

**$\mathcal{PT}$ -symmetric slowing down of decoherence**Bartłomiej Gardas,<sup>1,2</sup> Sebastian Deffner,<sup>1,3,4</sup> and Avadh Saxena<sup>1,4</sup><sup>1</sup>*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*<sup>2</sup>*Institute of Physics, University of Silesia, 40-007 Katowice, Poland*<sup>3</sup>*Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21250, USA*<sup>4</sup>*Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

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We investigate  $\mathcal{PT}$ -symmetric quantum systems ultraweakly coupled to an environment. We find that such open systems evolve under  $\mathcal{PT}$ -symmetric, purely dephasing and unital dynamics. The dynamical map describing the evolution is then determined explicitly using a quantum canonical transformation. Furthermore, we provide an explanation of why  $\mathcal{PT}$ -symmetric dephasing-type interactions lead to a critical slowing down of decoherence. This effect is further exemplified with an experimentally relevant system, a  $\mathcal{PT}$ -symmetric qubit easily realizable, e.g., in optical or microcavity experiments.

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*Introduction.* Symmetry is one of the most important and profound concepts in physics [1,2] which explains the *modus operandi* of many complex physical and biological systems [3]. It expresses how systems remain unaffected by perturbations [4]. Therefore, a violation of symmetry (or its breakdown [5]) constitutes an irreplaceable source of valuable information regarding the properties of physical systems [6–8]. There is an abundance of useful transformations providing the necessary ingredients to understand and investigate quantum systems. Among them, there are two of special physical significance: the time-reversal operation  $\mathcal{T}$  [9] and parity—a mirror-reflection symmetry— $\mathcal{P}$  [10]. These two transformations are both Hermitian and independent of each other, i.e.,  $[\mathcal{P}, \mathcal{T}] = 0$ . Systems that are invariant under the joint  $\mathcal{PT}$  operation are called  $\mathcal{PT}$ -symmetric [11]. Such effectively open systems exhibit dynamics with balanced loss and gain [12,13]. Recent results have proven to be of great theoretical [14–17] and experimental [18–21] importance, and  $\mathcal{PT}$ -symmetric quantum systems have been realized in many different setups, such as optical [22], optomechanical [23], or microcavity-based experiments [12,24].

Contemporary studies have revealed that important (non)equilibrium properties and thermodynamic relations also hold for  $\mathcal{PT}$ -symmetric quantum systems, e.g., the Carnot theorem [14,25] and the Jarzynski equality [26,27]. Nevertheless, to further advance our understanding of  $\mathcal{PT}$ -symmetric quantum systems, the next natural step is to understand decoherence [28]. This is particularly important when one wants to store and process information in quantum systems [29–31].

A comprehensive description of the system's dynamics requires tracing out the environmental degrees of freedom. Unfortunately, except for a few analytically solvable models, finding such reduced dynamics  $\rho_S(t)$  has proven to be extremely complicated, and often impossible, even for Hermitian systems [32]. Recently, it has been shown that all  $\mathcal{PT}$ -symmetric quantum systems that admit a real spectrum can be represented in a physically equivalent way by Hermitian Hamiltonians [33]. One would therefore expect them to be influenced by decoherence in a similar manner. In this Rapid Communication, however, we demonstrate features that are

unique to  $\mathcal{PT}$ -symmetric systems, resulting from the way they interact with their environment. In particular, we investigate a  $\mathcal{PT}$ -symmetric quantum system coupled ultraweakly to a Hermitian environment [34]. Our motivation is twofold: First, very weak coupling guarantees that no heat is exchanged between the system and environment [35,36]. This leads to a phenomenon known as pure decoherence or dephasing [37,38]. Only quantum information is allowed to enter or leave the system so that any effect caused solely by decoherence can be quantified easily. Finally, following the Ockham's razor principle [39], Hermiticity of the environment is assumed for the sake of simplicity and transparency of our description.

Under these assumptions, we find that such open systems evolve under  $\mathcal{PT}$ -symmetric, purely dephasing and unital dynamics. The dynamical map describing the evolution is then determined explicitly using a quantum canonical transformation. Therefore, as an immediate consequence of dephasing and unital dynamics, we find the validity of the Jarzynski equality [40]. Furthermore, we explain how a  $\mathcal{PT}$ -symmetric dephasing channel leads to a *critical slowing down of decoherence*. This effect is exemplified using an experimentally relevant example, a  $\mathcal{PT}$ -symmetric qubit. Such a two-level system can be realized, e.g., in optics [19] or in a microcavity [21]. In particular, the development of practical architectures for quantum computer systems with minimal or suppressed decoherence is appealing [41,42]. We will see that  $\mathcal{PT}$ -symmetric qubits are thus significantly better suited than standard, Hermitian qubits [43].

*Pure decoherence in  $\mathcal{PT}$ -symmetric quantum systems.* Consider a  $\mathcal{PT}$ -symmetric quantum system  $S$  interacting with its environment  $B$ . The composite system  $S + B$  can be described by the following Hamiltonian,

$$H = H_S \otimes \mathbb{I}_B + \mathbb{I}_S \otimes H_B + H_I, \quad (1)$$

where  $H_S$  and  $H_B$  are the Hamiltonians of the system and the environment, respectively, and  $H_I$  describes the interaction between them. In the following, we assume the usual form of the interaction  $H_I = V_S \otimes V_B$ , where both  $H_B$  and  $V_B$  are Hermitian yet  $V_S$  is  $\mathcal{PT}$ -symmetric. Typical examples include  $\mathcal{PT}$ -symmetric resonators coupled weakly to the rest of the (Hermitian) universe [44]. A particularly interesting example

arises when  $V_S = g(H_S)$ , where  $g$  is an arbitrary function. Since  $[H_S, H_I] = 0$ , there is no energy exchange between the system and its environment, i.e.,  $\langle H_S \rangle$  remains constant during the evolution. Therefore, any effect of the environment on the system leads to pure decoherence [45]. Without any loss of generality, we further assume that  $g(H_S) = H_S$ .

Henceforth, we focus on  $\mathcal{PT}$ -symmetric quantum systems and show how to construct their reduced dynamics in the presence of pure decoherence. To this end, we notice that if the spectrum of the system is real, a Hermitian transformation  $T$  such that  $h_S = TH_S T^{-1}$  is Hermitian can always be found [14]. Moreover, since  $h_S$  is Hermitian, we also have  $H_S^\dagger = T^2 H_S T^{-2}$ , which will be crucial for our analysis. We will prove this shortly. Here, we only note that in order to change Hermiticity, such a transformation cannot be unitary. However,  $T$  preserves (canonical) commutation relations (e.g., between  $x$  and  $p$ :  $[x, p] = i\hbar$ ) and therefore will be regarded as a quantum canonical transformation [46,47]. More importantly, canonical transformations do *not* change the expectation values of observables,  $\langle O_S \rangle_{\mathcal{PT}} = \langle O_S \rangle_H$ , where  $o_S = T O_S T^{-1}$ .

Now, applying the canonical transformation  $T$  to the Hamiltonian (1) yields

$$h = h_S \otimes \mathbb{I}_B + \mathbb{I}_S \otimes H_B + h_I, \quad h_I = T H_I T^{-1}, \quad (2)$$

where  $T$  acts nontrivially only on the system of interest. Since the two systems are now Hermitian, their composed dynamics is described by the Liouville–von Neumann equation of motion,  $i\dot{\rho}(t) = [h, \rho(t)]$ , whose unique solution can be written as [32]

$$\rho(t) \rightarrow U(t)\rho(0)U(t)^\dagger, \quad U(t) = \exp(-iht). \quad (3)$$

At any given time  $t$ , the reduced system's dynamics is determined by tracing out the environmental degrees of freedom (see, e.g., Ref. [48]). Thus, one can write

$$\rho_S(t) = \text{tr}_B\{U(t)\rho_S(0) \otimes \Omega_B U(t)^\dagger\}, \quad (4)$$

where  $\Omega_B$  is the initial state of the environment and  $\text{tr}_B\{\cdot\}$  denotes the partial trace [48]. Note that the two systems are uncorrelated at  $t = 0$  [49]. This requirement is crucial for the map  $\Phi$ ,  $\rho_S(t) = \Phi[\rho_S(0)]$ , to be well defined [50]. However, this is not difficult to fulfill experimentally [51].

Since  $\Omega_B$  is a density operator, it can be expressed as  $\Omega_B = \sum_\alpha p_\alpha |\alpha\rangle\langle\alpha|$ , where  $p_\alpha$  denotes the probability of finding the environment in state  $|\alpha\rangle$ . As a result, the reduced dynamics (4) can be rewritten using the so-called operator-sum representation [52],

$$\rho_S(t) = \sum_i K_i(t)\rho_S(0)K_i(t)^\dagger, \quad (5)$$

where the Kraus operators  $K_i(t) = \sqrt{p_\alpha}\langle\beta|U(t)|\alpha\rangle$  satisfy  $\sum_i K_i(t)^\dagger K_i(t) = \mathbb{I}_S$ . To simplify notation we have combined the two indices  $\alpha, \beta$  into  $i$ . Moreover, Eq. (5) defines a *unital* map, i.e.,  $\Phi[\mathbb{I}_S] = \mathbb{I}_S$  [40]. Indeed, since  $h_S$  commutes with  $h$ , we also have  $[K_i(t), K_i(t)^\dagger] = 0$  [53].

The operator-sum representation in Eq. (5) provides the most general description of decoherence and dissipation for Hermitian quantum systems, which results from the interaction with the environment. It is often referred to as a quantum

channel, i.e., a map that is completely positive and trace preserving (CPTP) [54]. When there is only one Kraus operator, the evolution is unitary [55]. Multiplying Eq. (5) from both sides by  $T^{-1}$  and  $T$ , respectively, yields

$$\rho_S(t) = T^{-1}\rho_S(t)T = \sum_i L_i(t)\rho_S(0)R_i(t), \quad (6)$$

where the left,  $L_i(t)$ , and right,  $R_i(t)$ , Kraus operators read

$$L_i(t) = T^{-1}K_i(t)T, \quad R_i(t) = T^{-1}K_i(t)^\dagger T. \quad (7)$$

We see immediately that they fulfill  $\sum_i L_i(t)R_i(t) = \mathbb{I}_S$ . The last equality assures that the  $\mathcal{PT}$ -CPTP map (7) is unital as well [56–58]. Therefore,  $\mathcal{PT}$ -symmetric, purely dephasing and unital dynamics preserve the Jarzynski equality [59]. Similar conclusions have been drawn recently for  $\mathcal{PT}$ -symmetric Schrödinger dynamics [27]. Note that when the dynamics is unitary, then  $L(t) = U(t)$  and  $R(t) = U(-t)$ , where  $U(t)$  satisfies the Schrödinger equation. We emphasize, however, that  $U(t)^\dagger \neq U(-t)$ .

In summary, Eq. (6) provides the most general description of open  $\mathcal{PT}$ -symmetric quantum systems. This is our main result. To this end, we followed the following recipe: First, one transforms the  $\mathcal{PT}$ -symmetric Hamiltonian into its Hermitian representation by using a quantum canonical transformation. Next, after solving the corresponding equation of motion, the inverse map is applied to obtain the final solution [60]. Our approach is generic and can be applied to, e.g., Lindblad master equations [61,62] or quantum Brownian motion [63,64]. Also, our strategy is not restricted just to Markovian dynamics [65]. However, for the present purposes we have chosen a model without heat exchange between the system and environment [45].

*Canonical transformation.* As we have seen, to obtain the reduced dynamics for an open  $\mathcal{PT}$ -symmetric system, one needs to construct a canonical transformation  $T$  that restores Hermiticity [14]. To this end, we assume that all energies of  $H_S$  are real and experimentally accessible. For the sake of simplicity, we also assume that the spectrum of  $H_S$  is discrete and nondegenerate. Therefore, there exists a basis  $|E_n\rangle$  in which all energies  $E_n$  can be measured. Hence,  $VH_S V^{-1} = \sum_n E_n |E_n\rangle\langle E_n|$ , where  $\langle E_n | E_m \rangle = \delta_{nm}$  and all energies  $E_n$  are real. Now, the canonical transformation  $T$  can be calculated as  $T = \sqrt{V^\dagger V}$ . Note that since  $H_S$  is not Hermitian,  $V$  is not unitary (i.e.,  $V^\dagger \neq V^{-1}$ ). To show this elegant and simple result, we first notice that  $H_S$  can also be rewritten as [66]

$$H_S = \sum_n E_n |\psi_n\rangle\langle\phi_n|. \quad (8)$$

The new eigenstates  $|\psi_n\rangle = V^{-1}|E_n\rangle$  and  $\langle\phi_n| = \langle E_n|V$  form a biorthonormal basis [67,68]. That is to say, the following orthogonality and completeness relations hold:  $\langle\psi_n|\phi_m\rangle = \delta_{nm}$  and  $\sum_n |\psi_n\rangle\langle\phi_n| = \mathbb{I}_S$ . Biorthonormality also means that  $|\psi_n\rangle, |\phi_n\rangle$  are the left and right eigenstates of  $H_S$ , respectively. The corresponding eigenenergy reads  $E_n$ . Since  $H_S$  is  $\mathcal{PT}$ -symmetric, it follows that [69]

$$\mathcal{P}H_S\mathcal{P} = \mathcal{T}H_S\mathcal{T} = H_S^\dagger. \quad (9)$$

From the last equation we have  $\mathcal{P}|\psi_n\rangle = e^{i\theta_n}|\phi_n\rangle$ , where  $\theta_n = 0, \pi$ . Now,  $T$  can be decomposed as  $T^2 = \mathcal{PC}$ , where the charge conjugation  $\mathcal{C}$  reads [69]

$$\mathcal{C} = \sum_n |\psi_n\rangle\langle\psi_n| \mathcal{P} \quad (\text{note } \mathcal{C}^2 = \mathbb{I}_S). \quad (10)$$

By construction, the charge conjugation commutes with the system Hamiltonian  $H_S$ , and thus from Eq. (9) it follows immediately that  $T^2 H_S T^{-2} = H_S^\dagger$  [70]. Finally,

$$h_S^\dagger = T^{-1}(T^2 H_S T^{-2})T = h_S. \quad (11)$$

In conclusion, the canonical map  $T$  indeed transforms a  $\mathcal{PT}$ -symmetric Hamiltonian  $H_S$  into a Hermitian one,  $h_S$ . The main results (6) and (7) hold for all  $\mathcal{PT}$ -symmetric quantum systems that admit real spectra [71].

*Critical slowing down of decoherence.* The remainder of our work is dedicated to studying an experimentally relevant example [21]. Consider a  $\mathcal{PT}$ -symmetric qubit [72],

$$H_S = \alpha\sigma_+\sigma_- + \gamma\sigma_+ + \text{H.c.} = \begin{pmatrix} \alpha & \gamma \\ \gamma^* & \alpha^* \end{pmatrix}, \quad (12)$$

where both  $\alpha$  and  $\gamma$  can be complex parameters, whereas  $\sigma_+$  and  $\sigma_-$  are the raising and lowering fermionic operators. This simple model has been extensively studied in the literature [11,27,73]. Moreover, it has also been realized experimentally both in optics [19] and semiconductor microcavities [21]. We assume the system (12) to be coupled to a bosonic heat bath at the inverse temperature  $\beta$  via a dephasing interaction. That is to say,

$$H_B = \sum_n \omega_n a_n^\dagger a_n, \quad V_B = \sum_n g_n (a_n + a_n^\dagger), \quad (13)$$

where  $a_n, a_n^\dagger$  are the bosonic creation and annihilation operators, respectively [74]. They obey the canonical commutation relation  $[a_n, a_m^\dagger] = \delta_{nm}$ . The bath's eigenmodes  $\omega_n$  and coupling constants  $g_n$  are assumed to be real. We emphasize that the above bosonic Hamiltonians are Hermitian [39]. Nevertheless, they do *not* commute, i.e.,  $[H_B, V_B] \neq 0$ . This results in nontrivial dynamics and decoherence.

In the following, we explicitly construct the Hermitian representation of Hamiltonian (12). Without any loss of generality, we can choose  $\alpha$  to be purely imaginary, i.e.,  $\alpha \rightarrow i\alpha$ ; we will also set  $\gamma = 1$ . Then, as long as  $|\alpha| \leq 1$ , the spectrum of  $H_S$  is real. It consists of two eigenvalues,  $E_{1,2} = \mp\sqrt{1 - \alpha^2}$ . Simple calculations show that [14]

$$T = U^\dagger \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} U, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix}, \quad (14)$$

where  $s_{1,2} = \sqrt{2(1 \pm \alpha)}$  and  $U$  is unitary. Therefore, the corresponding Hermitian Hamiltonian reads

$$h_S = E_1 \sigma_x = \begin{pmatrix} 0 & E_1 \\ E_1 & 0 \end{pmatrix}, \quad (15)$$

where  $\sigma_x$  is the Pauli- $x$  matrix. The resulting model describes the paradigmatic spin-boson system with effective couplings  $g_n E_1$  [75].

In what follows, we assume the initial state of the environment to be the Gibbs state,  $\Omega_B = \exp(-\beta H_B)/Z$ , where

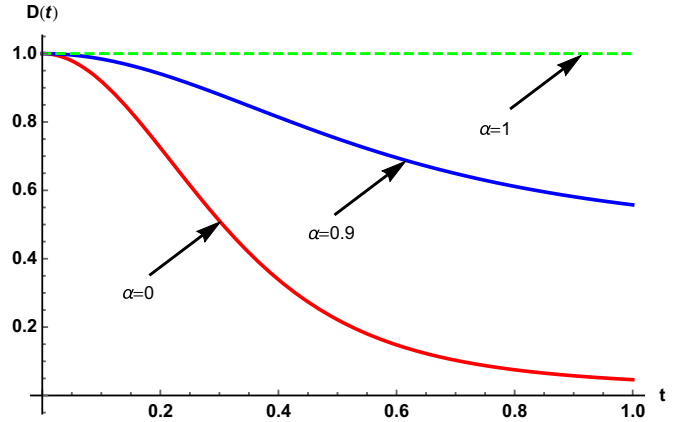


FIG. 1. Critical slowing down of decoherence. The decoherence function  $D(t)$  decays more gradually as  $\alpha$  increases. Eventually, it becomes constant at the critical point  $\alpha = 1$ . The red line corresponds to a Hermitian system ( $\alpha = 0$ ). The parameters are  $\mu = -0.5$ ,  $\beta = 0.5$ , and  $J_0 = \omega_c = 1$ . For negative  $\alpha$  the situation is symmetric [see, e.g., Eq. (18)].

$Z = \text{tr}\{\exp(-\beta H_B)\}$  is the partition function [76]. The reduced dynamics  $\varrho(t) = [\varrho_{ij}(t)]_{2 \times 2}$  can be obtained exactly [77]. Indeed, we have [78]

$$\begin{aligned} \varrho_{11}(t) &= \frac{1}{2} - \text{Re}[\varrho_{12}(0)e^{-iE_1 t}]D(t), \\ \varrho_{12}(t) &= \varrho_{11}(0) - \frac{1}{2} + i \text{Im}[\varrho_{12}(0)e^{-iE_1 t}]D(t). \end{aligned} \quad (16)$$

Moreover,  $\varrho_{22}(t) = 1 - \varrho_{11}(t)$  and  $\varrho_{21}(t) = \varrho_{12}(t)^*$  [79]. Above, symbols  $\text{Re}(z)$  and  $\text{Im}(z)$  denote the imaginary and real parts of a complex number  $z$ , respectively, whereas the decoherence function  $D(t) = \exp[-E_1^2 \gamma(t)]$  quantifies decoherence [80]. Information regarding the environment is encoded in the temperature-dependent function  $\gamma(t)$ ,

$$\gamma(t) = \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} (1 - \cos \omega t) \coth\left(\frac{\hbar\beta\omega}{2}\right), \quad (17)$$

where  $J(\omega) = \sum_n |g_n|^2 \delta(\omega - \omega_n)$  is the spectral density that characterizes the environment. Typical examples include  $J(\omega) = J_0 \omega^{1+\mu} \exp(-\omega/\omega_c)$  for some predefined constants  $J_0$ ,  $\mu$ , and  $\omega_c$  [81]. For example, when  $\mu = 0$  (Ohmic case) and  $\beta\omega_c \gg 1$ , in the long-time limit, the decoherence function behaves as (exponential relaxation [78])

$$D(t) \sim \exp[-\pi J_0(1 - \alpha^2)t/\beta]. \quad (18)$$

The reduced dynamics (16) can also be expressed by using the Kraus representation directly [78]. The reduced dynamics for the original  $\mathcal{PT}$ -symmetric qubit (12) can now be calculated as  $\rho(t) = T^{-1}\varrho(t)T$ , where  $T$  is given by Eq. (14).

Since  $D(t) \rightarrow 0$ , from Eq. (16) it is evident that the environment will eventually destroy the coherent dynamics of the system. However, this process can be controlled by changing  $\alpha$  [cf. Eq. (18)]. Indeed, as depicted in Fig. 1, decoherence becomes slower [i.e.,  $D(t)$  decays more gradually] as  $\alpha$  increases. Moreover, when  $\alpha \rightarrow 1$ , the decoherence process becomes completely suppressed [82]. However, when the system is Hermitian, i.e.,  $\alpha \rightarrow 0$ , decoherence becomes severe and quickly destroys any coherence.

To explain this phenomenon we notice that when  $\alpha > 1$ , all eigenvalues of  $H_S$  are complex. Therefore,  $\alpha = 1$  can be seen as a critical point separating two physically distinct regimes. As  $E_1 \rightarrow 0$  when  $\alpha \rightarrow 1$ , it takes longer for the system to complete one oscillation (in the Hilbert space) in close proximity to the critical point. Precisely at that point the dynamics “freezes out” completely. This critical slowing down also affects decoherence (critical slowing down of decoherence) because of the effective coupling strengths  $g_n E_1$  that also depend on  $\alpha$ . Setting  $g_n = 1/E_1$  removes the  $\alpha$  dependence from the interaction and assists decoherence [83]. At the critical point,  $\rho(t) \rightarrow \infty$ , however,  $\langle O_S(t) \rangle = \text{tr}\{\rho(0)o_S\}$  is finite. Therefore, when  $\alpha \rightarrow 1$ , expectation values are determined only by the initial condition and remain unchanged. This is due to  $h_S \rightarrow 0$ , the “freezing out” of the dynamics.

A similar dynamical behavior manifesting itself through the freezing-out scenario has already been observed in closed quantum systems. The one-dimensional (1D) Ising model [84], where the Kibble-Zurek mechanism [85,86] can be applied, provides one example; another example is the Landau-Zener problem [87] of a two-level quantum system that supports the Kibble-Zurek mechanism [88].

*Summary.* We have investigated a  $\mathcal{PT}$ -symmetric quantum system coupled to an external environment. To this end, we have considered a particular scenario where there is no heat exchange between these two systems and only quantum information is allowed to enter or leave the system. This phenomenon is known as pure decoherence or dephasing.

We have shown how to derive the reduced dynamics using a quantum canonical transformation.

Moreover, we have studied an experimentally relevant example, namely, a  $\mathcal{PT}$ -symmetric qubit. Such a system can be realized, e.g., in optics [19] and microcavities [21]. In contrast to Hermitian qubits, this system exhibits a phenomenon that we identified as a critical slowing down of decoherence. As we have argued, such behavior is characteristic of every open  $\mathcal{PT}$ -symmetric and any quantum system whose spectrum can be divided in two physically different regimes [24]. When a system approaches the critical point separating these two regimes, its dynamics “freezes out.” This critical slowing down also affects decoherence due to the dephasing interaction. Concluding, this behavior suggests that  $\mathcal{PT}$ -symmetric qubits may be more robust against decoherence and therefore are better suited as components in quantum computers [42,43,89].

Experimental setups that are sensitive enough to detect the  $\mathcal{PT}$ -symmetry breaking (i.e., the critical point) should also be able to capture the critical slowing down [24]. Thus, a critical slowing down of decoherence can be testable as well, provided there is no heat exchanged with the environment. Such induced dephasing, however, can be realized experimentally [90].

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