Extreme self-compression of laser pulses in the self-focusing mode resistant to transverse instability

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We show that a self-focusing mode for intense laser pulses comprising less than about ten optical cycles is resistant to the transverse modulational instability. For such pulses, a method of pulse self-compression based on adiabatic decrease of the duration of a solitonlike wave packet and transverse cumulating of energy during the self-focusing in Kerr-like media with anomalous dispersion is proposed. This method can be used for generation of high-quality, high-energy, few-cycle pulses down to a single-cycle duration.

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I. INTRODUCTION

Since the 1960s, it has been a well-established fact that transverse modulational instability plays a crucial role in intense laser pulse propagation in nonlinear Kerr-like media [1,2]. This implies a fundamental constraint on the generation and application of high-power pulses. The physics behind this effect lies, in particular, in the conventional picture of the instability when a comparatively long pump pulse and growing field perturbations propagate synchronously with the same group velocity, allowing formation of multifilament structures.

There are several ways to suppress filamentation instability. First, there is no instability in the field of partially coherent radiation if the correlation radius is less than the characteristic scale for which the instability growth rate is maximal [3]. Second, in media with inertial nonlinear response [4] filamentation instability in the field of a plane quasimonochromatic wave has no pronounced transverse scale with a maximum instability increment. In the present paper we show that dangerous filamentation instability may be suppressed in the case of short laser pulses for which synchronization between the pulses and the growing field perturbations may be disturbed. This may occur for intense laser pulses of ultrashort durations when the nonlinear dependence of the group velocity on light intensity comes into play. The origin of this suppression is connected to drifting of growing perturbations to the rear edge of the pulse where the intensity is low, thus stopping their amplification.

As a result, the well-known mechanism of laser-pulse self-compression in a nonlinear medium with anomalous dispersion [5–7] could be essentially intensified due to the possibility of the transverse energy cumulation of pulses with power greatly exceeding the self-focusing threshold. This could open a way for generation of high-energy few-cycle optical pulses, which is a challenge in contemporary laser physics that has important implications for high-field science and for many other extreme light applications as well [8].

The goal of this paper is twofold. First, we would like to draw the reader's attention to a self-focusing mode that is resistant to the filamentation instability for intense laser pulses comprising less than about ten optical cycles. Second, based on this filamentation-free self-focusing mode, a method of self-compression of high-power pulses down to a few-cycle duration or even generating a single-cycle pulse is proposed. The strategy of obtaining such energetic few-cycle pulses is as follows. At the input of the nonlinear Kerr medium with anomalous dispersion, a laser pulse should comprise less than about ten optical cycles and must be solitonlike with the beam width corresponding to a given pulse energy. Such a pulse will be smoothly compressed as a whole, reaching a single-cycle duration at the output.

II. SIMPLIFIED STABILIZATION MODEL

To start with, we consider a simplified model in order to argue that ultrashort laser pulses of few-cycle durations can be stable against transverse modulation instabilities, which contradicts the previous statement of laser-beam filamentation [1]. The latter is based on the plane-wave approach or solitonic pulse instability considered within the framework of the nonlinear Schrödinger equation [2]. In this case, transversely modulated wave perturbations move with the same group velocity as the pump pulse and therefore grow synchronously, thus giving rise to beam filamentation. As shown in Ref. [2], the maximum growth rate is typical of perturbations with the transverse characteristic scales comparable to the soliton width and being of the order of magnitude given in the plane-wave approximation, i.e., the growth rate along the propagation path $\Gamma = \frac{1}{2}n_2kI$, where $k = \omega/c$ is the laser wave number, the nonlinear refractive index is written as $n = n_0 + (1/2)n_2I$, n_0 is the linear refractive index, and I is the laser intensity. However, in the case of a shorter soliton pulse and therefore a more intense peak power, additional nonlinear effects, such as the dependence of the group velocity on intensity, should be taken into account, which in the first approximation modifies the nonlinear Schrödinger equation into the so-called derivative nonlinear Schrödinger (DNLS) equation in dimensionless variables

$$i\partial_z\Psi + \partial^2_{\tau\tau}\Psi + |\Psi|^2\Psi - 2i|\Psi|^2\partial_\tau\Psi + \Delta_\perp\Psi = 0. \quad (1)$$

Here $z \to 2z/|k_2|\omega^2$, $\tau \to (\tau - z/v_{\mathrm{gr}})/\omega$ is time in the moving frame of reference, and v_{gr} is group velocity, with $\mathcal{E} = \Psi\sqrt{c\omega|k_2|/n_2}$, $r_\perp \to r_\perp\sqrt{c/|k_2|\omega^3}$, and $k_2 = \partial^2k/\partial\omega^2$. This equation neglects the important effects such as high-order dispersions and space-time focusing, which can modify the modulational instability as shown in [9]. However, it should be noted that taking into account high-order dispersions does not lead to the stabilization of the instability.

The stability of a plane wave relative to perturbations within Eq. (1) may be analyzed as it was, for instance, in [1]. The resulting expression for the growth rate of the field perturbations $\Psi(z,\tau,r_\perp) = [u_0 + v(z,\tau,r_\perp)]e^{iu_0^2z+2iu_0^2\tau}$ in the laser pulse reference frame is written as

$$\Gamma = -2iu_0^2\Omega \pm \sqrt{(\Omega^2+\kappa_\perp^2)(2u_0^2-\Omega^2-\kappa_\perp^2)}. \quad (2)$$

Here a perturbation $v \propto e^{\Gamma z + i\Omega\tau + i\kappa_\perp r_\perp}$ and u_0 is the amplitude of the pump field. It is clear from the expression (2) that in addition to the exponential growth of perturbations at $0 \leqslant \sqrt{\Omega^2+\kappa_\perp^2} \leqslant \sqrt{2}u_0$ that is described by the second term in the expression (2) and coincides with expression derived in Ref. [10], the structure of the perturbations v includes pump perturbation drift with speed determined by the imaginary part of the increment Γ. Consequently, the homogeneous solution persists to be unstable. However, the instability changes its type and becomes convective with group velocity $\partial\,\mathrm{Im}\Gamma/\partial\Omega = -2u_0^2$. Hence, for laser pulses with a duration less than a certain value $\tau_p < \tau_{\rm cr}$, filamentation instability has no time to develop.

Let us estimate the pulse duration $\tau_{\rm cr}$ at which the growing perturbations are stabilized as a result of drift to the rear part of the pulse where they cannot be amplified further. In fact, this is exactly the case of solitonlike pulses. Indeed, Eq. (1) has the soliton solution $\Psi(\tau) = \mathcal{A}(\tau)e^{i\phi(\tau)+ihz}$ [11], where $\mathcal{A}^2 = 4h/[1+\alpha\cosh(2\sqrt{h}\tau)]$ and $\phi = \frac{1}{2}\int_{-\infty}^{\tau}\mathcal{A}^2(\xi')d\tau'$, with $\alpha = \sqrt{1+4h}$, $h = \frac{\mathcal{A}_m^2}{2}(1+\frac{\mathcal{A}_m^2}{2})$, and $\mathcal{A}_m$ the soliton amplitude. These solitons propagate as a nonlinear field structure with a group velocity that does not depend on intensity. As follows from the formula for the phase, a distinctive feature of the solution $\mathcal{A}$, in contrast to the nonlinear Schrödinger equation solitons, is the significantly strong frequency modulation in the laser pulse. This means that for the problem of interest, dangerous perturbations slip down to the rear side of the pulse, as their group velocity depending on the soliton intensity is less than the soliton velocity. Thus, making a simple estimation that the "slipping" length $z = z_\star$, at which the perturbation v shifts by half the pulse duration, i.e., $2u_0^2 z_\star \simeq \tau_{\rm cr}/2$, is less than about 15–20 times the perturbation growth length $1/\Gamma$, for which, as follows from Eq. (2), the maximum increment is $\Gamma_{\max} = u_0^2$, we can arrive at the following inequality for the pulse duration (in dimensional units):

$$\tau_p \lesssim \tau_{\rm cr} = 20\pi/\omega, \quad (3)$$

i.e., the pulse duration should be less than about ten optical cycles, for which the transverse modulational instability can be suppressed. Moreover, this suppression does not depend on the media properties or pulse amplitude.

To confirm the results of qualitative analysis we will further consider the results of numerical simulations using Eq. (1). As we are interested in the problem of structural stability, to simplify the analysis we will restrict it to the 2D+1 case, as the result weakly depends on the dimension of the problem. The results of numerical simulation of the dynamics of the input distribution of the wave packet $\Psi(x,\tau) = \Psi_0 \exp[-(\tau/\sqrt{2}\tau_p)^2 - (x/\sqrt{2}a_x)^8]$ are presented in Fig. 1 for two different cases demonstrating different evolution of the filamentation instability. The evolution of the pulse intensity $|\Psi(z,x,\tau)|^2$ in the case when the fourth term in Eq. (1) responsible for pulse steepening is neglected is shown in Fig. 1(a). The pulse evolution with nonlinear dispersion taken into account is shown for comparison in Fig. 1(b). As follows from Fig. 1(a), as a result of filamentation instability, the wave packet splits into separate beams in the transverse direction [1,2]. Here the level lines are shown by the green color. However, if the fourth term in Eq. (1) is taken into consideration, the instability is stabilized. It is clear from Fig. 1(b) that the inhomogeneities shift to the rear part of the pulse and cease to grow. This is the significant difference between the considered mode and the laser-pulse evolution within the framework of an ordinary nonlinear Schrödinger equation.

III. THREE-DIMENSIONAL WAVE MODEL

To describe adequately the self-action of ultrashort laser pulses including few-cycle pulses in a medium with Kerr-type nonlinearity ($\mathcal{P}_{\rm nl} = \chi^{(3)}|\mathcal{E}|^2\mathcal{E}$), we turn from the DNLS equation directly to the wave equation for an actual electric field

$$\frac{\partial^2\mathcal{E}}{\partial z^2} + \Delta_\perp\mathcal{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t}\varepsilon(t-t')\mathcal{E}(t')dt' = \frac{4\pi\chi^{(3)}}{c^2}\frac{\partial^2|\mathcal{E}|^2\mathcal{E}}{\partial t^2}.$$

To study the nonlinear few-optical-cycle pulse propagation, two approaches based on the reflection-free approximation have been considered. The first one is based on the nonlinear envelope equation extended to the few-cycle regime; it uses the Taylor expansion of the propagation constant around the central frequency [12]. Another approach developed in [13] assumes making calculations in Fourier space but using a one-way propagation constant $k(\omega)$. However, by taking into account that for the majority of media, the linear dielectric permeability in the transparent spectral region, especially in the mid-IR frequency band, can be written as follows: $\varepsilon(\omega) = \varepsilon_0 - \omega_D^2/\omega^2$. It should be noted that in this case the anomalous dispersion coefficient is $k_2 = \frac{1}{c}\frac{\partial^2}{\partial\omega^2}(\omega\sqrt{\varepsilon}) = -\frac{\omega_D^2}{\omega^3 c\sqrt{\varepsilon_0}}$. We use the approach developed in [14,15], which can be considered as a modification of the model [16]. In this case, with the use of the dispersion equation, a nonlinear wave equation for the real electric field can be written in dimensionless variables as (for details see [14,15,17])

$$\frac{\partial^2 u}{\partial z\partial\tau} + u + \frac{\partial^2}{\partial\tau^2}(|u|^2u) = \Delta_\perp u. \quad (4)$$

Here $u = (\mathcal{E}_x + i\mathcal{E}_y)\sqrt{4\pi\chi^{(3)}}\omega_0/\omega_D$ is a complex variable for a real-valued electric field $\vec{\mathcal{E}} = \{\mathcal{E}_x,\mathcal{E}_y\}$, $z \to z2\sqrt{\varepsilon_0}\omega_0 c/\omega_D^2$, $\tau = \omega_0(t - z\sqrt{\varepsilon_0}/c)$, ω_0 is the characteristic carrier frequency, and $r_\perp \to r_\perp c/\omega_D$.

To confirm the possibility of stabilizing the self-focusing modulation instability, let us turn to the results of numerical simulation in the framework of (4). Figure 2 presents the results of propagation of two different Gaussian laser pulses comprising 30 [Fig. 2(a)] and 10 [Fig. 2(b)] optical cycles with an initial noise level of about 10^{-4} of pulse amplitude. It can be easily seen that the process of beam self-focusing demonstrated in the (x,y) plane occurs in both cases together with the strong pulse compression shown in the (y,τ) plane. However, in the case in Fig. 2(a), the pulse is shortening simultaneously with development of the filamentation instability [(x,y) plane] cross section, i.e., the beam splits into separate filaments. For the shorter pulse, though, as follows from Fig. 2(b), the

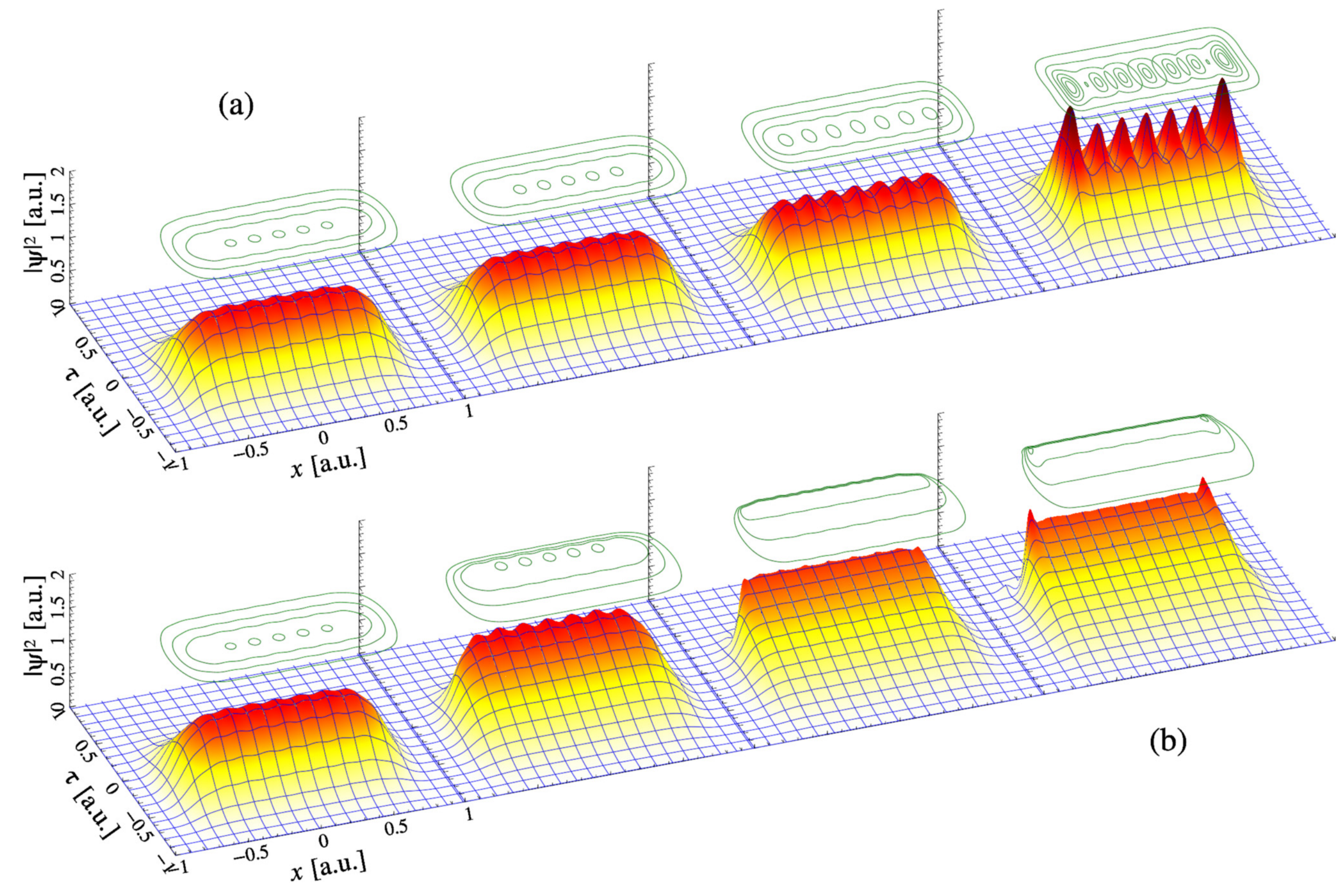

FIG. 1. Dynamics of wave packet intensity $|\Psi(z,x,\tau)|^2$ in Eq. (1) for two cases: (a) in the absence of nonlinear dispersion (b) with nonlinear dispersion taken into account. The green curve shows level lines.

spatial structure of the pulse remains smooth in the process of adiabatic shortening of the pulse duration [(x,y) plane] cross section. So transverse instability can be suppressed and no violation of the beam symmetry is observed. Correspondingly, we perform a thorough numerical investigation of the pulse dynamics for axisymmetric beams.

In what follows we consider the problem of self-compression of laser pulses that are stretched strongly in the transverse direction and have a dispersion length ($z_{\rm dis} \propto \tau_p^2/|k_2|$) much shorter than the diffraction length ($z_{\rm dif} \propto a_\perp^2$). A soliton-type distribution will be specified along the longitudinal coordinate. As a result of radiation self-focusing, amplification of the field u_m in the near-axis region of the beam will take place and the nonlinear length will decrease correspondingly ($L_{\rm nl} \propto 1/u_m^2$). Since a soliton is a nonlinear formation, in which the nonlinearity and dispersion of the medium balance each other, an increase in the intensity leads to an adiabatic decrease in the duration of the wave packet. Obviously, if a soliton pulse is both self-focused and self-compressed as a whole, the spatiotemporal distribution of the output pulse will be a high-quality pulse, i.e., we can efficiently compress high-energy pulses to a few-cycle duration.

IV. SPATIOTEMPORAL ADIABATIC COMPRESSION

For a rigorous procedure we restrict our consideration to a circularly polarized field for which a class of stable soliton solutions [14] was found for the case when spatial effects are insignificant ($\Delta_\perp \equiv 0$). It should be noted that soliton solutions exist for any polarization [17], including a linear polarization for which odd harmonic generation is taken into account [18].

The wave solitons of (4) can be represented as a family of two-parameter solutions having the form $u(z,\tau) = \sqrt{\gamma}G(\xi)\exp[i\omega_s(\tau+\gamma z)+i\phi(\xi)]$, where ω_s is the characteristic carrier frequency, γ is the parameter determining the group velocity of the soliton, and $\xi = \omega_s(\tau - \gamma z)$. The soliton envelope $G(\xi)$ and the nonlinear phase $\phi(\xi)$ obey the following equations:

$$\frac{d\phi}{d\xi} = \frac{G^2(3-2G^2)}{2(1-G^2)^2}, \qquad \int_{G_m}^{G} \frac{\pm(1-3G^2)dG}{G\sqrt{\delta^2 - F(G^2)}} = \xi - \xi_0, \tag{5}$$

where $F(G^2) = G^2[3/2(1+\delta^2) - (4-5G^2)/4(1-G^2)^2]$, G_m is the maximum amplitude of the soliton, and ξ_0 is the integration constant corresponding to the position of the maximum of the soliton envelope. It can be seen from (5) that the solutions for the soliton envelope $G(\xi)$ depend only on the parameter $\delta^2 = 1/\omega_s^2\gamma - 1$ and exist at $0 \leqslant \delta \leqslant \delta_{\rm cr} \equiv \sqrt{1/8}$. The critical value gives the single-cycle soliton [14]. It should be noted that the existence of the limiting soliton with the shortest duration is defined by the constraint $\int_{-\infty}^{\infty} u\, d\tau = 0$, which is one of the integrals of Eq. (4). At $\delta = \delta_{\rm cr}$ the shortest duration is equal to $\tau_s^* = 2.31\omega_s^{-1}$. Note that Eq. (4) can be

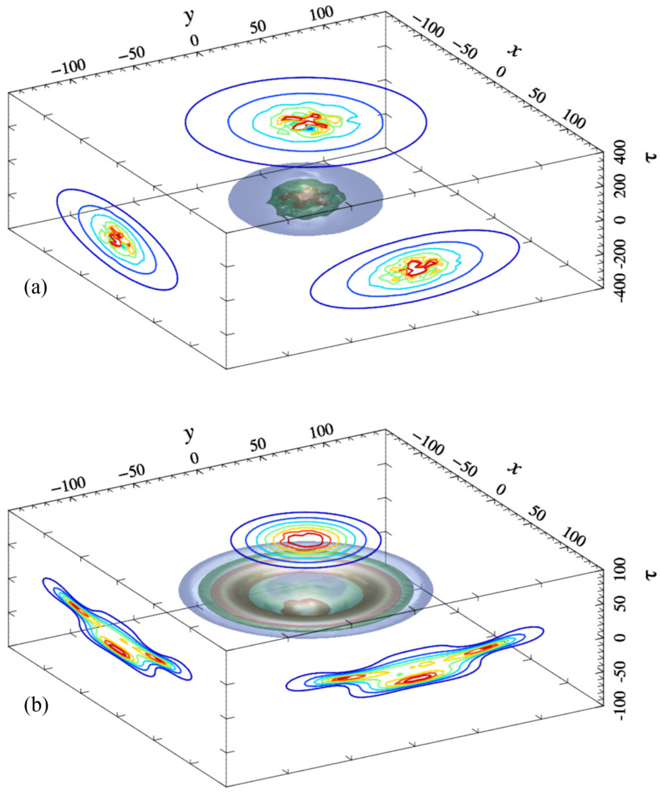

FIG. 2. Filamentation of the circularly polarized field $|u(z,x,y,\tau)|$ in the framework of (4) for two different initial durations in the process of radiation self-focusing: (a) $\tau_p = 30T_0$ and (b) $\tau_p = 10T_0$, where T_0 is the field period. Initial pulses have Gaussian form $u_0 = 0.6\exp(i\tau - \frac{x^2+y^2}{4000} - \frac{\tau^2}{2\tau_p^2})$ with noise level 10^{-4}. The coordinates x, y, and τ are dimensionless according to Eq. (4).

reduced to Eq. (1) for $\delta \ll 1$, where the group velocity can be expressed as

$$v_{\rm gr} \simeq \frac{c}{\sqrt{\varepsilon_0}}\left(1 - \gamma\frac{c}{\sqrt{\varepsilon_0}}\right), \quad \gamma \simeq \frac{\omega_D^2}{2\sqrt{\varepsilon_0}c\omega_s^2}$$

and is determined only by media properties and the carrier frequency ω_s.

We specify the soliton-type distribution and the Gaussian distribution in the longitudinal and transverse directions, respectively, with the beam width a as the initial distribution of the laser pulse:

$$u = \sqrt{\gamma}G(\tau)\exp\left(i\tau + i\phi(\tau) - \frac{r^2}{2a^2}\right). \tag{6}$$

As it was stated above, the ideal case is single-soliton compression. The results of simulation when the initial longitudinal distribution of the field is taken as a fundamental soliton (5) with $\delta_0 = 0.03$, $\omega_s = 1$, and $a = 400$ are presented in Fig. 3. In this case, the duration of the wave packet corresponds to ten optical cycles ($\tau_p^{\rm in} = 10T_0$, where T_0 is the field period). It should be also noted that the longitudinal scale is much shorter than the transverse one.

One can see in Fig. 3(a) that the self-focusing of the beam in the transverse direction is accompanied by the monotonic decrease of the soliton duration. The evolution of the pulse

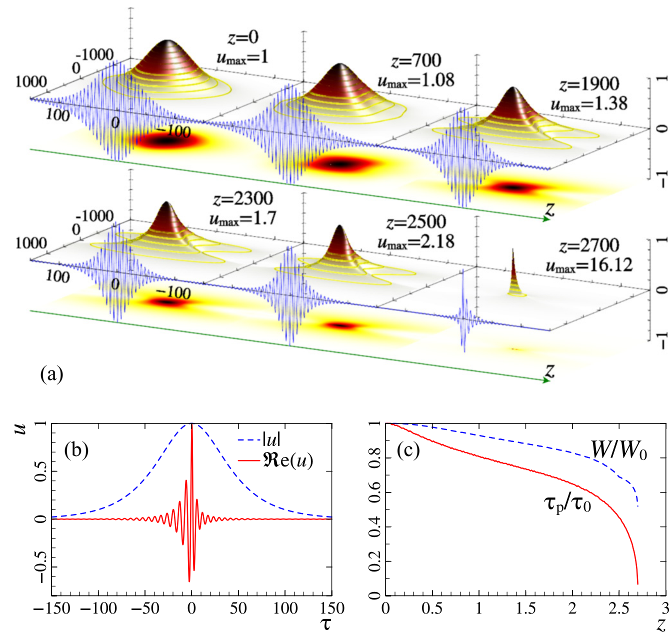

FIG. 3. (a) Dynamics of the circularly polarized field $|u(z,\tau,r)|$ with the initial profile (6) for $\delta = 0.03$, $\omega_s = 1$, and $a = 400$. The blue line shows the pulse evolution on the beam axis for one of the field components. (b) The blue dashed line is the distribution of the field envelope of the input laser pulse on the beam axis and the red line is the distribution of the field of the compressed pulse on the beam axis. (c) Pulse duration and energy of the compressed pulse (red line and blue dashed line, respectively) as functions of the z coordinate. The coordinates r, z, and τ are dimensionless according to Eq. (4).

field on the beam axis is depicted by the blue line. It can be seen that the beam size has decreased significantly due to the self-focusing effect. Based on the generalized lens approximation, it can be shown that in the case of laser pulses whose dispersion length is less than the diffraction one, the rate of shortening along the longitudinal coordinate is higher than along the transverse coordinate. Therefore, when the pulse is maximally compressed down to the field period, the beam in the transverse direction still remains wide, i.e., transverse collapse does not occur. This is clearly shown in Fig. 3(a). The analysis demonstrates that the average beam size decreases by three times, from 400 to 133, while the intensity in the compressed pulse increases by 260 times, but it is still lower than the ionization threshold.

The dashed line in Fig. 3(b) shows the initial distribution of the envelope of the pulse $|u|$ on the beam axis and the distribution of the field in the compressed pulse for $u_x = \mathrm{Re}(u)$ at the output of the nonlinear medium $z = 2700$. It can be seen that the laser pulse at the half-intensity level is compressed by a factor of 14, from $\tau_p^{\rm in} = 10T_0$ to $\tau_p^{\rm out} = 0.71T_0$, which corresponds to a duration slightly shorter than the field oscillation period. In this case, the rms duration $\tau_{\rm pulse}$ of the pulse calculated on the basis of the second moment is $\tau_{\rm pulse} = 1.1T_0$. It is shown that the pulse duration $\tau_{\rm pulse}$ averaged with respect to the transverse distribution is 1.5 times longer than the wave-packet pulse duration on the beam axis. It should be emphasized that the longitudinal scale always remains shorter than the transverse one during pulse propagation, i.e., no symmetrization of the wave-packet distribution occurs and therefore the pulse self-focusing is not replaced by the regime of spherically symmetric collapse.

The solid red line in Fig. 3(c) shows the dependence of the pulse duration on the evolution variable z, which is determined at the half-intensity level normalized with respect to the initial value. It can be seen from the plot that the pulse duration dependence is two scale. The dashed line in Fig. 3(d) shows the share of the energy contained in the compressed pulse as a function of the evolution variable z. One can see in the figure that the compressed pulse contains over 55% of the initial energy and the peak power of the compressed pulse increases by a factor of 5 as the pulse duration decreases by ten times.

V. CONCLUSION

It has been proved numerically that laser pulses with a duration less than about ten optical cycles are able to propagate in the filamentation resistant regime due to the nonlinear dispersion of the medium. This allows us to propose a method of self-compression of high-energy laser pulses under the conditions of self-focusing in the medium with anomalous group-velocity dispersion. It has been shown that when the dispersion length is shorter than the diffraction length, the pulse is compressed as a whole in all spatiotemporal domains, i.e., the soliton-type distribution shortens monotonically in the process of self-focusing. Thus, for a given pulse energy we can choose such an input beam size for which the longitudinal soliton profile will adiabatically compress along all dimensions down to single-cycle duration. In this case we are able to keep the peak laser intensity below the ionization threshold.

Let us make some estimates for realistic experimental realization of the proposed laser pulse compression. The typical initial parameters [19] of an appropriate laser pulse are a wavelength of 4 μm, an initial duration of $\tau_{\text{in}} = 100$ fs, an energy of $W_{\text{in}} = 15$ mJ, and an initial beam radius of 0.15 cm. This corresponds to the power $P_{\text{in}} = 15$ GW. At this typical critical self-focusing the power is $P_{\text{cr}} \simeq 20$ MW for this frequency range (for example, a YAG plate has such a critical power). Most media in this frequency range have anomalous group-velocity dispersion, which is a prerequisite for self-compression of the wave packet in the process of the radiation self-focusing. As a result, the laser-pulse duration will decrease to $\tau_{\text{out}} \simeq 7$ fs and the pulse energy will become about $W_{\text{out}} \simeq 7$ mJ ($P_{\text{out}} = 100$ GW) after passing the YAG plate with a thickness of 1.5 cm. This case corresponds to a pulse duration smaller than the initial field period.

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