqrt{2}}T_{2e}^*$ and $\omega T_{2e}^* \ll 1$ [4]. Therefore, the uncertainty of our hybrid sensor is $\sqrt{N}$ times smaller than that of the conventional one.

Note that our scheme can be interpreted as an application of quantum Zeno effect (QZE) [39–46] to quantum metrology. Quantum Zeno effect is one of the fascinating phenomena where a decay process is suppressed by performing frequent projective measurements. It is known that QZE can be observed if the survival probability $P_s(t)$ shows a quadratic decay as $P_s(t) \simeq 1 - \Gamma^2 t^2$ for $\Gamma t \ll 1$ where Γ denotes a decay rate of the system. When one performs N projective measurements with a time interval $\tau = \frac{t}{N}$, the success probability of projecting the state in the excited level for all N measurements

is $P(t,N) \simeq (1-\Gamma^2\tau^2)^N \simeq 1-\Gamma^2\frac{t^2}{N}$, and therefore one can increase the success probability as one increases the number of the measurements. It is known that QZE occurs in a system to show a quadratic decay while QZE cannot be observed for exponential decay process [39]. The QZE has been applied to enhance the sensitivity of a multipartite entanglement sensor where Greenberger-Horne-Zeilinger states or spin squeezed states are required to detect the magnetic fields [47–49]. We use a similar concept to construct a hybrid magnetic-field sensor with a single electron spin and a nuclear spin. In our case, the free-evolution time of the sensor is set to be in a time region when the decay process is quadratic, and the effect of the dephasing of the electron spin is suppressed.

However, to observe a quadratic decay behavior after each transfer process, we need to eliminate a correlation between the system and the environment. In QZE, projective measurements play a role of this resetting process, and so one can observe a quadratic decay of the system after each measurement. In contrast, we cannot eliminate the correlation by measuring our system, because such projective measurement destroys the phase information acquired by the magnetic field. This means that we need to wait until the correlation between the electron spin and the environment disappears. The correlation time of the environment around the NV spins has been measured as $\tau_c \simeq 25\ \mu$s via dynamical decoupling experiment [38]. Therefore, our scheme involves a waiting time $\tau_{\rm w}$ which should be sufficiently larger than 25 μs.

We discuss useful cases of our approach using a hybrid system. Our approach could be relatively slow compared with the conventional approach. The necessary time for a single cycle of the detection is limited by the long waiting time as $t_{\rm our} \simeq N\tau_{\rm w}$.

In contrast, in the conventional scheme, the time for a single cycle is $t_c = (\tau_{\rm p} + \frac{1}{\sqrt{2}}T_{2\rm e}^* + \tau_{\rm M})$ where $\tau_{\rm p}$, and $\tau_{\rm M}$ denote a preparation time of the initial state and a measurement time, respectively. The typical time scales of $\tau_{\rm p}$, $T_{2\rm e}^*$, and $\tau_{\rm M}$ are a few microseconds, hundreds of nanoseconds, and a few microseconds [34,36,38]. Since one needs to wait for $\tau_{\rm w}$ at each transfer process, it takes longer for a single cycle in our scheme than the conventional one. If we fix the total time T for the sensing and try to minimize the uncertainty of the estimated value, our sensor may not be so sensitive as the conventional one, because M becomes smaller. However, if we fix the number of measuring the spin M and try to minimize the uncertainty, our sensor is superior to the conventional one. Actually, this is the case when we use this magnetic-field sensor at low temperature or on photosensitive materials such as biological tissues [50]. In order to readout the electron spin, it is necessary to irradiate the optical laser which generates heat and could damage the surface of the materials to be measured. In such circumstance, we need to restrict the number of measurements M to avoid heating or damage [50]. In our sensor, we can decrease the uncertainty of the estimated value by transferring the information from the electron spin to the nuclear spin.

IV. GATE IMPERFECTION

Imperfection of gate operations is the primary source of errors in our scheme. Especially, we perform $2N$ C-NOT gates and one SWAP gate, the imperfection of which would decrease the efficiency of our scheme. Suppose that we are subject to depolarizing noise. Here, the state becomes an identity operator with a probability ϵ when we implement the two-qubit gate. Since we perform $(2N+1)$ two-qubit gates, we obtain the following state after the final SWAP gate:

$$\rho_N = (1-\epsilon)^{2N+1}\left(\frac{1}{2}|0\rangle_{\rm e}\langle 0| + \frac{e^{i\alpha\sqrt{N}\omega T_{2e}^* - \alpha^2}}{2}|0\rangle_{\rm e}\langle 1| + \frac{e^{-i\alpha\sqrt{N}\omega T_{2e}^* - \alpha^2}}{2}|1\rangle_{\rm e}\langle 0| + \frac{1}{2}|1\rangle_e\langle 1|\right)\otimes|0\rangle_{\rm n}\langle 0| + \{1-(1-\epsilon)^{2N+1}\}\frac{\hat{\mathbb{1}}_{\rm en}}{4}. \quad (8)$$

If we have $\alpha\sqrt{N}\omega T_2^* \ll 1$, we obtain $\delta\omega \simeq \frac{1}{(1-\epsilon)^{2N+1}e^{-\alpha^2}\alpha\sqrt{N}T_2^*}\frac{1}{\sqrt{M}}$. This means that, if the error rate of each two-qubit gate operation is below 0.1%, we can perform hundreds of such gate operations without significant degradation of the fidelity so that the uncertainty can be an order of magnitude smaller than that of the conventional one. Since the sensitivity for the conventional scheme is calculated as $|\delta\omega_{\rm conv}|_{\rm opt} = \sqrt{2}e^{\frac{1}{2}}\frac{1}{\sqrt{M}T_{2e}^*}$, the ratio between them is calculated as $r = \frac{|\delta\omega_{\rm conv}|_{\rm opt}}{|\delta\omega|} = \sqrt{2}e^{\frac{1}{2}}(1-\epsilon)^{2N+1}e^{-\alpha^2}\alpha\sqrt{N}$. For a given ϵ, we can maximize $r = \frac{|\delta\omega_{\rm conv}|_{\rm opt}}{|\delta\omega|_{\rm opt}}$ to choose the optimum N and α and plot this in Fig. 3. As long as the error rate is below 7.5%, we can achieve an enhancement over the conventional strategy.

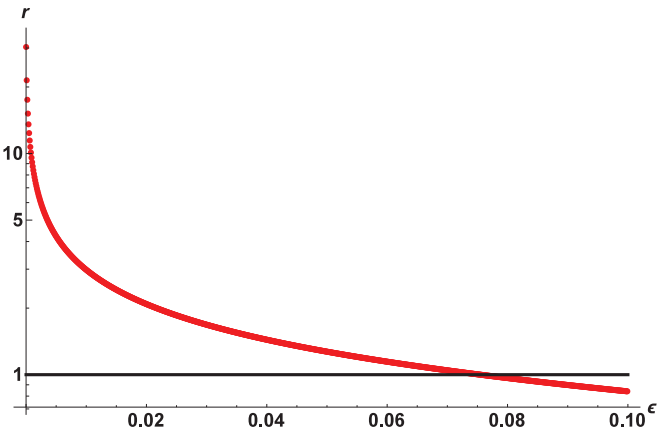

FIG. 3. We plot a relative sensitivity $r = \frac{|\delta\omega_{\rm conv}|_{\rm opt}}{|\delta\omega|_{\rm opt}}$ against a gate error ϵ where $|\delta\omega|_{\rm opt}$ ($|\delta\omega_{\rm conv}|_{\rm opt}$) denotes the sensitivity of our scheme (the conventional scheme). If the gate error is below 0.1%, the sensitivity of our sensor is one order of magnitude better than the conventional one.

V. CONCLUSION

In conclusion, we propose a hybrid magnetic-field sensor using an electron spin and a nuclear spin. The electron spin strongly interacts with the target magnetic field while the nuclear spin has a long coherence time. We have found that, by combining the best of two worlds, we can construct an efficient

magnetic-field sensor with a sensitivity far beyond that of a simple NV center.

Note added: Recently, we became aware of a related work of quantum sensing with a quantum memory [51]. This work uses both an electron spin of the NV center and the associated ^{14}N nuclear spin to sense temporal changes in magnetic field where the effect of static magnetic fields is removed by refocusing [51]. This technique is applied to detect a ^{13}C nuclear spin where radio frequency pulses are applied to flip the target nuclear spin [51]. On the other hand, our scheme described in this paper utilizes the electron spin and nuclear spin to improve the sensitivity of detecting static magnetic fields, which is different from the scheme in [51].

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