Quantum state transfer in optomechanical arrays

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Quantum state transfer between distant nodes is at the heart of quantum processing and quantum networking. Stimulated by this, we propose a scheme where one can achieve quantum state transfer with a high fidelity between sites in a cavity quantum optomechanical network. In our lattice, each individual site is composed of a localized mechanical mode which interacts with a laser-driven cavity mode via radiation pressure, while photons hop between neighboring sites. After diagonalization of the Hamiltonian of each cell, we show that the system can be reduced to an effective Hamiltonian of two decoupled bosonic chains, and therefore we can apply the well-known results in quantum state transfer together with an additional condition on the transfer times. In fact, we show that our transfer protocol works for any arbitrary joint quantum state of a mechanical and an optical mode. Finally, in order to analyze a more realistic scenario we take into account the effects of independent thermal reservoirs for each site. By solving the standard master equation within the Born-Markov approximation, we reassure both the effective model and the feasibility of our protocol.

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I. INTRODUCTION

For quantum information processing purposes one often needs to transfer a quantum state from one site to another [1]. This is the central goal in quantum networking schemes. To this end, a wide range of physical systems able to carry information is used. For instance, photons interacting with atoms through cavity quantum electrodynamics (QED) [2] could be used to transfer quantum states between distant atoms.

Although photons are individual quantum carriers themselves, several promising technologies for the implementation of quantum information processing rely on collective phenomena to transfer quantum states, such as optical lattices [3] and arrays of quantum dots [4], just to name a few. It is therefore, an interesting goal to find physical systems that provide robust quantum data bus (QDB) linking different quantum processors.

In recent years, extensive theoretical research has been carried out on the topic of state transfer in quantum networks, and many of them have been conducted in several different systems and architectures [5].

Interestingly, a plethora of results has been obtained based on qubit-state transfer through spin chains considering different types of neighbor (site-site) couplings [6,7], as well as errors and detrimental effects arising from network imperfections and/or nonidealities [8–10].

On the other hand, optical lattices constitute a promising platform for quantum information processing, where both the coherent transport of atomic wave packets [11] and the evolution of macroscopically entangled states [12] have been achieved. Furthermore, significant advances have been made in engineered (passive) quantum networks, where the adjustment of static parameters leads to quantum information tasks, such as entanglement generation and state transfer [13].

Motivated by all of the aforementioned quantum systems towards quantum networking and/or processing, we present the state transfer of quantum information in optomechanical cavity systems—*a promising growing field, where "weak" light-matter interactions (trilinear radiation pressure interaction) take place leading to interesting quantum effects* [14].

Specifically, we show that information encoded on polariton states, i.e., photonic-phononic combined excitations, can be transferred from one site to another. Additionally, the use of polariton states allows us to include both the degrees of freedom—the quantized electromagnetic radiation field and the mechanical mode. Furthermore, polaritons permit undemanding manipulations with an external laser field. In fact, quantum state transfers of polaritonic qubits (photonic-atomic excitations) in a coupled cavity system have been studied [15].

We would like to stress that recent works on networks of coupled optomechanical cells [16] and light storage [17] have been introduced. Also, collective effects such as synchronization [18], quantum phase transitions [19], and generation of entanglement [20] have been proposed in the optomechanical field. It is appropriate to mention that a multimode optomechanical setup [21,22] allows stronger nonlinearities than a single mode that can be used for quantum information processing, such as generating single photons and to perform controlled gate operations.

Moreover, earlier studies of quantum state transfer in optomechanical systems relied on some sort of external control in the realm of active small networks [23,24] or quantum state transfer only between mechanical modes [25]. The most straightforward approach in this context pertains to a sequence of SWAP gates, which ensure the successive transfer of the state between neighboring sites. While intuitively simple, active networks are considered to be very susceptible to errors—*which are accumulated in each operation applied*

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during the transfer, as well as to dissipation and detrimental effects due to decoherence [5].

However, alternative strategies are based on the idea of eigenmode-mediated state transfer, rely on a perturbative coupling, and ensure resonance between the common frequency of the sender and the receiver and a single normal mode of the QDB [26] or a tunneling-like mechanism, described by a two-body Hamiltonian, which allows either a bosonic or a fermionic state to be transferred directly from the sender to the receiver, without populating the QDB [27].

In this paper, we envisage the quantum state transfer from a sender to a receiver in an array of optomechanical cells. There, each cell is composed of a localized mechanical mode that interacts with a laser-driven cavity mode via radiation pressure, and therefore photons can hop between neighboring sites.

In addition, we show how to design the parameters that allow us perfect state transfer of an arbitrary quantum state. In fact, two-way simultaneous communication for different pairs of sites without mutual interference is possible. We stress that the linearization of the nonlinear optomechanical Hamiltonian does not constitute a major restriction. For example, for driven optomechanical systems in the strong single-photon regime, we can transfer information encoded in polariton states arising from ion-trap-like Hamiltonians [28] as well as in dark states in optomechanical systems [29].

Finally, we illustrate the effectiveness of our protocol when each cell is in contact with a thermal environment and under the red-sideband regime.

II. THE MODEL

We consider a one-dimensional array of N optomechanical cells, each of these cells consists of a mechanical mode of angular frequency ω_m^n coupled via radiation pressure to a cavity mode of angular frequency ω_r^n. In addition, we consider an external laser driving the optical mode at angular frequency ω_p^n, as schematically depicted in Fig. 1(a).

Following the standard linearization procedure for driving optical modes in optomechanical cavities, we can recast the following Hamiltonian (in units of Planck constant, i.e., $\hbar = 1$):

$$\hat{H}_n^L = -\Delta_p^n \hat{a}_n^\dagger \hat{a}_n + \omega_m^n \hat{b}_n^\dagger \hat{b}_n - G_n(\hat{b}_n + \hat{b}_n^\dagger)(\hat{a}_n + \hat{a}_n^\dagger), \quad (1)$$

where the mechanical (optical) mode of the nth cell is associated with the bosonic operator $\hat{b}_n$ ($\hat{a}_n$), $\Delta_p^n = \omega_p^n - \omega_r^n$ is the pump detuning from the cavity resonance, g_n corresponds to the single-photon coupling rate, and $G_n = \alpha_n g_n$ is the effective optomechanical coupling strength proportional to the laser amplitude.

Here the cells are coupled by evanescent coupling between nearest-neighbors cavities with hoping strength J_n, an

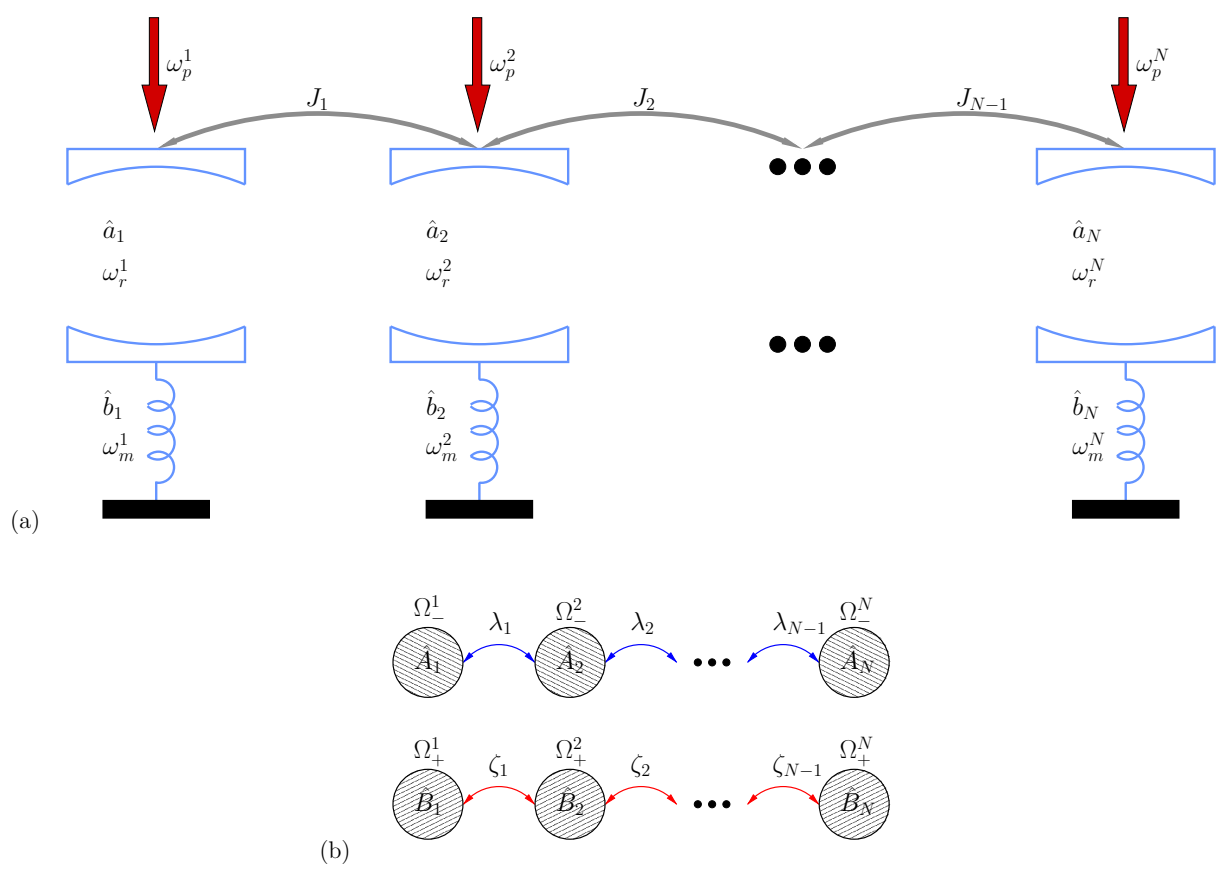

FIG. 1. (a) Sketch of an array of N optomechanical cells. Each of these cells consists of a mechanical mode of frequency ω_m^n coupled via radiation pressure to a cavity mode of frequency ω_r^n. The optical mode is driven by a laser at frequency ω_p^n and cells are coupled by evanescent coupling between nearest-neighbors cavities with hoping strength J_n. (b) Schematic of the effective model: two decoupled bosonic chains with polaritonic energies Ω_-^n and Ω_+^n and neighbor-site hopping λ_n and ζ_n, respectively.

interaction described by

$$\hat{H}_I = \sum_{n=1}^{N-1} J_n(\hat{a}_n^\dagger \hat{a}_{n+1} + \hat{a}_{n+1}^\dagger \hat{a}_n). \tag{2}$$

As seen from the above Hamiltonian $\hat{H}_n^L$ (with $\Delta_p^n < 0$), we can readily notice two linearly coupled quantum harmonic oscillators. To obtain the relevant decoupled effective Hamiltonian, we proceed to diagonalization of the Hamiltonian using the usual Bogoliubov transformation [30], leading to the eigenvalues

$$(\Omega_\pm^n)^2 = \frac{(\Delta_p^n)^2 + (\omega_m^n)^2}{2} \pm \frac{1}{2}\sqrt{\left[(\Delta_p^n)^2 - (\omega_m^n)^2\right]^2 - 16G_n^2\Delta_p^n\omega_m^n} \tag{3}$$

and normal mode operators

$$\begin{aligned} \hat{A}_n &= \sin\theta_n(\alpha_n^+\hat{a}_n + \alpha_n^-\hat{a}_n^\dagger) + \cos\theta_n(\beta_n^+\hat{b}_n + \beta_n^-\hat{b}_n^\dagger), \\ \hat{B}_n &= \cos\theta_n(\eta_n^+\hat{a}_n + \eta_n^-\hat{a}_n^\dagger) - \sin\theta_n(\mu_n^+\hat{b}_n + \mu_n^-\hat{b}_n^\dagger), \end{aligned} \tag{4}$$

where the coefficients are

$$\begin{aligned} \alpha_n^\pm &= \frac{\Omega_+^n \pm |\Delta_p^n|}{2\sqrt{\Omega_+^n|\Delta_p^n|}}, \quad \beta_n^\pm = \frac{\Omega_+^n \pm \omega_m^n}{2\sqrt{\Omega_+^n\omega_m^n}}, \\ \eta_n^\pm &= \frac{\Omega_-^n \pm |\Delta_p^n|}{2\sqrt{\Omega_-^n|\Delta_p^n|}}, \quad \mu_n^\pm = \frac{\Omega_-^n \pm \omega_m^n}{2\sqrt{\Omega_-^n\omega_m^n}}, \end{aligned} \tag{5}$$

and we have defined $\sin\theta_n = s_n/\sqrt{t_n}$ and $\cos\theta_n = c_n/\sqrt{t_n}$ with

$$\begin{aligned} s_n &= \tfrac{1}{4}\left[(\Delta_p^n)^2 - (\omega_m^n)^2 + (\Omega_+^n)^2 - (\Omega_-^n)^2\right], \\ c_n &= G_n\sqrt{\Delta_p^n\omega_m^n}, \quad t_n = \sqrt{c_n^2 + s_n^2}. \end{aligned} \tag{6}$$

Therefore, the total Hamiltonian $\hat{H} = \hat{H}_n^L + \hat{H}_I$ in the polariton basis can be rewritten as

$$\tilde{H} = \sum_{n=1}^{N} \Omega_+^n \hat{A}_n^\dagger \hat{A}_n + \Omega_-^n \hat{B}_n^\dagger \hat{B}_n + \sum_{n=1}^{N-1}(\lambda_n \hat{A}_n^\dagger \hat{A}_{n+1} + \zeta_n \hat{B}_n^\dagger \hat{B}_{n+1} + \text{H.c.}), \tag{7}$$

with the effective tunneling strengths

$$\lambda_n = J_n \sin\theta_n \sin\theta_{n+1}(\alpha_n^+\alpha_{n+1}^+ + \alpha_n^-\alpha_{n+1}^-)$$

and

$$\zeta_n = J_n \cos\theta_n \cos\theta_{n+1}(\eta_n^+\eta_{n+1}^+ + \eta_n^-\eta_{n+1}^-).$$

It is important to point out that in deriving the above expression, terms like $A_i^\dagger A_{i+1}^\dagger$ and $A_i^\dagger B_{i+1}$ have been neglected due to the usual rotating-wave approximation (RWA), which remains valid for

$$\Omega_\mp^n, |\Omega_\mp^n - \Omega_\mp^{n+1}| \gg \sqrt{\sum_{n=1}^{N}\langle\hat{a}_n^\dagger\hat{a}_n\rangle + \langle\hat{b}_n^\dagger\hat{b}_n\rangle}(\lambda_n + \zeta_n). \tag{8}$$

Now, it is straightforward to observe under the above mapping that the original full Hamiltonian of a unidimensional array of optomechanical cells becomes equivalent to a Hamiltonian of two distinct bosonic chains, this Hamiltonian being the central result of this paper [scenario schematically illustrated in Fig. 1(b)]. Because of the effective structure achieved above, i.e., two independent chains, we are now in position to take advantage of the well-known results on quantum state transfer.

As known from any state transfer scheme, the set of coupling parameters $\{\lambda_n, \zeta_n\}$ as well as energies $\Omega_\mp^n$ defines the transfer time τ. Here, we point out that our protocol requires that the transfer times for both polaritons $\hat{A}^\dagger\hat{A}$ and $\hat{B}^\dagger\hat{B}$ have to be the same or at least an odd multiple of each other.

To illustrate this point, we consider the red-detuned regime $\Delta_p^n \approx -\omega_m^n$; thus the Hamiltonian (1) can be simplified as

$$\hat{H}_n^{\text{red}} = \omega_m^n(\hat{a}_n^\dagger\hat{a}_n + \hat{b}_n^\dagger\hat{b}_n) - G_n(\hat{b}_n\hat{a}_n^\dagger + \hat{b}_n^\dagger\hat{a}_n). \tag{9}$$

To obtain the diagonal form of the above expression, we consider the operators

$$\hat{A}_n = \frac{(\hat{a}_n + \hat{b}_n)}{\sqrt{2}}, \quad \hat{B}_n = \frac{(\hat{a}_n - \hat{b}_n)}{\sqrt{2}}, \tag{10}$$

with eigenvalues $\omega_A^n = \omega_m^n + G_n$ and $\omega_B^n = \omega_m^n - G_n$, respectively.

For the strongly off-resonant regime ($G_n \gg J_n$) together with the RWA, we can recast the following polariton Hamiltonian:

$$\hat{H} = \sum_{n=1}^{N}\left(\omega_A^n\hat{A}_n^\dagger\hat{A}_n + \omega_B^n\hat{B}_n^\dagger\hat{B}_n\right) + \sum_{i=1}^{N-1}\frac{J_n}{\sqrt{2}}(\hat{A}_n^\dagger\hat{A}_{n+1} + \hat{B}_n^\dagger\hat{B}_{n+1} + \text{H.c.}). \tag{11}$$

Now we proceed to choose a set of parameters that allows quantum state transfer. For instance, a straightforward set can be found in Ref. [5] corresponding to $\omega_m^n = \omega_m$, $G_n = G$, and $J_n = (J/\sqrt{2})\sqrt{n(N-n)}$, which provides the same transfer time for each chain $\tau_A = \tau_B = \pi/J$.

Therefore, regardless of a relative phase depending on ω_A and ω_B which is fixed and known, and hence can be amended, any optomechanical state can be transferred only, ensuring the $G \gg J$ regime together with $J_n = (J/\sqrt{2})\sqrt{n(N-n)}$.

However, we stress that any other protocol could have been chosen for this purpose, for example, schemes based on eigenmodes, where one of many possibilities that permit quantum state transfer is the following set of parameters: $J_1 = J_{N-1} = \lambda \ll J_k = J \ll G$ ($k = 2\ldots N-2$, N being an odd number), $\omega_m^n = \omega_m$, and $G_n = G$.

On the other hand, in resonant schemes [26] the shorter transfer time possible corresponds to $\tau_A = \tau_B = (\pi/\lambda)\sqrt{2(N+1)}$, and for the tunneling-like protocol [27] with the same parameters and conditions $\omega_m^1 = \omega_m^N = \omega_m + \delta$ and $\lambda \ll \delta \ll J$, we obtain transfer times $\tau_A = \tau_B = N\pi\delta/2\lambda^2$.

Finally, it is worth stressing that the effect of a phononic hop term between neighboring sites only changes the strength of λ_n and ζ_n.

III. DISSIPATIVE MECHANISMS

In this section, in a step towards a more realistic model we take into account decoherence and dissipation. To fulfill this goal, we employ the standard formalism for open quantum systems, i.e., we solve the dynamics of the optomechanical array using the master equation in Lindblad form within the Born-Markov approximation.

Furthermore, we numerically investigate the effectiveness of our model computing the fidelity for the state transfer considering engineered hop couplings between cells where each cell is considered in the red-sideband regime.

The master equation for the composite coupled system is given as

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= -i[\hat{H},\hat{\rho}] \\ &\quad + \sum_{n=1}^{N} \frac{\kappa_n}{2}(1+\overline{n}_c)\mathcal{D}[\hat{a}_n]\hat{\rho} + \frac{\kappa_n}{2}\overline{n}_c\mathcal{D}[\hat{a}_n^\dagger]\hat{\rho} \\ &\quad + \frac{\gamma_n}{2}(1+\overline{n}_m)\mathcal{D}[\hat{b}_n]\hat{\rho} + \frac{\gamma_n}{2}\overline{n}_m\mathcal{D}[\hat{b}_n^\dagger]\hat{\rho}, \qquad (12)\end{aligned}$$

where $\hat{H} = \sum_{n=1}^{N} \hat{H}_n^{\rm red} + \hat{H}_I$ and the Lindblad term

$$\mathcal{D}[\hat{O}] = 2\hat{O}\rho\hat{O}^\dagger - \rho\hat{O}^\dagger\hat{O} - \hat{O}^\dagger\hat{O}\rho \qquad (13)$$

takes into account the dissipative mechanisms of the optics (mechanics) in contact with a thermal reservoir with the occupation number $\overline{n}_c$ ($\overline{n}_m$), where the photon (phonon) decay rate is given by $\kappa_n(\gamma_n)$.

Needless to say the first nontrivial quantum network in passive schemes is composed of four sites. Hence, for computational time purposes, we exemplify our findings considering an array of four cells where the couplings fulfill $J_n = (J/\sqrt{2})\sqrt{n(N-n)}$ and $\omega_m^n = \omega_m$.

To validate the polariton Hamiltonian (11), we present the closed evolution, computed from the full Hamiltonian (9) of the transfer fidelity at time $\tau = \pi/J$ as a function of G/J (see Fig. 2). We note the detrimental effects of the non-RWA terms cease to $G > 10J$, as expected by Eq. (8), whence we may infer that for the superposition of coherent states $|\alpha\rangle$, such as Schrodinger cat states with mean occupation numbers $|\alpha|^2$, we need to secure at least $G > |\alpha| \times 10J$ to achieve a high-fidelity state transfer.

In order to compute the fidelity of the transferred quantum state, we solved the closed quantum system dynamics (running the simulation in QuTiP [31]) considering the sender initially in the state $|\phi_+\rangle = (1/\sqrt{2})(|1,0\rangle + |0,1\rangle)$ or $|\Phi_+\rangle = (1/\sqrt{2})(|2,0\rangle + |0,2\rangle)$ (we have used the following notation $|a,b\rangle = |a\rangle_{\rm optics} \otimes |b\rangle_{\rm mechanics}$), where all the other cells are in the vacuum state.

Moreover, for our illustrative red-sideband detuning regime ($-\Delta_p \approx \omega_m \gg \gamma,\kappa$) a well-known stability condition [32] given by $G < (1/2)\sqrt{\omega_m^2 + (\gamma^2+\kappa^2)/4}$ comes into sight, and therefore it must be observed throughout the quantum state transfer protocol. On the other hand, in order to achieve a fidelity value close to unity, G has to be $G \approx (J/4) \times 10^2$ (as seen in Fig. 2). The effects of both the stability condition (being an upper bound for G) and the effectiveness of the fidelity [$\mathcal{F}(\tau) \to 1$] have as a result the limitation of the maximum coupling strength $J_{n=N/2} = NJ/4$ and consequently the maximum number of cells.

In Fig. 3, we compute the fidelity for the transfer of an initial quantum state given by $|\phi_+\rangle$ as a function of κ/J for two different mechanical phonon bath occupation numbers, $\overline{n}_m = 100$ and $\overline{n}_m = 1$, where we have used the following currently experimental parameters in optomechanical crystals [33] in the GHz regime: $\omega_m/2\pi = 3.68 \times 10^9$ Hz, $\gamma/2\pi = 35 \times 10^3$ Hz, $\overline{n}_c = 0.005$, and $G = (J/4) \times 10^2 = 5 \times 10^9$. The high fidelity shown in Fig. 3 up to $J = 10\kappa$ is an expected result, since $k_B T < \hbar\omega_m$, and the threshold for coherent operations takes place when $\max(J_n) = \max(\gamma_n,\kappa_n)$. Thus, to achieve transfer fidelities close to unity for an array with $N = 100$ cells (with the same set of parameters considered above), we can then estimate the cavity linewidth as $\kappa \sim 10^5$ Hz.

Finally, we point out that the hopping coupling reported in Ref. [18] is in the range of THz. Hence, to achieve the inequality $J < G$ within the stability region, we should

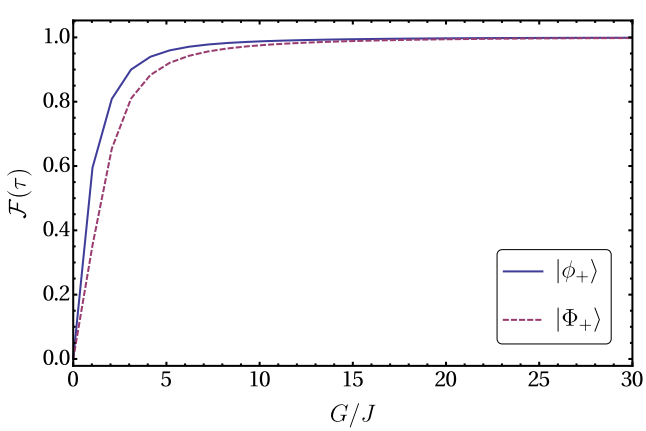

FIG. 2. The figure shows the dynamics of the transfer fidelity at time $\tau = \pi/J$ as a function of G/J. The states $|\phi_+\rangle = (1/\sqrt{2})(|1,0\rangle + |0,1\rangle)$ and $|\Phi_+\rangle = (1/\sqrt{2})(|2,0\rangle + |0,2\rangle)$ correspond to the initial states of the sender.

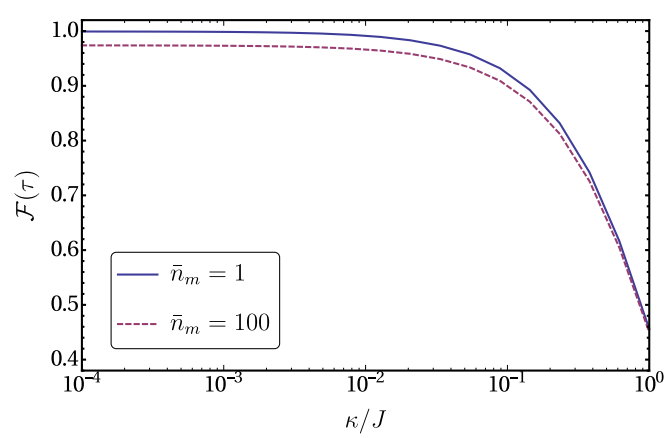

FIG. 3. We illustrate the fidelity of the transfer process ($\tau = \pi/J$) as a function of κ/J (log-axis) for two mechanical phonon bath occupation numbers: $\overline{n}_m = 100$ and $\overline{n}_m = 1$. To exhibit our findings, we consider the following feasible parameters in optomechanical crystals [33] in the microwave regime: $\omega_m/2\pi = 3.68 \times 10^9$ Hz, $\gamma/2\pi = 35 \times 10^3$ Hz, $\overline{n}_c = 0.005$, and $G = (J/4) \times 10^2 = 5 \times 10^9$.

engineer optomechanical arrays with larger lattice spacing and/or mechanical modes with frequencies above THz. This is, of course, a challenging experimental scenario.

IV. CONCLUSION

We have proposed a theoretical proposal for quantum state transfer in optomechanical arrays. Our proposal relies on a general scheme illustrated by polariton transformation of the linearized Hamiltonian (7) that allow us to obtain an effective Hamiltonian of two decoupled bosonic networks.

The central result of the present paper is the derivation of the polariton Hamiltonian (7), where we can bring in previous results from quantum state transfer protocols in bosonic networks. Specifically, we can apply any type of quantum state transfer scheme with an extra additional condition, namely, that the rate between the transfer times of both decoupled polaritonic chains must be an odd number. Furthermore, we analyze the effects of dissipation and a possible experimental implementation of our proposal in the red-sideband regime with experimental accessible parameters.

It is also important to point out that—*although not reported explicitly in this work*—the linearization of the nonlinear optomechanical Hamiltonian does not constitute a major restriction. For instance, for driven optomechanical systems in the strong single-photon regime, we can transfer information encoded both in polariton states arising from ion-trap-like Hamiltonians [28] and in dark states in optomechanical systems [29].

Before concluding we should stress one of the possible practical uses of the type of state transfer that we have proposed. In general, the generation of non-Gaussian states of both light and mechanics is difficult, as well as entangled states between the light and mechanics. One could then adopt the strategy of generating such states at one physical cell (say, called a non-Gaussian or entangler node) where, perhaps, one has built in enhanced nonlinearities or extra control on the parameters of a system. Then one can simply transfer such states to a distant location—where they will be exploited—through the optomechanical array as outlined by us. Moreover, even though we used a one-dimensional (1D) array in this work, any other topology might be consider, such as lattice (2D) or crystal (3D) setups. In addition, other interesting aspects to study are the "pretty good state transfer" schemes in Ref. [5] and the generation of long -distance quantum entanglement between sites [27].

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