Chirped solitary pulses for a nonic nonlinear Schrödinger equation on a continuous-wave background

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(Received 3 February 2016; published 8 June 2016)

A class of derivative nonlinear Schrödinger equation with cubic-quintic-septic-nonic nonlinear terms describing the propagation of ultrashort optical pulses through a nonlinear medium with higher-order Kerr responses is investigated. An intensity-dependent chirp ansatz is adopted for solving the two coupled amplitude-phase nonlinear equations of the propagating wave. We find that the dynamics of field amplitude in this system is governed by a first-order nonlinear ordinary differential equation with a tenth-degree nonlinear term. We demonstrate that this system allows the propagation of a very rich variety of solitary waves (kink, dark, bright, and gray solitary pulses) which do not coexist in the conventional nonlinear systems that have appeared so far in the literature. The stability of the solitary wave solution under some violation on the parametric conditions is investigated. Moreover, we show that, unlike conventional systems, the nonlinear Schrödinger equation considered here meets the special requirements for the propagation of a chirped solitary wave on a continuous-wave background, involving a balance among group velocity dispersion, self-steepening, and higher-order nonlinearities of different nature.

DOI: 10.1103/PhysRevA.93.063810

I. INTRODUCTION

Propagation of solitons in optical waveguides has been the subject of intense investigations in recent years [1–4]. These particlelike excitations are stabilized by a balance of dispersion and nonlinearity [5]. Importantly, when a fiber is operated in the anomalous dispersion regime, it is possible to excite optical solitons that propagate without distortion by canceling the effect of group velocity dispersion (GVD) through self-phase modulation [6]. The distinction between solitary wave and soliton solutions is that when any number of solitons interact they do not change form and the only outcome of the interaction is a phase shift [7].

The cubic nonlinear Schrödinger (NLS) equation is widely used for descriptions of the propagation of picosecond pulses in Kerr media [8]. However, for the propagation of a subpicosecond or femtosecond pulses, the higher-order effects should be taken into account [9] and one needs to describe the problem by various generalizations of the NLS equation. The simplest correction to the cubic nonlinear term is the quintic nonlinearity, which can be obtained in many optical materials, such as semiconductors, semiconductor doped glasses, polydiacetylene toluene sulfonate (PTS), chalcogenide glasses, and some transparent organic materials [10]. It is relevant to mention that the measurement of fifth- and seventh-order nonlinearities of several glasses has recently been achieved using a spectrally resolved two-beam coupling technique [11].

Interest in analyzing higher-order Kerr response nonlinearities has grown rapidly in recent years. They have been theoretically argued to provide the main mechanism in filament stabilization [12,13], instead of the plasma defocusing [14]. Furthermore, higher-order nonlinearities have recently been introduced to describe the experimental observation of a saturation (and subsequent sign inversion) of the nonlinear correction to the refractive index at high optical intensities in gases [15]. In addition, it has recently been proved that common optical media, such as air or oxygen, which can be described by focusing Kerr and higher-order nonlinearities of alternating signs, can support the propagation of new localized light structures called fermionic light solitons and liquid light states [16].

Recently, much interest has been focused on the propagation of nonlinearly chirped solitons in cubic [17–19] and cubic-quintic materials [20]. These chirped pulses possess extensive applications in pulse compression or amplification and thus they are particularly useful in the design of fiber-optic amplifiers, optical pulse compressors, and solitary-wave-based communications links [20]. A challenging problem is the search for chirped femtosecond solitons in nonlinear media which exhibits not only third- and fifth-order nonlinearities but even seventh- and ninth-order nonlinearity. Noting that the interplay between dispersion and higher-order nonlinearities can give rise to several important phenomena in optical systems, it is worth including novel localized structures with interesting properties. Moreover, studies of propagating solitons in NLS models involving higher-order Kerr nonlinearities are much more important than the ones for the simplified NLS equation. It is worth noting that the finding of exact solutions, especially soliton solutions for higher-order NLS equations, is highly challenging and helpful to understand the widely different physical phenomena described by the envelope equations.

In a recent work, localized propagating pulses of the nonic derivative nonlinear Schrödinger (DNLS) equation have been constructed via two integrals of motion [21]. As a particular result, a bright solitary pulse on a zero background is obtained in the absence of the seventh-order nonlinearity (see Eq. (12) in Ref. [21]). Such a bright localized pulse with intensity varying as the square root of the hyperbolic secant is derived in the long-wave limit where a pair of invariants vanishes. The aim here is to extend the investigation of localized pulses to a structure with nonvanishing boundary conditions which is determined in the presence of all physical effects of the model. One of the most useful results of the present work is the derivation of an exact chirped solution which describes two different types of solitary pulses on a cw background by using an appropriate *ansatz* in the presence of competing cubic, quintic, septic, and nonic nonlinearities. A form of chirping which is directly proportional to the pulse intensity and depends upon the self-steepening term characterizes these solutions. We note that finding exact and novel solitary wave solutions for a DNLS equation under the nonvanishing boundary conditions is an interesting and challenging work from both a theoretical and practical point of view. Exact bright and dark solitary pulses on a zero background are also derived in the absence of the seventh-order nonlinearity.

The paper is organized as follows: The model of the equation with cubic-quintic-septic-nonic nonlinearity and a self-steepening term which governs the propagation of ultrashort optical pulses in a nonlinear media exhibiting a higher-order Kerr effect is discussed in Sec. II. In Sec. III, a newly proposed *ansatz* is introduced to solve the nonlinear equation describing the pulse amplitude dynamics and obtain exact chirped bright and gray solitary pulse solutions on a cw background. We also investigate the stability of the solitary wave solution under some violation of the parametric conditions. Also, novel bright solutions on a zero background are explored under certain parametric conditions. In Sec. IV, results for a kink solitary pulse solution are also presented in the absence of the seventh-order nonlinearity. Furthermore, the nonlinear chirp associated with each of these optical pulses is determined. Section V is a summary of the main results.

II. THE MODEL

We consider the evolution of a slowly varying envelope A as modeled by a higher-order DNLS equation involving ninth-order nonlinearity [21]:

$$iA_t + \lambda A_{xx} + \mu|A|^2A + i\alpha|A|^2A_x + \upsilon|A|^4A + \delta|A|^6A + \sigma|A|^8A = 0, \tag{1}$$

where $A(x,t)$ is the complex envelope of the electric field, and λ, μ, α, υ, δ, and σ are the parameters related to GVD, cubic nonlinearity, self-steepening, quintic nonlinearity, septic nonlinearity, and nonic nonlinearity, respectively. In temporal waveguides, the coordinates t and x are typically the propagation distance and the retarded time. We note that the higher-order NLS Eq. (1) without the self-steepening term (i.e., $\alpha = 0$) has been recently used to describe the propagation of self-guided light beams in common optical media such as air or oxygen [16]. It should be noted that Eq. (1) corresponds to the standard NLS equation in which the dispersion effect is restricted to the second order. This restriction may appear at first glance as a too-severe approximation if we consider ultrashort pulses that would require taking into account the effects of higher-order dispersion. But the dramatic progress made in recent years in the manufacturing of optical fibers allows us to adjust the dispersion coefficients in very wide ranges of parameters and even to minimize or cancel the value of some dispersion coefficients. Equation (1) does not take into account the noninstantaneous response of the material and mainly the Raman effect. However, it is well known that, at the first order, the Raman effect is not detrimental to the existence of the soliton, since it simply causes a shift of the soliton in the frequency domain [22,23]. The Raman effect should be taken into account if the temporal width of the soliton is relatively small (typically in the subpicosecond range). More specifically, in the case of a standard Telecom fiber, the higher-order Raman effects can be considered as negligible if the spectral width of the solitons that are to be generated is clearly smaller than 13.2 THz. Obviously, as the quantities used in this work are dimensionless, a very refined design of optical fiber work will be necessary for approaching the conditions corresponding to Eq. (1). It is of interest to determine exact solitary wave solutions of Eq. (1) with nonlinear chirp. According to the chirped solution to the higher-order NLS models [17–20], the complex envelope traveling-wave solutions with nonlinear chirping to Eq. (1) in that form takes the compact expression

$$A(x,t) = \rho(\xi)e^{i[\chi(\xi)-\Omega t]}, \tag{2}$$

where ρ and χ are real functions of the traveling coordinate $\xi = x - ut$. Here u is given in terms of the group velocity of the wave packet as $u = 1/V$ in an optical fiber setting. The corresponding chirp is given by $\delta\omega(t,x) = -\frac{\partial}{\partial x}[\chi(\xi) - \Omega t] = -\chi'(\xi)$.

Substituting Eq. (2) into Eq. (1) and separating the real and imaginary parts, we find the pair of coupled equations in ρ and χ,

$$\Omega\rho + u\chi'\rho + \lambda\rho'' - \lambda\chi'^2\rho + \mu\rho^3 - \alpha\chi'\rho^3 + \upsilon\rho^5 + \delta\rho^7 + \sigma\rho^9 = 0, \tag{3}$$

and

$$-u\rho' + 2\lambda\rho'\chi' + \lambda\rho\chi'' + \alpha\rho^2\rho' = 0. \tag{4}$$

To solve these coupled equations, we adopt an ansatz that depends quadratically on the wave amplitude:

$$\chi' = p\rho^2 + q. \tag{5}$$

Accordingly, the resultant chirp takes the form $\delta\omega(t,x) = -(p\rho^2 + q)$, where p and q are the nonlinear and constant chirp parameters, respectively. This indicates that the chirp associated with propagating pulses is intensity dependent and includes both linear and nonlinear contributions. Further substitution of the ansatz (5) into Eq. (4) gives the relations of q and p as

$$p = -\frac{\alpha}{4\lambda}, \quad q = \frac{u}{2\lambda}. \tag{6}$$

As seen from the first relation of (6), the nonlinear chirp parameter depends on GVD and the self-steepening coefficients. Therefore the variation of these coefficients allows effective control of the amplitude of chirping.

Now, using Eqs. (5) and (6) in Eq. (3), one obtains

$$\rho'' + a_1\rho + a_2\rho^3 + a_3\rho^5 + a_4\rho^7 + a_5\rho^9 = 0, \tag{7}$$

where

$$a_1 = \frac{4\Omega\lambda + u^2}{4\lambda^2}, \quad a_2 = \frac{2\lambda\mu - u\alpha}{2\lambda^2}, \quad a_3 = \frac{16\lambda\upsilon + 3\alpha^2}{16\lambda^2},$$
$$a_4 = \frac{\delta}{\lambda}, \quad a_5 = \frac{\sigma}{\lambda}. \tag{8}$$

Multiplying the hyperelliptic Eq. (7) by ρ_ξ and integrating with respect to ξ, we get

$$\left(\frac{d\rho}{d\xi}\right)^2 + a_1\rho^2 + \frac{a_2}{2}\rho^4 + \frac{a_3}{3}\rho^6 + \frac{a_4}{4}\rho^8 + \frac{a_5}{5}\rho^{10} + 2E = 0, \tag{9}$$

where E is an arbitrary constant of integration which corresponds to the energy of the anharmonic oscillator [24,25].

III. EXACT SOLUTIONS

Equation (9) is a first-order nonlinear ordinary differential equation with a tenth-degree nonlinear term describing the dynamics of field amplitude in a nonlinear media with higher-order Kerr responses. Generally speaking, it is difficult to give the closed-form solution of Eq. (9). To the best of our knowledge, exact analytic solutions to this equation have not been reported. In what follows, we propose an efficient ansatz to obtain exact solitary pulse solutions of bright and gray types for this equation in the most general case, when all the coefficients have nonzero values.

A. Chirped solitary pulses on a cw background

Here we are interested in finding exact analytic solutions that describe chirped solitary pulse propagation on a cw background for Eq. (1) in the presence of competing higher-order nonlinearities.

First, one can see that the transformation $\rho(\xi) = \sqrt{\psi(\xi)}$ transforms Eq. (9) into a new one possessing a sixth-degree nonlinear term, namely,

$$\left(\frac{d\psi}{d\xi}\right)^2 + 4a_1\psi^2 + 2a_2\psi^3 + \frac{4a_3}{3}\psi^4 + a_4\psi^5 + \frac{4a_5}{5}\psi^6 + 8E\psi = 0. \tag{10}$$

As seen, the range of power in the resulting Eq. (10) covers from 1 to 6. At the limit $a_2 = a_4 = E = 0$, Eq. (10) recovers to the auxiliary equation with even nonlinearity powers, which has been successfully applied to exactly solve certain nonlinear evolution equations (see Refs. [26–28]). In this context, Choudhuri and Porsezian [25] presented new types of bright and dark solitary wave solutions for the higher-order NLS equation with a quintic non-Kerr term. In Ref. [29], exact traveling-wave solutions for some important models have been derived based on an auxiliary equation incorporating the highest order, a fourth-degree nonlinear term. No exact solution exists for such problems when $a_i \neq 0\,(i = 1,\ldots,5)$ and $E \neq 0$ in (10). As this is clearly a difficult task, it has in the past been necessary to restrict the treatment to an equation with a fourth-degree nonlinear term or an equation with at most a sixth-degree nonlinear term exhibiting even power-law expansion.

In the following we look for the solitary wave solutions whose asymptotic values are nonzero when the traveling-wave variable approaches infinity ($|\xi| \to \infty$) and make the ansatz

$$\psi(\xi) = \beta + w\,\text{sech}^{1/2}(\eta\xi), \tag{11}$$

where β, w, and η are parameters to be determined. In this ansatz, β can be regarded as a fixed parameter to account for the homogeneous background associated with the solitary pulse while η represents the pulse width.

Substituting the ansatz (11) into Eq. (10) and matching powers of sech, we can find the following parameters:

$$\beta = -\frac{5a_5}{24}, \tag{12}$$

$$\eta^2 = \frac{16a_5\beta^4}{5}, \tag{13}$$

$$w = \pm\beta, \tag{14}$$

and

$$a_1 = \frac{14a_5}{5}\beta^4,\ a_2 = -8a_5\beta^3,\ a_3 = 9a_5\beta^2,$$
$$a_4 = -\frac{24a_5}{5}\beta, \text{ and } \quad E = -\frac{2a_5}{5}\beta^5. \tag{15}$$

Thus, the exact chirped solitary pulse solution on a continuous-wave background of Eq. (1) is of the form

$$A(x,t) = \left\{\beta \pm \beta\text{sech}^{1/2}\left[\sqrt{\frac{16a_5\beta^4}{5}}(x-ut)\right]\right\}^{1/2} \times e^{i[\chi(\xi)-\Omega t]}. \tag{16}$$

The resultant chirp associated with this propagating envelope can be obtained readily as

$$\delta\omega(t,x) = -p\left(\beta \pm \beta\text{sech}^{1/2}\left[\sqrt{\frac{16a_5\beta^4}{5}}(x-ut)\right]\right) - q. \tag{17}$$

The set of relations in (15) shows that all the coefficients a_i ($i = 1,\ldots,4$) including the energy E are dependent only upon the parameter a_5. The latter is related to the physical parameters σ and λ associated with nonic nonlinearity and GVD, respectively [see Eq. (8)]. Thus, the ninth-order nonlinearity is crucial to the existence of a chirped solitary pulse solution (16) for the present model. Because the pulse width η in (13) needs to be real for the existence of a solitary solution (11), one must require $a_5 > 0$ which yields the condition $\frac{\sigma}{\lambda} > 0$.

Physically, Eq. (16) describes the evolution of two different types of solitary pulses on a cw background for the nonic DNLS Eq. (1). This includes the first solution, with the sign – corresponding to a bright pulse, and the second one, with sign + for a gray pulse (i.e., dark pulse having nonzero minimum intensity). In these propagating envelope solutions, parameter β [determined by Eq. (12)] decides the strength of the background in which these solutions propagate. The amplitude profiles of typical bright and gray solitary pulses are shown in Figs. 1(a) and 1(b) using the following values for the model parameters: $\lambda = 1.6001$, $\mu = -2.6885$, $\upsilon = 0.1174$, $u = -30.1280$, $\alpha = 53.6$, and $\sigma = 12.9846$. This type of pulses

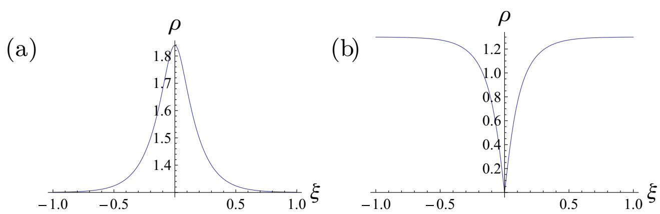

FIG. 1. The amplitude profiles of typical (a) bright and (b) gray solitary pulses. Here we have used the following values for the model parameters: $\lambda = 1.6001$, $\mu = -2.6885$, $\upsilon = 0.1174$, $u = -30.1280$, $\alpha = 53.6$, and $\sigma = 12.9846$.

typically exists due to a balance among cubic, quintic, septic, and nonic nonlinearities, GVD, and self-steepening. The corresponding chirping for the bright and gray solitary pulses are shown in Figs. 2(a) and 2(b), respectively (for $x = 0$). In order to check the stability of the solution we have shown the evolution of the typical intensity profile of the bright solitary wave pulse as computed from Eq. (16) for the condition $\frac{\sigma}{\lambda} > 0$, here we have used the following values for the model parameters: $\lambda = 1.6001$, $\mu = -2.6885$, $\upsilon = 0.1174$, $u = -30.1280$, $\alpha = 53.6$, and $\sigma = 12.9846$. We have seen in Fig. 3(a) a stable pulse for the condition $\frac{\sigma}{\lambda} > 0$. But if this condition violates, i.e., for $\frac{\sigma}{\lambda} < 0$, we have shown the evolution of the intensity profile of the bright solitary wave pulse in Fig. 3(b) using the model parameters $\lambda = 1.6001$, $\mu = -2.6885$, $\upsilon = 0.1174$, $u = -30.1280$, $\alpha = 0.37085$, and $\sigma = -2.59677$. On the contrary of the earlier we notice an unstable pulse.

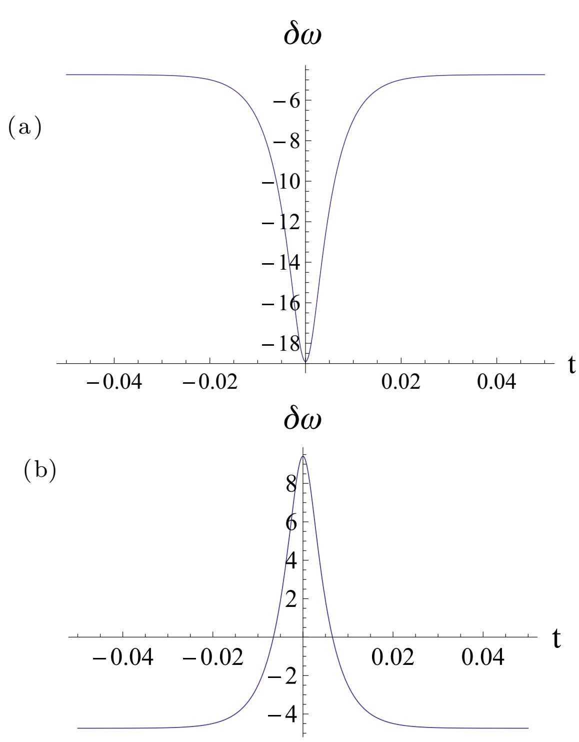

FIG. 2. Chirping profiles for the (a) bright [Fig. 1(a)] and (b) gray [Fig. 1(b)] solitary pulses.

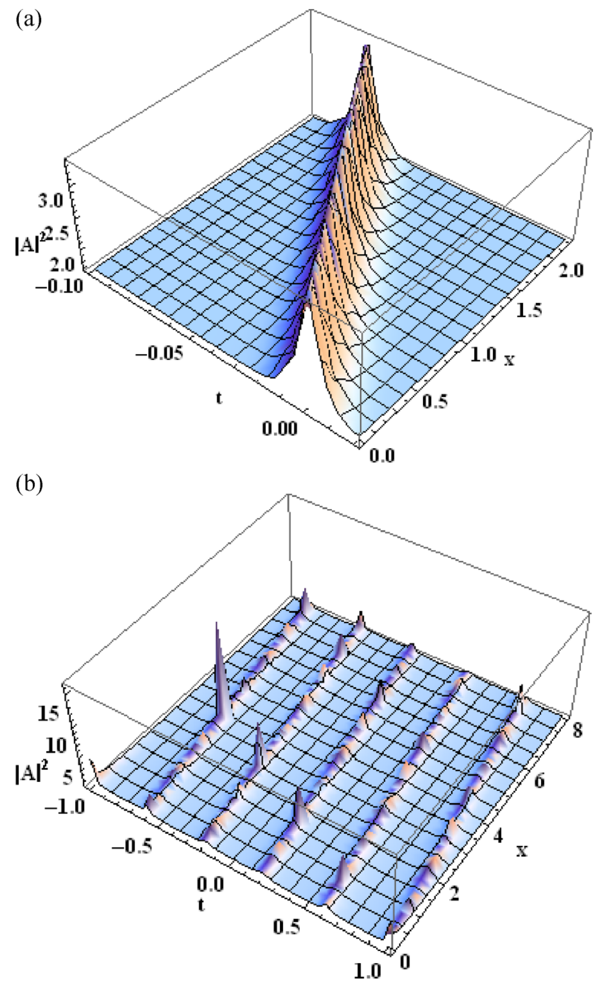

FIG. 3. The evolution of the typical intensity profile of the bright solitary wave pulse as computed from Eq. (16) for the condition (a) $\frac{\sigma}{\lambda} > 0$, where we have used the following values for the model parameters: $\lambda = 1.6001$, $\mu = -2.6885$, $\upsilon = 0.1174$, $u = -30.1280$, $\alpha = 53.6$, and $\sigma = 12.9846$; whereas in (b) $\frac{\sigma}{\lambda} < 0$ and the model parameters we have used in this case are the following: $\lambda = 1.6001$, $\mu = -2.6885$, $\upsilon = 0.1174$, $u = -30.1280$, $\alpha = 0.37085$, and $\sigma = -2.59677$.

B. Chirped bright solitary pulse on a zero background

We now investigate new types of chirped bright solitary pulse solutions on a zero background of the DNLS equation (1), when the coefficient of the seventh-order nonlinearity approaches zero [$\delta \to 0$ which leads to $a_4 = 0$]. We consider the case in which the cubic nonlinearity is related to self-steepening and GVD terms by the relationship $\mu = u\alpha/2\lambda$ [this yields $a_2 = 0$]. We have to find an exact bright pulse solution of Eq. (10) with zero energy ($E = 0$) as

$$\psi(\xi) = \frac{D}{\sqrt{1 + r\cosh(\gamma\xi) + s\sinh(\gamma\xi)}}, \tag{18}$$

where

$$D^2 = -\frac{6a_1}{a_3}, \quad \gamma^2 = -16a_1, \quad r^2 = 1 + s^2 - \frac{36a_1a_5}{5a_3^2}, \tag{19}$$

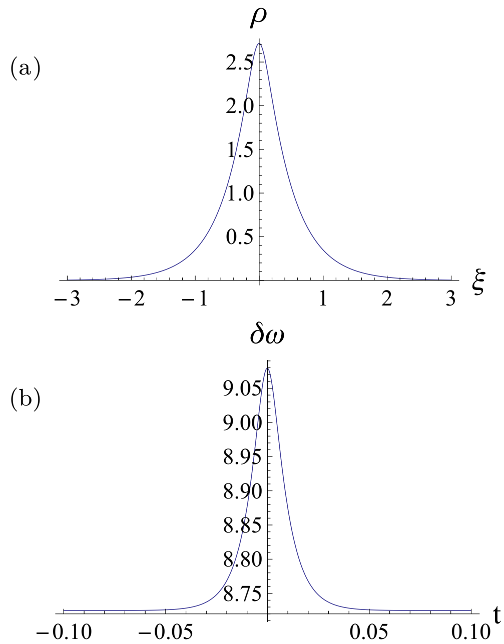

FIG. 4. Profiles for (a) the amplitude of a bright solitary pulse and corresponding (b) chirp profile on a zero background using the following values for the model parameters: $\lambda = 1.600\,1$, $\mu = -2.688\,5$, $\upsilon = 0.117\,4$, $\alpha = 0.308\,14$, $u = -27.921\,5$, $\Omega = -130$, and $\sigma = 0.01$.

provided that $a_1 < 0$ and $a_3 > 0$. It follows from the third relation in Eq. (19) that condition $r^2 > 0$ implies that the parameter s may only take values from the interval $s^2 > \frac{36a_1a_5}{5a_3^2} - 1$.

Based upon the above finding, we determine the solution of Eq. (1) as

$$A(x,t) = \left(\frac{D^2}{1 + r\cosh[\gamma(x-ut)] + s\sinh[\gamma(x-ut)]}\right)^{1/4} \times e^{i[\chi(\xi)-\Omega t]}. \tag{20}$$

Then, accordingly, the chirping takes the form

$$\delta\omega(t,x) = -p\left(\frac{D^2}{1 + r\cosh(\gamma\xi) + s\sinh(\gamma\xi)}\right)^{1/2} - q. \tag{21}$$

The typical profiles for the amplitude and chirping (for $t = 0$) are shown in Figs. 4(a) and 4(b), respectively, using the following values for the model parameters: $\lambda = 1.600\,1$, $\mu = -2.688\,5$, $\upsilon = 0.117\,4$, $\alpha = 0.308\,14$, $u = -27.921\,5$, $\Omega = -130$, and $\sigma = 0.01$. It is worth noticing that this physically interesting solitary pulse solution for the DNLS Eq. (1) essentially exists due to a balance among GVD, self-steepening, and competing cubic-quintic-nonic nonlinearities.

IV. A KINK SOLITARY WAVE

Our last result in this subsection concerns the finding of kink solitary wave solutions with nontrivial phase chirping to model (1) in the absence of the seventh-order nonlinearity (i.e., $\delta = 0$). To the best of our knowledge, exact analytical kink-type solutions of DNLS equations with a nonic nonlinearity, such as Eq. (1), have not been previously presented. This type of propagating structures are of particular importance, especially in nonlinear optics [30,31].

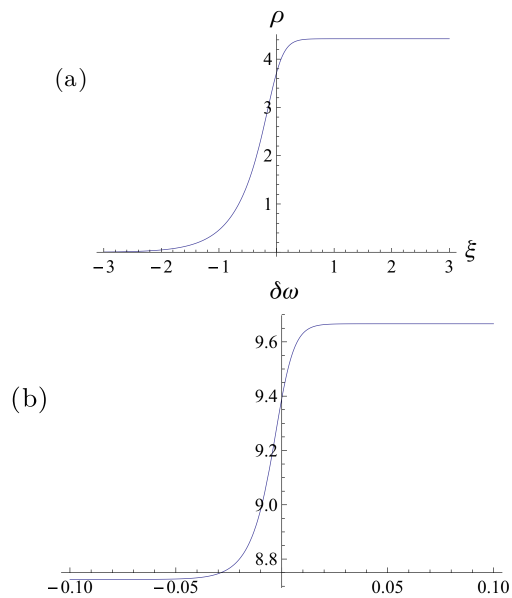

FIG. 5. Profiles for (a) amplitude of the kink solitary pulse and (b) corresponding chirp profile using the following values for the model parameters: $\lambda = 1.600\,1$, $\mu = -2.688\,5$, $\upsilon = 0.117\,4$, $\alpha = 0.308\,14$, $u = -27.921\,5$, $\Omega = -130$, and $k = 4.525\,71$.

For zero energy ($E = 0$), we have to find the solution of Eq. (10) as

$$\psi(\xi) = \left\{-\frac{3a_1}{a_3}[1 \pm \tanh(k\xi)]\right\}^{1/2}, \tag{22}$$

where

$$k = 2(-a_1)^{1/2}, \tag{23}$$

$$a_5 = \frac{5a_3^2}{36a_1}, \tag{24}$$

while

$$\delta = 0, \quad \mu = \frac{u\alpha}{2\lambda}. \tag{25}$$

Thus, the chirped kink solitary solution for the nonic model (1) is given by

$$A(x,t) = \left\{-\frac{3a_1}{a_3}[1 \pm \tanh(2\sqrt{-a_1})(x-ut)]\right\}^{1/4} \times e^{i[\chi(\xi)-\Omega t]}. \tag{26}$$

The corresponding chirping takes the form

$$\delta\omega(t,x) = -p\left(-\frac{3a_1}{a_3}[1 \pm \tanh(k\xi)]\right)^{1/2} - q. \tag{27}$$

It should be noted that the above kink solitary wave solutions are based on the corresponding constraint condition (24) which presents the balances among the competing quintic-nonic nonlinearities, GVD and self-steepening, and with a cubic nonlinearity subject to the constraint expressed by the second equation in (25). Unlike the conventional dark solitary wave in Kerr media, the amplitude of the kink solution (26) may approach nonzero when the variable x approaches infinity. The typical profiles for the amplitude and chirping (for $x = 0$) are shown in Figs. 5(a) and 5(b), respectively. It is worth mentioning that obtaining analytical solitary pulse solutions to a higher-order DNLS equation incorporating ninth-order nonlinearity is a physically important finding from both theoretical and experimental points of view. For instance, these exact solutions allow one to calculate certain important physical quantities analytically as well as serving as diagnostics for simulations. Taking applications into account, it was recently shown that the NLS equation combining cubic, quintic, septic, and nonic nonlinearities and without higher-order dispersion effects can accurately describe wave propagation in common optical media such as air or oxygen [16]. In addition, the investigation of the higher-order Kerr effect turns out to be necessary to obtain a good understanding of filamentation [32]. Consequently, exact solutions such as those presented here can be helpful for recognizing physical phenomena described by extended NLS equations involving higher-order Kerr terms up to the nonic nonlinearity.

V. CONCLUSION

In this paper, we have considered a derivative nonlinear Schrödinger equation which includes not only third- and fifth-order nonlinearities but even seventh- and ninth-order nonlinearities. The model governs the propagation of femtosecond pulses in nonlinear media with higher-order Kerr nonlinear response. It is shown that the evolution equation for the pulse amplitude satisfies a first-order nonlinear ordinary differential equation with a tenth-degree nonlinear term. Exact chirped bright and gray solitary pulse solutions on a continuous-wave background of the nonic model have been derived for the first time. We also investigated the stability of the solitary wave solution under some violation of the parametric conditions. We have shown that a small violation of the parametric condition related to σ and λ greatly influences the stable propagation of the bright pulse. Furthermore, new types of bright and dark solitary pulse solutions have been obtained in the absence of the seventh-order nonlinearity. The parametric conditions for these nonlinearly chirped pulses to exist were presented. We have found that the resultant chirp, including linear and nonlinear contributions, is directly proportional to the pulse intensity and depends upon the self-steepening term. These analytical results suggest potential applications in areas such as optical fiber compressors, optical fiber amplifiers, and optical communications.

A natural issue is the extension of the present setting into the nonic nonlinear Schrödinger equation with varying coefficients. One of the most important exact solutions of NLS models with variable coefficients is the so-called "nonautonomous soliton solutions," which are potentially useful for various applications in optical soliton telecommunications due to their special properties. The existence of these novel pulses has recently been studied for the cubic-quintic nonlinear Schrödinger equation with distributed coefficients [33] and the dispersive cubic-quintic Gross-Pitaevskii equation [34]. However, no attempt is made regarding nonautonomous soliton solutions of the nonic model with time-dependent coefficients. Such an important issue is indeed our next aim.

As a final remark, we would like to emphasize that the prospect for practical applications of soliton structures that we have obtained in the present study are not envisaged in the short term. Without being too speculative, we believe that in the long term such prospects are possible, but probably in modified versions and not necessarily with the solitons in the theoretical versions that we have presented in this work. Indeed, it is important to keep in mind that, although the conventional optical soliton could not be used in its theoretical version (with a temporal profile with hyperbolic secant shape) for long-distance transmissions, this soliton was the inspiration that led to the development of his practical version, namely, the dispersion-managed soliton. This practical version has been used to significantly increase the transmission capacities in Telecom lines using a binary modulation format. From this point of view, the results presented in this manuscript constitute a step forward in the exploration of new types of soliton structures in dispersive and nonlinear waveguides. Much of our work is currently ongoing on the stability analysis of the soliton solutions obtained in the present study and some of the solutions obtained in previous related works.

ACKNOWLEDGMENTS

K.P. acknowledges IFCPAR, DST, and NBHM of the Government of India for financial support through major projects.

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