

Chiral quantum optics with V-level atoms and coherent quantum feedbackPierre-Olivier Guimond,^{1,2} Hannes Pichler,^{1,3,4} Arno Rauschenbeutel,⁵ and Peter Zoller^{1,2}¹*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*²*Institute for Theoretical Physics, University of Innsbruck, A-6020, Innsbruck, Austria*³*ITAMP, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA*⁴*Physics Department, Harvard University, 17 Oxford Street, Cambridge, Massachusetts 02138, USA*⁵*Vienna Center for Quantum Science and Technology, Atominstitut, Vienna University of Technology, 1020 Vienna, Austria*

(Received 24 June 2016; published 16 September 2016)

We study the dissipative dynamics of an atom in a V-level configuration driven by lasers and coupled to a semi-infinite waveguide. The coupling to the waveguide is chiral, in that each transition interacts only with the modes propagating in a given direction, and this direction is opposite for the two transitions. The waveguide is terminated by a mirror which coherently feeds the photon stream emitted by one transition back to the atom. First, we are interested in the dynamics of the atom in the Markovian limit where the time delay in the feedback is negligible. Specifically, we study the conditions under which the atom evolves towards a pure “dark” stationary state, where the photons emitted by both transitions interfere destructively thanks to the coherent feedback, and the overall emission vanishes. This is a single-atom analog of the quantum dimer, where a pair of laser-driven two-level atoms is coupled to a unidirectional waveguide and dissipates towards a pure entangled dark state. Our setup should be feasible with current state-of-the-art experiments. Second, we extend our study to non-Markovian regimes and investigate the effect of the feedback retardation on the steady state.

DOI: [10.1103/PhysRevA.94.033829](https://doi.org/10.1103/PhysRevA.94.033829)**I. INTRODUCTION**

The ability to engineer the coupling between quantum optical systems and photonic baths allows for many applications in quantum information [1,2], such as the preparation of single- or many-body quantum states via the dissipative emission of photons [3–5]. During the last decade, tremendous experimental progress has been made to efficiently couple atoms (either real or artificial) to one-dimensional waveguides [6–11], for instance, with real atoms coupled to optical fibers [12,13] or photonic structures [14–16]. For the last few years there has been a strong interest towards the implementation of *chiral* couplings between atoms and waveguides, where by “chiral” we mean that the coupling depends on the propagation direction of the photons in the waveguide. Several recent experiments have demonstrated such chiral couplings between quantum emitters and guided light fields, for instance, using atoms coupled to the evanescent field of whispering-gallery-modes bottle microresonators [17] and tapered optical fibers [18], or quantum dots in photonic nanostructures [19,20].

Chiral couplings allow for the formation of entangled states as *pure* steady states of the dynamics of laser-driven open systems via the dissipative emission of photons [21–23]. In particular, it was shown in Ref. [24] that an ensemble of atomic two-level systems (TLS) driven by classical fields and coupled to a unidirectional waveguide eventually stops emitting photons in the waveguide under the right conditions on the driving fields. The atoms then form EPR-correlated pairs [see Fig. 1(a)] where the photons emitted by the first atom are coherently absorbed by the second one. The atomic pair, called quantum *dimer*, now forms a pure *dark* state $\rho(t) \rightarrow |D\rangle\langle D|$ which is decoupled from the waveguide. This steady state is of the form $|D\rangle \propto |gg\rangle + \alpha(|eg\rangle - |ge\rangle)$, where $|g\rangle$ and $|e\rangle$ are the ground and excited states of each TLS and α depends on the setup parameters. Only the excited components of the

state contribute to the photon radiation in the open waveguide, and their total contribution vanishes thanks to their opposite sign.

While the long-term goal is to observe this phenomenon in the laboratory, the purpose of this work is to explore the possibility of experimenting analogous physics with a single atom, which would be achievable at the current state of the technology. The system of interest is represented in Fig. 1(b), and is constituted of a single atom coherently driven by its quantum feedback and by external lasers. The atom has a V-level configuration where the transitions are coupled to the guided modes propagating in opposite directions. The first question we want to address is thus whether the dynamics of the two atoms of Fig. 1(a) can be mimicked by the interaction of one atom with its mirror image. The pumping to a dark state in both setups can be achieved only if the coupling to external nonguided modes γ' is negligible. Recent experiments coupling atoms to waveguides (albeit without featuring chiral couplings) have reported β factors of $\beta \equiv \gamma/(\gamma + \gamma') \approx 0.5$ [25,26], with γ the coupling to the guided modes. On the other hand, very high β factors of $\beta = 0.98$ have been reached using quantum dots as artificial atoms [27]. As an alternative way to increase the coupling strength between the atom and the guided modes, we consider the setup of Fig. 1(c), where the atom is coupled with a rate g to a cavity which preserves chiral coupling [17] and is itself strongly coupled to the guided modes with a rate κ . In the bad-cavity regime $g \ll \kappa$, the atom undergoes the same dynamics as in Fig. 1(b), with $\gamma \approx (2g)^2/\kappa$ and an increased β factor [16,28].

Moreover, our system is a very simple example of coherent quantum feedback [29], and from a theory viewpoint the effect of the delay in the feedback has recently attracted a lot of interest [30–35]. In general, one of the requirements for the existence of pure atomic states is that the photon number between the mirror and the atom is negligible, otherwise, the atom

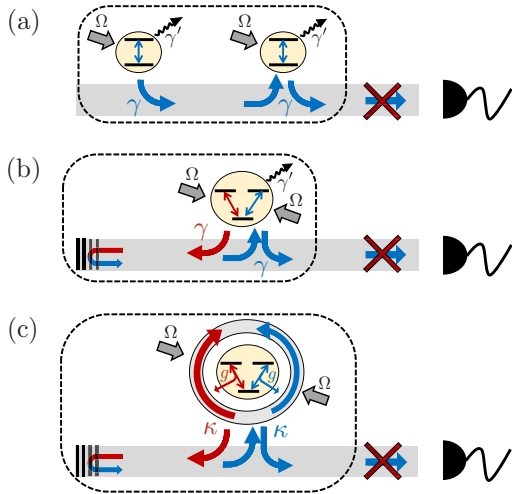


FIG. 1. (a) A pair of two-level systems (TLS) coupled to a unidirectional waveguide. The TLSs are driven by classical fields with Rabi frequencies Ω and are additionally coupled to the guided modes with a rate γ and to nonguided modes with a rate γ' which we will first assume to be negligible. Under specific conditions, the photons emitted by the left TLS are fully coherently absorbed by the right one. Remarkably, the whole system then relaxes towards a pure dark state, where the photon emission vanishes [24]. See Sec. II. (b) Atom in a V-level configuration driven with a Rabi frequency Ω and coupled to a bidirectional waveguide, where the two transitions couple to the guided modes propagating in opposite directions, represented with red and blue arrows, with the same rate γ . The photon stream propagating to the left is fed back to the system, which now acts as its own coherent absorber. Under the appropriate conditions, the system relaxes towards a pure dark state, analogously to the case described in (a). See Secs. III A and III B. (c) The cooperativity can be increased by coupling the atom with a rate g to a cavity which preserves chiral coupling [17] and is also coupled with a rate κ to the guided modes. In the bad-cavity regime $g \ll \kappa$, the system reproduces the physics described in (b), with an effective atom-waveguide coupling of $\gamma \approx (2g)^2/\kappa$ and a negligible decoherence rate γ' . See Sec. III C.

would become entangled with these photons. This assumption is equivalent to the usual *Markovian* approximation, where the retardation effects in the effective atomic dynamics, induced by the finite photon travel time, are neglected in order to derive an atomic master equation [22,23,36]. The second question we want to address is thus how this finite delay affects the properties of the steady state. In order to do so, an approach has been developed in Ref. [31], which employs matrix-product states (MPS) techniques [37] to track the entangled state of the atom and of the photons, and dynamically solves the quantum stochastic Schrödinger equation [38] (QSSE).

The paper is organized as follows. In Sec. II, we briefly review the physics of the quantum dimer formation with two-level atoms. In Sec. III, we address the dynamics of our feedback system in the Markovian limit. We derive the master equation and analyze the conditions under which the system dissipates towards a pure dark state. We also show that by coupling the atom to the cavity of Fig. 1(c) one can increase the β factor. In Sec. IV, we investigate how the steady-state properties are modified by the retardation effects

of a non-Markovian coherent feedback. Finally, in Sec. V we discuss some experimental considerations such as the effect of the coupling to external nonguided modes and the effect of an imperfect chiral coupling to the waveguide.

II. DIMERIZATION OF AN ATOMIC CHAIN

To provide the basis for the understanding of the physics of our feedback system [Fig. 1(b)], we first review the formation of entanglement in *cascaded* many-body two-level atoms [39,40], i.e., in an ensemble of atoms coupled to a unidirectional waveguide [41].

Let us first consider two atoms driven by classical fields near resonance and coupled with a rate γ to the guided modes, as represented in Fig. 1(a). The atoms are separated by a distance d along the waveguide. We consider the ideal case where the coupling to the guided modes is perfectly chiral and the coupling to nonguided modes γ' is negligible. Due to the unidirectionality of the waveguide, atom 1, on the left, does not feel the presence of atom 2, on the right. The second atom, however, is continuously driven by the coherent photon emission of the first one. If we neglect the travel time of the photons between both atoms (Markovian approximation), we can derive a master equation for the atoms, which reads as (with $\hbar = 1$) [24]

$$\frac{d\rho}{dt} = -i[H_S, \rho] + 2\gamma \mathcal{D}[\sigma_{\text{tot}}^-] \rho, \quad (1)$$

where \mathcal{D} is the Lindblad superoperator

$$\mathcal{D}[a]\rho \equiv a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}. \quad (2)$$

The Hamiltonian reads as

$$\begin{aligned} H_S = & -(\delta_1|e_1\rangle\langle e_1| + \delta_2|e_2\rangle\langle e_2|) \\ & - \frac{\Omega}{2}(\sigma_1^- + e^{i\phi'}\sigma_2^- + \text{H.c.}) \\ & + i\frac{\gamma}{2}(e^{i\phi}\sigma_1^+\sigma_2^- - e^{-i\phi}\sigma_2^+\sigma_1^-), \end{aligned} \quad (3)$$

where $|g_i\rangle$ and $|e_i\rangle$ are the ground and excited states of atom $i = 1, 2$, and $\sigma_i^- = |g_i\rangle\langle e_i|$. The first two rows of Eq. (3) are the laser driving terms, with $\delta_i = \bar{\omega} - \omega_i \ll \omega_i$ the detuning between the laser frequency $\bar{\omega}$ and the transition frequency for atom i . The drivings of the two atoms have the same Rabi frequency Ω and a relative phase ϕ' . The last row is the dipole-dipole interaction induced by the exchange of photons through the waveguide. The phase acquired by a photon traveling between both atoms is $\phi = -\bar{\omega}d/c$ where c is the speed of light in the waveguide. The unidirectionality of the problem lies in this dipole-dipole term, which is asymmetric under the exchange of the labels 1 and 2. The last term of Eq. (1) is the collective decay of the ensemble at a super-radiant rate 2γ , where the collective jump operator is

$$\sigma_{\text{tot}}^- = (\sigma_1^- + e^{i\phi}\sigma_2^-)/\sqrt{2}. \quad (4)$$

Although two phases appear in Eq. (3), only their difference $\phi - \phi'$ affects the dynamics. We can thus gauge ϕ' away in Eqs. (3) and (4) by redefining the state $|e_2\rangle \rightarrow e^{i\phi'}|e_2\rangle$ and the phase $\phi \rightarrow \Delta\phi = \phi - \phi'$. We now make the assumption that $\Delta\phi$ is a multiple of 2π , which we will refer to as the *commensurability condition*. Let us consider two scenarios. In the first scenario, the atoms are driven by different lasers

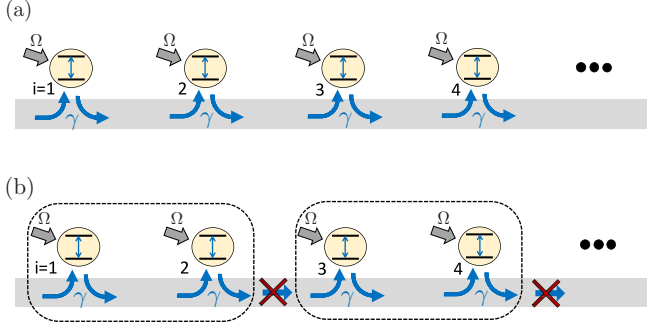


FIG. 2. (a) Ensemble of laser-driven two-level atoms coupled to a unidirectional waveguide. (b) As time evolves, the atoms form pairs. These quantum dimers are in a pure dark state and do not exchange or radiate photons.

outside of the waveguide. The relative laser phase ϕ' can be independently tuned, and should be set to ϕ in order to satisfy the commensurability. In the second scenario, the atoms are driven by the same laser propagating *inside* the fiber. In that case, ϕ' is no longer an independent parameter and is equal to ϕ by definition. Here, the commensurability is thus automatically satisfied.

We are interested in the formation of pure atomic steady states $\rho(t) \rightarrow |D_{12}\rangle\langle D_{12}|$, which have to be in particular disentangled from the waveguide. This requires that, for these states, the system should effectively decouple from the waveguide and stop radiating photons, hence, their appellation of “dark” states. In terms of Eq. (1), this means that the jump operator σ_{tot}^- , which induces decoherence, should annihilate the steady state. This restricts us to states of the form

$$|D_{12}\rangle = \frac{1}{\sqrt{1 + |\alpha|^2}} (|g_1 g_2\rangle + \alpha |S_{12}\rangle), \quad (5)$$

where $|S_{12}\rangle = (|e_1 g_2\rangle - |g_1 e_2\rangle)/\sqrt{2}$ is the “singlet” state. In order to be a steady state of the Hamiltonian H_S , one can show that the detunings have to be opposite ($\delta_1 = -\delta_2$), which yields

$$\alpha = -\sqrt{2} \frac{\Omega}{i\gamma + 2\delta_1}. \quad (6)$$

Remarkably, this result can be easily generalized to the situation represented in Fig. 2(a), where an array of $2N$ atoms is coupled to the waveguide. Provided $\delta_1 = -\delta_2$, the first two atoms will be pumped to the dark state $|D_{12}\rangle$ since they are not influenced by the presence of the other atoms thanks to the directionality of the problem. As their state converges towards this steady state, the photon stream between atoms 2 and 3 vanishes, and finally the third and fourth atoms do not feel the presence of the first two ones either, which allows them to evolve towards their dark state $|D_{34}\rangle$, provided $\delta_3 = -\delta_4$. By iterating this argument, the steady state of the atomic array will factorize into

$$|D\rangle = |D_{12}\rangle \otimes |D_{34}\rangle \otimes \dots \quad (7)$$

This is represented in Fig. 2(b). Each atomic pair in a dark state is called quantum *dimer*, and the factorization process is the dimerization of the atomic chain.

III. MARKOVIAN QUANTUM FEEDBACK

In this section we analyze the dynamics of the atom of our coherent feedback system [Fig. 1(b)] in the Markovian limit where the feedback retardation effects are neglected, and we expose the similarities and the differences with the quantum dimers of the previous section. In Sec. III A, we derive the master equation for the atom in the ideal case where the coupling to nonguided modes is negligible. In Sec. III B, we study the dissipative evolution of the system towards the steady state and derive the conditions under which this state is pure. In Sec. III C, we take the nonguided modes into account and we discuss the possibility of increasing the cooperativity in this setting by coupling the atom to a cavity in the bad-cavity regime.

A. Master equation

The system consists of a single laser-driven atom in a V-level configuration. The atom is coupled to a semi-infinite waveguide terminated by a mirror located at a distance d from the atom, as represented in Fig. 1(b). We denote the atomic ground state by $|g\rangle$, and the excited states by $|e_1\rangle, |e_2\rangle$. The transition operators are given by $\sigma_i^- = |g\rangle\langle e_i|$ ($i = 1, 2$). Additionally, the σ_1 transition is exclusively coupled to the guided modes propagating towards the mirror whereas the σ_2 transition is coupled to the modes propagating outwards. We assume that the dispersion relation of the waveguide is approximately linear around the laser frequency $\bar{\omega}$ over a relevant bandwidth $\theta \ll \bar{\omega}$ (i.e., $\omega \approx c|k|$ with ω the mode frequency, k the wave number along the propagation axis, and c the speed of light in the waveguide).

The Hamiltonian for the waveguide is thus given by (with $\hbar = 1$)

$$H_B = \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \omega b_\omega^\dagger b_\omega, \quad (8)$$

where b_ω annihilates a photon with frequency ω , and $[b_\omega, b_{\omega'}^\dagger] = \delta(\omega - \omega')$. The Hamiltonian for the driven atom reads as

$$H_a = \omega_1 |e_1\rangle\langle e_1| + \omega_2 |e_2\rangle\langle e_2| - \frac{\Omega}{2} (e^{i\bar{\omega}t} \sigma_1^- + e^{i\bar{\omega}t} e^{i\phi'} \sigma_2^- + \text{H.c.}), \quad (9)$$

where ω_i are the transition frequencies, Ω is the Rabi frequency which we assume real and positive without loss of generality, and ϕ' is the relative driving phase. We used a rotating wave approximation (RWA), which is valid for $\Omega, |\delta_i| \ll \bar{\omega}$, where $\delta_i = \bar{\omega} - \omega_i$ is the detuning between the laser and the transition frequencies. The atom-waveguide interaction Hamiltonian in the RWA is given by

$$H_{\text{int}} = i \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega g(\omega) b_\omega^\dagger (\sigma_1^- + \sigma_2^- e^{i(\pi-2\omega d/c)}) - \text{H.c.}, \quad (10)$$

where $g(\omega)$ is the atom-waveguide coupling and the phase factor is the feedback photon phase which accounts for the propagation over a distance $2d$ and for the π shift due to the mirror reflexion. Analogously to the case of Sec. II, the only physically relevant phase here is the phase difference $\pi - 2\omega d/c - \phi'$ between the feedback and the driving phases,

hence, we will gauge ϕ' away by redefining the state $|e_2\rangle \rightarrow e^{i\phi'}|e_2\rangle$. We will also assume that the coupling is approximately independent of the frequency over the relevant bandwidth $[\bar{\omega} - \theta, \bar{\omega} + \theta]$ and replace $g(\omega) \rightarrow \sqrt{\gamma/2\pi}$.

In order to derive the master equation for the atom, we move to an interaction picture with respect to the waveguide Hamiltonian H_B and to a frame rotating with the laser frequency $\bar{\omega}$ for the atomic transitions. In this picture, the total Hamiltonian now reads as $H_{\text{tot}}^I = H_a^I + H_{\text{int}}^I$, where

$$H_a^I = -\delta_1|e_1\rangle\langle e_1| - \delta_2|e_2\rangle\langle e_2| - \frac{\Omega}{2}(\sigma_1^- + \sigma_2^- + \text{H.c.}) \quad (11)$$

and

$$H_{\text{int}}^I = i\sqrt{\frac{\gamma}{2\pi}} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega b_\omega^\dagger(e^{-i(\bar{\omega}-\omega)t} \sigma_1^- + e^{-i(\bar{\omega}-\omega)(t-\tau)} e^{i\Delta\phi} \sigma_2^-) - \text{H.c.}, \quad (12)$$

with $\tau = 2d/c$ the time delay of the quantum feedback and $\Delta\phi = \pi - \bar{\omega}\tau - \phi'$ the phase difference which we will restrict to the interval $[-\pi, \pi]$ for convenience. From now on, we will drop the superscript I and always refer to the Hamiltonians in this picture. The state of the system comprising the atom and the waveguide at time t $|\Psi(t)\rangle$ is related to the initial state $|\Psi_0\rangle$ by a unitary operator $U(t)$ such that $|\Psi(t)\rangle = U(t)|\Psi_0\rangle$, which satisfies the Schrödinger equation $i\frac{d}{dt}U(t) = H_{\text{tot}}U(t)$. In the Heisenberg picture, the waveguide operators $b_\omega(t) = U^\dagger(t)b_\omega U(t)$ satisfy the Heisenberg equation

$$\frac{d}{dt}b_\omega(t) = \sqrt{\frac{\gamma}{2\pi}} [e^{-i(\bar{\omega}-\omega)t} \sigma_1^-(t) + e^{-i(\bar{\omega}-\omega)(t-\tau)} e^{i\Delta\phi} \sigma_2^-(t)], \quad (13)$$

where $\sigma_i^-(t) = U^\dagger(t)\sigma_i^- U(t)$. On the other hand, the Heisenberg equation for an operator $a(t)$ acting on the atomic subspace is

$$\begin{aligned} \frac{d}{dt}a &= -i[a, H_a] + \sqrt{\frac{\gamma}{2\pi}} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega b_\omega^\dagger(t) [a, e^{-i(\bar{\omega}-\omega)t} \sigma_1^-(t) + e^{-i(\bar{\omega}-\omega)(t-\tau)} e^{i\Delta\phi} \sigma_2^-(t)] \\ &\quad - \sqrt{\frac{\gamma}{2\pi}} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega [a, e^{i(\bar{\omega}-\omega)t} \sigma_1^+(t) + e^{i(\bar{\omega}-\omega)(t-\tau)} e^{-i\Delta\phi} \sigma_2^+(t)] b_\omega(t). \end{aligned} \quad (14)$$

Note that, in order to simplify the notation, we remove the time dependence of a whenever it should be understood as $a(t)$. Formally integrating Eq. (13) and inserting the corresponding expression into Eq. (14), we obtain

$$\begin{aligned} \frac{da}{dt} &= -i[a, H_a] + \sqrt{\gamma}[a, \xi^\dagger(t)\sigma_1^-(t) - \text{H.c.}] + \sqrt{\gamma}[a, \xi^\dagger(t-\tau)e^{i\Delta\phi}\sigma_2^-(t) - \text{H.c.}] \\ &\quad + \frac{\gamma}{2\pi} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \int_0^t dt' e^{-i(\bar{\omega}-\omega)(t-t')} \sigma_1^+(t') [a, \sigma_1^-(t) + e^{i(\bar{\omega}-\omega)\tau} e^{i\Delta\phi} \sigma_2^-(t)] \\ &\quad - \frac{\gamma}{2\pi} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \int_0^t dt' [a, \sigma_1^+(t) + e^{-i(\bar{\omega}-\omega)\tau} e^{-i\Delta\phi} \sigma_2^+(t)] e^{i(\bar{\omega}-\omega)(t-t')} \sigma_1^-(t') \\ &\quad + \frac{\gamma}{2\pi} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \int_0^t dt' e^{-i(\bar{\omega}-\omega)(t+\tau-t')} e^{-i\Delta\phi} \sigma_2^+(t') [a, \sigma_1^-(t) + e^{i(\bar{\omega}-\omega)\tau} e^{i\Delta\phi} \sigma_2^-(t)] \\ &\quad - \frac{\gamma}{2\pi} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \int_0^t dt' e^{i(\bar{\omega}-\omega)(t+\tau-t')} e^{i\Delta\phi} [a, \sigma_1^+(t) + e^{-i(\bar{\omega}-\omega)\tau} e^{-i\Delta\phi} \sigma_2^+(t)] \sigma_2^-(t'), \end{aligned} \quad (15)$$

where $\xi(t) = \frac{1}{\sqrt{2\pi}} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega b_\omega(0) e^{i(\bar{\omega}-\omega)t}$ are the quantum noise operators. We now perform a Born-Markov treatment where the integration over ω of the phase factors generates Dirac delta functions of t' , which allows to evaluate the integral in t' . This approximation is valid if $\sigma_i^-(t') \approx \sigma_i^-(t)$ for $t' \in [t - 1/\theta, t + 1/\theta]$, which requires $\gamma, \Omega, |\delta_i| \ll \theta$. This gives rise to terms such as

$$\begin{aligned} &\int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \int_0^t dt' e^{-i(\bar{\omega}-\omega)(t-t')} \sigma_1^+(t') [a, \sigma_1^-(t)] \\ &\approx 2\pi \int_0^t dt' \delta(t-t') \sigma_1^+(t') [a, \sigma_1^-(t)] \\ &= \pi \sigma_1^+(t) [a, \sigma_1^-(t)], \end{aligned} \quad (16)$$

and

$$\begin{aligned} &\int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega \int_0^t dt' e^{-i(\bar{\omega}-\omega)(t-t'-\tau)} e^{i\Delta\phi} \sigma_1^+(t') [a, \sigma_2^-(t)] \\ &\approx 2\pi \int_0^t dt' \delta(t-t'-\tau) e^{i\Delta\phi} \sigma_1^+(t') [a, \sigma_2^-(t)] \\ &= 2\pi e^{i\Delta\phi} \sigma_1^+(t-\tau) [a, \sigma_2^-(t)]. \end{aligned} \quad (17)$$

For now we are interested in the Markovian limit where τ is set to 0^+ . This requires that the delay is much shorter than the typical evolution time of the system, i.e., $\gamma, \Omega, |\delta_i| \ll 1/\tau$, which we will assume from now on. We then obtain the

quantum Langevin equation for the atomic operators

$$\begin{aligned}
 \frac{d}{dt}a = & -i[a, H_a] + \sqrt{\gamma}[a, \xi^\dagger(t)\sigma_1^-(t) - \text{H.c.}] \\
 & + \sqrt{\gamma}[a, \xi^\dagger(t)e^{i\Delta\phi}\sigma_2^-(t) - \text{H.c.}] \\
 & + \frac{\gamma}{2}\{\sigma_1^+(t)[a, \sigma_1^-(t)] - [a, \sigma_1^+(t)]\sigma_1^-(t)\} \\
 & + \frac{\gamma}{2}\{\sigma_2^+(t)[a, \sigma_2^-(t)] - [a, \sigma_2^+(t)]\sigma_2^-(t)\} \\
 & + \gamma\{e^{i\Delta\phi}\sigma_1^+(t)[a, \sigma_2^-(t)] - [a, \sigma_2^+(t)]e^{-i\Delta\phi}\sigma_1^-(t)\}.
 \end{aligned} \quad (18)$$

Let us write the expectation value of this equation for an initial state $|\Psi_0\rangle$ where the waveguide is in the vacuum state. In that case, $\xi(t)|\Psi_0\rangle = \langle\Psi_0|\xi^\dagger(t) = 0$, and we get

$$\begin{aligned}
 \frac{d\langle a \rangle}{dt} = & -i\langle [a, H_a] \rangle + \frac{\gamma}{2}\langle \sigma_1^+(t)[a, \sigma_1^-(t)] - [a, \sigma_1^+(t)]\sigma_1^-(t) \rangle \\
 & + \frac{\gamma}{2}\langle \sigma_2^+(t)[a, \sigma_2^-(t)] - [a, \sigma_2^+(t)]\sigma_2^-(t) \rangle \\
 & + \gamma\langle e^{i\Delta\phi}\sigma_1^+(t)[a, \sigma_2^-(t)] - [a, \sigma_2^+(t)]e^{-i\Delta\phi}\sigma_1^-(t) \rangle.
 \end{aligned} \quad (19)$$

We now move to the Schrödinger picture and express the average terms in Eq. (19) as

$$\langle a \rangle = \langle\Psi_0|a|\Psi_0\rangle = \text{Tr}_a\{\text{Tr}_w[a(0)|\Psi(t)\rangle\langle\Psi(t)|]\}, \quad (20)$$

where Tr_w denotes the trace over the waveguide modes, and Tr_a the trace over the atomic states. The atomic density matrix ρ is obtained from the full density matrix $|\Psi(t)\rangle\langle\Psi(t)|$ by tracing over the waveguide modes: $\rho(t) = \text{Tr}_w[|\Psi(t)\rangle\langle\Psi(t)|]$. Notice that every operator appearing in Eq. (19) acts only on the atomic Hilbert space, and can thus be taken out of the waveguide trace Tr_w . The average terms thus read as $\langle a \rangle = \text{Tr}_a[a(0)\rho(t)]$, and by using the cyclic property of the trace, one finally obtains the master equation

$$\frac{d\rho}{dt} = -i[H_a + H_{\text{dd}}, \rho] + \gamma\mathcal{D}[\sigma_1^- + \sigma_2^-]\rho, \quad (21)$$

where the effective dipole-dipole interaction term reads as

$$H_{\text{dd}} \equiv i\frac{\gamma}{2}(e^{i\Delta\phi}\sigma_1^+\sigma_2^- - e^{-i\Delta\phi}\sigma_2^+\sigma_1^-). \quad (22)$$

We will denote the effective Hamiltonian as $H_{\text{eff}} = H_a + H_{\text{dd}}$. In Fig. 3(a), we show the level scheme of the system along with the different terms of the master equation.

It is convenient to introduce the following states:

$$\begin{aligned}
 |S\rangle &= \frac{1}{\sqrt{2}}(|e_1\rangle - e^{-i\Delta\phi}|e_2\rangle), \\
 |T\rangle &= \frac{1}{\sqrt{2}}(|e_1\rangle + e^{-i\Delta\phi}|e_2\rangle),
 \end{aligned} \quad (23)$$

and the corresponding operators $\sigma_S^- = |g\rangle\langle S|$ and $\sigma_T^- = |g\rangle\langle T|$. The master equation can then be expressed as

$$\frac{d\rho}{dt} = -i[H_{\text{eff}}, \rho] + 2\gamma\mathcal{D}[\sigma_T^-]\rho, \quad (24)$$

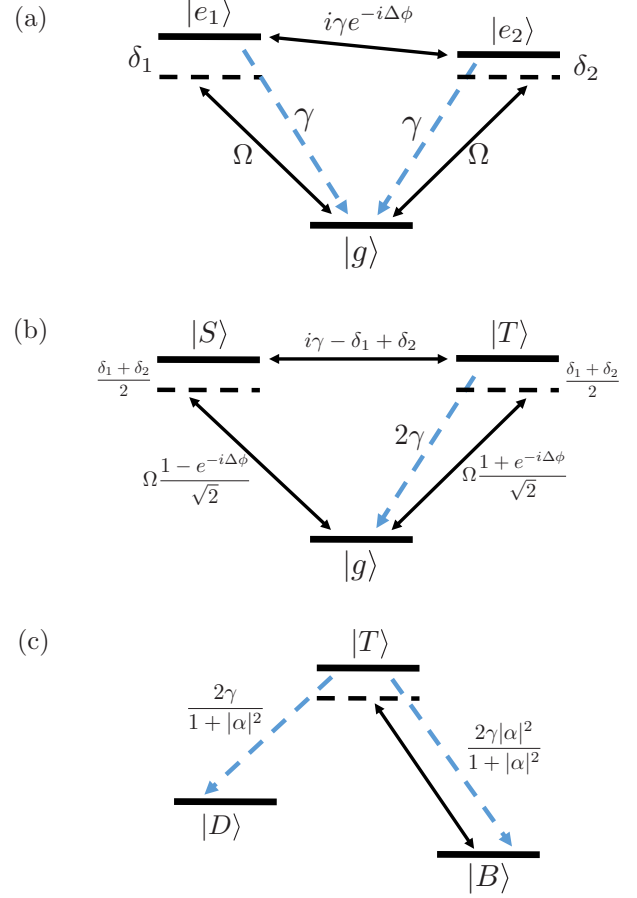


FIG. 3. Representation of the atom's dynamics in different basis. The coherent component is represented in solid black arrows and the incoherent component in dashed blue ones. (a) $\{|g\rangle, |e_1\rangle, |e_2\rangle\}$ basis. The dynamics is given by Eq. (21). The transitions are driven with a Rabi frequency Ω and undergo a dipole-dipole interaction. (b) $\{|g\rangle, |S\rangle, |T\rangle\}$ basis. The dynamics is given by Eq. (24). The state $|T\rangle$ deexcites at a rate 2γ and is coherently coupled to the other two states. When $\Delta\phi = 0$, the state $|S\rangle$ is decoupled from the ground state. (c) $\{|D\rangle, |B\rangle, |T\rangle\}$ basis. The dynamics coherently couples the bright component $|B\rangle$ to the decaying state $|T\rangle$, which incoherently jumps to the other states via photon emission. Eventually, the system is pumped into the dark state $|D\rangle$.

where the Hamiltonian now reads as

$$\begin{aligned}
 H_{\text{eff}} = & -\frac{\delta_1 + \delta_2}{2}(|S\rangle\langle S| + |T\rangle\langle T|) \\
 & + \left(\frac{i\gamma - \delta_1 + \delta_2}{2}\sigma_S^+\sigma_T^- + \text{H.c.} \right) \\
 & - \frac{\Omega}{2} \left(\frac{1 - e^{-i\Delta\phi}}{\sqrt{2}}\sigma_S^- + \frac{1 + e^{-i\Delta\phi}}{\sqrt{2}}\sigma_T^- + \text{H.c.} \right).
 \end{aligned} \quad (25)$$

We see from Eq. (24) that the state $|T\rangle$ is *super-radiant* as it decays with a rate 2γ , while the state $|S\rangle$ is *subradiant* as it does not spontaneously decay. Although the latter state does not generate radiation, it is not a dark state since it is unstable, due to the dipole-dipole term. We illustrate the dynamics of the atom in Fig. 3(b).

Notice that the master equation of Eq. (24) is very similar to the equation for the cascaded two-level atoms of Sec. II [Eq. (1)], where the state $|g_1g_2\rangle$ is now replaced by $|g\rangle$, the state $|e_1g_2\rangle$ by $|e_1\rangle$, and the state $|g_1e_2\rangle$ by $|e_2\rangle$. The double-excited state $|e_1e_2\rangle$, however, does not have an equivalent in our feedback setup.

B. Formation of dark states

Due to the dissipation of the open system via photon emission, the system relaxes towards a steady state which is in general mixed. However, depending on the settings of the driving laser, this steady state can be a pure state $|D\rangle$. The two conditions for the existence of such a state are [42]

- (1) $[H_{\text{eff}}, |D\rangle\langle D|] = 0$,
- (2) $\sigma_T^- |D\rangle = 0$.

The first condition implies that the state is stationary, i.e., an eigenstate of H_{eff} . The second condition requires that no incoherent stochastic jump occurs, which is a requirement for the state to be pure. Since the jump operator is σ_T^- , such a state belongs to the manifold spanned by $|g\rangle$ and $|S\rangle$. We can then write

$$|D\rangle = \frac{1}{\sqrt{1+|\alpha|^2}}(|g\rangle + \alpha|S\rangle) \quad (26)$$

for some α , in analogy with Eq. (5). Requiring this state to be an eigenstate of H_{eff} provides two constraints. Denoting the projector on the $\{|g\rangle, |S\rangle\}$ subspace $P = |g\rangle\langle g| + |S\rangle\langle S|$, the first one is $(1 - P)H_{\text{eff}}|D\rangle = 0$ which means that the coupling between $|D\rangle$ and $|T\rangle$ must vanish, and yields

$$\alpha = -\frac{\Omega}{\sqrt{2}} \frac{1 + e^{i\Delta\phi}}{i\gamma + \delta_1 - \delta_2}. \quad (27)$$

The second one reads as $PH_{\text{eff}}|D\rangle \propto |D\rangle$ which means that $|D\rangle$ is an eigenstate of the effective Hamiltonian restricted to the P subspace, and yields

$$\alpha^2\Omega(e^{-i\Delta\phi} - 1)/\sqrt{2} + \alpha(\delta_1 + \delta_2) + \Omega(1 - e^{i\Delta\phi})/\sqrt{2} = 0. \quad (28)$$

The existence of a solution for α satisfying these two constraints strongly depends on the phase difference $\Delta\phi$. In what follows, we will consider two different regimes, namely, when the *commensurability condition* $\Delta\phi = 0$ is satisfied, and when it is not. Note that if one drives the atom through the waveguide, $\phi' = \pi - \bar{\omega}\tau$ and the commensurability condition is automatically satisfied. In order to tune $\Delta\phi$ to different values, the driving must thus be done using fields outside of the waveguide. We now show that dark states can arise in both regimes and that they display different properties.

I. $\Delta\phi = 0$

In Sec. II, we have assumed this condition satisfied. Indeed, one can show [22] that for the quantum dimers, any deviation from this phase induces a coherent coupling between the $|S_{12}\rangle$ state from Eq. (5) and the double-excited state $|e_1e_2\rangle$. Since the latter state is not destroyed by the Lindblad jump operator, the commensurability is a necessary condition for a dark steady state. In our case, if we assume this condition satisfied, Eq. (28) simply becomes $\delta_1 + \delta_2 = 0$, signifying that the state $|S\rangle$ is not

detuned from $|g\rangle$. The $|S\rangle$ fraction is then given by

$$\alpha = -\sqrt{2} \frac{\Omega}{i\gamma + 2\delta_1} \quad (29)$$

which increases *linearly* with the Rabi frequency. For any Ω , there exists a dark state as the unique steady state of the dynamics. Note that this is the same expression as Eq. (6), which shows the similarities between the physics of our system and the quantum dimer.

We define the *bright* state $|B\rangle = \frac{1}{\sqrt{1+|\alpha|^2}}(\alpha^*|g\rangle - |S\rangle)$, which is bright in the sense that, contrary to the dark state, it is coupled to $|T\rangle$ which eventually decays by emitting photons. The three states $\{|T\rangle, |D\rangle, |B\rangle\}$ form an orthonormal basis on the atomic Hilbert space, and the dynamics is represented in Fig. 3(c), where it is clear that $|D\rangle$ is the steady state. On this basis, the decay from $|T\rangle$ to $|g\rangle$ generates an effective decay to $|D\rangle$ with a rate $\gamma_{\text{eff}} = 2\gamma/(1 + |\alpha|^2)$ and to $|B\rangle$ with a rate $|\alpha|^2\gamma_{\text{eff}}$. The time necessary to reach the steady state is roughly given by $2\pi/\gamma_{\text{eff}}$, which grows quadratically with Ω . The Hamiltonian from Eq. (24) now takes the simple form

$$H_{\text{eff}} = \left(i\frac{\gamma}{2} + \delta_1\right) \sqrt{1 + \frac{2\Omega^2}{|i\gamma + 2\delta_1|^2}} |T\rangle\langle B|. \quad (30)$$

2. $\Delta\phi \neq 0$

The fact that the state $|e_1e_2\rangle$ from the quantum dimer setup has no equivalent in our system allows to construct dark states even when the commensurability is not satisfied, i.e., the feedback photons are out of phase with the driving. Inserting Eq. (27) into (28), a relation between the variables of the system can be obtained. We require Ω to be real, and find that the detunings must satisfy $\delta_1 - \delta_2 = 0$, which is the exact opposite condition as for the $\Delta\phi = 0$ case. From now on, we will assume this condition to be satisfied, and we will denote $\delta = \delta_1 = \delta_2$. The requirement for obtaining a dark state then becomes

$$\Omega/\gamma = \sqrt{\frac{1}{1 + \cos(\Delta\phi)} - \frac{2\delta/\gamma}{\sin(\Delta\phi)}}. \quad (31)$$

This dark state can be interpreted as follows [see Fig. 3(b)]. In contrast to the case $\Delta\phi = 0$, the two states $|g\rangle$ and $|S\rangle$ are coupled by the coherent part of the dynamics. Since a pure steady state has to be an eigenstate of this coherent evolution, it restricts the possibility to the two dressed states $|\pm\rangle = (|g\rangle + \alpha_{\pm}|S\rangle)/\sqrt{1 + |\alpha_{\pm}|^2}$, where

$$\alpha_{\pm} = -\frac{\Omega \frac{1 - e^{i\Delta\phi}}{\sqrt{2}}}{\delta \pm \text{sgn}(\Delta\phi) \sqrt{\delta^2 + \left|\Omega \frac{1 - e^{i\Delta\phi}}{\sqrt{2}}\right|^2}}. \quad (32)$$

Equation (31) then states the condition under which the couplings to the state $|T\rangle$ from the components of the state $|g\rangle$ and the state $|S\rangle$ in the state $|+\rangle$ interfere destructively, rendering $|+\rangle$ a dark eigenstate of the coherent dynamics with eigenvalue

$$E_+ = -\delta + \frac{\gamma}{2} \frac{1 - \cos(\Delta\phi)}{\sin(\Delta\phi)}. \quad (33)$$

In particular, for $\delta = 0$ we find $|\alpha| = 1$, which is now *independent* of the Rabi frequency. Moreover, from Eq. (31)

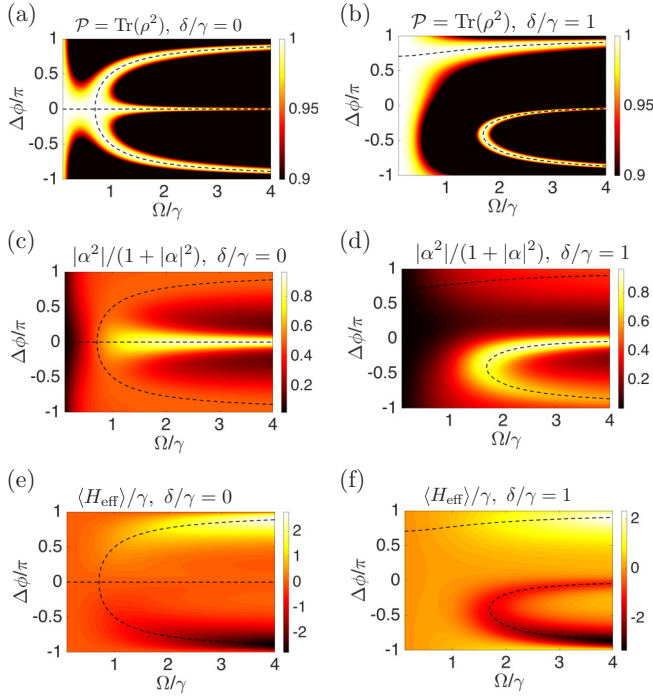


FIG. 4. Properties of the steady state, as a function of the Rabi frequency Ω and the phase difference $\Delta\phi$. The master equation (24) was solved exactly by vectorizing ρ . In dashed black lines, we plot the curves along which we predicted dark states. In the first column, $\delta = 0$, whereas $\delta = \gamma$ in the second one. In the first row, we plot the purity, which is exactly 1 along the black lines. In the second row, we plot the excitation probability, which is the occupancy of $|S\rangle$. In the third row, we plot the expectation value of H_{eff} in units of γ .

we note that this dark state appears only above a critical Rabi frequency $\Omega_c = \gamma/\sqrt{2}$.

The bright state $|B\rangle$ is identified as the other dressed state $|-\rangle$ with the energy

$$E_- = -\frac{\gamma}{2} \frac{1 - \cos(\Delta\phi)}{\sin(\Delta\phi)}. \quad (34)$$

On the $\{|T\rangle, |D\rangle, |B\rangle\}$ basis, the coherent part of the dynamics of the system, represented by the black arrow in Fig. 3(c), is now governed by the Hamiltonian

$$H_{\text{eff}} = -\delta|T\rangle\langle T| + E_+|D\rangle\langle D| + E_-|B\rangle\langle B| + (J_{TB}|T\rangle\langle B| + \text{H.c.}), \quad (35)$$

where the coupling can be expressed as

$$J_{TB} = i\frac{\gamma}{2}\sqrt{2 + \delta/E_-}. \quad (36)$$

In particular, if $\Delta\phi \rightarrow \pi$ then $E_{\pm} \rightarrow \pm\infty$ whereas J_{TB} remains finite. In this limit, we see that all three states effectively decouple, hence, the time necessary to reach the steady state $|D\rangle$ diverges. For other values of $\Delta\phi$, however, this time remains finite.

We now look at the whole parameter range for Ω and $\Delta\phi$. In Fig. 4, we plot some of the properties of the steady state as a function of Ω/γ and $\Delta\phi$. When $\delta = 0$, the horizontal dashed black line corresponds to the case $\Delta\phi = 0$. The other dashed curve is a plot of Eq. (31). In Fig. 4(a), we see that for any given

phase $\Delta\phi \neq (0, \pi)$ there exists a unique Rabi frequency Ω for which the steady state is dark. By adiabatically increasing the laser intensity from zero, one would thus observe a dip in the intensity of the photon emission in the waveguide, which is the signature of the dark state. Conversely, for $\Omega \leq \Omega_c = \gamma/\sqrt{2}$ the only possible phase is $\Delta\phi = 0$ whereas for $\Omega > \Omega_c$ three different values lead to a dark state. Figure 4(c) shows that, as predicted, the occupation of $|S\rangle$ along the $\Delta\phi = 0$ line rapidly converges to 1 when Ω/γ increases, whereas along the other curve it remains constant at $\frac{1}{2}$. In Fig. 4(e), we see that the energy of the dark state has the same sign as $\Delta\phi$. When the detuning becomes nonzero [Figs. 4(b), 4(d), and 4(f)], both black curves merge into two bands, which are now separated by a phase gap.

C. Coupling to a cavity in the bad-cavity regime

In the previous sections we have always assumed that the coupling of the atom to nonguided modes γ' was negligible compared to the coupling to the guided ones γ . This is currently not the case when working with real atoms, where $\beta = \gamma/(\gamma + \gamma') \lesssim 0.5$ [26] in setups where the atom-waveguide coupling is not chiral, and $\beta \approx 0.025$ [28] in setups featuring chiral couplings. A way to increase the effective atom-waveguide cooperativity is depicted in Fig. 1(c), where the atom couples with a rate g to a cavity which preserves chiral coupling [17]. The cavity is resonant with the laser and is coupled to the guided modes with a rate κ .

In a frame rotating at the laser frequency, the master equation for the system consisting of atom and cavity is given by

$$\frac{d\rho}{dt} = -i[H_a, \rho] + \mathcal{L}'\rho + \mathcal{L}_{\text{int}}\rho + \mathcal{L}_{\text{cav}}\rho, \quad (37)$$

where the loss to nonguided modes from the atom is given by

$$\mathcal{L}'\rho = \gamma'(\mathcal{D}[\sigma_1^-]\rho + \mathcal{D}[\sigma_2^-]\rho), \quad (38)$$

the coupling between the cavity and the atom is described by

$$\mathcal{L}_{\text{int}}\rho = -ig[\sigma_T^+ a_T + \sigma_S^+ a_S + \text{H.c.}, \rho] \quad (39)$$

and the free evolution of the cavity modes is given by

$$\mathcal{L}_{\text{cav}}\rho = \frac{\kappa}{2}[a_S^\dagger a_T - a_T^\dagger a_S, \rho] + 2\kappa\mathcal{D}[a_T]\rho + \kappa'(\mathcal{D}[a_1]\rho + \mathcal{D}[a_2]\rho). \quad (40)$$

Here, κ' is the intracavity loss rate due to absorption and coupling to nonguided modes, a_1 and a_2 are the annihilation operator for the cavity modes coupled, respectively, to the σ_1 and σ_2 transitions, and we have defined the T and S cavity modes $a_{T/S} = (a_1 \pm e^{i\Delta\phi} a_2)/\sqrt{2}$ in analogy with the σ_T^- and σ_S^- atomic operators. In order to write the master equation, we assumed that the time delay τ is much smaller than the relevant time scale of the system, namely here $1/\kappa$.

As detailed in Appendix A, in the bad-cavity regime $g \ll \kappa$ the cavity can be adiabatically eliminated [43], and the density matrix for the atom ρ_a , obtained from ρ by tracing over the cavity modes, is governed by the following master equation:

$$\frac{d\rho_a}{dt} = -i[H_a, \rho_a] + 2\gamma\mathcal{D}[\sigma_T^-]\rho_a + \frac{\gamma}{2}[\sigma_S^+ \sigma_T^- - \sigma_T^+ \sigma_S^-, \rho_a] + (\gamma' + \gamma\kappa'/\kappa)(\mathcal{D}[\sigma_1^-]\rho_a + \mathcal{D}[\sigma_2^-]\rho_a), \quad (41)$$

where $\gamma = (2g)^2\kappa/(\kappa + \kappa')^2$ is identified as the effective atom-waveguide coupling and $\Delta\phi$ has been redefined with an additional π shift, and hence now reads as $\Delta\phi = -\bar{\omega}\tau - \phi'$. This shift is reminiscent of the cavity and the fact that a resonant photon entering it will leave with a π shift [44]. The dynamics of Eq. (41) is equivalent to the case without a cavity [Eq. (24)], with an additional coupling to nonguided modes whose rate is identified as $\gamma'_{\text{tot}} = \gamma' + \gamma\kappa'/\kappa$, so that the ratio between the respective couplings to the nonguided and to the guided modes is

$$\frac{\gamma'_{\text{tot}}}{\gamma} = \frac{\gamma'}{\gamma} + \frac{\kappa'}{\kappa}. \quad (42)$$

In order to have a low ratio, one needs $\kappa' \ll \kappa$ [in which case $\gamma \approx (2g)^2/\kappa$] and $\gamma' \ll \gamma$.

IV. BEYOND THE MARKOVIAN APPROXIMATION

In this section, we study the dynamics of our setup beyond the Markovian regime, when the retardation effects in the coherent quantum feedback become important. We are thus interested in the regime where the time delay τ is non-negligible compared to the relevant time scales, namely, $1/\Omega$, $1/\gamma$, $1/|\delta_i|$. Due to the constant driving, a non-negligible photon number is now present between the atom and the mirror, which results in entanglement between these photons and the atom, and in a retarded dipole-dipole interaction between the two transitions of the atom. The previous master equation treatment is no longer valid [31,45] and one needs to employ numerical methods in order to track the entangled states of the electromagnetic field and of the atom. Recently, techniques have been developed to study the dynamics of photonic circuits consisting of quantum optical systems coupled via waveguides, where the time delay in the interaction can be significant [31]. The state of the system comprising the atom and the photonic field is approximated using matrix-product states (MPS) methods [37], which are well suited to account for the entanglement in our system. We provide a brief description of the method in Appendix B. In this section, we first neglect the effect of the nonguided modes and assume a perfect chiral coupling.

In Fig. 5, we show the evolution of the populations of the atomic reduced density matrix as a function of time. Up to time $t = \tau = 10/\gamma$, the system evolves freely, as if the mirror was not present. The solution is thus given by a Rabi oscillation between the ground state $|g\rangle$ and the state $(|e_1\rangle + |e_2\rangle)/\sqrt{2}$ with a Rabi frequency $\sqrt{2}\Omega$, and a dissipation induced by photon emission into the waveguide. At time τ , the system starts interacting with the feedback, and for $\Delta\phi = 0$, the feedback photons previously emitted by the σ_1 transition are perfectly in phase with the photons emitted by the σ_2 transition. This generates a constructive photon interference which amplifies the emission via a super-radiance process. This is demonstrated by the sudden dip in the excitation of $|e_2\rangle$ and the bump for the ground state. Consequently, a fraction of this ground-state bump will be transferred to the occupation of $|e_1\rangle$ by the laser driving which leads to the bump in the excitation of $|e_1\rangle$. A similar process can be distinguished for $t = 2\tau$, after which the system becomes very close to the steady state.

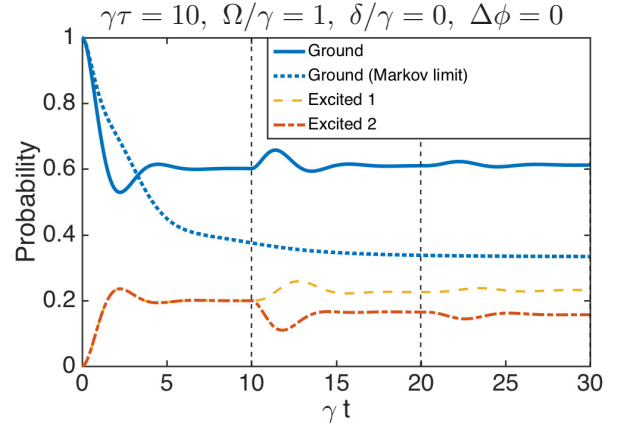


FIG. 5. Atomic occupation probabilities as a function of time for a long delay $\tau = 10/\gamma$, with $\Omega = \gamma$, $\delta = 0$, and $\Delta\phi = 0$. For reference we plot the ground-state occupation in the Markovian limit.

In Fig. 6, we show how some of the steady-state properties are affected by an increasing time delay τ . In Figs. 6(a) and 6(b), we see that the purity is locally maximal along the lines given by $\Delta\phi = \phi_D + (E_+ - E_-)\tau$, where ϕ_D is one of the values of $\Delta\phi$ for which the steady state is dark in the Markovian limit, and E_+ and E_- are the energies

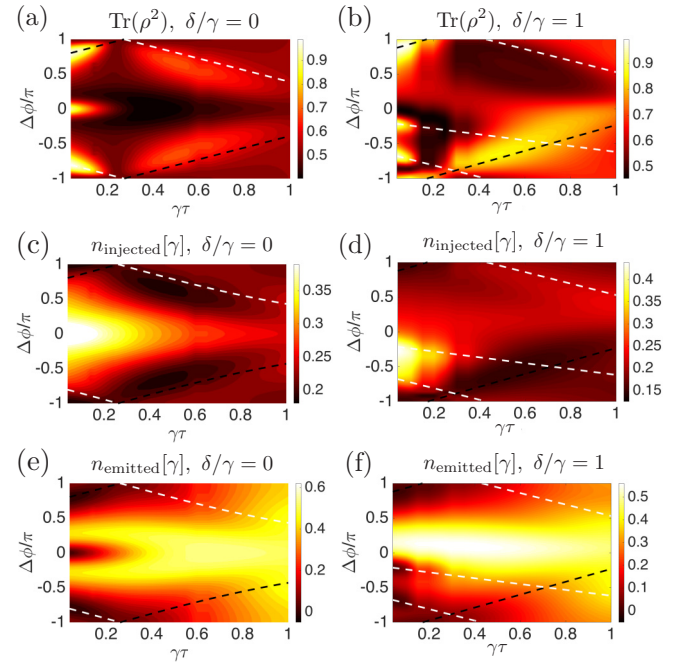


FIG. 6. Properties of the steady state as a function of the phase difference $\Delta\phi$ and the propagation time $\gamma\tau$, for $\Omega = 2\gamma$ and $\delta = 0$ (first column) or $\delta = \gamma$ (second column). In the first row, we plot the purity of the reduced density matrix of the atom. In the second row, we plot the number of photons injected inside the feedback loop per unit time $1/\gamma$ via the σ_1 transition. In the third row, we plot the number of photons emitted in the waveguide, outside of the feedback loop, per unit time $1/\gamma$. In dashed lines, we plot $\phi_D + (E_+ - E_-)\tau$, where ϕ_D is one of the values of $\Delta\phi$ for which the steady state is dark in the Markovian limit, and E_+ and E_- are the energies of the corresponding dark and bright states, given by Eqs. (33) and (34), in black for $\Delta\phi > 0$ and in white for $\Delta\phi < 0$.

of the corresponding dark and bright states, which are given by Eqs. (33) and (34). For $\delta = 0$, the phase of the solution $\phi_D = 0$ is not shifted. The other two solutions are shifted symmetrically and thus cross at $\Delta\phi = \pm\pi$. The purity decreases locally around this point, which indicates that the steady state is now mixed. Figures 6(c) and 6(e) show that the photon number in the waveguide is, however, still very low (albeit nonzero). For larger values of $\gamma\tau$, the photon emission increases along the dashed lines, therefore, the purity vanishes. The fact that the photon emission in the dark state with $\phi_D = 0$ increases much more rapidly with $\gamma\tau$ than for the other dark states is related to the fact that the component of the dark state which lies in the excited manifold, given by α , is higher, as can be seen from Fig. 4(c). This leads to a higher number of photons injected in the feedback loop, as represented in Fig. 6(c), hence, even a small delay generates an important number of feedback photons which are entangled with the system. Similar results are obtained with $\delta = \gamma$ in Figs. 6(b), 6(d), and 6(f). Along the black line, the dark state is much more robust to the increase of the delay than for the white lines, which is due to the fact that the value of α is lower [see Fig. 4(d)].

V. EXPERIMENTAL CONSIDERATIONS

To conclude this work, we discuss some experimental considerations for the physical implementation of our system.

A. Effect of nonguided modes

We now consider the case where the atom can spontaneously deexcite by emitting a photon in the nonguided modes with a rate γ' . This coupling will effectively decrease the purity of the steady state. In Fig. 7, we investigate the robustness of the dark states. In the Markovian regime, this is done by adding a decoherence term $\gamma'(\mathcal{D}[\sigma_1^-]\rho + \mathcal{D}[\sigma_2^-]\rho)$ to the master equation [Eq. (24)]. Figure 7(a) shows that in this regime, increasing Ω diminishes the purity. Two effects are in play

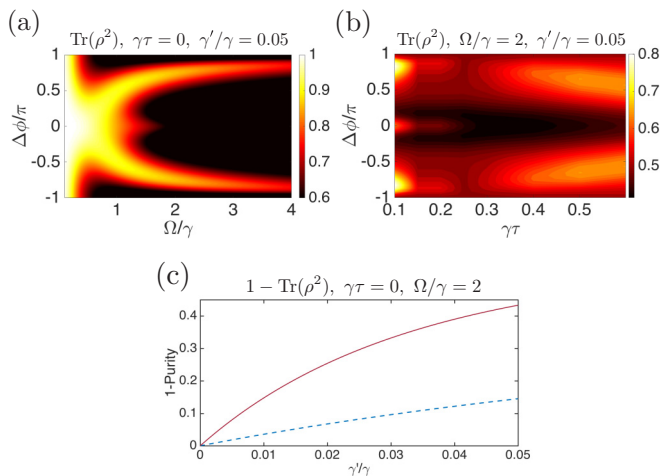


FIG. 7. Effect of the coupling to nonguided modes. (a) Purity of the steady state in the Markovian limit, as a function of Ω/γ and $\Delta\phi$, for $\gamma'/\gamma = 5\%$ and $\delta = 0$. (b) Purity of the steady state as a function of $\gamma\tau$ and $\Delta\phi$, for $\gamma'/\gamma = 5\%$ and $\delta = 0$. (c) Impurity of the steady state in the Markovian limit for $\Omega/\gamma = 2$, where $\Delta\phi$ is set to the dark state phases ($\Delta\phi = 0$ in solid red, $\Delta\phi \neq 0$ in dashed blue).

here. For the dark state with $\Delta\phi = 0$, we saw from Eq. (29) that α increases proportionally with the Rabi frequency. Since α is the excited fraction of the dark state, which can spontaneously emit in the nonguided modes, increasing Ω lowers the purity. For the dark states with $\Delta\phi \neq 0$, we saw that the time required to reach the steady state diverges as $\Delta\phi$ approaches $\pm\pi$. As this time becomes large compared to the decoherence time $1/\gamma'$, the purity decreases. In Fig. 7(b), we show the effect in the non-Markovian regime. As the delay τ increases, the effect on the purity is not significantly increased, and the shifting of the phase with the delay is still very recognizable. In Fig. 7(c), we show the purity as a function of γ' for $\Omega = 2\gamma$. The dark state with $\Delta\phi \neq 0$ (in dashed blue) is more robust than the one with $\Delta\phi = 0$ (in solid red) which already for $\gamma'/\gamma = 2\%$ has a purity of $\mathcal{P} \approx 0.75$.

B. Imperfection of the chiral coupling

Finally, we consider the case where the coupling between the atom and the waveguide modes is not perfectly chiral. Each transition of the atom σ_1 and σ_2 now couples to the guided modes propagating in both directions. Let us define the directionality of the coupling as $\eta = \gamma_{\text{pref}}/(\gamma_{\text{pref}} + \gamma_{\text{imp}})$, where γ_{pref} is the decay rate in the preferred direction and $\gamma_{\text{imp}} = \gamma - \gamma_{\text{pref}}$ the decay rate in the other direction. We assume that both transitions have the same directionality, although their preferred directions are opposite. In the definition of the atom-waveguide interaction Hamiltonian [Eq. (10)], the creation of a photon propagating towards the mirror is now associated with the atomic operator $\sigma_L^- = \sqrt{\eta}\sigma_1^- + \sqrt{1-\eta}\sigma_2^-$ instead of σ_1^- . For a photon propagating outwards, the atomic operator is $\sigma_R^- = \sqrt{1-\eta}\sigma_1^- + \sqrt{\eta}\sigma_2^-$ instead of σ_2^- .

In the Markovian regime, the master equation [Eq. (24)] is now modified in the following way. First, the relative laser phase ϕ' can no longer be gauged away and is instead an independent parameter which we will set to zero. Second, the Lindblad jump operator is redefined as $\sigma_T^- = \sigma_L^- + e^{i\Delta\phi}\sigma_R^-$, and undergoes a dipole-dipole interaction with the redefined operator $\sigma_S^- = \sigma_L^- - e^{i\Delta\phi}\sigma_R^-$.

In Fig. 8, we study the effect of this imperfect directionality. For the dark state with $\Delta\phi = 0$, the directionality does not

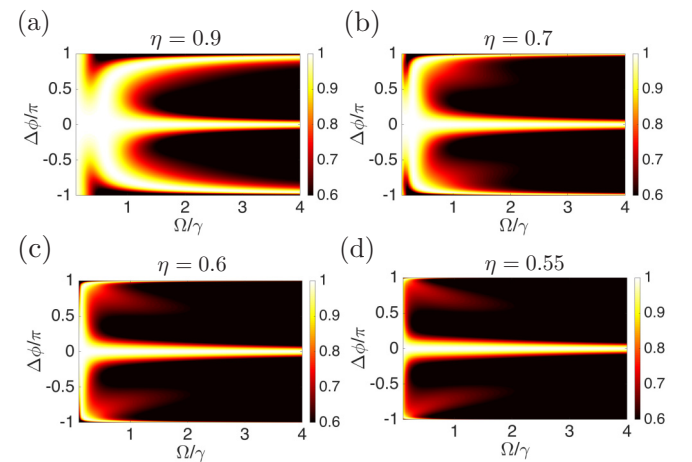


FIG. 8. Effect of the directionality η of the system on the purity of the steady state for $\delta = 0$ in the Markovian limit.

noticeably alter the purity. This situation is in fact analogous to the dimerization of two-level atoms coupled to a bidirectional waveguide, where now the atomic pair couples to the left-moving guided modes with a rate γ_L and to the right-moving ones with a rate γ_R . This system has been investigated in Ref. [22], where the authors have shown that as long as $\gamma_L \neq \gamma_R$, the atoms dimerize to form a unique pure steady state. In our system, γ_L is identified as γ_{pref} and γ_R as γ_{imp} .

For $\Delta\phi \neq 0$, on the contrary, the dark state vanishes as $\eta \rightarrow 0.5$. However, we see that the purity of the steady state is not drastically altered for small variations of η . For example, for $\eta = 0.9$, the results are still very similar to the predictions of the scenario with a perfect directionality $\eta = 1$.

VI. CONCLUSION

In this paper, we have studied the dissipative dynamics of an atom in a V-level configuration exhibiting a chiral coupling with a semi-infinite waveguide, where the atomic transitions are coupled to the modes propagating in opposite directions. The atom is coherently driven by lasers and by its own quantum feedback. In the Markovian limit, i.e., when the retardation effects of the quantum feedback are neglected, we have shown that the dynamics can lead to a situation where the atom is in a pure steady state and no photons are emitted in the waveguide. The properties of this steady state crucially depend on the phase acquired by the quantum feedback.

If this phase is a multiple of 2π , the atom mimics the dynamics of a pair of laser-driven two-level atoms coupled to a unidirectional waveguide in a cascaded setup [24]. In the steady state of the dissipative dynamics, these two-level atoms form a pure EPR-entangled pair which effectively decouples from the waveguide, and the excited fraction of this steady state grows linearly with the driving intensity. The current effort in the development of chiral atom-waveguide couplings in different platforms [18,20] should allow the observation of these atomic pairs in the future. In the meantime, we have shown that a single atom can experience analogous physics which could be achieved with current state-of-the-art technology.

If the phase of the quantum feedback is different than 2π , we have shown that the pure steady state of the atom exhibits very different properties, where the driving strength is dependent on the feedback phase and the excited fraction of the steady state is now a constant. We have also extended our study to non-Markovian regimes, where the retardation effects in the quantum feedback become important, and we have shown that this delay induces a shift in the feedback phase.

ACKNOWLEDGMENTS

The work at Innsbruck is supported by the ERC Synergy Grant UQUAM, the Austrian Science Fund through SFB FOQUS, the EU FET Proactive Initiative SIQS and RYQS, and the Army Research Laboratory Center for Distributed Quantum Information via the project SciNet. H.P. was supported by the NSF through a grant for the Institute for Theoretical Atomic, Molecular, and Optical Physics at Harvard University and the Smithsonian Astrophysical Observatory.

APPENDIX A: ADIABATIC ELIMINATION OF THE CAVITY MODES

We consider the setup of Sec. III C and we derive the master equation for the atom in the regime $g \ll \kappa$ starting from Eq. (37). The projection operator for the adiabatic elimination of the cavity modes is

$$\mathcal{P}\rho \equiv \text{Tr}_c(\rho) \otimes |0\rangle\langle 0|, \quad (\text{A1})$$

where Tr_c denotes the trace over the cavity modes and $|0\rangle$ is the vacuum state on the cavity subspace. A second-order perturbation expansion of the master equation projected on the \mathcal{P} space provides an equation for the atomic density matrix $\rho_a \equiv \text{Tr}_c(\rho)$ [38]:

$$\begin{aligned} \frac{d\rho_a}{dt} = & -i[H_a, \rho_a] + \mathcal{L}'\rho \\ & + \underbrace{\text{Tr}_c\{\mathcal{P}[\mathcal{L}_{\text{int}}(-\mathcal{L}_{\text{cav}})^{-1}\mathcal{L}_{\text{int}}(\rho_a \otimes |0\rangle\langle 0|)]\}}_{\mathcal{L}_{\text{eff}}\rho_a}. \end{aligned} \quad (\text{A2})$$

Using the fact that $\lim_{t \rightarrow \infty} e^{\mathcal{L}_{\text{cav}}t} = |0\rangle\langle 0|$ (the unique steady state of \mathcal{L}_{cav} is the vacuum) and $\text{Tr}_c[\mathcal{L}_{\text{int}}(\rho_a \otimes |0\rangle\langle 0|)] = 0$, the third term of the right-hand side can be rewritten

$$\mathcal{L}_{\text{eff}}\rho_a = \int_0^\infty dt \text{Tr}_c[\mathcal{L}_{\text{int}}e^{\mathcal{L}_{\text{cav}}t}\mathcal{L}_{\text{int}}(\rho_a \otimes |0\rangle\langle 0|)]. \quad (\text{A3})$$

We now move to a picture where the operators evolve with \mathcal{L}_{cav} . For two operators O_1 and O_2 acting on the cavity subspace, we will write $\langle 0|O_1(t)O_2(0)|0\rangle$ to denote the vacuum correlations $\text{Tr}_c[O_1e^{\mathcal{L}_{\text{cav}}t}(O_2|0\rangle\langle 0|)]$. Using this notation and the explicit form of \mathcal{L}_{int} from Eq. (39), the expression from Eq. (A3) can be rewritten

$$\begin{aligned} \mathcal{L}_{\text{eff}}\rho_a = & \sum_{i,k \in \{S,T\}} g^2 \int_0^\infty dt \langle 0|a_i(t)a_k^\dagger(0)|0\rangle [\sigma_k^- \rho_a \sigma_i^+ \\ & + \langle 0|a_k(0)a_i^\dagger(t)|0\rangle [\sigma_i^- \rho_a \sigma_k^+]. \end{aligned} \quad (\text{A4})$$

We thus have to obtain the expressions for $\langle 0|a_i(t)a_k^\dagger(0)|0\rangle$ and $\langle 0|a_k(0)a_i^\dagger(t)|0\rangle$. Using the expression of Eq. (40) for the Liouvillian, we first solve the equations of motion for $\langle a_T(t) \rangle$ and $\langle a_S(t) \rangle$ for an arbitrary density matrix ρ . These equations are obtained by noting that $\frac{d}{dt}\langle a(t) \rangle = \frac{d}{dt}\text{Tr}[a\rho(t)] = \text{Tr}(a\mathcal{L}_{\text{cav}}\rho)$. Using the expression of \mathcal{L}_{cav} from Eq. (40), we get

$$\frac{d}{dt}\langle a_T(t) \rangle = -\left(\kappa + \frac{\kappa'}{2}\right)\langle a_T(t) \rangle - \frac{\kappa}{2}\langle a_S(t) \rangle, \quad (\text{A5})$$

$$\frac{d}{dt}\langle a_S(t) \rangle = \frac{\kappa}{2}\langle a_T(t) \rangle - \frac{\kappa'}{2}\langle a_S(t) \rangle, \quad (\text{A6})$$

whose solution reads as

$$\begin{aligned} \langle a_T(t) \rangle = & e^{-(\kappa+\kappa')t/2}\langle a_T(0) \rangle \\ & - \frac{\kappa t}{2}e^{-(\kappa+\kappa')t/2}[\langle a_T(0) \rangle + \langle a_S(0) \rangle], \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \langle a_S(t) \rangle = & e^{-(\kappa+\kappa')t/2}\langle a_S(0) \rangle \\ & + \frac{\kappa t}{2}e^{-(\kappa+\kappa')t/2}[\langle a_T(0) \rangle + \langle a_S(0) \rangle]. \end{aligned} \quad (\text{A8})$$

We now apply the quantum regression theorem [38] to obtain the vacuum correlations $\langle 0|a_i(t)a_k^\dagger(0)|0\rangle$. For example,

by choosing $\rho = a_S^\dagger |0\rangle\langle 0|$, Eq. (A7) provides

$$\langle 0|a_T(t)a_S^\dagger(0)|0\rangle = -\frac{\kappa t}{2}e^{-(\kappa+\kappa')t/2}. \quad (\text{A9})$$

The other terms are similarly obtained, and the integral in Eq. (A4) can be performed, which provides the effective Liouvillian

$$\begin{aligned} \mathcal{L}_{\text{eff}}\rho_a &= 2\gamma\mathcal{D}[\sigma_S^-]\rho_a + \frac{\gamma}{2}[\sigma_T^+\sigma_S^- - \sigma_S^+\sigma_T^-, \rho_a] \\ &+ \gamma\frac{\kappa'}{\kappa}(\mathcal{D}[\sigma_1^-]\rho_a + \mathcal{D}[\sigma_2^-]\rho_a), \end{aligned} \quad (\text{A10})$$

where we define $\gamma = (2g)^2\kappa/(\kappa + \kappa')^2$. In the limit $\kappa' \rightarrow 0$, this Liouvillian describes the coupling of the atom to the guided modes [Eq. (24)], if we exchange the labels of the states $|T\rangle = (|e_1\rangle + e^{i\Delta\phi}|e_2\rangle)/\sqrt{2}$ and $|S\rangle = (|e_1\rangle - e^{i\Delta\phi}|e_2\rangle)/\sqrt{2}$. This can be done by redefining the phase $\Delta\phi$ with an additional π shift, in which case the master equation becomes Eq. (41).

APPENDIX B: MATRIX-PRODUCT STATE ALGORITHM IN THE NON-MARKOVIAN REGIME

1. Quantum stochastic Schrödinger equation

We provide here a description of the numerical method developed in [31] that we use to study our feedback system in non-Markovian regimes. Let us start from the interaction picture Hamiltonian given by Eqs. (11) and (12) and define the Fourier transform operators

$$b(t) = \frac{1}{\sqrt{2\pi}} \int_{\bar{\omega}-\theta}^{\bar{\omega}+\theta} d\omega b_\omega e^{i(\bar{\omega}-\omega)t}. \quad (\text{B1})$$

Their commutation relations can be approximated by a Dirac delta function $[b(t), b^\dagger(t')] \approx \delta(t - t')$ on time scales much larger than the photon correlation time $1/\theta$. The interaction Hamiltonian [Eq. (12)] becomes

$$H_{\text{int}}(t) = i\sqrt{\gamma}[b^\dagger(t)\sigma_1^- + b^\dagger(t - \tau)e^{i\Delta\phi}\sigma_2^- - \text{H.c.}], \quad (\text{B2})$$

and provides a quantum stochastic Schrödinger equation [38] $\frac{d|\Psi\rangle}{dt} = [H_a + H_{\text{int}}(t)]|\Psi\rangle$ for the system comprising the atom and the waveguide.

2. Time discretization

We discretize time into time steps of length Δt which we take much smaller than the relevant time scales $1/\gamma$, $1/\Omega$, $1/|\delta_i|$, but much larger than the photon correlation time $1/\theta$. For a given Δt we define the quantum noise increments

$$\Delta B_k = \int_{t_k}^{t_{k+1}} dt b(t), \quad (\text{B3})$$

where $t_{k+1} = t_k + \Delta t$, and $[\Delta B_k, \Delta B_{k'}^\dagger] \approx \Delta t \delta_{k,k'}$. In this stroboscopic view, the photons are separated into discrete time bins, which consist of a bosonic Fock space with the corresponding annihilation operators given by the operator $\Delta B_k/\sqrt{\Delta t}$. The Fock basis for each time bin k is denoted $\{|i_k\rangle, i_k = 1, 2, \dots\}$, where $|i_k\rangle = \frac{(\Delta B_k^\dagger/\sqrt{\Delta t})^{i_k}}{\sqrt{i_k!}}|\text{vac}\rangle$ and i_k is interpreted as the number of photons in the time bin. We denote

the state of the system consisting of the atom and all the time bins as $|\Psi(t)\rangle$.

The evolution between two successive discrete times t_k and t_{k+1} is given by a unitary operator U_k , such that $|\Psi(t_{k+1})\rangle = U_k|\Psi(t_k)\rangle$. Using the Hamiltonian from Eq. (B2), this operator reads as

$$\begin{aligned} U_k &= \mathcal{T} \exp[-iH_a\Delta t + \sqrt{\gamma}(\Delta B_k^\dagger\sigma_1^- \\ &+ \Delta B_{k-m}^\dagger e^{i\Delta\phi}\sigma_2^- - \text{H.c.})], \end{aligned} \quad (\text{B4})$$

where \mathcal{T} denotes the time ordering of the $b(t)$ operators appearing if one replaces the ΔB operators by their definition [Eq. (B3)]. To first order in Δt , we will neglect this time ordering. During each time step k , we see that the atom interacts only with the time bins k and $k - m$, where we have defined $m = \lfloor \tau/\Delta t \rfloor$. The first one physically corresponds to the photons emitted towards the mirror, whereas the second one corresponds to the delayed interaction with the feedback photons.

We assume that the initial state is of the form $|\Psi(t=0)\rangle = |\psi_a\rangle \otimes_{p=1}^\infty |\phi_p\rangle$, where $|\psi_a\rangle$ denotes the initial state of the atom and $|\phi_p\rangle$ the initial state of the time bin p (namely, the vacuum state in our case), hence the system is initially fully disentangled. After an evolution up to time t_k , the entanglement grows and in general the system is of the form $|\Psi(t_k)\rangle = |\psi(t_k)\rangle \otimes_{p=k}^\infty |\phi_p\rangle$, where $|\psi(t_k)\rangle$ denotes the state of the system comprising the atom and the time bins up to $p = k - 1$. We work in the basis $|i_a, i_{k-1}, i_{k-2}, \dots, i_1\rangle$, where $i_a \in \{g, e_1, e_2\}$ labels the atomic states. On this basis, $|\psi(t_k)\rangle$ is decomposed as

$$|\psi(t_k)\rangle = \sum_{i_a, \{i_p\}} \psi_{i_a, i_{k-1}, i_{k-2}, \dots, i_1} |i_a, i_{k-1}, i_{k-2}, \dots, i_1\rangle. \quad (\text{B5})$$

3. Matrix-product state algorithm

The matrix-product state (MPS) [37] approach consists in writing the amplitude as the trace of a product of matrices

$$\psi_{i_a, i_{k-1}, i_{k-2}, \dots} = \text{Tr}(A[a]^{i_a} A[k-1]^{i_{k-1}} A[k-2]^{i_{k-2}} \dots), \quad (\text{B6})$$

where each $A[p]^{i_p}$ is a matrix of finite dimensions $D_p \times D_{p-1}$. The bond dimension D_p represents the entanglement between the different components of the system, more precisely between the two parties formed by a bipartite cut of the time bins between bins p and $p + 1$. The objects $A[p]$ are thus tensors with two bond indices encoding the entanglement, and one physical index i_p . In our case, we use open boundary conditions, meaning that $D_a = D_0 = 1$ at each time t_k . Note that by setting a boundary D_{max} for the bond dimensions, the numerical cost is bounded by NdD_{max}^2 for N bins with a physical dimension d . This can be much lower than the usual exponential complexity d^N . As discussed in Ref. [31], D_{max} needs to increase exponentially with the delay $\gamma\tau$ due to long-range correlations between the bins which increase the entanglement entropy. However, given a fixed $\gamma\tau$, the entropy remains constant once the system reaches the steady state, meaning that the numerical cost increases only linearly with time.

The algorithm then consists in updating the tensors at each time step in the following way. We first extend the definition of

the system by one time bin. Formally, this amounts to writing

$$\psi_{i_a, i_k, i_{k-1}, \dots} = \text{Tr}(A[k]^{i_k} A[a]^{i_a} A[k-1]^{i_{k-1}} \dots), \quad (\text{B7})$$

where $A[k]^{i_k} = \delta_{i_k, 0}$ is a 1×1 matrix, as this new time bin is still completely disentangled from the rest of the system. The Kronecker delta signifies that the state of the time bin is the vacuum state. The next step is to apply the unitary evolution from Eq. (B4). U_k can be seen as a tensor with six physical indices, three of which are to be contracted with the physical indices of the MPS tensors $A[a]$, $A[k]$ and $A[k-m]$ to obtain the evolved state. Notice that $A[a]$ and $A[k]$ are successive tensors in the state representation [Eq. (B7)], which corresponds to a short-range interaction between the tensors. On the other hand, the interaction with $A[k-m]$ is more involved, as it implies long-range interactions, hence,

all the bins between $A[k]$ and $A[k-m]$ must be updated in order to account for the entanglement increase arising from this interaction. Various methods for dealing with these interactions exist [46–48]. Our algorithm is described in details in the Supplemental Material of [31] and employs a method proposed in [49,50]. It consists in exchanging recursively $m-1$ times the state of the $(k-m)$ th bin with that of the $(k-m+1)$ th bin in order to obtain an MPS description of the state where all three interacting tensors are successive tensors. We then merge these three tensors into a tensor with three physical indices i_k, i_a, i_{k-m} , and locally apply the unitary tensor U_k by contracting these indices. The resulting tensor can finally be brought back into single bin tensors by applying singular value decompositions (SVD), and by exchanging the positions of $A[k]$ and $A[a]$, the system is put back in the form of Eq. (B6), with $k \rightarrow k+1$.

-
- [1] H. J. Kimble, *Nature (London)* **453**, 1023 (2008).
 [2] A. Reiserer and G. Rempe, *Rev. Mod. Phys.* **87**, 1379 (2015).
 [3] H. Weimer, M. Muller, I. Lesanovsky, P. Zoller, and H. P. Buchler, *Nat. Phys.* **6**, 382 (2010).
 [4] F. Verstraete, M. M. Wolf, and J. Ignacio Cirac, *Nat. Phys.* **5**, 633 (2009).
 [5] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Buchler, and P. Zoller, *Nat. Phys.* **4**, 878 (2008).
 [6] I. C. Hoi, A. F. Kockum, L. Tornberg, A. Pourkabirian, G. Johansson, P. Delsing, and C. M. Wilson, *Nat. Phys.* **11**, 1045 (2015).
 [7] A. F. van Loo, A. Fedorov, K. Lalumiere, B. C. Sanders, A. Blais, and A. Wallraff, *Science* **342**, 1494 (2013).
 [8] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Y. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, *Science* **327**, 840 (2010).
 [9] P. Lodahl, S. Mahmoodian, and S. Stobbe, *Rev. Mod. Phys.* **87**, 347 (2015).
 [10] R. Riedinger, S. Hong, R. A. Norte, J. A. Slater, J. Shang, A. G. Krause, V. Anant, M. Aspelmeyer, and S. Gröblacher, *Nature (London)* **530**, 313 (2016).
 [11] K. Fang, M. H. Matheny, X. Luan, and O. Painter, *Nat. Photon.* **10**, 489 (2016).
 [12] D. Reitz, C. Sayrin, R. Mitsch, P. Schneeweiss, and A. Rauschenbeutel, *Phys. Rev. Lett.* **110**, 243603 (2013).
 [13] R. Yalla, M. Sadgrove, K. P. Nayak, and K. Hakuta, *Phys. Rev. Lett.* **113**, 143601 (2014).
 [14] A. Goban, C. L. Hung, S. P. Yu, J. D. Hood, J. A. Muniz, J. H. Lee, M. J. Martin, A. C. McClung, K. S. Choi, D. E. Chang, O. Painter, and H. J. Kimble, *Nat. Commun.* **5**, 3808 (2014).
 [15] J. D. Thompson, T. G. Tiecke, N. P. de Leon, J. Feist, A. V. Akimov, M. Gullans, A. S. Zibrov, V. Vuletić, and M. D. Lukin, *Science* **340**, 1202 (2012).
 [16] T. G. Tiecke, J. D. Thompson, N. P. de Leon, L. R. Liu, V. Vuletić, and M. Lukin, *Nature (London)* **508**, 241 (2014).
 [17] C. Junge, D. O’Shea, J. Volz, and A. Rauschenbeutel, *Phys. Rev. Lett.* **110**, 213604 (2013).
 [18] R. Mitsch, C. Sayrin, B. Albrecht, P. Schneeweiss, and A. Rauschenbeutel, *Nat. Commun.* **5**, 5713 (2014).
 [19] I. J. Luxmoore, N. A. Wasley, A. J. Ramsay, A. C. T. Huijssen, R. Oulton, M. Hugues, S. Kasture, V. G. Achanta, A. M. Fox, and M. S. Skolnick, *Phys. Rev. Lett.* **110**, 037402 (2013).
 [20] I. Söllner, S. Mahmoodian, S. L. Hansen, L. Midolo, A. Javadi, G. Kiršanskė, T. Pregolato, H. El-Ella, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, *Nat. Nanotechnol.* **10**, 775 (2015).
 [21] T. Ramos, H. Pichler, A. J. Daley, and P. Zoller, *Phys. Rev. Lett.* **113**, 237203 (2014).
 [22] H. Pichler, T. Ramos, A. J. Daley, and P. Zoller, *Phys. Rev. A* **91**, 042116 (2015).
 [23] T. Ramos, B. Vermersch, P. Hauke, H. Pichler, and P. Zoller, *Phys. Rev. A* **93**, 062104 (2016).
 [24] K. Stannigel, P. Rabl, and P. Zoller, *New J. Phys.* **14**, 063014 (2012).
 [25] A. Goban, C.-L. Hung, J. D. Hood, S.-P. Yu, J. A. Muniz, O. Painter, and H. J. Kimble, *Phys. Rev. Lett.* **115**, 063601 (2015).
 [26] J. D. Hood, A. Goban, A. Asenjo-Garcia, M. Lu, S.-P. Yu, D. E. Chang, and H. J. Kimble, *Proc. Nat. Acad. Sci. U.S.A.* (2016).
 [27] M. Arcari, I. Söllner, A. Javadi, S. Lindskov Hansen, S. Mahmoodian, J. Liu, H. Thyrrstrup, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, *Phys. Rev. Lett.* **113**, 093603 (2014).
 [28] C. Sayrin, C. Junge, R. Mitsch, B. Albrecht, D. O’Shea, P. Schneeweiss, J. Volz, and A. Rauschenbeutel, *Phys. Rev. X* **5**, 041036 (2015).
 [29] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, 2010).
 [30] A. L. Grimsmo, *Phys. Rev. Lett.* **115**, 060402 (2015).
 [31] H. Pichler and P. Zoller, *Phys. Rev. Lett.* **116**, 093601 (2016).
 [32] N. Német and S. Parkins, *Phys. Rev. A* **94**, 023809 (2016).
 [33] U. Alvarez-Rodriguez, A. Perez-Leija, I. L. Egusquiza, M. Gräfe, M. Sanz, L. Lamata, A. Szameit, and E. Solano, *arXiv:1606.05143v1*.
 [34] E. Sánchez-Burillo, L. Martin-Moreno, J. J. García-Ripoll, and D. Zueco, *arXiv:1602.05603v2*.
 [35] M. Laakso and M. Pletyukhov, *Phys. Rev. Lett.* **113**, 183601 (2014).
 [36] D. E. Chang, L. Jiang, A. V. Gorshkov, and H. J. Kimble, *New J. Phys.* **14**, 063003 (2012).
 [37] U. Schollwöck, *Rev. Mod. Phys.* **77**, 259 (2005).

- [38] C. W. Gardiner and P. Zoller, *Quantum Noise: A Handbook of Quantum Stochastic Methods with Applications to Quantum Optics* (Springer, Berlin, 2004).
- [39] H. J. Carmichael, *Phys. Rev. Lett.* **70**, 2273 (1993).
- [40] C. W. Gardiner, *Phys. Rev. Lett.* **70**, 2269 (1993).
- [41] C. W. Gardiner and P. Zoller, *The Quantum World of Ultra-Cold Atoms and Light Book II: The Physics of Quantum-Optical Devices* (Imperial College Press, London, 2015).
- [42] B. Kraus, H. P. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, *Phys. Rev. A* **78**, 042307 (2008).
- [43] J. I. Cirac, *Phys. Rev. A* **46**, 4354 (1992).
- [44] J. Volz, M. Scheucher, C. Junge, and A. Rauschenbeutel, *Nat. Photonics* **8**, 965 (2014).
- [45] R. Doll, M. Wubs, P. Hänggi, and S. Kohler, *Phys. Rev. B* **76**, 045317 (2007).
- [46] T. Koffel, M. Lewenstein, and L. Tagliacozzo, *Phys. Rev. Lett.* **109**, 267203 (2012).
- [47] J. Haegeman, C. Lubich, I. Oseledets, B. Vandereycken, and F. Verstraete, [arXiv:1408.5056v2](https://arxiv.org/abs/1408.5056v2).
- [48] M. P. Zaletel, R. S. K. Mong, C. Karrasch, J. E. Moore, and F. Pollmann, *Phys. Rev. B* **91**, 165112 (2015).
- [49] J. Schachenmayer, I. Lesanovsky, A. Micheli, and A. J. Daley, *New J. Phys.* **12**, 103044 (2010).
- [50] M. C. Bănuls, R. Orús, J. I. Latorre, A. Pérez, and P. Ruiz-Femenía, *Phys. Rev. A* **73**, 022344 (2006).