Collective excitability, synchronization, and array-enhanced coherence resonance in a population of lasers with a saturable absorber

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In this article we present a numerical study of the collective dynamics in a population of coupled semiconductor lasers with a saturable absorber, operating in the excitable regime under the action of additive noise. We demonstrate that temporal and intensity synchronization takes place in a broad region of the parameter space and for various array sizes. The synchronization is robust and occurs even for a set of nonidentical coupled lasers. The cooperative nature of the system results in a self-organization process which enhances the coherence of the single element of the population too and can have broad impact for detection purposes, for building all-optical simulators of neural networks and in the field of photonics-based computation.

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I. INTRODUCTION

Excitability is a fascinating phenomenon occurring in many nonlinear systems but mostly studied in biology [1], especially in relation to brain dynamics since it lies at the heart of the neuronal cells spiking activity [2]. It occurs when a system originally at an equilibrium stationary state is triggered by a perturbation, the magnitude of which exceeds a given threshold, and subsequently undergoes a big excursion in phase space associated with the generation of a spike in the time trace of one or more of its dynamical variables. After a characteristic refractory time the system comes back to the initial state and is ready to be excited again. In nonlinear optics excitability has been observed experimentally in many different systems such as active photonic crystals [3], lasers with feedback [4], semiconductor amplifiers [5], CO_2 [6], and semiconductor lasers with a saturable absorber [7]. Semiconductor lasers are versatile coherent light sources with many well-established technological applications. In the presence of an intracavity saturable absorber, they support the existence of cavity solitons [8] being a promising tool towards an all optical information processing and storage. They exhibit excitability when sufficiently big perturbations to the (close to threshold) nonlasing solution induce the generation of huge light pulses [9]. The excitable nature of such systems is due to the peculiar structure of their phase space which has the *off* solution attractor located close to a saddle-point bifurcation into a limit cycle. When strong enough perturbations make the close to threshold laser reach the saddle-point bifurcation, a big and well-defined excursion of the emitted intensity takes place: the laser is switched on and produces a high intensity pulse. The stimulated process depletes the gain and the laser comes back to the original *off* state. The recovery time of the depleted gain sets the characteristic scale of the refractory temporal interval which separates the generation of two consecutive pulses in the presence of persistent perturbations (noise). It has been demonstrated that a laser with a saturable absorber in the excitable regime exhibits coherence resonance, i.e., there exists a specific value of the noise amplitude which makes the system response the most coherent and regular; in particular it minimizes the jitter in the pulse train [9]. Among the striking features of the semiconductor laser with a saturable absorber in the excitable regime, we mention the existence of temporally excitable spatially localized cavity solitons, the recently theoretically demonstrated possibility of generating pulsating excitable solitons, and the potentiality for pulse reshaping, due to the fact that the output pulses features are independent of the intensity and duration of the triggering perturbations [10,11]. Even more interestingly some studies have demonstrated the possibility of exploiting excitability for coincidence detection and for creating optical switches [12] but also to achieve spatiotemporal pattern recognition [13,14]. Synchronization of coupled linear and nonlinear oscillators is a universal and ubiquitous phenomenon which occurs in a wealth of physical, chemical, and biological systems ranging from simple coupled pendula to nonlinear chemical reactions, including coupled lasers, neurons populations, cardiac dynamics, electrical circuits, and fireflies flocks [15,16]. A broad literature can be found on the experimental and analytical study of coupled lasers, ranging from injection-locking [17], to lasers with different frequencies [18], and to chaotic synchronization [19], as well as on synchronization of coupled excitable systems of different nature, for instance neuronal models [2,20,21]. Nevertheless a full characterization of the collective and self-organized dynamics of coupled excitable lasers under the action of random noisy perturbations, to the best of our knowledge, is still missing. Only some particular cases have been studied such as the two coupled semiconductor lasers sharing the same saturable absorber [22,23] and some configurations with very specific connectivity, low number of elements, and well-defined pulselike external perturbation [13,14].

In this article we present the first study of the collective dynamics of an array of coupled semiconductor lasers with a saturable absorber, described by the Yamada model [9,24], under the action of independent additive noise perturbations. We have characterized the dynamical properties of the array as a function of various parameters such as the intensity of the noise, the strength of the coupling, and the number of coupled oscillators. Our study demonstrates that the system self-organizes in such a way that synchronization of the pulses generated by different lasers, both in intensity and in time,

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takes place and reaches a maximum value in some specific configurations. Furthermore the cooperative nature of the interacting lasers results in a beautiful example of *array-enhanced coherence resonance* [21,25]. The performances of the single laser are improved; in particular its dynamics can result in a more stable and predictable output, which minimizes the jitter and the pulses' peak intensity fluctuations. Additionally the array-enhanced coherence resonance has the effect of increasing the synchronization inducing an effect that we define as *array-enhanced synchronization*.

II. THE MODEL

Initially we have considered a one-dimensional array of n coupled identical semiconductor lasers with a saturable absorber. The ith laser's dimensionless dynamical variables, complex electric-field amplitude F_i, inversion G_i, and absorption Q_i obey the following set of coupled nonlinear equations (the dot denotes the temporal derivative):

$$\begin{aligned} \dot{F}_i &= \frac{1}{2}(G_i - Q_i - 1)F_i + \sigma_i + \frac{K}{2}[F_{i+1} + F_{i-1} - 2F_i], \\ \dot{G}_i &= \gamma_i(A_i - G_i - I_i G_i), \\ \dot{Q}_i &= \gamma_i(B_i - Q_i - a_i Q_i I_i), \end{aligned} \tag{1}$$

where $I_i = |F_i|^2$ is the field intensity, A_i is the bias current of the gain, a_i is the differential absorption relative to the differential gain, B_i is the background absorption, while γ_i is the absorber and gain decay rate that, being much smaller than the unit, gives to the field amplitudes F_i the role of the slow variables of the system. σ_i is a delta-correlated Gaussian noise term with $\langle \sigma_i(t_1)\sigma_j(t_2)\rangle = \sqrt{2D}\delta(t_1 - t_2)\delta_{ij}$ which provides the necessary perturbations to the close to threshold *off* solution in order to induce the excitable behavior. The phenomenological coupling term K describes the exchange of radiation between first-neighbor lasers scaled to the intensity damping rate of the single laser (see [26] for the uncoupled model's normalization details). The interaction can be physically implemented using semitransparent mirrors with the desired transmittance, which couple light from one laser cavity to the adjacent ones. Periodic conditions at the array boundaries have been assumed. During all the study the value of K has been chosen to be constant along the whole array (this assumption allows us to consider it as real valued without loss of generality). We notice that the presence of the coupling term in the field equations introduces an effective discrete one-dimensional Laplace operator which acts as a diffusion, spreading the local electric-field intensity gradients which form in the laser array due to the stochastic fluctuations induced by the noise. In order to study the synchronization properties of the array we have first considered various numbers of coupled identical lasers described by the following set of parameters: $A_i = 6.5$, $B_i = 5.8$, $a_i = 1.8$, and $\gamma_i = 10^{-3}\ \forall i$. With $K = 0$ the single laser exhibits excitability for $A_i \in [6.06,6.8]$ [9].

III. TEMPORAL DYNAMICS

In the presence of coupling ($K \neq 0$), we observe the emergence of two interesting features (see Fig. 1). First of all we observe a modification of the single laser's dynamics: the pulse train's repetition rate decreases with respect to the uncoupled situation. This fact can be explained considering that the coupling acts in a diffusive fashion, redistributing the energy of the strongest noisy spikes forming in a given laser to its neighbors. Since excitability requires a particularly high perturbation in order to be triggered, if the noise is not too strong, the presence of coupling makes the crossing of the threshold less likely, with a consequent reduction of the repetition rate of the pulse train. The second feature is the manifestation of temporal synchronization, as it can be clearly qualitatively observed from the intensity temporal series depicted in Fig. 1. To investigate in a quantitative way the synchronization properties of the lasers, we have numerically simulated the system's dynamics for a total time $T = 100\,000$ and repeated the simulation for a set of pairs (K,D) chosen, respectively, in the interval [0,2] and [0.001,0.35]. The choice of weak-coupling limit is motivated by the fact that if K is too large Eqs. (1) lose their validity because the single lasers cannot be any longer regarded as independent. This procedure allowed us to obtain a map of the interesting physical observables in the (K,D) plane. In order to take into account the stochasticity of the system, for each quantity of interest a map obtained from an average of ten independent realizations has been plotted. The white regions in all the following plots in the (K,D) plane correspond to situations where less than ten pulses were observed.

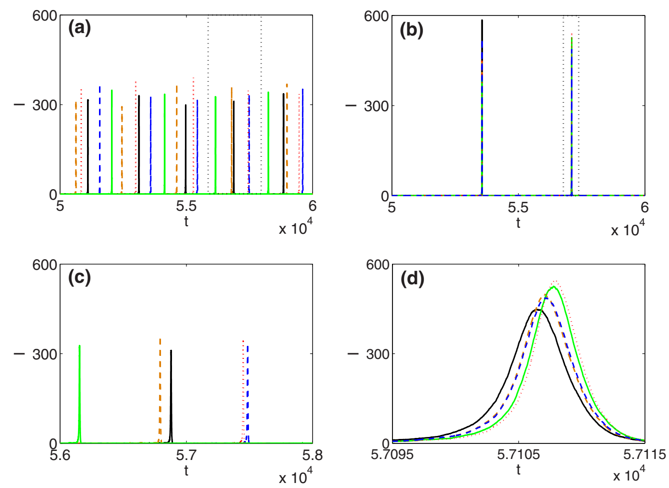

FIG. 1. (a) A time trace of the intensity of five uncoupled lasers, different colors (gray scales or line styles) corresponding to different lasers, obtained for $D = 0.015$ and $K = 0$. (b) A simulation done with the same parameters as in (a) but with $K = 0.2$ shows a clear example of temporal synchronization. (c) and (d) Zooms of the regions indicated by a dashed-line box for the uncoupled (a) and coupled (b) configuration, respectively. Both the abscissa and the ordinate in all the plots are in dimensionless units.

IV. ARRAY-ENHANCED COHERENCE RESONANCE

Before providing a characterization of the synchronization of the coupled lasers spiking activity, we give a description of the beneficial effects that the laser array has on the behavior of the single laser, in terms of pulse train jitter

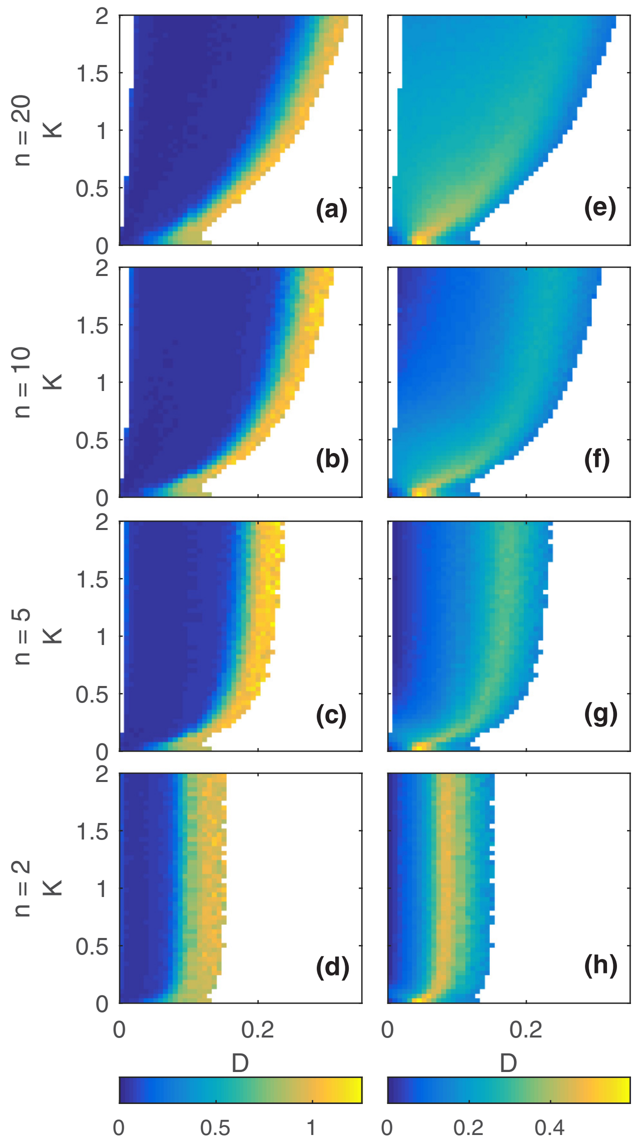

FIG. 2. The normalized jitter J, panels (a)–(d), and intensity peak standard deviation σ_I, panels (e)–(h), are plotted as functions of coupling strength K and noise intensity D in left and right columns, respectively, for values of n indicated on the figures.

reduction and of the increased uniformity of the pulses peak intensity. To this purpose we plot in the (K,D) plane the single laser jitter defined as $J = \sigma_T/\langle T\rangle$, where $\langle T\rangle$ is the average temporal interval between two consecutive pulses and σ_T is the corresponding standard deviation (see Fig. 2). By increasing K, the high coherence region [i.e., the region of low jitter depicted in dark blue (dark gray) in Figs. 2(a)–2(d)] along the D direction becomes broader and broader till reaching a saturation regime where increments of K do not lead to any further significative increase of the region. This is particularly evident for $n = 2$ and 5. For larger n, saturation is reached for higher coupling values but the low jitter region is sensibly broader: due to the collective dynamics of the whole system low jitter is achieved for a bigger set of noise amplitudes D. We are in the presence of a clear example of array-enhanced coherence resonance [21,25]. The array-enhanced coherence resonance exhibits as well a saturation process with respect to increasing n; indeed the case of $n = 20$ shows only minor differences from the case of $n = 10$. A measure of the regularity of the single laser intensity emission can be provided by the standard deviation of the pulse intensity distribution σ_I normalized to the mean pulse intensity. From Fig. 2 we can conclude that when the single laser emission is coherent in time then it is coherent in intensity too. Concerning the most extreme regimes in the excitability region we can comment that for low noise values the lasers do not emit high intensity pulses (white regions on the left in Fig. 2). For high values of D—yellow (light gray) regions of Figs. 2(a)–2(d)—the intensity traces exhibit relatively strong fluctuations between the giant pulses and no "clean" refractory time is present anymore. Nevertheless, high intensity pulses with large signal-to-noise ratio induced by the excitable nature of the lasers are still present. For very intense noise [white region on the right in Figs. 2(a)–2(h)] no clear excitable pulses can be observed.

V. SYNCHRONIZATION OF IDENTICAL LASERS

To characterize the temporal synchronization properties we used the method described in [21] which associates to each element of the array a phase value at each instant of the temporal evolution. The phase of the ith array element is defined as [27]

$$\phi_i(t) = \frac{t - \tau_k}{\tau_{k+1} - \tau_k} + 2k\pi \tag{2}$$

where τ_k is the time of the kth firing event, i.e., the position in time of the kth pulse. In order to describe the degree of synchronization the following quantity is considered:

$$s_i = \sin\left(\frac{\phi_i - \phi_{i+1}}{2}\right)^2, \tag{3}$$

which after the spatiotemporal average yields the S indicator that provides a measure of the synchronization degree:

$$S = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(\frac{1}{n}\sum_{i=1}^{n} s_i\right) dt. \tag{4}$$

The maximum synchronization is described by $S = 0$ while in complete absence of synchronization $S = 0.5$. In Figs. 3(a)–3(d) we have depicted $\log_{10}(S)$ as a function of coupling strength and noise amplitude for different values of n. Synchronization is a multifaceted concept and the temporal aspect constitutes only one side of the phenomenon. Synchronization in the intensity of the pulses emitted by different lasers has been characterized by computing the linear correlation coefficient ρ_I over the peak intensities of twin pulses in the $(K\text{-}D)$ plane. ρ_I has been calculated for the ith laser by considering, for each of its generated pulses, the linear correlation coefficient between the intensity of the pulse and the intensity of the closest (in time) pulse emitted by the $(i+1)$th laser, then averaging over all the pulses emitted by the ith laser and finally over all the lasers.

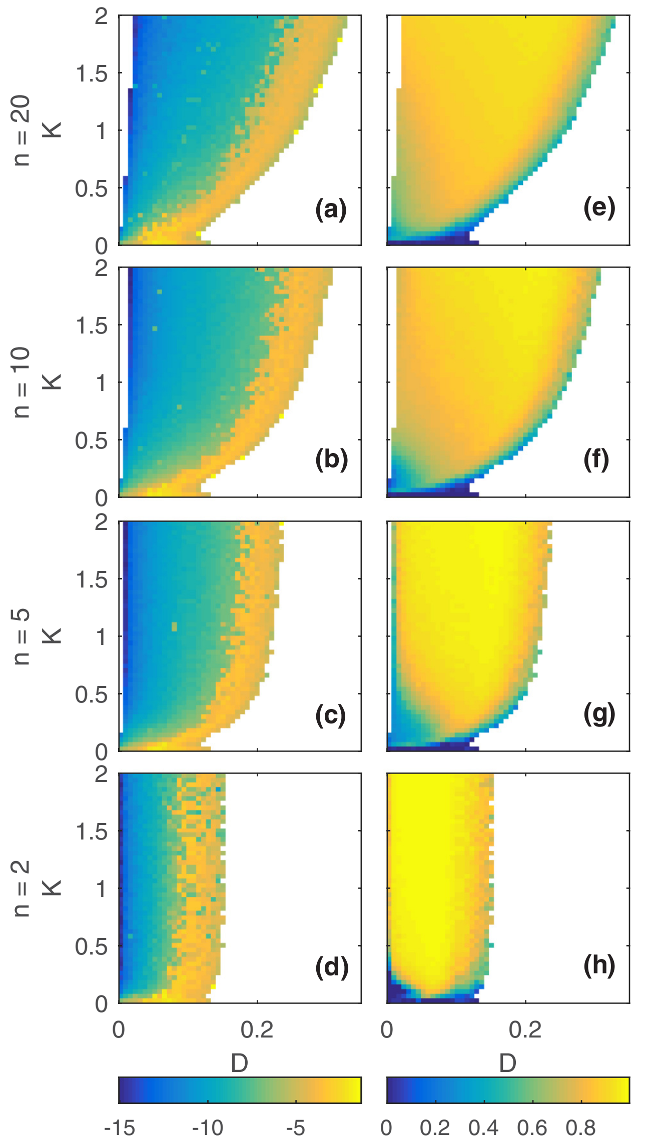

FIG. 3. $\log_{10}(S)$ is plotted in the $(K\text{-}D)$ plane for various values of n, panels (a)–(d). The minimal synchronization theoretically achievable corresponds to $\log_{10}(S) \approx -0.69$. The intensity linear correlation coefficient ρ_I is depicted, panels (e)–(h).

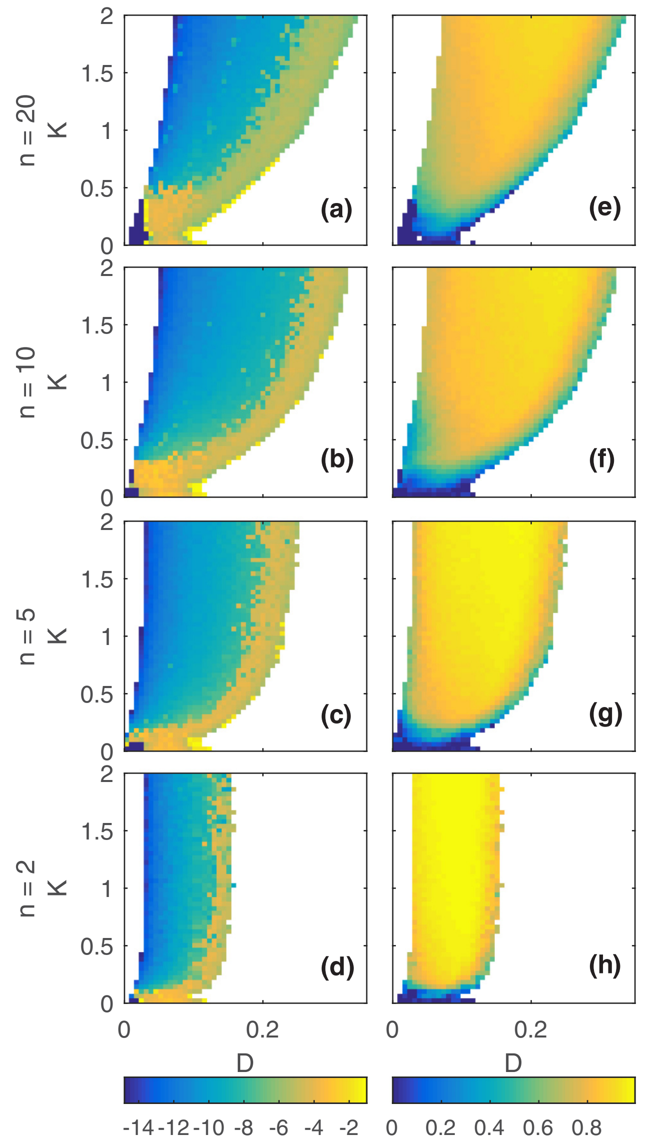

FIG. 4. The equivalent of Fig. 3 but for a population of nonidentical lasers. The large inhomogeneity among different lasers implies, as intuitively expected, the necessity of a stronger coupling in order to achieve the same degree of synchronization compared to the case of identical lasers.

An inspection of Fig. 3 reveals a striking feature. Indeed, analogously to the case of array-enhanced coherence resonance, strong temporal synchronization [that we can phenomenologically identify as starting from the green (medium gray) regions $\log_{10}(S) \approx -7$] takes place for a larger interval of values of D if the number of lasers is increased. A sufficiently strong coupling is needed if the arrays are very big. We define this phenomenon *array-enhanced synchronization*. Concerning the intensity synchronization ρ_I exhibits an almost homogeneous plateau for all the considered cases showing a good intensity synchronization in the parameters region where temporal synchronization takes place. It is interesting to notice how the synchronization in pulse intensity is maximal in the center of the excitable region, while the regularity in the pulse intensity requires low values of noise to be optimal [minimum of σ_I in Figs. 2(e)–2(h)].

VI. SYNCHRONIZATION OF NONIDENTICAL LASERS

In order to test the robustness of synchronization we have performed a characterization using the same parameters as in the case of identical lasers, but with the pump current of the ith laser, A_i, being a uniformly distributed random number generated in the interval [6.3,6.7]. Figure 4 shows that, qualitatively, the results of identical lasers are recovered for nonidentical lasers too, but with an important difference: a larger value of the coupling strength is necessary to achieve a good degree of synchronization, and for increasing population sizes the synchronization region is strongly reduced compared to the case of identical lasers.

Understanding the properties of the synchronization in a population of different lasers can be relevant for experimental studies of the phenomenon because of the unavoidable fluctuations of real-world systems where a certain degree of inhomogeneity cannot be avoided or to explore more complex scenarios. The improvement of the single laser performances, such as jitter reduction and more regular intensity emission, has been achieved for an array of nonidentical elements too, and as it happens for synchronization the inhomogeneity introduces a less regular behavior.

VII. CONCLUSIONS

In conclusion, we have demonstrated array-enhanced coherence resonance and synchronization in a one-dimensional array of coupled excitable semiconductor lasers with a saturable absorber, described by the Yamada model. The study has been conducted using realistic parameters and offers the possibility of an experimental verification which could be realized with currently available technological facilities. Other interesting properties could be revealed by considering networks with a higher dimensionality, more elaborated connectivities between the elements, and delayed coupling. We believe that our study of the collective dynamics in a population of excitable lasers can have impact on future technology: arrays of coupled excitable lasers could become an important tool for detection and measurement purposes; in particular we have demonstrated that the features of collective excitability could be tailored by varying the number of coupled elements and their parameters. Populations of coupled excitable lasers are good candidates to constitute the corner stone in the construction of integrated all-optical simulators of neural networks due to their fast dynamics and possible scalability, once synaptic plasticity will be included [28–30]; furthermore they could become the building blocks towards the development of ultrafast self-organized computational networks [31].

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