Atom-assisted quadrature squeezing of a mechanical oscillator inside a dispersive cavity

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(Received 11 December 2015; published 17 August 2016)

We present a hybrid optomechanical scheme to achieve dynamical squeezing of position quadrature of a mesoscopic mechanical oscillator, that can be externally controlled by classical fields. A membrane-in-the-middle setup is employed, in which an atom in Λ configuration is considered to be trapped on either side of the membrane inside the cavity. We show that a considerable amount of squeezing (beyond the 3-dB limit) can be achieved and maintained at a transient time scale that is not affected by the spontaneous emission of the atom. Squeezing depends upon the initial preparation of atomic states. Further, a strong effective coupling (larger than the relevant decay rates) between the atom and the oscillator can be attained by using large control fields that pump the atom and the cavity. The effects of cavity decay and the phononic bath on squeezing are studied. The results are supported by the detailed analytical calculations.

DOI: 10.1103/PhysRevA.94.023831

I. INTRODUCTION

Detection of quantum effects in mesoscopic harmonic oscillators (MHOs) has been in the focus of study for quite a long time. Such an oscillator is composed of a few billion atoms and therefore can be considered as a system of classical nature. Interestingly, at low temperature, it can be driven into a quantum state, e.g., a superposition of separated position eigenfunctions. Enormous attempts have been taken to reach the quantum regime in the MHO. A central thrust of this effort has been the development of ultrasensitive displacement measurement techniques. The measurement of position of an oscillator of mass m in the quantum regime is, however, limited by the standard quantum limit [SQL; $(\Delta x)_{\rm SQL} = \sqrt{\hbar/2m\omega_m}$], arising due to the intrinsic zero-point fluctuation, where ω_m is the natural frequency of the oscillator. In addition, the oscillator also gets perturbed by the measurement device itself in the quantum regime, leading to the back-action of the oscillator onto the measurement device. This increases the minimum limit of achievable uncertainty of the position to $\sqrt{2}(\Delta x)_{\rm SQL}$. To date, the best possible uncertainty that could have been achieved is $\sim 4(\Delta x)_{\rm SQL}$ [1] in a nanomechanical oscillator ($\omega_m = 1.35$ GHz), coupled to a single-electron transistor, while uncertainties $\sim 100(\Delta x)_{\rm SQL}$ [2] and $\sim 30(\Delta x)_{\rm SQL}$ [3] are also reported in lower-frequency oscillators. Back-action evading techniques with ideally infinite measurement precision have been proposed [4] and demonstrated [5] to achieve up to $\sim 1.3(\Delta x)_{\rm SQL}$.

Measurement of position below the SQL has seen a growing interest in recent times in the context of cavity optomechanical systems [6]. Generating nonclassical states like position-squeezed states in this system can lead to $(\Delta x) < (\Delta x)_{\rm SQL}$. Such a system consists of a single mode Febry-Perot cavity with one movable end mirror, in which the coupling between the cavity mode and the mechanical mode of the mirror is created due to the radiation pressure force. It has been considered as a test platform to explore possibilities of squeezing in mesoscopic oscillators. The radiation pressure force makes the coupling between the two modes, linear in x, the displacement of the mirror from its equilibrium position. Several proposals have been made to achieve position squeezing in such systems. For example, it can be obtained by pumping the cavity by a squeezed light source and thereafter transferring this squeezing to the oscillator mode through a state transfer protocol [7]. A two-mode cavity can be made equivalent to an engineered reservoir that can lead to squeezing of the oscillator via feedback [8]. It is also shown that short pulses can be used to obtain mechanical squeezing in the optical microcavity [9]. Such methods, however, require either a continuous source of squeezed light and high transfer efficiency at the quantum level or short pulses and thereby are not the most sought-after methods for squeezing.

A natural way of obtaining quadrature squeezing dynamically is to use a Hamiltonian that is quadratic in position quadrature $X = (b + b^\dagger)/\sqrt{2}$ or momentum quadrature $P = (b - b^\dagger)/i\sqrt{2}$, where b is the annihilation operator of the quantized oscillator. E.g., the Hamiltonian $H = \chi(b^2 + b^{\dagger 2})$, χ being equivalent to the squeezing parameter, that is similar to that of a degenerate parametric amplifier [10]. Position squeezing in the ground state of the oscillator in the presence of back-action would refer to $(\Delta X)^2 < (\Delta X)^2_{\rm SQL} = 1/2$. It was shown in [11] that if a mechanical oscillator is suspended inside a cavity (with both the mirrors fixed) at a position where frequency ω_c of the cavity sees a node or antinode (i.e., $\partial\omega_c/\partial x = 0$, a "membrane-in-the-middle" setup), the coupling becomes quadratic in the displacement of the oscillator. In such a system, squeezing can be obtained through a unitary evolution. Driving the cavity with two laser beams, the frequencies of which are detuned to either side of the cavity resonance by an amount equal to the mechanical frequency in such a setup, one can also obtain [12] a quadratic Hamiltonian. The squeezing property of a quadratic Hamiltonian is discussed in detail in [13] in the context of cavity optomechanical systems. It is shown that to obtain a large squeezing, one requires a large number of average thermal photons and a proper conditional measurement of photon numbers in the cavity. This is, however, limited by the cavity decay, as a large number of photons are prone to faster decay out of the cavity and it can lead to degradation of squeezing. To combat this dissipation, alternative methods have also been proposed, that require applying three coherent fields [14] or

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periodic intense pulses [15], as commonly used in dynamical decoupling techniques [16].

In all the above methods to obtain squeezing, one employs either a passive method such as feedback or a set of coherent fields or pulses. Further, though one can dynamically achieve squeezing through a quadratic Hamiltonian, the squeezing effect is not pronounced, because the optical cavity decays much faster than the oscillator. In this paper, we propose a scheme to obtain dynamical squeezing, in which the effect of cavity decay is minimized. This would be possible, if the cavity mode interacts dispersively with the system, while squeezing in the oscillator is governed by another auxiliary system, say, an atom. Specifically, we consider an atom-cavity-oscillator hybrid system, in which a coherently driven atom is coupled to the mechanical oscillator via their common coupling to the cavity mode in the membrane-in-the-middle setup. We choose the low-lying energy levels of the atom, which are immune to spontaneous emission. The cavity mode is adiabatically eliminated from the interaction to reduce the effect of cavity decay substantially. The amount of squeezing can be obtained and maintained in a controllable way within the transient time scale. The interesting features of this model are (a) the control fields, that drive the atomic transition and the cavity mode, control the degree of squeezing in the position quadrature of the oscillator and (b) the squeezing parameter depends upon the initial state of the atom. Note that cavity optomechanics mediated by a two-level system has also been proposed in [17], in which a Josephson-junction qubit strongly interacts directly with both a microwave cavity and the micromechanical oscillator. In contrast, in the present model, the cavity mode, instead of the atom, mediates the coupling between the atom and the oscillator and the intrinsic atom-cavity and oscillator-cavity couplings are weak in nature, that further can be controlled by external pumping fields.

The paper is organized as follows. We describe our hybrid system in Sec. II. We discuss how the effective atom-oscillator Hamiltonian can be obtained via adiabatic elimination of the cavity mode. We also derive the expression of the squeezing in the time domain as well as in the frequency domain. We discuss the effect of the cavity decay and the phonon bath on squeezing. Results are discussed in detail in Sec. III, along with comparison with other proposals on squeezing. We conclude the paper in Sec. IV.

II. A HYBRID MODEL

We consider a mechanical oscillator suspended inside an optical cavity, that has both the mirrors fixed (the "membrane-in-the-middle" setup) [11]. The dynamics of this system is governed by the following Hamiltonian (in $\hbar = 1$ unit):

$$H_1 = H_0 + H_{\rm cm} + H_{\rm pump}, \tag{1}$$

where

$$\begin{aligned} H_0 &= \omega_c a^\dagger a + \omega_m b^\dagger b, \\ H_{\rm cm} &= g_2 a^\dagger a (b + b^\dagger)^2, \\ H_{\rm pump} &= \epsilon (a^\dagger e^{-i\omega_l t} + \text{H.c.}). \end{aligned} \tag{2}$$

Here ω_c and ω_m are the frequencies of the cavity mode a and the mechanical oscillator mode b, respectively, g_2 defines the coupling between them, and ϵ is the amplitude of the coherent field of frequency ω_l that pumps the cavity mode. Note that the g_2 is proportional to the second-order derivative of ω_c with respect to the displacement x of the oscillator from its equilibrium position. This Hamiltonian is quadratic, as $H_{\rm cm}$ is proportional to the second order of the operators b and $b^\dagger$ of the oscillator.

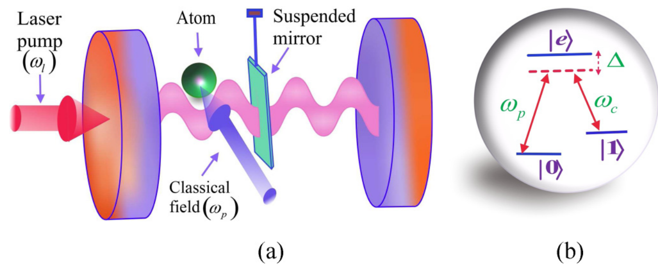

FIG. 1. (a) A membrane-in-the-middle setup, in which an atom is trapped and a mechanical oscillator is suspended inside a driven cavity. (b) Energy-level configuration of the atom.

In our hybrid model, we consider a single atom with three relevant energy levels $|0\rangle, |1\rangle, |e\rangle$ in Λ configuration, that is magneto-optically trapped inside the cavity on either side of the oscillator (see Fig. 1). The $|0\rangle \leftrightarrow |e\rangle$ transition is driven by a classical control field with frequency ω_p and the Rabi frequency Ω, while the cavity mode drives the $|1\rangle \leftrightarrow |e\rangle$ transition. The relevant Hamiltonian of the atom-cavity system can be written as

$$\begin{aligned} H_{\rm ac} &= \Omega e^{-i\omega_p t}|e\rangle\langle 0| + g_1 |1\rangle\langle e| a^\dagger + \text{H.c.}, \\ H_0^{\rm atom} &= \omega_{e0}|e\rangle\langle e| + \omega_{10}|1\rangle\langle 1|, \end{aligned} \tag{3}$$

where g_1 is the atom-cavity coupling constant and $H_0^{\rm atom}$ is the unperturbed Hamiltonian of the atom. In the reference frame, rotating with the pumping laser frequency ω_l, the total Hamiltonian $H = H_1 + H_{\rm ac} + H_0^{\rm atom}$ of the hybrid system reduces to

$$H^{(1)} = H_0^{(1)} + H_{\rm ac}^{(1)} + H_{\rm cm} + H_{\rm pump}^{(1)}, \tag{4}$$

where

$$\begin{aligned} H_0^{(1)} &= \delta a^\dagger a + \omega_m b^\dagger b + \omega_{e0}|e\rangle\langle e| + \omega_{10}|1\rangle\langle 1|, \\ H_{\rm ac}^{(1)} &= (\Omega e^{-i\omega_p t}|e\rangle\langle 0| + g_1|1\rangle\langle e| a^\dagger e^{-i\omega_l t} + \text{H.c.}), \\ H_{\rm pump}^{(1)} &= \epsilon(a + a^\dagger), \end{aligned} \tag{5}$$

and $\delta = \omega_c - \omega_l$ is the cavity pump detuning. Next, in the interaction picture with respect to the Hamiltonian $H_0^{\rm atom}$, the Hamiltonian $H^{(1)}$ takes the following form:

$$H^{(2)} = H_0^{(2)} + H_{\rm ac}^{(2)} + H_{\rm cm} + H_{\rm pump}^{(1)}, \tag{6}$$

where

$$\begin{aligned} H_0^{(2)} &= \delta a^\dagger a + \omega_m b^\dagger b, \\ H_{\rm ac}^{(2)} &= (\Omega e^{i\Delta t}|e\rangle\langle 0| + g_1|1\rangle\langle e| e^{i(\Delta+\delta)t} a^\dagger + \text{H.c.}), \end{aligned} \tag{7}$$

and $\Delta = \omega_{e0} - \omega_p = \omega_{e1} - \omega_c$ is the common detuning of the laser field and the cavity mode from the respective one-photon transition.

A. Effective Hamiltonian

We consider the large detuning limit $\Delta \gg \Omega, g_1$. With this approximation, the excited state $|e\rangle$ can be eliminated adiabatically [18] and the three-level atom reduces to an effective two-level atom, with the relevant energy levels $|0\rangle$ and $|1\rangle$. The Hamiltonian then can be written as

$$H^{(3)} = H_0^{(2)} + H_{\mathrm{ac}}^{(3)} + H_{\mathrm{cm}} + H_{\mathrm{pump}}^{(1)}, \tag{8}$$

where

$$H_{\mathrm{ac}}^{(3)} = -\frac{\Omega g_1}{\Delta}(|0\rangle\langle 0| + |1\rangle\langle 1|a^\dagger a) - \frac{\Omega g_1}{\Delta}(|0\rangle\langle 1|a + \mathrm{H.c.}) - \delta|1\rangle\langle 1|. \tag{9}$$

The first term in Eq. (9) above represents the Stark shifts of the ground states of the atom due to its coupling to the control field and the cavity field, while the second term describes the dispersive coupling between the atom and the cavity mode.

The Heisenberg equation of motion of the cavity mode a can be written as

$$\dot{a} = -i[a, H^{(3)}] = -i\left[\delta a + \epsilon - \frac{\Omega g_1}{\Delta}|1\rangle\langle 1|a - \frac{\Omega g_1}{\Delta}|1\rangle\langle 0| + g_2(b + b^\dagger)^2 a\right]. \tag{10}$$

In the limit, $\delta \gg \frac{\Omega g_1}{\Delta}, g_2$, we can adiabatically eliminate the cavity mode a by choosing $\dot{a} \approx 0$. This leads us to the following operator identity:

$$a \approx \frac{1}{\delta}\left(\frac{\Omega g_1}{\Delta}|1\rangle\langle 0| - \epsilon\right). \tag{11}$$

Thus, the final effective Hamiltonian becomes

$$H^{(4)} = \omega_m b^\dagger b + \hat{g}(b + b^\dagger)^2, \tag{12}$$

where

$$\hat{g} = \frac{g_2}{\delta^2}\left[\left(\frac{\Omega g_1}{\Delta}\right)^2 |0\rangle\langle 0| - \epsilon\frac{\Omega g_1}{\Delta}(|0\rangle\langle 1| + |1\rangle\langle 0|) + \epsilon^2\right]. \tag{13}$$

Clearly, Eq. (12) depends quadratically on the position quadrature (proportional to $b + b^\dagger$) of the oscillator. This is the desired form of the Hamiltonian to obtain squeezing through a unitary evolution. This is also equivalent to the Hamiltonian that gives rise to the quantum optical spring effect [19], in which decay of the atom or the cavity mode is now effectively eliminated.

Further, $\hat{g}$ defines an atomic operator, indicating that by suitably choosing the initial state of the atom one can control the squeezing parameter. This can be further revealed by rewriting $\hat{g}$ in the eigenbasis of its atomic part as

$$\hat{g} = \frac{g_2}{\delta^2}[\lambda_1|e_1\rangle\langle e_1| + \lambda_2|e_2\rangle\langle e_2| + \epsilon^2], \tag{14}$$

where $\lambda_{1,2} = \frac{1}{2}\frac{\Omega g_1}{\Delta}[\frac{\Omega g_1}{\Delta} \pm \sqrt{(\frac{\Omega g_1}{\Delta})^2 + 4\epsilon^2}]$ and $|e_{1,2}\rangle$ are the corresponding eigenstates. If the atom is prepared in one of these eigenstates $|e_i\rangle$ $(i = 1,2)$, the effective coupling constant takes the form $g_{\mathrm{eff}} = (g_2/\delta^2)[\lambda_i + \epsilon^2]$. Therefore one can obtain desired squeezing by a suitable choice of Ω and ϵ. Note that one could also achieve squeezing if the oscillator is parametrically driven so that the coupling constant becomes a sinusoidal function of time. This is usually done in a movable-mirror setup, in which the frequency of the cavity pump laser is suitably modulated [12,20], while in the present hybrid model, one does not require any modulation.

B. Squeezing

We assume that the oscillator is in thermal equilibrium with the phononic environment at a temperature T. The state of the oscillator is described by the density matrix $\rho = \sum_n p_n|n\rangle\langle n|$, where $p_n = (1 - \exp[-\hbar\omega_m/k_B T])\exp(-n\hbar\omega_m/k_B T)$ is the probability that the oscillator is in the phonon number state $|n\rangle$ and k_B is the Boltzmann constant. To identify squeezing in position, we calculate the time-dependent uncertainty of the relevant quadrature $X = (b + b^\dagger)/\sqrt{2}$. In the Heisenberg picture, the operator b evolves as

$$b(t) = \exp[iH^{(4)}t]b\exp[-iH^{(4)}t] = rb(0) + sb^\dagger(0), \tag{15}$$

where

$$r = \cos(qt) - \frac{ik}{q}\sin(qt), \quad s = -\frac{2ig_{\mathrm{eff}}}{q}\sin(qt), \tag{16}$$

with $k = 2g_{\mathrm{eff}} + \omega_m$ and $q = \sqrt{k^2 - 4g_{\mathrm{eff}}^2}$. The uncertainty of the position quadrature X is therefore given by

$$\langle\Delta X(t)\rangle^2 = \frac{V}{2}\left[1 - \frac{4g_{\mathrm{eff}}}{4g_{\mathrm{eff}} + \omega_m}\sin^2(qt)\right], \tag{17}$$

where $V = \coth(\hbar\omega_m/2k_B T)$. This clearly represents a time-dependent squeezing with respect to the thermal state [12], as $\langle\Delta X\rangle^2 \leqslant \langle\Delta X\rangle^2_{g_{\mathrm{eff}}=0} = V/2$. At $qt = \pi/2$, the uncertainty becomes minimum as

$$(\langle\Delta X(t)\rangle^2)_{\min} = \frac{V}{2}\frac{\omega_m}{4g_{\mathrm{eff}} + \omega_m}, \tag{18}$$

that can be further minimized by increasing g_{eff}. From the above equation, the relative squeezing can be expressed in decibel units as [21]

$$\begin{aligned} S_{\mathrm{therm}} &= -10\log_{10}\left[\langle\Delta X(t)\rangle^2_{\min}/\langle\Delta X\rangle^2_{g_{\mathrm{eff}}=0}\right] \\ &= -10\log_{10}\left[1 - \frac{4g_{\mathrm{eff}}}{4g_{\mathrm{eff}} + \omega_m}\right] \\ &= 10\log_{10}[4(g_{\mathrm{eff}}/\omega_m) + 1], \end{aligned} \tag{19}$$

that does not depend upon the temperature.

The squeezing usually refers to the uncertainty less than that for a vacuum state. In the present case, the position squeezing with respect to the SQL corresponds to $(\langle\Delta X(t)\rangle^2)_{\min} < (\Delta X)^2_{\mathrm{SQL}} = 1/2$. This translates into the following condition for squeezing, for an oscillator of natural frequency ω_m at an equilibrium temperature T:

$$\frac{g_{\mathrm{eff}}}{\omega_m} > \frac{1}{4}\left[\coth\left(\frac{\hbar\omega_m}{2k_B T}\right) - 1\right] = \frac{\bar{n}}{2}, \tag{20}$$

where

$$\bar{n} = [\exp(\hbar\omega_m/k_B T) - 1]^{-1} \tag{21}$$

is the average phonon number in the mechanical oscillator. In decibel units, the squeezing relative to the SQL can be

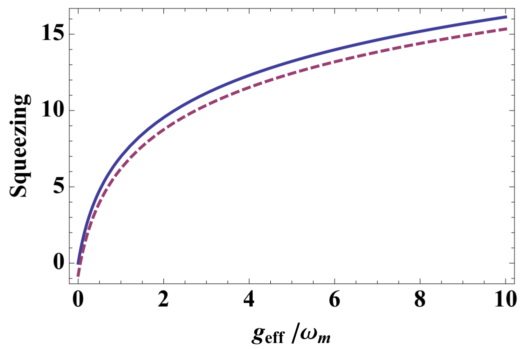

FIG. 2. Variation of squeezing S_{therm} [Eq. (19), solid line] and S_{SQL} (in decibel) [Eq. (22), dashed line] with coupling constant g_{eff}/ω_m for an oscillator with frequency 50 MHz at a temperature $T = 1$ mK. Note that below $g_{\text{eff}} \lesssim 0.05\omega_m$ the thermal effect dominates and gives rise to no squeezing with respect to vacuum, as $S_{\text{SQL}} < 0$.

expressed as

$$S_{\text{SQL}} = -10\log_{10}\left[\langle(\Delta X(t))^2_{\min}/\langle\Delta X\rangle^2_{\text{SQL}}\right]$$
$$= 10\log_{10}\left[\{4(g_{\text{eff}}/\omega_m)+1\}\tanh\left(\frac{\hbar\omega_m}{2k_BT}\right)\right]. \quad (22)$$

Clearly the squeezing depends upon the coupling constant g_{eff} and the temperature T. Lower temperature and larger coupling constant help to have larger squeezing. In Fig. 2, we show how the maximum attainable squeezing S_{therm} and S_{SQL} varies with g_{eff} when the atom is prepared in one of the eigenstates $|e_i\rangle$. This further demonstrates that ground-state cooling is not necessary for mechanical squeezing [22].

It should be borne in mind that in the traditional models of quadratic coupling, the coupling strength g_2 between the cavity mode and the oscillator is much smaller than the linear coupling strength. This leads to the achievable squeezing only of the order of 1.8 dB [13]. In the present case, we consider a hybrid model, which provides us a flexibility to increase the effective coupling to a much larger value, leading to larger squeezing. For example, for $g_{\text{eff}} = 0.5\omega_m$ [23] for an oscillator at an equilibrium temperature $T = 1$ mK with the natural frequency $\omega_m = 2\pi \times 50$ MHz [24], we have a squeezing $S_{\text{therm}} = 10\log 3 = 4.77$ dB with respect to the thermal state and $S_{\text{SQL}} \approx 4.69$ dB with respect to SQL, beating the standard 50% squeezing ($\equiv$ 3 dB) limit for a bosonic system coupled to a thermal bath [25]. Further, the squeezing can be enhanced by moderately increasing ϵ and therefore g_{eff} (see Fig. 2). This effectively leads to the strong-coupling limit (when g_{eff} is larger than the decay rates of the atom and the oscillator), that would be useful to drive the mechanical oscillator and that could be achieved just by using large classical pump fields.

C. Effect of cavity decay

We emphasize that as long as the cavity mode is large detuned from the external field and the relevant atomic transition $|1\rangle \leftrightarrow |e\rangle$, its decay does not substantially affect the squeezing, especially in the transient time scale. To understand this qualitatively, we incorporate the decay rate κ of the cavity mode through an effective non-Hermitian Hamiltonian given by $H_{\text{decay}} = H^{(3)} - i\kappa a^\dagger a$ [10]. Then Eq. (11) takes the form

$$a \approx \frac{1}{\delta - i\kappa}\left(\frac{\Omega g_1}{\Delta}|1\rangle\langle 0| - \epsilon\right). \quad (23)$$

Clearly, for $\delta \gg \kappa$, the effect of the photon decay can be negligible. For further analysis, we employ the standard Markovian master equation, as follows:

$$\dot\rho = -i[H^{(3)},\rho] - \kappa(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a). \quad (24)$$

Here, we have considered the Hamiltonian $H^{(3)}$, that has been derived *before* the adiabatic elimination of the cavity mode. Note that for an optical cavity the average number of the photons is negligible at low temperature (a few mK or less) and therefore the cavity is considered to couple to a vacuum bath in Eq. (24). The atom is initially prepared to be in one of the eigenstates $|e_i\rangle$, while the mechanical oscillator is prepared in a state $\rho_{\text{mech}} = \sum_m p_m|m\rangle\langle m|$, which is at thermal equilibrium with the phonon bath at a temperature T. Considering that the cavity contains no photon at the time $t = 0$, we have solved the above equation in the basis $|k\rangle|n\rangle|m\rangle$ ($|k \in 0,1\rangle$, $|n\rangle$, and $|m\rangle$ represent the atomic states, photon number state, and the phonon number state, respectively) and have calculated the position uncertainty of the mechanical oscillator, by calculating the time-dependent reduced density matrix of the oscillator. We have observed that in the transient time scale (i.e., $\omega_m t \lesssim 1$), this uncertainty can be made lower than the standard quantum limit. More importantly, for large detunings $\delta \gg \kappa$, the minimum achievable uncertainty in position quadrature becomes negligibly affected by the cavity decay. We demonstrate this effect in Fig. 3 using the presently available experimental parameters, as in [24].

It must be borne in mind that, as required for adiabatic elimination of the cavity mode, δ should be much larger than $\Omega g_1/\Delta$ and g_2. In that case, to obtain a larger effective coupling strength g_{eff} and therefore, larger squeezing, one needs to

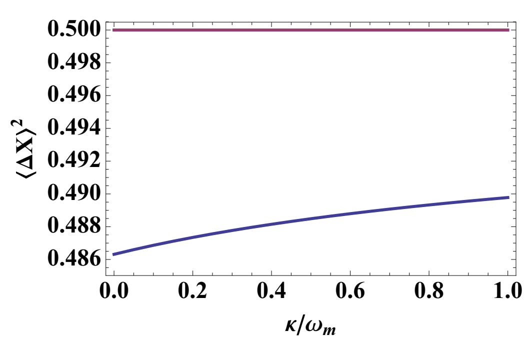

FIG. 3. Variation of the minimum achievable uncertainty (blue line) and the standard quantum limit of uncertainty (red line) in the position quadrature with the decay rate κ/ω_m of the cavity mode. We choose a mechanical oscillator with frequency $\omega_m = 2\pi \times 50$ MHz at an equilibrium temperature $T = 10\,\mu$K. The other parameters chosen are $\Omega = 1$, $g_1 = 1$, $g_2 = 0.5$, $\delta = 5$, $\Delta = 5$, $\epsilon = 0.75$, all in the unit of ω_m. The uncertainty is measured at $\omega_m t \lesssim 1$. The atom is assumed to be prepared in the state $|e_1\rangle$. Clearly, the variation of the uncertainty with respect to κ remains negligible. Note that the Markovian master equation is valid when κ is much smaller than ω_m [13].

increase ϵ and to optimally choose the ratio ϵ/δ [refer to the expression $g_{\rm eff} = (g_2/\delta^2)(\lambda_i + \epsilon^2)$].

D. Squeezing spectrum

As the evolution of the atom is effectively confined in the ground-state manifold, the spontaneous emission may be ignored in the present study. Further the cavity mode, after its adiabatic elimination, does not significantly affect the squeezing, as discussed in the previous subsection. In the present model, the primary source of decoherence is the coupling of the mechanical oscillator to the thermal phononic bath at a temperature T, due to which the position uncertainty increases and becomes proportional to $V = \coth(\hbar\omega_m/2k_BT)$ [see Eq. (17) for $g_{\rm eff} = 0$].

The effect of decoherence of the mechanical oscillator can be further analyzed in terms of squeezing spectrum, where we introduce another annihilation operator $c(\omega)$ for the bosonic bath. The oscillator-bath interaction can be described by the following Hamiltonian [26]:

$$\begin{aligned} H_{\rm tot} &= H^{(4)} + H_{\rm bath} + H_I, \\ H_{\rm bath} &= \int d\omega\ \ \omega\ \ c^\dagger(\omega)c(\omega), \\ H_I &= i\int d\omega K(\omega)[c^\dagger(\omega)b - c(\omega)b^\dagger], \end{aligned} \tag{25}$$

where $K(\omega)$ is the frequency-dependent coupling constant. In this case, the Heisenberg equation of motion of the mode b can be written as

$$\dot{b} = -i[b,H^{(4)}] - \frac{\gamma}{2}b + \sqrt{\gamma}b_I(t), \tag{26}$$

where we have chosen $K(\omega) = \sqrt{\gamma}$ as in the case of white bath, γ being the decay constant of the oscillator, and

$$b_I(t) = \int_{-\infty}^{\infty} d\omega \exp[-i\omega(t-t_0)]c(\omega,t_0). \tag{27}$$

The solution of Eq. (26) can be obtained in the frequency domain, through the Fourier transform

$$b(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}b(t)dt. \tag{28}$$

We find that

$$b(\omega) = \frac{\sqrt{\gamma}[-2ig_{\rm eff}b_I^\dagger(-\omega) - \{i(\omega+2g_{\rm eff}+\omega_m) - \frac{\gamma}{2}\}b_I(\omega)]}{\left(i\omega - \frac{\gamma}{2}\right)^2 + \omega_m(4g_{\rm eff} + \omega_m)}. \tag{29}$$

Therefore, by noting that $X(\omega) = [b(\omega) + b^\dagger(\omega)]/\sqrt{2}$, the position quadrature fluctuation $\langle X(\omega),X(\omega')\rangle = \langle X(\omega)X(\omega')\rangle - \langle X(\omega)\rangle\langle X(\omega')\rangle$ can be easily obtained. Using the following relations for the bath at thermal equilibrium at a temperature T,

$$\begin{aligned} \langle b_I^\dagger(\omega)b_I(-\omega')\rangle &= \bar{n}(\omega)\delta(\omega+\omega'), \\ \langle b_I(\omega)b_I^\dagger(-\omega')\rangle &= [\bar{n}(\omega)+1]\delta(\omega+\omega'), \\ \langle b_I(\omega)\rangle &= \langle b_I^\dagger(\omega)\rangle = 0, \end{aligned} \tag{30}$$

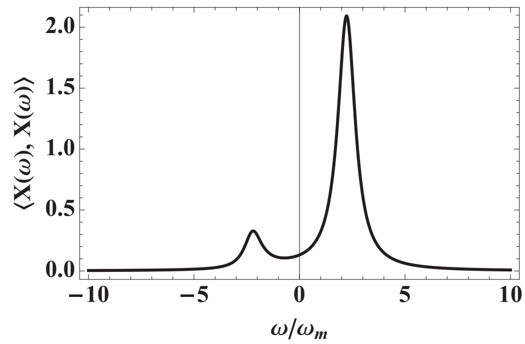

FIG. 4. Variation of the position uncertainty $\langle X(\omega),X(\omega)\rangle$ with the frequency ω/ω_m. We have chosen $\bar{n} = 0$ (corresponding to a temperature of 10 μK of a 50-MHz oscillator and $\gamma = g_{\rm eff} = \omega_m$.

where $\bar{n}$ is given by Eq. (21), we find that

$$\begin{aligned} \langle X(\omega),X(\omega)\rangle &= \frac{\gamma}{2}\frac{P}{Q}, \\ P &= (\bar{n}+1)\left\{\left(\frac{\gamma}{2}\right)^2 + (\omega + 2g_{\rm eff} + \omega_m)^2\right\} \\ &\quad + \bar{n}\left\{\left(\frac{\gamma}{2}\right)^2 + (\omega - 2g_{\rm eff} - \omega_m)^2\right\} \\ &\quad - 2g_{\rm eff}\{\omega + (2\bar{n}+1)\omega_m\}, \\ Q &= \left[\left(\frac{\gamma}{2}\right)^2 + \omega_m(4g_{\rm eff}+\omega_m) - \omega^2\right]^2 + (\omega\gamma)^2. \end{aligned} \tag{31}$$

In Fig. 4, we display the spectrum of position uncertainty. It exhibits two maxima at the critical frequencies $\omega_{\rm crit} = \pm\sqrt{\omega_m(4g_{\rm eff}+\omega_m) - \gamma^2/4}$, where Q becomes minimum. Note that the above variance (31) decreases as $g_{\rm eff}$ increases, referring to squeezing at a particular frequency ω (see Fig. 5). We also find that as the decay rate γ of the oscillator increases, the uncertainty increases at $\omega = \omega_{\rm crit}$. This suggests that the decoherence degrades squeezing.

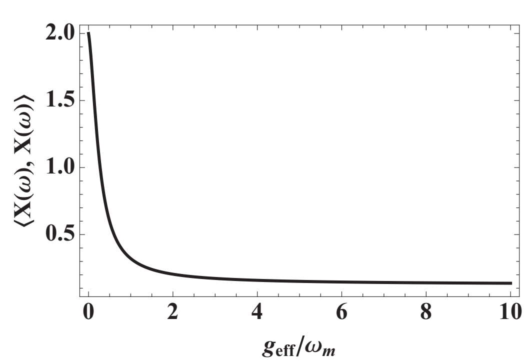

FIG. 5. Variation of the position uncertainty $\langle X(\omega),X(\omega)\rangle$ with the effective coupling constant $g_{\rm eff}/\omega_m$ at the frequency $\omega = \omega_m$. We have chosen $\bar{n} = 0$ and $\gamma = \omega_m$.

III. DISCUSSION

As discussed above, the atom-assisted squeezing can be generated at a time scale $\lesssim 1/\omega_m$ and is least affected by the decay of the cavity mode, in the large cavity-pump detuning limit. In addition, the squeezing can be controlled dynamically using external classical fields Ω and ϵ, used for driving the atomic transition and the cavity mode, respectively. As clear from Eq. (17), the squeezing depends upon the effective atom-oscillator coupling constant $g_{\mathrm{eff}} = (g_2/\delta^2)(\lambda_i + \epsilon^2)$. Hence, for negligible cavity driving ($\epsilon \to 0$), the squeezing primarily depends upon the driving field Ω. However, in the adiabatic limit, $\lambda_i \ll 1$ and therefore the squeezing cannot be increased substantially to a large value. On the other hand, for larger values of ϵ, the squeezing can be increased to a larger extent, as $g_{\mathrm{eff}} \approx (g_2/\delta^2)\epsilon^2$.

Note that as ϵ increases, the average photon number $\langle a^\dagger a\rangle$ of the cavity becomes nonzero [see Eq. (11)]. However, in the limit of large detuning δ, $\dot{a}$ remains negligible as it inversely varies as δ, leading to $d(a^\dagger a)/dt \approx 0$. The cavity photon would decay through the interaction with its own thermal bath and this decay does not influence the atom-oscillator interaction, as an effect of this adiabatic elimination. This means that as long as large detuning is maintained, the condition for adiabatic elimination of the cavity mode remains valid and eventually the cavity decay does not substantially affect the transient squeezing. This essentially means that to obtain a larger g_{eff}, there must be a tradeoff between the values of ϵ and δ.

We also note that an alternative way of achieving larger g_{eff} could be to consider the microwave cavities (with angular frequency $\sim$ a few GHz) and Rydberg atoms that have negligible decay rates [27]. Such cavities have a decay rate of approximately a few kHz which is much smaller than the frequency ω_m ($\sim$ a few MHz) of the mechanical oscillator. Therefore the effect of cavity decay on the transient dynamics of squeezing becomes negligible. This would allow one to increase ϵ to a larger extent and therefore to achieve a larger g_{eff}.

Squeezing has also been considered by Jähne *et al.* [7], who had driven the cavity with a 8–10-dB squeezed light and thereafter transferred this squeezing to the mechanical oscillator to obtain 5 dB mechanical squeezing for strong coupling $\sim 0.1\omega_m$. In contrast, our technique does not rely upon such constraints. Just by pumping the cavity using a highly detuned field, one can achieve a squeezing as large as > 5 dB. Further, Asjad *et al.* [15] had used a cavity driven with a pulsed laser and obtained a squeezing $\sim$10 dB using open-loop feedback control, for an effective coupling $10^{-8}\omega_m$. This mechanism is, however, limited by requirement of high power short optical nanosecond pulses. In our case, a cw pump laser would suffice to achieve squeezing. Girvin and coworkers [12] had proposed to drive the cavity with two fields at different frequencies, but of equal strengths, for a coupling $0.1\omega_m$. This may lead to certain squeezing; however, it is constrained to work in the resolved sideband limit only (ω_m is much larger than the cavity decay rate). Our proposal does not require one to work in this condition, as the cavity mode is adiabatically eliminated.

IV. CONCLUSION

In conclusion, we consider a hybrid atom-optomechanical system with the membrane-in-the-middle setup. An atom is trapped inside the cavity and dispersively interacts with the cavity mode, leading to squeezing in the position quadrature of the mechanical oscillator at a transient time scale. This squeezing is independent of spontaneous emission of the atom. We show that for large detuning of the cavity pump field, the position uncertainty remains almost unaffected even for a large decay rate $\kappa \lesssim \omega_m$. We also discuss how the squeezing depends upon the initial preparation of the atomic state. The squeezing can further be enhanced by increasing g_{eff}, which can controlled externally by the classical fields that drive the atom and the cavity mode. As an example, we show that a squeezing of $S_{\mathrm{SQL}} = 4.69$ dB of the oscillator with respect to SQL can be attained for a strong coupling $g_{\mathrm{eff}} = 0.5\omega_m$, that beats the standard 50% squeezing (= 3 dB) limit. We have also analytically derived the squeezing spectrum that exhibits two maxima, the width of which increases by larger decay rate of the oscillator.

ACKNOWLEDGMENT

We acknowledge a research grant from Science and Engineering Research Board, Department of Science and Technology, Government of India (Grant No. EMR/2014/000872) during this work.

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