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Resonant two-photon annihilation of an electron-positron pair in a pulsed electromagnetic wave

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Two-photon annihilation of an electron-positron pair in the field of a plane low-intensity circularly polarized pulsed electromagnetic wave was studied. The conditions for resonance of the process which are related to an intermediate particle that falls within the mass shell are studied. In the resonant approximation the probability of the process was obtained. It is demonstrated that the resonant probability of two-photon annihilation of an electron-positron pair may be several orders of magnitude higher than the probability of this process in the absence of the external field. The obtained results may be experimentally verified by the laser facilities of the international megaprojects, for example, SLAC (National Accelerator Laboratory), FAIR (Facility for Antiproton and Ion Research), and XFEL (European X-Ray Free-Electron Laser).

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I. INTRODUCTION

In a series of works (see, for example, the papers [1–12], the review [13], and the monograph [14]) the resonances in the second-order processes in the fine-structure constant have been investigated for the monochromatic wave field. The analysis of quantum-electrodynamics processes of the second order in the fine-structure constant in an arbitrary intensities field is complicated by computational difficulties and a cumbersome form of results. Therefore we restrict our consideration to the case when the intensity of the wave meets the condition

$$\eta = \frac{|e|a}{m} \ll 1. \tag{1}$$

Here η is the classical relativistic-invariant parameter characterizing the intensity of the wave, which is equal to the ratio of the work done by the field within the wavelength to the rest energy of the electron; e,m are the charge and the mass of an electron; $a = F/\omega$; F and ω are the amplitude of the electric-field strength and the frequency of the wave.

For the second-order processes in the fine-structure constant in the pulsed electromagnetic field the intermediate particle may fall within the mass shell as well (see, for example, the papers [8,15–23], the review [21], and the monographs [23,24]). The amplitude can be represented as the sum of partial components:

$$S = \sum_{l,l'} S_{l,l'}, \tag{2}$$

where l is the number of photons absorbed ($l > 0$) of the wave or emitted ($l < 0$) in the wave during the whole process, and $|l'|$ is the number of photons absorbed ($l' > 0$) or emitted ($l' < 0$) in one of the vertices. The possibility of resonance of the partial process is determined by the parameter (introduced in the works [16,21,23–25])

$$\beta = \frac{q^2 - m^2}{4(kq)}\varphi_{\mathrm{imp}}, \quad \varphi_{\mathrm{imp}} = \omega t_{\mathrm{imp}}, \tag{3}$$

where q is the four-momentum of an intermediate particle, which corresponds to "exact" laws of four-momentum conservation (as in the case of a monochromatic plane wave); t_{imp} is the pulse duration in the laboratory frame of reference. As shown in the works [16,19–21,23–28] partial processes for which the condition

$$\beta \lesssim 1 \tag{4}$$

can be satisfied are resonant processes, and opposite condition $\beta \gg 1$ determines nonresonant processes.

In the case of the low-intensity plane monochromatic electromagnetic wave [Eq. (1)] and not too large integers $|l - l'|$, $|l'|$ the probability of partial processes is proportional to

$$\mathrm{W}_{l,l'}\varphi_{\mathrm{imp}}^2\eta^{2(|l-l'|+|l'|)} \times \begin{cases} 1, & \beta \lesssim 1 \\ \beta^{-2}, & \beta \gg 1 \end{cases}. \tag{5}$$

So the probability of resonant processes is significantly higher than the probability of nonresonant processes.

In the works [26,29–32] the process of one-photon annihilation of an electron-positron pair in the pulsed light field was investigated theoretically. It was shown that the cross-section is significantly larger than for two-photon pair annihilation in vacuum [33,34]. Nonresonant two-photon annihilation of an electron-positron pair in the field of a moderately strong circularly polarized wave is investigated theoretically in the work [35]. In this work, we study the resonant two-photon annihilation of an electron-positron pair in a low-intensity [Eq. (1)] pulsed electromagnetic wave.

The organization of this paper is as follows. In Sec. II an amplitude of the process of two-photon annihilation of an electron-positron pair in the external field is presented as the sum of partial components. Each partial amplitude is characterized by the number of photons absorbed ($l > 0$) of the wave or emitted ($l < 0$) in the wave during the whole process and the number of photons absorbed ($l' > 0$) or emitted ($l' < 0$) in one of the vertices. It is shown that the

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main resonant process with an electron intermediate state is the process with $l=0,l'=1$ (and $l=0,l'=-1$ for the positronic intermediate state). The kinematics of the resonance are analyzed in detail. Section III is devoted to finding the amplitude of the process in the resonant approach. In Sec. IV we obtain the resonant probability of the two-photon annihilation process in the pulsed plane wave. The resonant probability of the two-photon annihilation process in the pulsed plane wave compares with the probability two-photon annihilation of an e^+-e^- pair in the absence of external field.

In the relativistic system of units, $\hbar=c=1$ and the standard metrics for four-vectors $(ab)=a_0b_0-\mathbf{ab}$ for two arbitrary four-vectors $a=(a_0,\mathbf{a}),b=(b_0,\mathbf{b})$ will be used throughout this paper.

II. RESONANT KINEMATICS

Let us choose the four-potential of the pulsed plane wave in following form:

$$A(\phi)=g(\phi)A_{\rm mono}(\varphi), \tag{6}$$

$$A_{\rm mono}(\varphi)=a(e_x\cos\varphi+\delta' e_y\sin\varphi), \tag{7}$$

$$\varphi=(kx)=\omega t-\mathbf{kr},\quad \phi=\varphi/\varphi_{\rm imp}, \tag{8}$$

where φ is the wave phase; $x=(t,\mathbf{r})$ is the four-radius vector; $e_{x,y}=(0,\mathbf{e}_{x,y})$ and $k=(\omega,\mathbf{k})$ are the wave polarization four-vector and the four-momentum of the photon of the external wave, while $k^2=0$, $e_{x,y}^2=-1$, $(e_{x,y}k)=0$; $\delta'=\pm1$ corresponds to the circularly polarized wave; and $g(\phi)$ is the envelope function of the external wave four-potential, which has to satisfy the additional conditions $g(0)=1$ and $g\to0$ at $|\phi|\gg1\quad(|\varphi|\gg\varphi_{\rm imp})$.

The following condition is satisfied in the range of the optical frequency and picosecond pulse durations:

$$\varphi_{\rm imp}\gg1. \tag{9}$$

Thus, the spectral density of the four-potential (6) represents a sharp peak with an amplitude in order with $\varphi_{\rm imp}$ and a width in order with $\varphi_{\rm imp}^{-1}$. Therefore it is possible to consider the quantity ω as the quasimonochromatic field frequency. For concretization of the theoretical computing, we choose the wave envelope function in the Gaussian form (see, for example, the works [17,21,22,24,25,36], and many others):

$$g(\phi)=\exp(-4\phi^2). \tag{10}$$

The amplitude of the two-photon annihilation process in an external field has the form

$$S=-ie^2\int d^4xd^4x'\overline{\Psi}_{\bar p}(x)\gamma^\mu G(x,x')\gamma^\nu\Psi_p(x')\times[A_\mu^*(k_1x)A_\nu^*(k_2x')+A_\nu^*(k_1x')A_\mu^*(k_2x)], \tag{11}$$

where $p=(E,\mathbf{p}),\bar p=(\bar E,\bar{\mathbf{p}})$ are the four-momentum of an initial electron and a positron; $k_{1,2}=\omega_{1,2}n_{1,2}$ are the wave four-vectors of formed photons; $n_{1,2}=(1,\mathbf{n}_{1,2})$; $\mathbf{n}_{1,2}$ are directions of formed photons propagation; γ^μ $(\mu=0,1,2,3)$ are the Dirac matrices; $A_\mu^*(k_{1,2}x)$ are the wave functions of formed photons; $\Psi_p(x),\Psi_{\bar p}(x)$ are the wave functions (Volkov wave functions) of the initial electron and positron in the field (6) [37]; and $G(x,x')$ is the Green's function of an intermediate particle in the field (6) [38]. The explicit forms of these function are shown, for example, in the work [8,16,17,22,23,25,36,39–43].

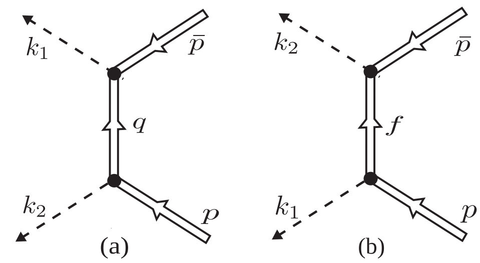

FIG. 1. Feynman diagram of two-photon annihilation of an e^--e^+ pair in the external field.

After integration of the spatial coordinates, in zeroth approximation over the parameters $\varphi_{\rm imp}^{-1}$, the amplitude (11) takes the form (2), where the partial amplitude is represented by the following expression:

$$S_{l,l'}=\frac{B}{\sqrt{\omega_1\omega_2}}\bar u_{\bar p}T_{l,l'}u_p(2\pi)^3\delta^{(2)}(\mathbf{p}_\perp+\bar{\mathbf{p}}_\perp-\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})\times\delta(p_-+\bar p_--k_{1-}-k_{2-}). \tag{12}$$

Here $u_p,\bar u_{\bar p}$ are the Dirac bispinors; $\mathbf{p}_\perp$ is the projection of the vector $\mathbf{p}$ on the plane of the wave polarization ($\bar{\mathbf{p}}_\perp$, $\mathbf{k}_{1\perp},\mathbf{k}_{2\perp}$ are defined similarly); $p_-=E-p_z$ is the difference between the zero component of the four-momenta and their projection along the wave propagation direction ($\bar p_-$, k_{1-},k_{2-} are defined similarly); $B=-ie^2/\sqrt{2\bar E2EV^2}$ is the normalization constant; V is the normalization volume; and $\omega_{1,2}$ are frequencies of formed photons.

In the expression (12) $T_{l,l'}=e_1^{*\mu}e_2^{*\nu}T_{l,l'}^{\mu\nu}$, where $\mu,\nu=0,1,2,3$ and summation over the mute indices ν,μ is assumed; e_1^*,e_2^* are the polarization four-vectors of formed photons; $T_{l,l'}^{\mu\nu}$ is determined by the formula

$$T_{l,l'}^{\mu\nu}=T_{l,l'}^{\mu\nu}(a)+T_{l,l'}^{\nu\mu}(b). \tag{13}$$

In the expression (13) $T_{l,l'}^{\mu\nu}(a)$ corresponds to the diagram (a) in Fig. 1, and $T_{l,l'}^{\nu\mu}(b)$ corresponds to the diagram (b). For the diagram (a) we have

$$T_{l,l'}^{\mu\nu}(a)=Q_{l-l'}^{\mu}(-\bar p,q,k_1)\frac{\hat q+m}{q^2-m^2}Q_{l'}^{\nu}(q,p,k_2), \tag{14}$$

where q is the four-momentum of an intermediate particle.

Note that in the pulsed field Eq. (7) does not hold a complete four-momentum conservation law. However, as in the case of an arbitrary plane-wave field there are conservation laws for the perpendicular components to the wave propagation direction of the momentum and difference between the zero component of the four-momenta and its projection along the wave propagation direction of the particles. Let us introduce the parameters l_*,l_*' which are determined by the equations

$$p+(l_*'-l_*)k=q+k_1,\quad \bar p+q=k_2+l_*'k \tag{15}$$

where as a result of the condition (1) the terms proportional to $\sim\eta^2$ are neglected. Excluding the intermediate particle momentum q from the system (15) we get

$$p + \bar{p} + l_* k = k_1 + k_2. \tag{16}$$

Note that $l_* \to l, l'_* \to l'$ corresponds to the case of a monochromatic wave and as a result of the condition (1) $|l - l_*| \sim \varphi_{\mathrm{imp}}^{-1} \ll 1$, $|l'_* - l'| \sim \varphi_{\mathrm{imp}}^{-1} \ll 1$.

The matrix $Q^{\nu}_{l'}$ in the expression (14) up to the terms $\sim\eta^2$ takes the forms

$$Q^{\nu}_{l'}(q,p,k_2) \approx \begin{cases} \gamma^{\nu} 2\pi\delta(\zeta_0) + \eta^2 M^{\nu}_0(q,p) f_2(\zeta_0), & l' = 0; \\ \eta^{|l'|} M^{\nu}_{l'}(q,p) f_{|l'|}(\zeta_{l'}), & l' = \pm 1, \pm 2. \end{cases} \tag{17}$$

Here

$$\zeta_{l'} = l' - l'_*. \tag{18}$$

The matrices $M^{\nu}_{l'}$ ($l' = 0, \pm 1, \pm 2$) in the expression (17) are given by

$$M^{\nu}_0(q,p) = -\frac{y_0^2}{4}\gamma^{\nu} + \frac{m^2\eta^2}{8(kp)(kq)}\hat{k}k^{\nu} + \frac{y_0}{2}(e^{-i\chi}a^{\nu}_{+} - e^{i\chi}a^{\nu}_{-}), \tag{19}$$

$$M^{\nu}_{\pm 1}(q,p) = \pm\frac{y_0}{2}e^{\mp i\chi}\gamma^{\nu} + a^{\nu}_{\mp}, \tag{20}$$

$$M^{\nu}_{\pm 2}(q,p) = \pm\frac{1}{2}y_0 e^{\mp i\chi}\left(\frac{1}{4}y_0 \times e^{\mp i2\chi}\gamma^{\nu} + a^{\nu}_{\mp}\right), \tag{21}$$

$$a^{\nu}_{\pm} = \frac{m}{2(kp)}(\hat{\varepsilon}_{\pm}k^{\nu} - \hat{k}\varepsilon^{\nu}_{\pm}) + \frac{m}{4}\beta_{-}\hat{\varepsilon}_{\pm}\hat{k}\gamma^{\nu}, \tag{22}$$

$$\varepsilon_{\pm} = e_x \pm i\delta' e_y, \tag{23}$$

$$y_0 = 2\sqrt{\frac{m^2}{2}\beta_{-}\left(1 - \frac{m^2}{2}\beta_{-}\right)}, \quad \beta_{-} \leqslant \frac{2}{m^2}, \tag{24}$$

$$\beta_{-} = \frac{1}{(kq)} - \frac{1}{(kp)}. \tag{25}$$

The expression for $T^{\nu\mu}_{l,l'}(b)$ we can obtain from the formula (14) by substituting $q \to f, k_1 \leftrightarrow k_2$. The functions $f_{n'}(\zeta_{l'})$ ($n' = 1,2; l' = 0, \pm 1, \pm 2$) in the expressions (17) have the form

$$f_{n'}(\zeta_{l'}) = \int_{-\infty}^{\infty} d\phi\, g^{n'}(\phi) \exp[i\varphi_{\mathrm{imp}} h(\phi)], \tag{26}$$

$$h(\phi) = \zeta_{l'}\phi + \frac{m^2\eta^2}{2}\beta_{-}\int_{-\infty}^{\phi} g^2(\phi')d\phi'. \tag{27}$$

Subsequently the intensity will be restricted by the more stringent condition than Eq. (1):

$$\eta^2 \lesssim \varphi_{\mathrm{imp}}^{-1}. \tag{28}$$

Using this condition we obtain an analytical form of the function $f_{n'}(\zeta_{l'})$:

$$f_{n'}(\zeta_{l'}) \approx \int_{-\infty}^{\infty} g^{n'}(\phi) \exp(i\varphi_{\mathrm{imp}}\zeta_{l'}\phi)d\phi = \frac{1}{2}\sqrt{\frac{\pi}{n'}}e^{-\frac{\varphi_{\mathrm{imp}}^2\zeta_{l'}^2}{16n'}}. \tag{29}$$

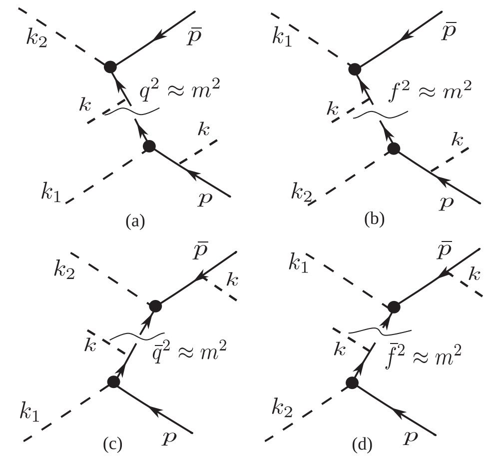

FIG. 2. Resonance of a two-photon annihilation of an electron-positron pair in the external field.

As a result of this expression the difference between l' and l'_* as well as l and l_* is small (l_*, $\zeta_{l'} \sim \varphi_{\mathrm{imp}}^{-1} \ll 1$).

In accordance with the condition (4) resonant behavior shows processes for which

$$q^2 \approx m^2, \quad f^2 \approx m^2. \tag{30}$$

The analysis of expressions (15) and (30) shows that the resonance is possible only for the processes $l - l' \leqslant 1$, $l' \geqslant 1$ (electronic intermediate state) and $l - l' \geqslant 1$, $l' \leqslant 1$ (positronic intermediate state). As a result of the low-intensity condition (1) the main resonant process with an electron intermediate state is the process with $l = 0, l' = 1$ (and $l = 0, l' = -1$ for the positronic intermediate state). In Fig. 2 the diagrams (a,b) correspond to the electron intermediate state, and the diagrams (c,d) correspond to the positronic intermediate state. The diagrams (a,b) as well as (c,d) do not interfere because they are corresponded to emission of final photons in different elements of the phase space. However, the diagrams (a,c) [and accordingly (b,d)] may interfere under certain conditions.

Let us analyze the conditions when resonances appear for the diagram (a). Resonant conditions for other diagrams are performed by substituting momentums of particles. Thus the resonant condition for the diagram (b) is performed by substituting $k_1 \leftrightarrow k_2, q \to f$; for diagram (c) $q \to -\bar{q}, k \to -k$; for diagram (d) $k_1 \leftrightarrow k_2$, $q \to -\bar{f}, k \to -k$;

For analyzing the resonant region we use the exact conservation of four-momentum:

$$q = p + k - k_1 = -\bar{p} + k_2 + k. \tag{31}$$

which corresponds to a consequence of two subprocesses: emission of the photon k_1 by an electron p with absorption of one wave photon and transition of an electron to intermediate state q and annihilation of a positron and an intermediate electron with emission of one wave photon and the photon k_2. The corresponding conservation laws have the forms

$$\begin{aligned} p + k &= q + k_1, \\ q + \bar{p} &= k_2 + k. \end{aligned} \tag{32}$$

From the expressions (31) we find the expressions for the resonant frequencies of formed photons:

$$\omega_1 = \frac{(kp)}{([p+k]n_1)}, \tag{33}$$

$$\omega_2 = \frac{(k\bar{p})}{([k-\bar{p}]n_2)}. \tag{34}$$

On account of positiveness of the frequency (34) it must hold inequality

$$\bar{u} = \frac{2(k\bar{p})}{m^2} > 1. \tag{35}$$

From this condition and within the range of optical frequency of the external field a positron energy must be ultrarelativistic and the angle between the positron momentum and direction of wave propagation is not small:

$$\bar{E} > E_{\lim} = \frac{m^2}{2\omega(1-\cos\bar{\theta}_i)} \gg m, \tag{36}$$

$$\bar{\theta}_i = \angle(\mathbf{k},\bar{\mathbf{p}}) \sim 1. \tag{37}$$

From the formula (34) it follows that the angle between the direction of the photon $\mathbf{k}_2$ propagation and the positron momentum $\bar{\theta}_2 = \angle(\mathbf{n}_2,\bar{\mathbf{p}})$ is small:

$$\bar{\theta}_2^2 < \left(\frac{m}{\bar{E}}\right)^2(\bar{u}-1) \sim \left(\frac{m}{\bar{E}}\right)^2 \ll 1. \tag{38}$$

Taking into account the conditions (36) and (38) the expression for the frequency (34) can be rewritten as

$$\omega_2 \approx \frac{\bar{u}\bar{E}}{\bar{u}-1-(\bar{u}\bar{\delta}_2)^2}, \quad \bar{\delta}_2^2 < \frac{\bar{u}-1}{\bar{u}^2}, \tag{39}$$

where $\bar{\delta}_2 = E_{\lim}\bar{\theta}_2/m$ is a relativistic parameter.

Note that the second photon frequency (39) is greater then the incident positron energy $\omega_2 > \bar{E}$.

Excluding q from the system (32) we find the conservation law which coincides with the case of the absence of the external field:

$$p + \bar{p} = k_1 + k_2. \tag{40}$$

Taking into account that there are two final particles in the studied process, which correspond to six unknown values (for example, these are two frequencies and 2×2 angles), the conservation law (40) allows to find four of them. As a result two values remain unknown. Let it be the polar and azimuthal angles of the second photon $(\bar{\theta}_2,\bar{\psi}_2)$ in the coordinate system in which the z axis is along the positron momentum. The resonant condition decreases their numbers by one. Thus, under the resonant condition there is a relationship between the angles $\bar{\theta}_2,\bar{\psi}_2$. For determination of this relationship we find the frequency ω_2 from the expression (40):

$$\omega_2 = \frac{(p+\bar{p})^2}{2([p+\bar{p}]n_2)}. \tag{41}$$

Equating the frequencies (34) and (41) we obtain the expression

$$(\bar{h}n_2) = 0, \tag{42}$$

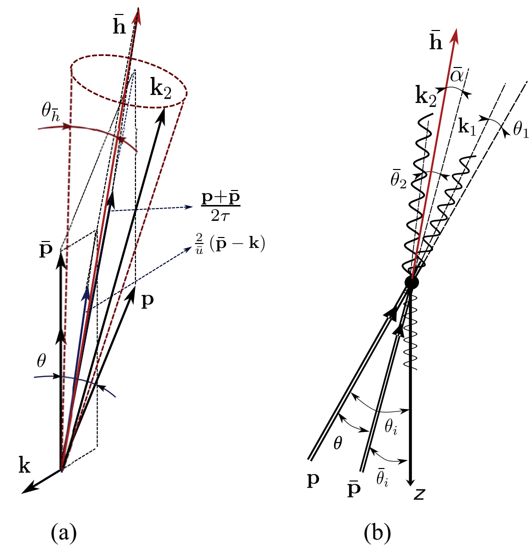

FIG. 3. The geometry of emission of the photon k_2 under resonance of the diagram (a) in Fig. 2: (a) geometric interpretation of the formula 47 and (b) collision geometry.

where four-vector h is determined by parameters of initial particles:

$$\bar{h} = (\bar{h}_0,\bar{\mathbf{h}}) = \frac{p+\bar{p}}{2\tau} + \frac{\bar{p}-k}{\bar{u}/2}. \tag{43}$$

Here

$$\tau = \frac{(p+\bar{p})^2}{4m^2} \approx \frac{(u+\bar{u})^2}{4\bar{u}u} + \frac{\bar{u}u}{4}\delta^2, \tag{44}$$

where the relativistic parameter δ and the invariant parameter u are given by

$$\delta = \frac{E_{\lim}\theta}{m}, \quad \theta = \angle(\mathbf{p},\bar{\mathbf{p}}), \tag{45}$$

$$u = \frac{2(kp)}{m^2}. \tag{46}$$

From the formula (42) it follows that

$$\cos\theta_{\bar{h}} = \frac{\bar{h}_0}{|\bar{\mathbf{h}}|}, \quad \angle(\bar{\mathbf{h}},\mathbf{n}_2) = \theta_{\bar{h}}. \tag{47}$$

The expression (47) indicates that under resonance of the diagram (a) through the electronic intermediate state the direction of emission of the second photon is one of the rulings of the cone, which is made by the vector $\bar{\mathbf{h}}$ (43) with the cone angle $\theta_{\bar{h}}$ (47) [see Fig. 3(a)]. Under the condition (38) the angle $\theta_{\bar{h}}$ is small:

$$\theta_{\bar{h}} \approx \frac{\sqrt{-\bar{h}^2}}{\bar{h}_0} = \frac{m}{E_{\lim}}\delta_{\bar{h}} \sim \frac{m}{E_{\lim}} \ll 1, \tag{48}$$

$$\delta_{\bar{h}} = \frac{\sqrt{\frac{\bar{u}u}{4\tau}-1}}{\bar{u}\left(\frac{\bar{u}+u}{4\tau}+1\right)}. \tag{49}$$

Note that $\bar{\delta}_2$ in the formula (39) can be written as

$$\bar{\delta}_2^2 \approx \delta_{\bar{\alpha}}^2 + \delta_{\bar{h}}^2 - 2\delta_{\bar{h}}\delta_{\bar{\alpha}}\cos\bar{\psi}_2. \tag{50}$$

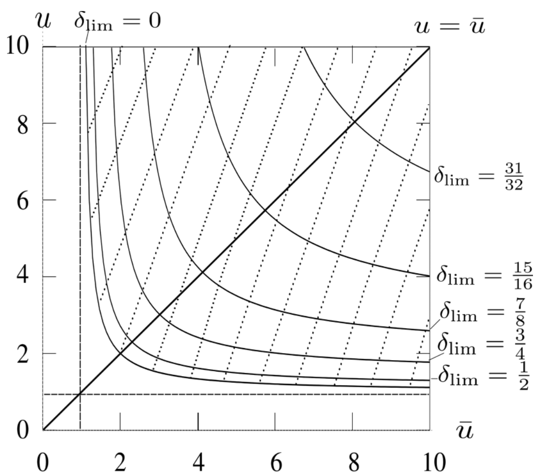

FIG. 4. The resonant region (shaded) on the plane $(\bar{u},u)$. The line $\bar{u} = u$ is corresponded to interference of the resonant amplitudes.

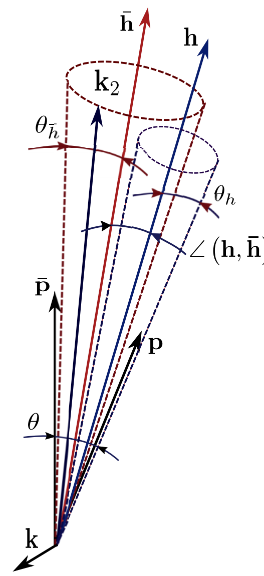

FIG. 5. Geometry of resonant annihilation through electronic and positronic intermediate states.

Here $\bar{\psi}_2$ is the azimuthal angle, in the coordinate system in which the z axis is along the vector $\bar{\mathbf{h}}$; the measure of angle $\bar{\psi}_2$ starts from the vector $\bar{\mathbf{p}}$; and $\delta_{\bar{\alpha}}$ is determined by

$$\delta_{\bar{\alpha}} = \frac{E_{\rm lim}\bar{\alpha}}{m} = \sqrt{\frac{4}{\bar{u}^2\left(2+\frac{u+\bar{u}}{2\tau}\right)} - \frac{1}{\bar{u}^2} + \delta_{\bar{h}}^2}, \tag{51}$$

where $\bar{\alpha} = \angle(\bar{\mathbf{p}},\bar{\mathbf{h}})$.

From Eq. (47) follows the condition $\bar{h}^2 \leqslant 0$, from which the additional condition of resonance follows:

$$\frac{4\tau}{u\bar{u}} \leqslant 1, \tag{52}$$

The analysis of the expression (52) leads to the inequality

$$0 \leqslant \delta^2 \leqslant \delta_{\rm lim}^2, \quad \delta_{\rm lim}^2 = 1 - \left(\frac{1}{\bar{u}} + \frac{1}{u}\right)^2. \tag{53}$$

The inequality (53) indicates that under resonant condition an electron moves within a narrow cone with a positron ($\theta \sim m/E_{\rm lim} \ll 1$), and also an electron energy must be ultrarelativistic:

$$E > \frac{\bar{E}}{\bar{u}-1} \gg m. \tag{54}$$

Figure 4 demonstrates the resonant region on the plane $(\bar{u},u)$ which is determined by the condition (53).

Taking into account the ultrarelativistic values of an electron [Eq. (54)] and a positron [Eq. (36)], and the condition (53), the parameters u [Eq. (46)] and $\bar{u}$ [Eq. (35)] can be written as

$$u \approx \frac{E}{E_{\rm lim}}, \quad \bar{u} \approx \frac{\bar{E}}{E_{\rm lim}}. \tag{55}$$

Note that the condition (52) is stronger than the condition (35). Moreover from the expression (52) it follows that the resonance through the electronic intermediate state is accompanied by the resonance through the electronic intermediate state. Arguing similarly as for the case of the resonance through the positronic intermediate state, for the resonance through the electronic intermediate state [diagram (c) in Fig. 2], we obtain that the direction of emission of the second photon is one of the rulings of the cone, which is made by the vector $\mathbf{h}$, with the cone angle θ_h. The expressions for $\mathbf{h}$ and θ_h we can obtain from the formulas (43) and (48) with the replacements $\bar{p} \leftrightarrow p$.

Note that the condition $h^2 \leqslant 0$ leads to the inequality (52). An angle between the vectors $\mathbf{h}$ and $\bar{\mathbf{h}}$ (see Fig. 5) is equal to

$$\angle(\mathbf{h},\bar{\mathbf{h}}) \approx \left(\frac{m}{E_{\rm lim}}\right)\delta_{\mathbf{h}\bar{\mathbf{h}}} \sim \frac{m}{E_{\rm lim}} \ll 1, \tag{56}$$

$$\delta_{\mathbf{h}\bar{\mathbf{h}}} = \frac{\delta}{1+\frac{u+\bar{u}}{4\tau}}. \tag{57}$$

Note also that for the resonance through the positronic intermediate state it is not necessary to obtain separate expressions for this case. All the necessary formulas are derived from the case of resonance through the positronic intermediate state with the replacements $\bar{E} \leftrightarrow E$ $(\bar{u} \longleftrightarrow u)$.

III. RESONANT AMPLITUDE

The resonance of the diagram (a) is corresponded to the partial amplitude $S_{1,-1}$ (12), in which

$$T_{1,-1}^{\mu\nu}(a) = M_1^{\mu}(-\bar{p},q)\omega \int_{-\infty}^{\infty} f_1(l_* - \zeta_1)\frac{\hat{q}+m}{q^2-m^2+i0} \times f_1(\zeta_1)d\zeta_1 M_{-1}^{\nu}(q,p), \tag{58}$$

where $|l_* - 1| \sim \varphi_{\rm imp}^{-1} \ll 1$.

Taking into account Eq. (10) the integration over $d\zeta_1$ in the expression (58) is derived analytically:

$$\int_{-\infty}^{\infty} f_1(l_* - \zeta_1)\frac{\hat{q}+m}{q^2-m^2+i0} f_1(\zeta_1)d\zeta_1 \approx \frac{\pi^2}{8(kq)}(\hat{q}+m)I(\beta,l_*). \tag{59}$$

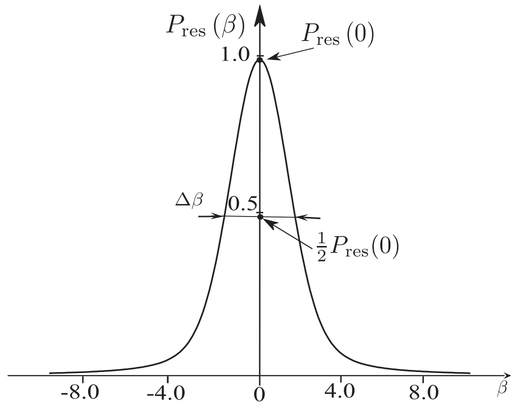

FIG. 6. Plot of function $P_{\rm res}$ [Eq. (64)] that determines the resonance profile vs resonance parameter β.

In the expression (59) the complex function $I(\beta,l_*)$ is given by

$$I(\beta,l_*)=\frac{\pi}{4(kq)}\left[\operatorname{erfi}\left(\frac{\sqrt{2}(\beta+l_*\varphi_{\rm imp}/4)}{2}\right)+i\right]\times\exp\left[-\frac{\varphi_{\rm imp}^2l_*^2+8(\beta+\varphi_{\rm imp}l_*/4)^2}{16}\right].\quad(60)$$

Note that in the nonresonant region when $\beta\gg1$ function $I(\beta,l_*)$ is asymptotically represented as

$$I(\beta,l_*)\approx\sqrt{\frac{\pi}{2}}\frac{1}{\beta}e^{-\frac{1}{32}l_*^2\varphi_{\rm imp}^2}\quad(61)$$

and it corresponds to Gaussian broadening of the frequency of final photons.

Taking into account Eq. (59) the expression (58) takes the following form:

$$T_{1,-1}^{\mu\nu}(a)=\frac{\omega\pi^2}{8(kq)}I(\beta,l_*)K_{1,-1}^{\mu\nu}(a),\quad(62)$$

$$K_{1,-1}^{\mu\nu}(a)=M_1^{\mu}(-\bar{p},q)(\hat{q}+m)M_{-1}^{\nu}(q,p).\quad(63)$$

Note that the expression for $K_{1,-1}^{\nu\mu}$ does not differ from the case of a monochromatic wave.

Note that the function $I(\beta,l_*)$ [Eq. (60)] determines the resonance profile [see the formula (72), works [16,21,23,25,44], and Fig. 6]:

$$P{\rm res}(\beta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}|I(\beta,l_*)|^2dl_*.\quad(64)$$

The resonant parameter β (3) for the diagram (a) can be written as

$$\beta\approx\frac{1}{2u_q}\frac{\bar{u}^2}{\left(\frac{\bar{u}-1}{\bar{u}}\right)-\bar{u}\delta_2^2}\left(1+\frac{u+\bar{u}}{4\tau}\right)\left(\delta_{\bar{h}}^2-\delta_{\bar{h}}'^2\right)\varphi_{\rm imp}\quad(65)$$

where the invariant parameter u_q and the relativistic parameter $\delta'_{\bar{h}}$ are given by

$$u_q=\frac{2(kq)}{m^2}=\frac{2(pk_1)}{m^2}\approx\bar{u}\frac{1+\bar{\delta}_2^2}{\bar{u}-1-\bar{\delta}_2^2},\quad(66)$$

$$\delta'_{\bar{h}}=\frac{E_{\rm lim}\theta'_{\bar{h}}}{m},\quad\theta'_{\bar{h}}=\angle(\bar{\mathbf{h}},\mathbf{k}_2).\quad(67)$$

Note that the angle $\theta'_{\bar{h}}$ is not limited by the relation (50). It can get any value and the angle $\theta'_{\bar{h}}=\theta_{\bar{h}}$ is corresponded to the resonance maximum.

IV. RESONANT PROBABILITY

In a resonant approach and in the absent interference of amplitudes the probability of the two-photon annihilation process in the pulsed plane wave (7) is given by

$$\begin{aligned}dW_{\rm res}^{(a)}&=2|S_{1,-1}|^2\frac{d^3k_1d^3k_2}{(2\pi)^6}\approx\frac{2|A|^2}{\omega_1\omega_2}\left(\frac{\omega\pi^2}{8(kq)}\right)^2|I(\beta,l_*)|^2\\&\times\frac{\varphi_{\rm imp}^4}{\omega^4}|\bar{u}_{-\bar{p}}K_{1,-1}u_p|^2V^2d^3k_1d^3k_2\\&\times[\delta^{(2)}(\mathbf{p}_\perp+\bar{\mathbf{p}}_\perp-\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})\\&\times\delta(p_-+\bar{p}_--k_{1-}-k_{2-})]^2.\end{aligned}\quad(68)$$

where $K_{1,-1}=e_{1\mu}^*e_{2\nu}^*K_{1,-1}^{\mu\nu}(a)$ [Eq. (63)]. Note that the resonant probability is doubled as a result of the identity of the final particles (photons) and the squared 3-δ functions are equal to

$$\begin{aligned}&[\delta^{(2)}(\mathbf{p}_\perp+\bar{\mathbf{p}}_\perp-\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})\delta(p_-+\bar{p}_--k_{1-}-k_{2-})]^2\\&\to\delta(\mathbf{p}_\perp+\bar{\mathbf{p}}_\perp-\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})\\&\times\delta(p_-+\bar{p}_--k_{1-}-k_{2-})\frac{\omega_2}{k_{2-}}\frac{V}{(2\pi)^9}.\end{aligned}\quad(69)$$

The integration over $d^2\mathbf{k}_{1\perp}$ is simple owing to the presence of the δ functions: $\delta^{(2)}(\mathbf{k}_{1\perp}-[\mathbf{p}_\perp+\bar{\mathbf{p}}_\perp-\mathbf{k}_{2\perp}])$. The integration over dk_{1z} is conducted taking into account that

$$\delta(p_-+\bar{p}_--k_{1-}-k_{2-})=\frac{\omega_1}{k_{1-}}\delta\big(k_{1z}-\big[E_z+\bar{E}_z-k_{2z}\big]\big).\quad(70)$$

We change the integration over $d\omega_2$ to dl_* taking into account that the range of variation of the parameter l_* is small ($|l_*|\sim\varphi_{\rm imp}^{-1}\ll1$):

$$d\omega_2\approx dl_*\frac{(kk_2)}{([p+\bar{p}]n_2)}.\quad(71)$$

Taking into account the expressions (69)–(73) after averaging over initial and summation over final polarizations of particles the resonant differential probability takes the form

$$\frac{dW_{\rm res}}{d\bar{\Omega}_2}=\frac{\pi^2e^4\eta^4\omega_2^2}{64V\bar{E}Ek_{2-}u_q^2([p+\bar{p}]n_2)}\varphi_{\rm imp}^2P_{\rm res}(\beta)H\tau_{\rm imp},\quad(72)$$

where $d\bar{\Omega}_2=\sin\bar{\theta}_2d\bar{\theta}_2d\bar{\psi}_2$ is the element of solid angle in which the second photon is emitted and we introduced the following notations:

$$H=-\left(f(\bar{\upsilon},u_q)f(\upsilon,u_q)+y(\bar{\upsilon},u_q)\cdot y(\upsilon,u_q)-\frac{2\upsilon\bar{\upsilon}}{u_q^2}\tau+\frac{\upsilon\bar{\upsilon}}{(1-\upsilon)(1-\bar{\upsilon})}\left[\frac{\bar{\upsilon}}{u_q}+\frac{\upsilon}{u_q}-2\frac{\upsilon\bar{\upsilon}}{u_q^2}\right]\right),\quad(73)$$

$$f(\bar{\upsilon},u_q)=2\frac{\bar{\upsilon}}{u_q}\left(1-\frac{\bar{\upsilon}}{u_q}\right)-\left(1+\frac{\bar{\upsilon}^2}{2(1-\bar{\upsilon})}\right),\quad(74)$$

$$y(\bar{\upsilon},u_q)=\frac{(2+\bar{\upsilon})(2\bar{\upsilon}-u_q)\bar{\upsilon}}{2u_q(\bar{\upsilon}-1)}. \tag{75}$$

The invariant parameters $\upsilon,\bar{\upsilon}$ are given by

$$\upsilon=\frac{(kk_1)}{(kp)}=\frac{(kn_1)}{(qn_1)}=1+\frac{\bar{u}}{u}(1-\bar{\upsilon}), \tag{76}$$

$$\bar{\upsilon}=\frac{(kk_2)}{(k\bar{p})}\approx\frac{\bar{u}}{\bar{u}-1-\bar{\delta}_2^2}. \tag{77}$$

Note that the term proportional to the zero power of η in the series expansion of the amplitude (11) determines the amplitude of two-photon annihilation of an electron-positron pair in the absence of external field. The corresponding differential probability is given by

$$\frac{dW}{d\bar{\Omega}_2}=\frac{2e^4}{\bar{E}EV}\left[-\left(\frac{m^2}{(\rho-m^2)}+\frac{m^2}{(s-m^2)}\right)^2 -\left(\frac{m^2}{\rho-m^2}+\frac{m^2}{s-m^2}\right)+\frac{1}{4}\left(\frac{s-m^2}{\rho-m^2}+\frac{\rho-m^2}{\rho-m^2}\right)\right] \times\frac{1}{\omega_2([p+\bar{p}]n_2)}T, \tag{78}$$

where the parameters ρ,s are given by

$$s=(-\bar{p}+k_2)^2,\quad \rho=(p-k_2)^2, \tag{79}$$

$$s-m^2=-m^2u_q,\quad \rho-m^2=-m^2(4\tau-u_q). \tag{80}$$

Using these notations the probability (78) can be written as

$$dW=\frac{2e^4}{\bar{E}EV}f(u_{12},u_2)\frac{1}{\omega_2([p+\bar{p}]n_2)}d\bar{\Omega}_2T, \tag{81}$$

where T is the observation time and introducing additional parameters:

$$u_{12}=\frac{(k_1k_2)}{(k_2\bar{p})}=1+\frac{1+(u\delta_2)^2}{1+(\bar{u}\bar{\delta}_2)^2}, \tag{82}$$

$$u_2=\frac{2(pk_2)}{m^2}\approx\frac{\bar{u}^2}{u}\frac{1+(u\delta_2)^2}{\bar{u}-1-(\bar{u}\bar{\delta}_2)^2}. \tag{83}$$

Here $\delta_2=E_{\lim}\theta_2/m$, where $\theta_2=\angle(\mathbf{p},\mathbf{n}_2)$. In the resonance region the parameter δ_2 takes values

$$\delta_2^2\approx\delta_\alpha^2+\delta_h^2+2\delta_\alpha\delta_h\cos\bar{\psi}_2, \tag{84}$$

where the expression for δ_α we can obtain from the formula (51) with the replacements $\bar{p}\leftrightarrow p$.

The ratio of the resonant probability of the two-photon annihilation process in the pulsed plane wave (7) to the probability two-photon annihilation of an e^+-e^- pair in the absence of external field is equal to

$$R=\frac{dW_{\rm res}}{dW} =\frac{\pi^2}{2^6(1-\cos\theta_i)u_q^2}\eta^4\varphi_{\rm imp}^2P_{\rm res}(\beta)\frac{H}{f(u_{12},u_2)}\frac{t_{\rm imp}}{T}. \tag{85}$$

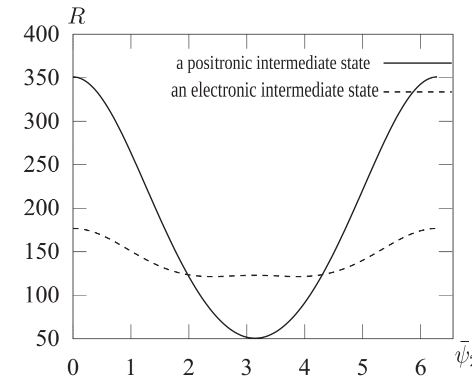

FIG. 7. Dependence of the ratio of the resonant differential probability of two-photon annihilation of an electron-positron pair at the resonance peak ($\beta=0$) to the differential probability of this process in the absence of the external field at $t_{\rm imp}/T=1$, $\eta=0.1$ ($I=4.04\times10^{16}$ W/cm^2), $\bar{u}=7,u=2$, $\varphi_{\rm imp}=2.29\times10^3$.

Figure 7 demonstrates the ratio (85) at the resonance peak ($\beta=0$) at $t_{\rm imp}/T=1$ and $\eta=0.1$ for the electronic (the dotted line) and positronic (the solid line) intermediate states. It is seen that the resonance probability can be significantly greater than the probability of this process in the absence of external field. Figure 8 demonstrates the dependences of the ratio of energies of the final photons $\omega_{1,2}$ to the threshold energy $E_{\lim}$ (36) on the azimuth angle ψ_2 at the same values of the parameters of initial particles as in Fig. 7. The values $\bar{u}=7$, $u=2,\varphi_{\rm imp}=2.29\times10^3$ correspond to the energies $\bar{E}\approx1225.4$ GeV, $E\approx350.1$ GeV, and the pulse duration $t_{\rm imp}\approx1.0\times10^{-12}$ s for the optical frequency $\omega=1.5$ eV ($E_{\lim}\approx175.1$ GeV). And these values correspond to the energies $\bar{E}\approx1.84$ GeV, $E\approx0{,}50$ GeV, and the pulse duration

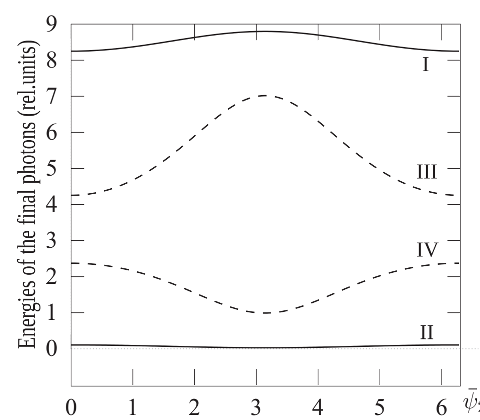

FIG. 8. Dependence energies (in units of the threshold energy $E_{\lim}$) of the final photons on the azimuth angle $\bar{\psi}_2$. Solid line I corresponds to the second photon frequency (ω_2) in the case of resonance through the positronic intermediate state. Solid line II corresponds to the first photon frequency ω_1 in the case of resonance through the positronic intermediate state. Dashed line III corresponds to the second photon frequency (ω_2) in the case of resonance through the electronic intermediate state. Dashed line IV corresponds to the first photon frequency ω_2 in the case of resonance through the electronic intermediate state.

$t_{\text{imp}} \approx 1.5 \times 10^{-15}$ s for free-electron laser with keV-order photon energy $\omega = 1.0$ keV ($E_{\text{lim}} \approx 0{,}26$ GeV).

V. CONCLUSION

Analysis of two-photon annihilation of an electron-positron pair in the field of the low-intensity pulsed electromagnetic wave has demonstrated that it may occur in the resonant region.

The resonance condition on the parameters of the initial particles are the following:

(1) Energies of a positron and an electron exceed a threshold and are ultrarelativistic: $\bar{E} > E_{\text{lim}} \gg m, E > E_{\text{lim}} \gg m$: $\bar{E}^{-1} + E^{-1} < E_{\text{lim}}^{-1}$.

(2) The angles between the positron momentum and direction of wave propagation and the electron momentum and direction of wave propagation are not small, $\theta_i, \bar{\theta}_i \sim 1$; the angle between the electron momentum and the positron momentum is small, $\theta \sim m/\bar{E} \ll 1$.

(3) Simultaneously the resonances are observed through positron and electronic intermediate states. However, these conditions do not interfere with the exception of a case when initial electron and positron energies are equal, when these two states can not be distinguished.

The resonance probability of the two-photon annihilation of an electron-positron pair in the presence of the field of the low-intensity pulsed electromagnetic wave can be greater than the probability in the absence of the external field by two orders of magnitude at the upper limit of the condition (28): $\eta^2 \sim \varphi_{\text{imp}}^{-1}$.

The obtained results may be experimentally verified by the laser facilities of the international megaprojects, for example, SLAC (National Accelerator Laboratory), FAIR (Facility for Antiproton and Ion Research), XFEL (European X-Ray Free-Electron Laser), and HIBEF (Helmholtz International Beamline for Extreme Fields).

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