Testing wave-function-collapse models using parametric heating of a trapped nanosphere

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We propose a mechanism for testing the theory of collapse models such as continuous spontaneous localization (CSL) by examining the parametric heating rate of a trapped nanosphere. The random localizations of the center of mass for a given particle predicted by the CSL model can be understood as a stochastic force embodying a source of heating for the nanosphere. We show that by utilizing a Paul trap to levitate the particle and optical cooling, it is possible to reduce environmental decoherence to such a level that CSL dominates the dynamics and contributes the main source of heating. We show that this approach allows measurements to be made on the time scale of seconds and that the free parameter λ_{csl} which characterizes the model ought to be testable to values as low as 10^{-12} Hz.

Dynamical reduction models—better known as collapse theories—seek to resolve the measurement problem by inserting a nonlinear stochastic term in the Schrödinger equation (SE). This would account for genuine collapses of superposition states. In these theories, so-called localization events occur at a frequency scaling with the mass of the system at hand. These are fundamentally different than environmental decoherence [1] and are invoked as the origin of wave-function collapse. Such modifications to the SE aim to provide a theory capable of describing phenomena at all scales and are designed to reproduce conventional quantum mechanics (CQM) when dealing with small masses and classical mechanics at the macroscopic scale.

One of the most celebrated models of dynamical reduction is the continuous spontaneous localization (CSL) model [2]. It is characterized by two parameters: a length r_c and a frequency λ_{csl}. The former provides a length scale above which reduction effects would be relevant, and the latter embodies the rate at which spatial superpositions of a single nucleon separated by a distance greater than r_c would collapse. While r_c is generally taken to be ≈ 100 nm, the value of λ_{csl} is the subject of uncertainties [3–5] and is currently taken to span a range from 10^{-16} [2] to 10^{-5} Hz [6]. A value of $10^{-8\pm2}$ Hz has been proposed [7], based on the process of image formation on photographic film. The heating rate of ultracold atoms was used to set a value of 10^{-7} Hz [8], while the maximum allowable heating rate of the intergalactic medium seems to be compatible with $10^{-10\pm2}$ Hz [9].

The effects of localization are mathematically very similar to those of decoherence [10], meaning that any experiment built to search for a signature of such a collapse mechanism must minimize the effects of decoherence as much as possible so as to better distinguish the hallmark of the former from that of the latter. However, for a given object, both its rate of localization and decoherence will increase proportionally to mass. To test collapse we must study objects large enough to have an appreciable localization rate, yet small

enough that decoherence does not dominate the dynamics. The scale at which this becomes possible is the so-called mesoscopic one, the liminal scale between the well-established quantum and classical regimes. Recently, Nimmrichter *et al.* have shown [11] that beyond a certain size collapse effects have a sublinear scaling with size, a result which we corroborate here and which clearly identifies the regime of interest.

Lately there have been a considerable number of proposals to test collapse theories, and the challenge of reducing decoherence manifests itself differently depending on the scheme [12–15]. Unfortunately many proposals do not include detailed analyses of conventional environmental noise, and as such, it is not clear what range of values of λ_{csl} they could probe. Matter-wave interferometric methods are an attractive means for such tests with a well-established range of testable values of λ_{csl} [16]. Such settings, however, require the preparation and detection of many identical particles, which makes implementation challenging. Optomechanical proposals [17] are attractive in comparison, because they in principle require only a line-shape measurement of the light scattered by an intracavity macroscopic oscillator and do not require ground-state cooling. However, as such line shape would be narrow (on the order of microhertz), very long times (in excess of months) would be required for the measurement. A recent proposal has been made based on dynamical decoupling [18], which is promising but may also be constrained by long testing times.

The concept of utilizing the energy gain of a harmonic oscillator to test CSL was first suggested by Adler [19] and more recently re-examined in Refs. [20–22]. Whilst this method is relatively straightforward and therefore attractive for implementation, the ability to test for CSL is very dependent on a detailed and realistic inclusion of conventional decoherence mechanisms. In order for the *classical* approach put forward, for instance, in Ref. [22], to be effective, the collapse mechanism must be able to induce the excitation of a rather substantial number of thermal phonons in a given oscillator. This is, in general, not the case for a large range of values of the parameters that characterize a given collapse model and such
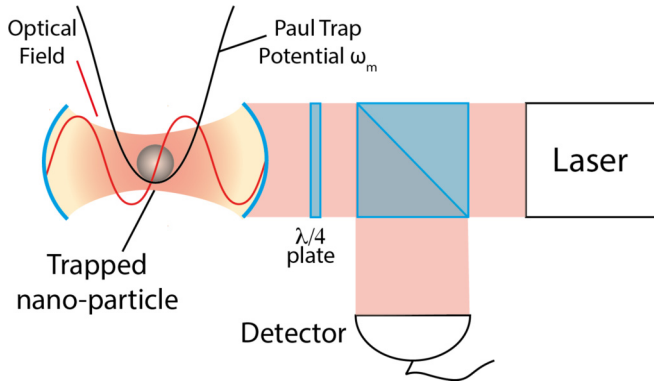
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FIG. 1. Schematic of the experiment, in which the particle is levitated by the electric field of the Paul trap and cooled by the optical cavity.

approaches can only assess the largest of their conjectured values.

In this Rapid Communication, we propose an experimentally viable way to explore CSL on the mesoscale by utilizing a cavity-cooled, single-charged nanosphere trapped in a Paul trap [23]. Measurements can be made with a single trapped particle in less than 100 s and, under optimal conditions, we find this scheme capable of probing λ_{csl} to values as low as 10^{-12} Hz, thus going significantly beyond the literature reported so far. Most important, our protocol allows for the discrimination between collapse effects and mischaracterized conventional noise ones—a challenge which has to our knowledge remained unaddressed so far.

We explore the possibility for an optomechanical test of a form of CSL described in Refs. [10,11,17], in which the effects of spontaneous localization are modeled as a δ-correlated stochastic noise source w_t. This approach is valid when the scale of spatial-superposition separations is less than r_c. The noise term w_t will occur in the dynamical equations of the system as an extra Langevin force [17,35]. Its effect on the dynamics of a mechanical oscillator would depend on the size and density of the object collapsing and the two parameters r_c and λ_{csl} characterizing the model. Conveniently, we can represent the effects of the localization process via a diffusion operator characterized by the coefficient D_{csl} and appearing in the master equation describing the particle in the same way as conventional heating sources. The diffusion coefficient takes the form [11]

$$D_{csl} = \frac{\hbar}{m\omega_m}\frac{\lambda_{csl}}{r_c^2}\alpha,\qquad(1)$$

where α is a geometry-dependent factor, which for a sphere is given by

$$\alpha = \left(\frac{m}{m_0}\right)^2\left[e^{-R^2/r_c^2}-1+\frac{R^2}{2r_c^2}(e^{-R^2/r_c^2}+1)\right]\frac{6r_c^6}{R^6}.$$

Here $m_0 = 1$ amu and R is the sphere's radius.

The protocol. We now describe the scheme that we propose to test the CSL model as shown in Fig. 1. We levitate a charged nanosphere in a hybrid trap consisting of a Paul trap and an optical cavity, and use them to cool its motion to a temperature corresponding to a low occupation number. We then turn off

the optical field (and hence the cooling) and let the dynamics evolve for a certain amount of time before measuring the energy of the oscillator again. A model including the effects of CSL predicts it will have heated more than one would expect due to conventional noise sources alone. If the measured energy matches that predicted by conventional noise sources, we will have provided evidence against CSL to within a certain range of λ_{csl}, whereas a higher measured energy would indicate some other dynamics at play, in favor of collapse theories.

We divide the procedure into two phases: a cooling phase and free evolution. For the purpose of testing CSL, it is the second phase that is important. In this period of free evolution the nanosphere is levitated using a single electric potential, which could be provided by a number of generic trap architectures. The mechanism of cavity cooling for nanoscale objects is well established [13,23–34], and relies on having the particle sit in two potential wells, one of which traps and one (or both) of which cool. Though these potentials are traditionally provided by an optical cavity populated with two distinct optical fields, Ref. [23] show that cooling is possible using a Paul trap in conjunction with a single-mode optical cavity. While both these systems are required for the cooling phase, the Paul trap alone suffices to levitate it.

This is appealing, because in the low-pressure scenario of a particle levitated solely by optical fields, the dominant source of heating is the scattering of cavity photons [29]. By using a hybrid trap we can cool the particle to a desired temperature and then turn off the optical field completely, leaving the particle suspended in the Paul trap alone. This ability to turn off the optical field without losing the particle means that we can do away with what would otherwise be the dominant cause of heating—optical scattering—and thus greatly reduce the conventional heating sources that would otherwise mask the CSL effects.

We emphasize that although we require cooling, we do not need to achieve the ground-state energy. Indeed, the simple comparison between the initial phonon number n_0 and the final one n_f after the period of free evolution will give us information on the heating rate [36].

The period of free evolution is governed by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}'$, where $\hat{H}_0 = \hbar\omega_m\hat{a}^\dagger\hat{a}$, ω_m is the secular frequency of the Paul trap, $\hat{a}^\dagger, \hat{a}$ are the creation and annihilation operators for the center-of-mass motion of the sphere respectively, and H' represents the interaction between system and environment. We can then solve the master equation $\dot{\rho} = -(i/\hbar)[\hat{H},\rho]$ for the oscillator [37]. The forms of coupling to noise sources in H' determine their effects on the master equation [32]. We have explored each noise source in detail, examining collisions with the background gas, black-body radiation, acoustic noise affecting the trap, Johnson and patch potential noise from the electrodes, micromotion from the trap's driving frequency, and anisotropy of the sphere [36]. We group these noise sources as momentum diffusion, occurring at rate D_p, position diffusion at rate D_q, and momentum dissipation at rate Γ, thus getting the dynamical equation [37]

$$\dot{\rho} = -\frac{i}{\hbar}[\hat{H}_0,\rho] - \sum_{j=1}^{2}D_j[\hat{X}_j,[\hat{X}_j,\rho]] - \Gamma[\hat{Q}_z,\{\hat{P}_z,\rho\}],\quad(2)$$
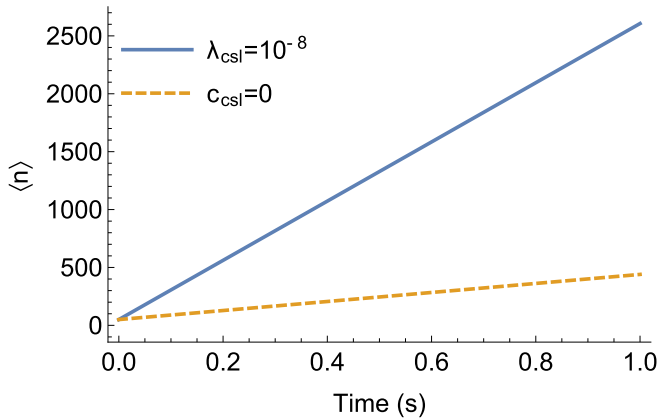
FIG. 2. Expected phonon number $\langle n(t) \rangle$ over 1 s with and without collapse effects. We have used $n_0 = 50$, $R = 100$ nm, a pressure of 10^{-12} mbar, $\omega_m = 5$ kHz, $\rho = 2300$ kg/m^3, $T_{\text{env}} = 4$ K, and an internal temperature of 65 K. The solid (dashed) line is for a CSL mechanism characterized by $\lambda_{\text{csl}} = 10^{-8}$,(0) Hz, i.e., the Adler value (no CSL mechanism).

where $\hat{X} = (\hat{Q}_z, \hat{P}_z)$ is the vector of quadratures of the nanosphere $\hat{Q}_z = \hat{a} + \hat{a}^\dagger$ and $\hat{P}_z = i(\hat{a}^\dagger - \hat{a})$, and $\boldsymbol{D} = (D_p, D_q)$. Our analysis of the various noise sources, including the possible effects of anisotropy of the nanosphere (and the consequent nonuniform distribution of the charge) finds all but gas collision and black-body radiation to be negligible [36], giving us $\Gamma = (\gamma_{\text{gas}} + \gamma_{\text{bb,e}} + \gamma_{\text{bb,a}})/4$ and $D_p = D_{\text{gas}} + D_{\text{bb}} + D_{\text{csl}}$, where D_{csl} is given in Eq. (1) and

$$D_{\text{gas}} = \frac{\gamma_g k_B T_{\text{env}}}{2\hbar\omega_m}, \quad D_{\text{bb}} = \frac{k_B(\gamma_{\text{bb,e}} T_{\text{int}} + \gamma_{\text{bb,a}} T_{\text{env}})}{2\hbar\omega_m}. \quad (3)$$

Here, $\gamma_{\text{gas}}, \gamma_{\text{bb,e}}, \gamma_{\text{bb,a}}$ are the damping constants related to gas collisions, black-body emission, and black-body absorption respectively. The environmental temperature is T_{env}, ω_m is the mechanical frequency of the nanosphere, and m is its mass.

Heating rate of a trapped particle. The mean occupation number (phonon number) of the trapped particle as a function of time $\langle n \rangle_t$ is determined via Eq. (2). Assuming a thermal state [24,32,38,39], the expression for the rate of change of the average phonon number simplifies to $\langle \dot{n}(t) \rangle = -\Gamma \langle n(t) \rangle + D_p$, which has the solution

$$\langle n(t) \rangle = e^{-\Gamma t}\left(n_0 - \frac{D_p}{\Gamma} \right) + \frac{D_p}{\Gamma}, \quad (4)$$

where n_0 is the initial average number. In Fig. 2 we plot the expected mean phonon number over the first second of free evolution, starting from an initial number of $n_0 = 50$, showing the heating when the CSL mechanism is included (solid blue line) and when it is not (dashed orange line). In our case, the diffusive terms in Eq. (4) dominate over the dissipative terms, resulting in an expression that is approximately linear, as seen in Fig. 2. The inclusion of the CSL mechanism results in a heating rate of the nanosphere motion of about 2500 phonons/s, which is in stark contrast with the $\lambda_{\text{csl}} = 0$ case, where only ~ 350 phonons/s are achieved for the parameters used in our simulations.

Differentiating CSL from conventional noise. If a final occupation number is recorded that agrees with the model of $\lambda_{\text{csl}} = 0$, we interpret it as falsifying a certain range of λ_{csl}.

However, if we measure a higher phonon number, we can infer that some extra heating process is present, possibly collapse effects. However, an objection could be made that the increased heating would more likely result from mischaracterizing the environmental noise sources present.

Therefore, an important requirement is how to correctly identify CSL and distinguish it from other noise sources. Indeed, this problem is generic to any test of collapse theories. We can address this problem by monitoring the conventional noise sources, such as black-body radiation and gas collisions, by varying the associated parameters and determining the effect they have on the heating rate. For example, if the heating is strongly dominated by CSL, then varying the pressure will have little effect over some range, while a gas-collision-dominated process results in an almost linear relation between pressure and heating rate.

Figure 3 summarizes such behaviors and displays the trend followed by the mean phonon number after 1 s when varying different parameters for the cases of $\lambda_{\text{csl}} = 10^{-8}$ Hz and for $\lambda_{\text{csl}} = 0$. In Fig. 3(a), we examine the effect of varying the background pressure when we include or exclude CSL. The difference in response between the theories is instructive. Without CSL the effect of an increasing pressure can be seen across the whole range, whereas for CSL there is a region of immunity where CSL dominates the dynamics. Likewise for the internal temperature depicted in Fig. 3(b), the point at which this significantly influences the heating is different for the two theories. The response to a varying mechanical frequency also takes a different shape, as seen in Fig. 3(c). Most interesting is the effect of a varying radius, shown in Fig. 3(d). Our findings agree with those of Ref. [11]: Objects must be large enough to have an appreciable collapse rate but small enough that decoherence does not dominate the dynamics in order for us to observe collapse effects. Such request is met for $R \in [10,100]$nm, roughly. For all of these we have compared the Adler value for $\lambda_{\text{csl}} = 10^{-8}$ Hz with
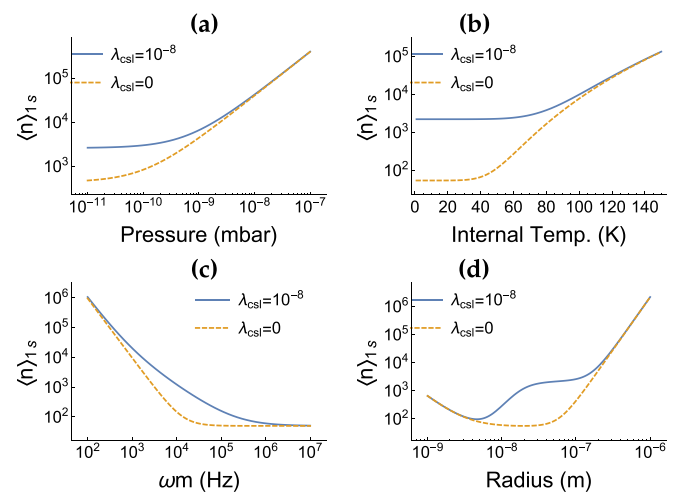

FIG. 3. Phonon number after 1 s as predicted with (blue line) and without (dashed orange line) collapse effects as we vary the pressure in panel (a), bulk temperature in panel (b), mechanical frequency in panel (c), and the radius of the sphere in panel (d). Except where a parameter is under investigation, we have used the same values as in Fig. 2.
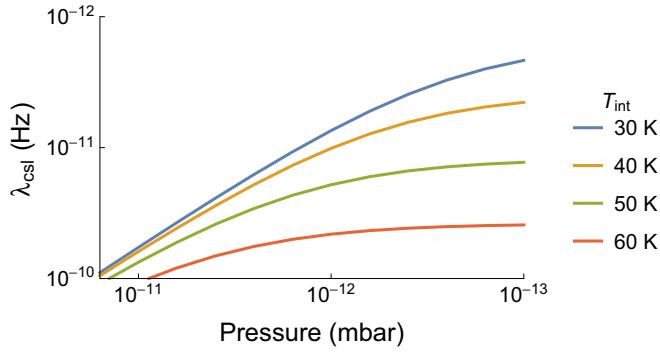
FIG. 4. Each line shows the lowest value of λ_{csl} which could be tested for a given internal temperature (30, 40, 50, and 60 K from top to bottom) as a function of environmental pressure. We take the lowest testable value to be the minimum for which $\langle n(100\,\text{s})\rangle_{\text{csl}}/\langle n(100\,\text{s})\rangle_{\text{qm}} \geqslant 1.2$ holds true, meaning that the we must expect that the effect of CSL must cause heating 20% greater than conventional noise alone over a period of 100 s, from a starting phonon number of $n_0 = 50$. The sphere radius and mechanical frequency have been optimized for each individual data point's temperature and pressure.

the case of no collapse effects. However, the same differences persist for any chosen nonzero value of λ_{csl}, though being more pronounced for higher collapse rates.

In order to find the best conditions for testing CSL it is necessary to numerically optimize all the parameters simultaneously. In Fig. 4, we show the range of λ_{csl} which can be probed for an illustrative set of experimental conditions. We see that, as one would expect, the testable range depends ultimately upon the conditions that can be achieved, most relevantly the minimum value of both environmental pressure and internal temperature. The value of the initial phonon number and evolution time also play a significant role: After enough time, the phonon ratio will tend to the heating rate ratio as $\langle n(\infty)\rangle_{\text{csl}}/\langle n(\infty)\rangle_{\text{qm}} \to \langle \dot{n}\rangle_{\text{csl}}/\langle \dot{n}\rangle_{\text{qm}}$, where the denominator in each refers to the expected value when we take $\lambda_{\text{csl}} = 0$. The time required for the phonon ratio to approximate the heating ratio in this way depends upon the initial phonon number and heating rates. A lower initial phonon number or longer evolution times would promote each plot in Fig. 4, as a given set of parameters would be capable of probing a lower value of λ_{csl}.

Experimental feasibility. Charged silica particles of 200 nm have been trapped and cooled to milli-Kelvin temperatures in a Paul trap using cavity cooling to milliKelvin temperatures [18], thus demonstrating the key experimental components required for our proposal. Pressures down to the 10^{-11} mbar range and internal particle temperatures in the 10-K range can be obtained for this setup using standard cryopumps. An important component of these experiments is the measurement of the oscillator energy following parametric heating by CSL and conventional noise. These can be carried out by performing a homodyne measurement of position as a function of time, which allows the determination of mean occupation number. Importantly, the light-cavity detuning can be nonperturbative during the measurement such that it neither cools nor heats the particle during this time. This ability to control the cooling rate also allows us to avoid giving strong kicks to the particle when we turn off the optical field after the particle is cooled to the desired energy.

Concluding remarks. We have shown that the parametric heating rate of a trapped nanosphere provides a viable mean of testing CSL. Central to the success of this scheme is the minimization of all sources of environmental decoherence. The two-stage cool-and-release protocol that we have illustrated allows us to exploit optical cooling while avoiding problematic scattering noise that would otherwise dominate the dynamics. We remark that, owing to established techniques and common experimental settings, the experiment can be performed rapidly and is repeatable upon a given nanosphere. Remarkably, the central experimental setup has already been demonstrated [23]. Our scheme can be used to distinguish CSL from other noise sources—an essential condition for inferring the existence of collapse effects from an experiment. Further, we have shown that the parameter range of CSL that is testable using our proposal is broad and can readily be expected to probe $\lambda_{\text{csl}} = 10^{10}$ ($\lambda_{\text{csl}} = 10^{-12}$) using a background pressure of 10^{-11} (10^{-13}) mbar and an internal temperature of 60 (20) K. Based on state of the art, values as low as $\lambda_{\text{csl}} = 10^{-8}$ could be tested imminently and $\lambda_{\text{csl}} = 10^{-12}$ plausibly in the next few years.

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