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Measuring the scrambling of quantum information

Brian Swingle,[1,2,*] Gregory Bentsen,[1] Monika Schleier-Smith,[1] and Patrick Hayden[1,2]
[1]*Department of Physics, Stanford University, Stanford, California 94305, USA*
[2]*Stanford Institute for Theoretical Physics, Stanford, California 94305, USA*

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We provide a general protocol to measure out-of-time-order correlation functions. These correlation functions are of broad theoretical interest for diagnosing the scrambling of quantum information in interacting quantum systems and have recently received particular attention in the study of chaos and black holes within holographic duality. Measuring them requires an echo-type sequence in which the sign of a many-body Hamiltonian is reversed. We illustrate our protocol by detailing an implementation employing cold atoms and cavity quantum electrodynamics to probe spin models with nonlocal interactions. To verify the feasibility of the scheme with current technology, we analyze the effects of dissipation in a chaotic kicked-top model. Finally, we propose a number of other experimental platforms where similar out-of-time-order correlation functions can be measured.

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Advances in the coherent manipulation of quantum many-body systems are enabling measurements of the dynamics of quantum information [1–4]. Notably, recent experiments [1] have corroborated the Lieb-Robinson bound, a fundamental speed limit on the propagation of signals even in nonrelativistic spin systems [5]. At the same time, new theoretical bounds have been derived from the study of black holes [6]. Consistent with their wide variety of extreme physical properties, black holes saturate several absolute limits on quantum information processing. They are the densest memories in nature [7]. They also process their information extremely rapidly [8,9] and reach a conjectured bound on the rate of growth of chaos [10].

That black holes process quantum information at all is demonstrated by the holographic principle [11,12]: a black hole in anti–de Sitter space is equivalent to a thermal state of a lower dimensional quantum field theory *without gravity* [13]. This means that certain quantum-mechanical systems [14,15] that might in principle be realizable in experiments [16] are dynamically equivalent to black holes in quantum gravity. A major open question is the extent to which familiar quantum many-body systems also behave like black holes. Besides potentially enabling experimental tests of the holographic principle, addressing this question will shed light on fundamental limits on quantum information processing.

For a quantum field theory to be the holographic dual of a black hole, its dynamics must be highly chaotic [10]. To quantify the rate of growth of chaos, recent work has explored an inherently quantum-mechanical version of the butterfly effect, namely, the growth of the commutator between two operators as a function of their separation in time [6]. This growth is indicative of a process known as scrambling [6,8,9,17], wherein a localized perturbation spreads across a quantum many-body system's degrees of freedom, thereby becoming inaccessible to local measurements. The time scale for scrambling is theoretically distinct from the relaxation time and has yet to be probed in any experiment.

In this Rapid Communication, we propose a broadly applicable protocol for measuring scrambling. We describe a concrete implementation with cold atoms coupled to an optical cavity, a versatile platform for engineering spin models with nonlocal interactions [18–22] that have the potential to exhibit fast scrambling dynamics at or near the chaos bound [8,23]. We show that a realistic measurement—including coupling to the environment—can distinguish between time scales for relaxation and scrambling in a globally interacting chaotic "kicked-top" model, a special case of the nonlocal models accessible in the cavity-QED setting. We also discuss prospects for measuring scrambling in local Hamiltonians using trapped ions, Rydberg atoms, or ultracold atoms in optical lattices.

We emphasize the generality of our approach because scrambling, while hitherto difficult to study outside the black-hole context, is of broad importance in quantum many-body dynamics. Probing scrambling in diverse physical systems could elucidate links between chaos and fast computation [24], reveal novel ways of robustly hiding quantum information in nonlocal degrees of freedom [17], uncover new bounds on transport coefficients [25–27], and offer insight into closed-system thermalization. While identifying mechanisms that promote scrambling could aid in designing many-body systems dual to black holes, measuring scrambling may also illuminate under what conditions holographic duality is useful for understanding more conventional many-body systems.

We access the scrambling time via the decay of an out-of-time-order (OTO) correlation function [6,28]

$$F(t) = \langle W_t^\dagger V^\dagger W_t V \rangle, \tag{1}$$

where V and W are commuting unitary operators and $W_t = U(-t) W U(t)$ is the Heisenberg operator obtained by time evolution $U(t) = e^{-iHt}$ (setting $\hbar = 1$) under a Hamiltonian H. Physically, F describes a gedanken experiment in which we are able to reverse the flow of time. We compare two quantum states obtained by either (1) applying V, waiting for a time t, and then applying W; or (2) applying W at time t, going back in time to apply V at $t = 0$, and then letting time resume its forward progression to t [29,30]. The correlator F measures the overlap between the two final states. In a many-body system with a nontrivial interaction Hamiltonian H, $F(t)$ diagnoses the spread of quantum information by measuring how quickly

*bswingle@stanford.edu

the interactions cause initially commuting operators V and W to fail to commute: $\langle |[W_t,V]|^2\rangle = 2(1-\mathrm{Re}[F])$.

Because F is always one in the absence of noncommutativity, it may be regarded as an intrinsically quantum-mechanical variant of the Loschmidt echo [31], a paradigmatic probe of chaos. The Loschmidt echo, which depends on time-ordered correlation functions, has been measured in several pioneering experiments [31–36]. Its decay can be related to the mean Lyapunov exponent of a corresponding chaotic classical system and to decoherence [37–39]. By comparison, the decay rates of OTO correlators depend not only on Lyapunov exponents [40] but also on the number of degrees of freedom: the higher the entropy, the slower the decay.

The growth of the scrambling time with entropy $\mathbb{S}$ can be understood from a model of $\mathbb{S}$ qubits (spins) evolving unitarily in discrete time [23]. Intuitively, if we allow arbitrary pairwise interactions, the fastest way to delocalize information is to apply random two-body unitaries between $\mathbb{S}/2$ random pairs of spins at each time step of length τ. A single time step suffices to make simple time-ordered autocorrelation functions decay, i.e., τ is the relaxation time. Scrambling, however, requires information in one spin to spread to all the spins. With information spreading exponentially fast to $2,4,\ldots,2^{t/\tau}$ spins, the scrambling time is then $t_* = \tau \log_2 \mathbb{S}$.

The relevance of the OTO correlator F for accessing the scrambling time [6,10,17,28] can be seen from the same random-circuit model. For operators W and V that perturb individual spins, the typical time for $\langle |[W_t,V]|^2\rangle$ to become order unity is t_* because W_t is supported on approximately $2^{t/\tau}$ spins. Similarly, for black holes in Einstein gravity, scrambling occurs exponentially fast and is accompanied by an initial growth $1-F \sim e^{t/\tau}/\mathbb{S}+\cdots$, where the relaxation time $\tau = 1/(2\pi T)$ is set by the temperature T [41]. The resulting decay time $t_* = \tau \ln \mathbb{S}$ is conjectured to be a fundamental bound for thermal states of time-independent Hamiltonians [10]. Identifying bounds on scrambling in time-dependent models or in nonthermal states is an open problem, which experimental study of OTO correlators will help to address.

A key capability required to measure the OTO correlator $F(t) = \langle W_t^\dagger V^\dagger W_t V\rangle$ is that of reversing the sign of the Hamiltonian. Obtaining full information about $F(t)$ additionally requires many-body interferometry, similar to schemes in Refs. [42–46]. A control qubit can be used to produce two branches in the many-body state [45–47]; measurements of the control qubit then reveal F [Fig. 1(a)]. Even without the control qubit, an alternative protocol [Fig. 1(b)] suffices to measure the magnitude of F, which quantifies the indistinguishability of two states obtained by applying V and W_t in differing order.

As perfect time reversal is impossible in practice, our protocol is the experimentally reasonable one: the Hamiltonian dynamics is reversed but dissipation is not. It is thus important to establish that observables obtained from this partial time reversal access the same physics as the analogous observables in the unitary protocol. We show that such a regime is achievable in the cavity model with current technology.

General protocol. Consider a quantum system $\mathcal{S}$ initialized in state $|\psi\rangle_{\mathcal{S}}$. Our goal is to measure the four-point function $F(t) = \langle W_t^\dagger V^\dagger W_t V\rangle$, where V and W are simple unitary operators acting on $\mathcal{S}$ which initially commute. For F to be

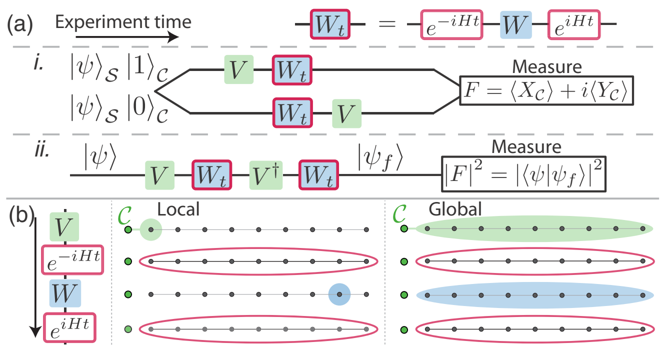

FIG. 1. (a) Protocols for measuring $F(t)$. (i) Given a control qubit $\mathcal{C}$, the *interferometric protocol* can measure F for the system $\mathcal{S}$ by applying different sequences of operators in the two interferometer arms. (ii) Without a control qubit, the *distinguishability protocol* can access $|F|^2$. (b) $F(t)$ can be measured with either local or global operators V,W, as shown for a spin chain in the upper branch of the interferometer, with control qubit $\mathcal{C}$ in green (left of chain).

nontrivial, the time evolution $U(t) = e^{-iHt}$ must be governed by a many-body Hamiltonian H containing interactions between different degrees of freedom. The Heisenberg operator $W_t = U(-t)WU(t)$ then grows in complexity as t increases and eventually fails to commute with V.

The *interferometric protocol* for measuring F employs a control qubit $\mathcal{C}$ initialized in state $|{+X}\rangle_{\mathcal{C}} = \frac{|0\rangle_{\mathcal{C}}+|1\rangle_{\mathcal{C}}}{\sqrt{2}}$. First apply the gate sequence [illustrated in Fig. 1(a)]

$$[1]:\ I_{\mathcal{S}} \otimes |0\rangle\langle 0|_{\mathcal{C}} + V_{\mathcal{S}} \otimes |1\rangle\langle 1|_{\mathcal{C}},$$
$$[2]:\ U(t)_{\mathcal{S}} \otimes I_{\mathcal{C}},$$
$$[3]:\ W_{\mathcal{S}} \otimes I_{\mathcal{C}},$$
$$[4]:\ U(-t)_{\mathcal{S}} \otimes I_{\mathcal{C}},$$
$$[5]:\ V_{\mathcal{S}} \otimes |0\rangle\langle 0|_{\mathcal{C}} + I_{\mathcal{S}} \otimes |1\rangle\langle 1|_{\mathcal{C}}$$

to prepare the state

$$\frac{(VW_t|\psi\rangle_{\mathcal{S}})|0\rangle_{\mathcal{C}} + (W_tV|\psi\rangle_{\mathcal{S}})|1\rangle_{\mathcal{C}}}{\sqrt{2}}. \tag{2}$$

Then measure the control qubit in the X and Y bases to find the real and imaginary parts of the OTO correlator

$$F = \langle X_{\mathcal{C}}\rangle + i\langle Y_{\mathcal{C}}\rangle, \tag{3}$$

where $X_{\mathcal{C}}$ and $Y_{\mathcal{C}}$ denote Pauli matrices acting on $\mathcal{C}$.

Even without a control qubit, it is possible to measure the magnitude of F using the *distinguishability protocol*. Initialize the system into state $|\psi\rangle$. Apply the gate sequence shown in Fig. 1(b) to prepare the state $|\psi_f\rangle = W_t^\dagger V^\dagger W_t V|\psi\rangle$. Finally, measure the projector $\Pi = |\psi\rangle\langle\psi|$. The result is $\langle\psi_f|\Pi|\psi_f\rangle = |F|^2$, which quantifies the distinguishability of the two branches and is expected to contain roughly the same time scales as F. As the projection Π onto an arbitrary many-body state can be challenging to implement, the distinguishability protocol requires a careful choice of the initial state $|\psi\rangle$.

Cavity-QED proposal. As a representative system amenable to probing the OTO correlator, we consider a collection of

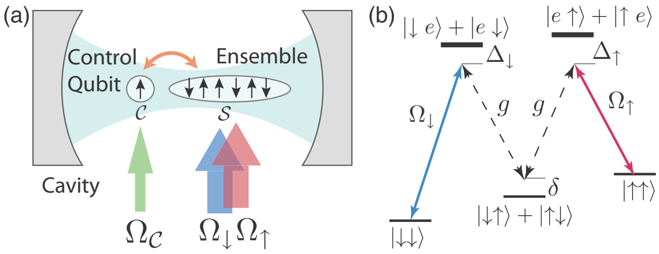

FIG. 2. Scheme for measuring out-of-time-order correlators. (a) Atomic ensemble $\mathcal{S}$ and control qubit $\mathcal{C}$ in an optical cavity are driven by control fields $\Omega_\uparrow,\Omega_\downarrow,\Omega_\mathcal{C}$. (b) Control fields $\Omega_{\uparrow,\downarrow}$ and cavity coupling g mediate pairwise interactions in the ensemble $\mathcal{S}$ via four-photon Raman transitions.

two-level atoms (spins) that interact via their mutual coupling to one or more modes of an optical cavity (Fig. 2). A drive laser incident from the side of the cavity generates interactions among all pairs of atoms it addresses. The sign of the interactions is set by the laser frequency, enabling access to the magnitudes of OTO correlators via the distinguishability protocol. To access the phase, a single individually addressable atom can serve as a control qubit for interferometry [47].

The cavity-mediated interactions within the ensemble generically take the form of a nonlocal spin model [18,20,21]

$$H = \sum_{ij} J_{ij} s_i^x s_j^x + \text{H.c.}, \tag{4}$$

where $\mathbf{s}_i$ is a pseudospin operator representing two internal atomic states (e.g., hyperfine states) $|s_i^z = \pm 1/2\rangle$. For N atoms at positions r_i with couplings $g_\alpha(r_i)$ to a set of degenerate cavity modes indexed by α, the spin-spin couplings are given by

$$J_{ij} = \sum_\alpha \frac{\Omega_\uparrow^*(r_i)\Omega_\downarrow(r_j)}{\Delta_\uparrow \Delta_\downarrow} \frac{g_\alpha(r_i) g_\alpha^*(r_j)}{\delta}, \tag{5}$$

where $\Omega_{\uparrow,\downarrow}$ are the Rabi frequencies of the drive fields, detuned by $\Delta_{\uparrow,\downarrow}$ from atomic resonance, and δ is the detuning of the two-photon transition mediated by the drive fields and cavity couplings g_α.

Key features of the light-mediated interactions are that their sign is controllable via the two-photon detuning δ [48], they can be switched on and off, and the full graph of interactions can depend on the atomic positions and the spatial structure of the cavity modes and control fields. Also, it is possible to produce noncommuting s^+s^- type interactions, to add fields in any direction, and to include time dependence in the Hamiltonian. This versatility allows for studying a range of many-body phenomena, from quantum glasses [20,21,49] to random circuit models that mimic the fast scrambling of black holes [23].

For ease of visualization, we focus here on globally interacting spin models obtained by coupling all atoms uniformly to a single cavity mode. Here, the Hamiltonian of Eq. (4) reduces to a "one-axis twisting" Hamiltonian $H_\text{twist} = \chi S_x^2$, where $\mathbf{S} = \sum_i \mathbf{s}_i$ and the total spin is $S = N/2$. By considering correlators where the operations V and W are global spin rotations, we restrict the dynamics to a space of permutation-symmetric states that are conveniently described by quasiprobability distributions on a Bloch sphere (Fig. 3).

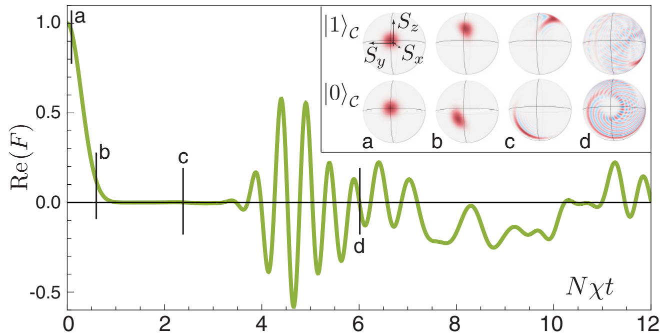

FIG. 3. Interferometric protocol for unitary S_x^2 dynamics at $N = 50$. For an initial coherent state $|\hat{y}\rangle$ and rotation angle $\phi = \pi/4$, Re[F] (green) exhibits decay at short times [(a),(b)], a quiescent period (c), and subsequent oscillations (d). Inset: states of the two interferometer arms at various times, illustrated by Wigner quasiprobability distributions.

To perform the controlled-V step in the interferometer of Fig. 1(a), we convert the control qubit state $|n\rangle_\mathcal{C}$ (with $n \in \{0,1\}$) into an n-photon state of the cavity, which produces a differential ac-Stark shift $\propto n$ between each of the ensemble atoms' two levels. The result is a collective controlled phase gate

$$Z_\phi^\mathcal{C} = I_\mathcal{S} \otimes |0\rangle\langle 0|_\mathcal{C} + e^{-i\phi S_z^\mathcal{S}} \otimes |1\rangle\langle 1|_\mathcal{C}, \tag{6}$$

where $S_z^\mathcal{S} = \sum_i s_i^z$. The rotation W, by contrast, is applied irrespective of the control qubit state.

Figure 3 shows calculated results of the interferometric protocol for the one-axis twisting model with an initial state $|\hat{y}\rangle = |S_y = S\rangle$ and rotations $V = W = e^{-i\phi S_z}$ with $\phi = \pi/4$. Such a large controlled rotation is neither necessary nor sustainable at higher atom numbers, as discussed below, but it illustrates in exaggerated form the processes controlling F. The initial decay of F corresponds to the collective spin's trajectory on the Bloch sphere diverging between the two interferometer arms, as shown by Wigner quasiprobabilty distributions [50] in Fig. 3. Later fluctuations in F correspond to the spread of the quantum state over the entire Hilbert space.

To further illuminate the physics of the OTO correlator, the S_x^2 Hamiltonian may be modified to produce a chaotic system. Periodically applying a rapid S_z rotation produces a kicked-top model that has been studied both theoretically and experimentally [51–53]. The stroboscopic dynamics are described by repeated application of the unitary operator $U = e^{-ikS_x^2/(2S)}e^{-ipS_z}$, where k measures the strength of interactions and p measures the size of the rotational kick. Following Haake *et al.* [51], we set $p = \pi/2$; then the corresponding classical model describes motion on the Bloch sphere which is regular for small k and chaotic for large k. The semiclassical limit is the limit of large S, whereas previous experimental work has studied the case $S = 3$ [52]. The cavity-QED implementation proposed here, where the spin is scalable from the small-S quantum regime to the semiclassical limit, provides an ideal testbed for probing the physics of the OTO correlator in a paradigmatic chaotic system.

We compare the OTO correlator $F(t)$ with a time-ordered correlator $G(t) = \langle V_t^\dagger V\rangle$, similar to a Loschmidt echo, for the

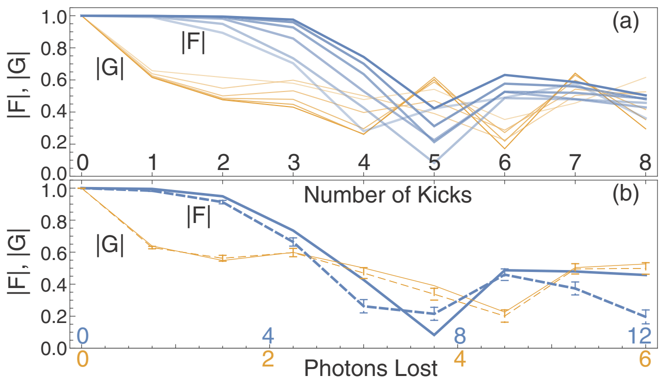

FIG. 4. Interferometric protocol for the kicked top. (a) Unitary time-ordered correlators $|G(t)|$ (thin yellow) and out-of-time-order correlators $|F(t)|$ (thick blue) for atom numbers $N = 50,100,200,300,400,500$ (light to dark), $k = 3$, $\phi = 1/\sqrt{N}$, and initial state $e^{-iS_y\pi/4}e^{-iS_z\pi/4}|\hat{x}\rangle$. (b) Unitary (solid) and dissipative (dashed) evolution of $|G(t)|$ (thin yellow) and $|F(t)|$ (thick blue) for $N = 100$. Dissipative evolution is calculated at $\eta = 100$ and $\delta = 10\kappa$ from 200 quantum trajectories; error bars are statistical. Horizontal axes show kick number (black) and mean number of photons lost by decay processes in measuring F (blue) and G (yellow) [54].

kicked top at several atom numbers $N = 2S$ in Fig. 4. We take V and W to be S_z rotations by a small angle $\phi = 1/\sqrt{2S}$, which is chosen so that we can expect to observe a separation of time scales between time-ordered and OTO correlators as $S \to \infty$ [54]. We plot both correlators for an initial coherent state $e^{-iS_y\pi/4}e^{-iS_z\pi/4}|S_x = S\rangle$ and kick strength $k = 3$, first assuming unitary dynamics. Even at finite N, the OTO correlator (blue) decays on a significantly longer time scale than the time-ordered correlator (yellow). While the decay time for the time-ordered correlator is roughly independent of N, the decay time for the OTO correlator grows as $\log(N)$. This scaling is consistent with a butterfly effect wherein the initial coherent spin state of solid angle $1/N$ exponentially expands on the Bloch sphere.

The measurement of F can be compromised by two forms of dissipation: leakage of photons from the cavity of linewidth κ; and decay from the atomic excited state of linewidth Γ. The fidelities of the controlled phase gate and of the time-reversed Hamiltonian are thus limited by the cooperativity $\eta = 4g^2/\kappa\Gamma$, where $2g$ is the vacuum Rabi frequency. For an ensemble of N atoms, the maximum achievable controlled phase rotation is of order $\sqrt{\eta/N}$, while observing the onset of chaos in the kicked top requires $\eta \gtrsim (k/2 \ln N)^2$ [54]. Thus, dissipation can be kept small at atom numbers $N \lesssim 10^2$ in a state-of-the-art strong coupling cavity with $\eta \sim 10^1$–10^2 [55,56], but it cannot be entirely neglected.

To verify that a realistic nonunitary evolution suffices to estimate the OTO correlator, we simulate measurements of F and G in the kicked-top model using quantum trajectories [54]. The results of the interferometric protocol are plotted in Fig. 4(b) for a cavity cooperativity $\eta = 100$ (dashed lines) and compared with the unitary case (solid lines). The early-time dissipative evolution is faithful to the unitary evolution, and the difference in time scales between F and G can easily be resolved. Fully investigating the dissipative effects, by experimental study of longer times and larger atom numbers, may shed new light on chaos and the quantum-to-classical transition in many-body systems.

Outlook. Observing the early-time physics of the OTO correlator in state-of-the-art cavities will allow for probing scrambling in diverse spin models with nonlocal interactions. Realizing a model known to have the scrambling properties of a black hole remains a highly nontrivial task. However, Kitaev has designed one such model, involving random four-fermion interactions [57], that is a close relative of random nonlocal spin models proposed to study quantum spin glasses [49] in multimode cavities [20,21]. Here, periodically modulating external fields or interactions might promote scrambling by melting glass order or by simulating multispin couplings [58].

While fast scrambling is a necessary condition for duality to a black hole, whether it is also a sufficient condition is an open question. If so, then the OTO protocol provides a sharp experimental test for the presence of a black hole by revealing a universal Lyapunov exponent that can be compared with the chaos bound [6,10]. A recently proposed cold-atom realization of Kitaev's model [59] could be a platform for validating such a test.

In other physical systems, measurements of the OTO correlator could reveal emergent low-energy bounds on information propagation [26] and distinguish between single-particle [60–62] and many-body [63,64] localization. Our protocol can be translated directly to trapped-ion simulations of transverse-field Ising models with tunable range [2,3,65]. Neutral Rydberg atoms also allow for engineering local spin models with either sign of interaction [66–68] and qubit-controlled rotations [46]. In optical-lattice implementations of Hubbard models, the sign of interactions can be controlled using Feshbach resonances, the sign of the hopping can be changed by modulating the lattice [69–71], and a controlled phase shift can be imprinted using an impurity atom [42,43] or a locally addressed control atom [72,73]. Alternatively, the distinguishability protocol can be performed by time-of-flight or *in situ* imaging for special initial states, e.g., superfluids or Mott insulators in two dimensions.

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