PHYSICAL REVIEW A **93**, 052505 (2016)

Effective Landau-Zener transitions in the circuit dynamical Casimir effect with time-varying modulation frequency

A. V. Dodonov,[1,2] B. Militello,[2,3] A. Napoli,[2,3] and A. Messina[2,3]

[1]*Institute of Physics, International Center for Condensed Matter Physics, University of Brasilia, 70910-900, Brasilia, Federal District, Brazil*

[2]*Dipartimento di Fisica e Chimica, Università degli Studi di Palermo, Via Archirafi 36, I-90123 Palermo, Italy*

[3]*Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Italy*

(Received 15 March 2016; published 9 May 2016)

We consider the dissipative single-qubit circuit QED architecture in which the atomic transition frequency undergoes a weak external time modulation. For sinusoidal modulation with linearly varying frequency we derive effective Hamiltonians that resemble the Landau-Zener problem of finite duration associated with a two- or multilevel systems. The corresponding off-diagonal coupling coefficients originate either from the rotating or the counter-rotating terms in the Rabi Hamiltonian, depending on the values of the modulation frequency. It is demonstrated that in the dissipationless case one can accomplish almost complete transitions between the eigenstates of the bare Rabi Hamiltonian even for relatively short durations of the frequency sweep. To assess the experimental feasibility of our scheme we solved numerically the phenomenological and the microscopic quantum master equations in the Markovian regime at zero temperature. Both models exhibit qualitatively similar behavior and indicate that photon generation from vacuum via effective Landau-Zener transitions could be implemented with the current technology on the time scales of a few microseconds. Moreover, unlike the harmonic dynamical Casimir effect implementations, our proposal does not require precise knowledge of the resonant modulation frequency to accomplish meaningful photon generation.

DOI: 10.1103/PhysRevA.93.052505

I. INTRODUCTION

The area of circuit quantum electrodynamics (circuit QED) offers unprecedented possibilities to manipulate *in situ* the properties of mesoscopic systems composed of superconducting artificial atoms interacting with the electromagnetic field inside the waveguide resonator on a chip [1–3]. Along with practical applications for quantum-information processing (QIP) this solid-state architecture allows for the experimental study of some of the most fundamental physical processes, such as the light-matter interaction at the level of a few photons. Due to the relative ease to achieve the *strong-coupling regime* in which the coherent light-matter coupling rate is much larger than the system damping and dephasing rates [1,4], this setup can probe novel phenomena with tiny coupling rates. One example is the implementation of the *dynamical Casimir effect* (DCE) and associated phenomena using actively controlled artificial atoms, which may serve both as the source and as the detector of modulation-induced radiation [5–11]. We recall that DCE is a common name ascribed to the processes in which photons are generated from vacuum due to the external time variation of boundary conditions for some field [12–15]. For the usual electromagnetic case this corresponds to the fast motion of a mirror or modulation of the dielectric properties of the mirror or intracavity medium [16]. Analogs of DCE were recently verified experimentally in the setups resembling a single mirror [17] and a lossy cavity [18] where the modulation of the boundary conditions was achieved by threading a time-dependent magnetic flux through superconductive quantum interference devices located inside the coplanar waveguide. These experiments stimulated new theoretical research on role of dynamical Casimir physics in quantum-information processing, quantum simulations, and engineering of nonclassical states of light and matter [19–24].

Recent studies have indicated that DCE could be implemented even using a single two-level atom (qubit) with time-dependent parameters, such as the transition frequency or the atom-field coupling strength [5,25–28]. Generation of excitation from vacuum occurs due to the counter-rotating terms in the Rabi Hamiltonian, which for many years had been neglected under the rotating-wave approximation (RWA). Moreover, single-atom DCE carries some new characteristics, such as the atom-field entanglement, saturation in the number of created photons (due to intrinsic nonlinearities associated with a nonharmonic spectrum of the composite system), and emergence of additional resonant modulation frequencies [27]. Since the ultra-strong-coupling regime [29–32] (for which the atom-field coupling rate is comparable to the cavity and atomic frequencies) is experimentally demanding, especially in nonstationary configurations, here we assume moderate values of the coupling strength, which in turn imply small transition rates for the phenomena originating from the counter-rotating terms. Hence the dissipation must be sufficiently weak, and the frequency of modulation must be tuned with high precision, typically on the order of 10–100 kHz for modulation frequencies in the gigahertz range, that ultimately should be determined experimentally or numerically. For the harmonic modulation of system parameters the observable quantities, e.g., the average photon number or the atomic excitation probability, exhibit an oscillating behavior as a function of time [5,28]. Therefore the duration of modulation must also be minutely adjusted to grasp a meaningful amount of excitations since the detection is typically carried out when the modulation has ceased.

In this paper we propose a simple scheme to implement the phenomena induced by the counter-rotating terms that does not require accurate knowledge of the resonance frequencies and is a little sensitive to the duration of the perturbation. Our method is based on the *adiabatic* variation of the modulation frequency of the atomic level splitting through the expected resonance when one can steadily create excitations from vacuum and other initial states without the aforementioned sporadic comebacks of the system to the initial state. This phenomenon can be understood in terms of some effective two- or multilevel Hamiltonians with constant nondiagonal couplings (dependent on the modulation depths) and time-dependent diagonal terms, which are responsible for *Landau-Zener* (LZ) processes.

By the way, Landau-Zener transitions [33–36] are very fundamental phenomena since they occur every time a two-level system is subjected to a time-dependent Hamiltonian whose bare energies change with time and cross at some instant in time. In its original form, the LZ model is characterized by bare energies changing linearly in time. Over the years, the original scheme has been extended in several directions: nonlinear time dependence of the diagonal matrix elements of the Hamiltonian [37,38], multilevel systems, even with n-fold crossings [39,40], and, of course, including environmental effects inducing dissipation and decoherence [41–44]. This ubiquitous model has been applied to describe similar dynamical situations in several physical scenarios. For example, it has been useful in such systems as spinorial Bose-Einstein condensates [45], Josephson junctions [46,47], in optical lattices with cold atoms [48], and even—in its modified version known as the "hidden crossing model" [49]—in classical optics [50]. Also in the context of circuit QED, Landau-Zener tunneling has been extensively used to suitably manipulate the state of a single qubit [41,51,52]. In our case, Landau-Zener processes naturally rise from the fact that the frequency sweep allows realizing a series of resonances between the (time-varying frequency) external perturbation and a series of couples of the Rabi-Hamiltonian eigenstates.

One possible downside of our method is the relatively long implementation times. Nonetheless, we demonstrate that it can work with the current dissipative parameters provided the modulation depth of the atomic transition frequency is on the order of a few percent of its bare value. Besides, we show that the states generated from vacuum can be quite different from the squeezed vacuum state (SVS) that is typically generated for strictly harmonic modulations [16,53], so our scheme may find new applications in QIP.

This paper is organized as follows. In Sec. II we deduce the effective Hamiltonians similar to the Landau-Zener physics in the resonant and dispersive regimes of atom-field interaction. In Sec. III we describe our approach to realistically account for the dissipation with thorough numerical analysis carried out in Sec. IV. Finally, in Sec. V we discuss the obtained results and present our conclusions.

II. EFFECTIVE LANDAU-ZENER SWEEPS

We consider a single qubit in nonstationary circuit QED. Our starting point is the Hamiltonian (we set $\hbar = 1$),

$$\hat{H}_R(t) = \omega_0 \hat{n} + \frac{\Omega(t)}{2}\hat{\sigma}_z + g_0(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-), \quad (1)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are cavity annihilation and creation operators and $\hat{n} = \hat{a}^\dagger\hat{a}$ is the photon number operator; $\hat{\sigma}_+ = |e\rangle\langle g|$, $\hat{\sigma}_- = |g\rangle\langle e|$, and $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ are the atomic ladder operators, where $|g\rangle$ ($|e\rangle$) denotes the atomic ground (excited) state. ω_0 is the cavity frequency, Ω is the atomic transition frequency, and g_0 is the atom-field coupling strength. For the sake of simplicity here we consider the external modulation of the atomic transition frequency as $\Omega(t) = \Omega_0 + \varepsilon_\Omega \sin[\eta(t)t + \phi_\Omega]$, although it was shown that for $g_0 \ll \omega_0, \Omega_0$ the weak modulation of any system parameter produces similar results. In this paper we suppose that the modulation frequency $\eta(t)$ may also slowly change as a function of time. It will be shown that in specific regimes we obtain the two-level or multilevel effective Landau-Zener physics, so initially unpopulated system states can be excited without undergoing oscillations back to the initial state.

For $\varepsilon_\Omega \ll \Omega_0$ and $\varepsilon_\Omega \lesssim g_0$ we can treat the external modulation as a perturbation that drives the transitions between the bare eigenstates of the Rabi-Hamiltonian $\hat{H}_R^{(0)}|R_i\rangle = E_i|R_i\rangle$, where

$$\hat{H}_R^{(0)} = \omega_0 \hat{n} + \frac{\Omega_0}{2}\hat{\sigma}_z + g_0(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-), \quad (2)$$

and E_i increases with the index i. Expanding the wave function corresponding to the Hamiltonian (1) as

$$|\psi(t)\rangle = \sum_i A_i(t) e^{-itE_i}|R_i\rangle, \quad (3)$$

the probability amplitudes obey the coupled differential equations,

$$i\dot{A}_j(t) = \frac{\varepsilon_\Omega \sin(\eta t + \phi_\Omega)}{2} \sum_i A_i(t) e^{-it(E_i - E_j)}\langle R_j|\hat{\sigma}_z|R_i\rangle. \quad (4)$$

Thus one can induce the coherent coupling between the Rabi dressed states $\{|R_i\rangle, |R_j\rangle\}$ by setting the modulation frequency roughly equal to $|E_i - E_j|$. The remaining rapidly oscillating terms can be neglected according to the RWA approach, which sets the criteria for the validity of the resulting effective equations. In addition, the RWA method introduces small intrinsic frequency shifts in the final equations, which slightly alter the resonant modulation frequency [26,27]. One of the advantages of the present scheme is that such frequency shifts are completely irrelevant for the experimental implementation.

For the weak qubit-field coupling considered here $g_0 \ll \omega_0$, we can find the approximate spectrum of the Rabi Hamiltonian by performing the unitary transformation [54],

$$\hat{U}_R = \exp[\Lambda(\hat{a}\hat{\sigma}_- - \hat{a}^\dagger\hat{\sigma}_+) + \xi(\hat{a}^2 - \hat{a}^{\dagger 2})\hat{\sigma}_z], \quad (5)$$

where $\Lambda \equiv g_0/\Delta_+$, $\xi \equiv \Lambda g_0/2\omega_0$, and $\Delta_\pm = \omega_0 \pm \Omega_0$. For the first order in Λ we get the Bloch-Siegert Hamiltonian [55],

$$\hat{H}_{\mathrm{BS}} = \hat{U}_R^\dagger \hat{H}_R^{(0)} \hat{U}_R = (\omega_0 + \delta_+\hat{\sigma}_z)\hat{n} + \frac{\Omega_0 + \delta_+}{2}\hat{\sigma}_z + g_0(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-). \quad (6)$$

Hence the approximate eigenvalues of $\hat{H}_R^{(0)}$ are

$$E_0 = -(\Omega_0 + \delta_+)/2, \quad (7)$$

$$E_{n>0,\pm} = \omega_0 n - \frac{\omega_0 + \delta_+}{2} \pm \frac{1}{2}\sqrt{[\tilde{\Delta}_-(n)]^2 + 4g_0^2 n}, \quad (8)$$

where $\delta_\pm \equiv g_0^2/\Delta_\pm$, $\tilde{\Delta}_-(n) \equiv \Delta_- - 2\delta_+ n$, and the integer n is the number of total excitations associated with $\hat{H}_{\mathrm{BS}}$. The approximate eigenstates of (2) are $|R_i\rangle = \hat{U}_R|\Upsilon_i\rangle$, where $|\Upsilon_i\rangle$

are the eigenstates of $\hat{H}_{\mathrm{BS}}$,

$$|\Upsilon_0\rangle = |g,0\rangle, \tag{9}$$

$$|\Upsilon_{n>0,+}\rangle = \sin\theta_n|g,n\rangle + \cos\theta_n|e,n-1\rangle, \tag{10}$$

$$|\Upsilon_{n>0,-}\rangle = \cos\theta_n|g,n\rangle - \sin\theta_n|e,n-1\rangle, \tag{11}$$

$$\theta_{n>0} = \arctan\left(\frac{\tilde{\Delta}_-(n) + \sqrt{[\tilde{\Delta}_-(n)]^2 + 4g_0^2 n}}{2g_0\sqrt{n}}\right). \tag{12}$$

Contrary to the standard studies on DCE and related resonant effects where the precise knowledge of the spectrum is necessary to accomplish the desired transitions [25,27,28,56], in this paper only an approximate knowledge of the spectrum is enough to achieve the phenomena of interest. Therefore the lowest-order eigenvalues and eigenstates derived above are sufficient for our purposes. The essence of our proposal is most clearly seen in specific regimes of the system parameters as illustrated below for the resonant and dispersive regimes.

A. Resonant regime

First we consider the *resonant regime* $\Delta_- = 0$ and assume a small number of excitations $\Lambda^2 n \ll 1$. For the initial system ground state $|R_0\rangle$ and the modulation frequency,

$$\eta = 2\omega_0 \pm g_0\sqrt{2} - \nu(t), \tag{13}$$

one can show [by neglecting the rapidly oscillating terms in Eq. (4)] that to the lowest order in Λ the dynamics is described by the effective Hamiltonian,

$$\hat{H}_i = E_0|R_0\rangle\langle R_0| + E_{2,\pm}|R_{2,\pm}\rangle\langle R_{2,\pm}| \pm \left(ig_0\frac{\sqrt{2}}{4}\frac{\varepsilon'_\Omega}{\Delta_+}e^{it[E_{2,\pm}-E_0-\nu(t)]}|R_0\rangle\langle R_{2,\pm}| + \text{H.c.}\right). \tag{14}$$

Here we defined the complex modulation depth $\varepsilon'_\Omega \equiv \varepsilon_\Omega e^{i\phi_\Omega}$, and $\nu(t)$ is a small time-dependent function $|\nu(t)| \ll g_0$ to be specified later. This Hamiltonian can also be obtained after cumbersome calculations using the method of Refs. [26,27], although in this paper it is derived just in a few lines [57]. Performing the time-dependent unitary transformation,

$$\hat{S}(t) = \exp\left\{-it\left[\left(E_0 + \frac{\nu(t)}{2}\right)|R_0\rangle\langle R_0| + \left(E_{2,\pm} - \frac{\nu(t)}{2}\right)|R_{2,\pm}\rangle\langle R_{2,\pm}|\right]\right\}, \tag{15}$$

we obtain the "interaction-picture" effective Hamiltonian,

$$\hat{H}_f \equiv -i\hat{S}^\dagger\frac{d\hat{S}}{dt} + \hat{S}^\dagger\hat{H}_i\hat{S} = \frac{V(t)}{2}(|R_{2,\pm}\rangle\langle R_{2,\pm}| - |R_0\rangle\langle R_0|) \pm (i\beta|R_0\rangle\langle R_{2,\pm}| + \text{H.c.}), \tag{16}$$

$$V(t) \equiv \nu(t) + t\dot{\nu}(t), \quad \beta = \frac{1}{\sqrt{2}}g_0\frac{\varepsilon'_\Omega}{2\Delta_+}. \tag{17}$$

Notice that this Hamiltonian only holds for the modulation frequency (13) and β is nonzero due to the presence of the counter-rotating terms in the Rabi-Hamiltonian (2).

When $\nu(t)$ is a linear function of time, Eq. (16) describes the standard two-level Landau-Zener problem, so for adiabatic variation of $V(t)$ across the avoided crossing one can transfer steadily the population from the initial state $|R_0\rangle$ to the final one $|R_{2,\pm}\rangle$. For the linear variation of $V(t)$ from $-\infty$ to $+\infty$ the probability of such a transition is $1 - \exp(-\pi\beta^2/|\dot{\nu}|)$, which is close to one provided we have $|\dot{\nu}| \ll \pi\beta^2$. However, Eq. (16) is only valid for small values of $|\nu|$, so our scheme corresponds to the finite duration Landau-Zener sweeps in which we have $|\nu(t)| \leqslant K|\beta|$, where K is on the order of 10. Nonetheless, as shown later, in this way we can achieve the desired transition with sufficiently high probability and compensate for the ignorance in the knowledge of the exact eigenvalues of the system Hamiltonian.

B. Dispersive regime

The *dispersive regime* is defined as $g_0\sqrt{n} \ll |\Delta_-|/2$ for all relevant values of n, and we assume the standard condition $|\Delta_-| \ll \omega_0$. Repeating the above reasoning one finds that for the initial ground state and the modulation frequency,

$$\eta = \Delta_+ - 2(\delta_- - \delta_+) + 4\alpha - \nu(t), \quad \alpha \equiv \frac{g_0^4}{\Delta_-^3}, \tag{18}$$

the interaction-picture effective Hamiltonian reads

$$\hat{H}_f = \frac{V(t)}{2}(|R_{2,-\mathcal{D}}\rangle\langle R_{2,-\mathcal{D}}| - |R_0\rangle\langle R_0|) - \mathcal{D}(i\beta|R_0\rangle\langle R_{2,-\mathcal{D}}| + \text{H.c.}), \tag{19}$$

$$\beta = g_0\frac{\varepsilon'_\Omega}{2\Delta_+}, \tag{20}$$

where $\mathcal{D} = \pm$, being the sign of $\Delta_-/|\Delta_-|$. Thus one can achieve the steady population transfer from $|R_0\rangle$ to $|R_{2,-\mathcal{D}}\rangle$, which corresponds approximately to the transition $|g,0\rangle \to |e,1\rangle$. For $\nu(t) = 0$ this behavior was previously named the anti-Jaynes-Cummings regime [5,6] or the blue-sideband transition [54].

On the other hand, for

$$\eta = 2\omega_0 + 2(\delta_- - \delta_+) - 4\alpha - \nu(t), \tag{21}$$

we obtain an analog of the DCE Hamiltonian in the presence of the Kerr nonlinearity [27],

$$\hat{H}_f = \sum_{n=0}^{n_{\max}}\left[\left(\frac{V - 2\alpha(n-2)}{2}\right)n|R_{n,\mathcal{D}}\rangle\langle R_{n,\mathcal{D}}| + \left(i\sqrt{\frac{(n+1)(n+2)}{2}}\beta|R_{n,\mathcal{D}}\rangle\langle R_{n+2,\mathcal{D}}| + \text{H.c.}\right)\right], \tag{22}$$

$$\beta = \delta_-\frac{\varepsilon'_\Omega}{\sqrt{2}\Delta_+}. \tag{23}$$

Here we defined $|R_{0,\mathcal{D}}\rangle \equiv |R_0\rangle$, and $n_{\max}$ denotes the limiting value for the validity of the dispersive regime. If $|\beta| \gtrsim |\alpha|$, we get the multilevel Landau-Zener problem in which one can couple several dressed states $|R_{2n,\mathcal{D}}\rangle \approx |g,2n\rangle$ at each avoided crossing (although the avoided crossings for different pairs of levels do not coincide exactly), so one can asymptotically create several photons from the initial vacuum state $|g,0\rangle$.

Our approach is not limited to the initial ground state. For example, for the initial state with a finite number of excitations one can apply the low modulation frequency,

$$\eta = |\Delta_- - 2\delta_+ m| + 2|\delta_-|m - 2|\alpha|m^2 - \nu(t). \tag{24}$$

For $\varepsilon_\Omega \lesssim \Delta_-$ we find the interaction-picture effective Hamiltonian,

$$\hat{H}_f = \sum_{n=1}^{n_{\max}} \left[\left(\frac{\mathcal{D}V - 2(\delta_- - \delta_+)(m-n) + 2\alpha(m^2-n^2)}{2} \right) \times (|R_{n,\mathcal{D}}\rangle\langle R_{n,\mathcal{D}}| - |R_{n,-\mathcal{D}}\rangle\langle R_{n,-\mathcal{D}}|) + \left(i\frac{\sqrt{n}}{2}\beta |R_{n,-\mathcal{D}}\rangle\langle R_{n,\mathcal{D}}| + \text{H.c.} \right) \right], \tag{25}$$

$$\beta = g_0 \frac{\varepsilon_\Omega'^{(\mathcal{D})}}{\Delta_-}, \tag{26}$$

where $\varepsilon'^{(+)} \equiv \varepsilon'$ and $\varepsilon'^{(-)} \equiv \varepsilon'^*$. The nondiagonal terms in the Hamiltonian (25) rely only on the rotating terms in the Rabi Hamiltonian, so the parameter Λ does not appear in the coupling coefficient β. For $\varepsilon_\Omega\sqrt{m} \ll g_0$ and $|\nu|_{\max} \lesssim 10|\beta|$ we achieve the coupling only between the states $\{|R_{m,\mathcal{D}}\rangle, |R_{m,-\mathcal{D}}\rangle\}$, that corresponds approximately to the steady transition $|g,m\rangle \to |e,m-1\rangle$. For larger variations of $|\nu|$ several dressed states may become successively coupled during the frequency sweep.

III. ACCOUNT OF DISSIPATION

For open quantum systems the dynamics must be described by the master equation (ME) for the system density operator,

$$d\hat{\rho}/dt = -i[\hat{H}_R(t), \hat{\rho}] + \hat{\mathcal{L}}\hat{\rho}, \tag{27}$$

where $\hat{\mathcal{L}}$ is the Liouvillian superoperator whose form depends on the details of system-reservoir interaction. In this paper we do not aim to develop a microscopic *ab initio* model for dissipation in nonstationary systems, instead we assess whether the effective Landau-Zener transitions discussed in the previous section could be implemented in real circuit QED architectures. So we use the simplest consistent dissipation approaches available in the literature to evaluate numerically the dynamics during the time scales of interest.

We consider independent reservoirs for different processes, such as dissipation and pure dephasing. Moreover, we assume that their correlation times are much shorter than the system relevant time scales, virtually zero so that we can treat the noise in the Markovian limit. Applying the Davies-Spohn theory [58], we can consider that in a given time window the bath sees the system as if it was governed by a time-independent Hamiltonian and if such a time window is larger than the typical correlation time of the bath, then the bath acts on the system as if it was governed by a time-independent Hamiltonian, time interval after time interval. If the transition frequencies of the system are all different for any pair of eigenstates of $\hat{H}_R(t)$, which is true for the examples discussed below, then at zero temperature the master equation reads [54]

$$\hat{\mathcal{L}}_R\hat{\rho} = \mathcal{D}\left[\sum_l \Phi^l |l\rangle\langle l|\right]\hat{\rho} + \sum_{l,k\neq l} \Gamma_\phi^{lk}\mathcal{D}[|l\rangle\langle k|]\hat{\rho} + \sum_{l,k>l} (\Gamma_\kappa^{lk} + \Gamma_\gamma^{lk})\mathcal{D}[|l\rangle\langle k|]\hat{\rho}, \tag{28}$$

where $\mathcal{D}[\hat{O}]\hat{\rho} \equiv \frac{1}{2}(2\hat{O}\hat{\rho}\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\hat{\rho} - \hat{\rho}\hat{O}^\dagger\hat{O})$ is the Lindbladian superoperator and we use the shorthand notation $|l\rangle$ to denote the time-dependent eigenstates of $\hat{H}_R(t)$, where the index l increases with the eigenenergy $\lambda_l(t)$. The time-dependent parameters of Eq. (28) are defined as $\Phi^l = [\gamma_\phi(0)/2]^{1/2}\sigma_z^{ll}$, $\Gamma_\phi^{lk} = \gamma_\phi(\Delta_{kl})|\sigma_z^{lk}|^2/2$, $\Gamma_\kappa^{lk} = \kappa(\Delta_{kl})|a^{lk}|^2$, and $\Gamma_\gamma^{lk} = \gamma(\Delta_{kl})|\sigma_x^{lk}|^2$. Here $\kappa(\varpi)$, $\gamma(\varpi)$, and $\gamma_\phi(\varpi)$ are the dissipation rates corresponding to the resonator and qubit dampings and dephasing noise spectral densities at frequency ϖ; we also defined the time-dependent quantities $\Delta_{kl} \equiv \lambda_k(t) - \lambda_l(t)$, $\sigma_z^{lk} = \langle l|\hat{\sigma}_z|k\rangle$, $a^{lk} = \langle l|(\hat{a} + \hat{a}^\dagger)|k\rangle$, and $\sigma_x^{lk} = \langle l|(\hat{\sigma}_+ + \hat{\sigma}_-)|k\rangle$.

From the lowest-order approximate expressions for the eigenvalues and the eigenstates [consider equations (7)–(12)], we see that for $\varepsilon_\Omega \ll \max\{g_0, \Delta_-\}$ the time-dependent eigenvalues and eigenstates are very close to the time-independent ones evaluated at the bare-qubit frequency Ω_0. In this paper we assume a small modulation depth and $g_0 \ll \omega_0, \Omega_0$, hence in the ME (28) we can use the lowest-order *time-independent* Rabi eigenvalues and eigenstates given by Eqs. (7)–(12) [59]. Moreover, we do not restrict our analysis to a specific model for the reservoirs' spectral densities and make the simplest assumption that the dissipation rates are zero for $\varpi < 0$ and take on constant values κ, γ, and γ_ϕ for $\varpi \geqslant 0$. Since our primary goal is to study the photon generation from vacuum, such an assumption is the most conservative with regard to the spurious generation of excitations due to dephasing [54,60,61]. Besides, eventual random fluctuations in the modulation frequency may be treated as additional dephasing noise [54,60], so the simultaneous inclusion of κ, γ, and γ_ϕ covers the most common experimental situations.

The numeric integration of the master equation (28) with the approximate Rabi eigenstates $|R_i\rangle$ is still cumbersome from a practical viewpoint, so we also evaluate the kernel $\hat{\mathcal{L}}_{\text{JC}}$ in which one uses the time-independent Jaynes-Cummings eigenvalues and eigenstates obtained by setting $\delta_+ = \Lambda = \xi = 0$ in Eqs. (7)–(12). It will be shown that in our examples, where $\Lambda \approx 0.02$, these two approaches give almost indistinguishable results, which are qualitatively similar to the prediction of the phenomenological "standard master equation" of quantum optics (in whose microscopic derivation one assumes that the qubit and resonator do not interact [62]),

$$\hat{\mathcal{L}}_{ph}\hat{\rho} = \kappa\mathcal{D}[\hat{a}]\hat{\rho} + \gamma\mathcal{D}[\hat{\sigma}_-]\hat{\rho} + \frac{\gamma_\phi}{2}\mathcal{D}[\hat{\sigma}_z]\hat{\rho}, \tag{29}$$

with constant dissipative rates κ, γ, and γ_ϕ. So the knowledge of regimes in which $\hat{\mathcal{L}}_{ph}$ provides the same results as $\hat{\mathcal{L}}_R$ may be important for future studies where the numerical evaluation of $\hat{\mathcal{L}}_R$ or $\hat{\mathcal{L}}_{\text{JC}}$ is prohibitively complicated.

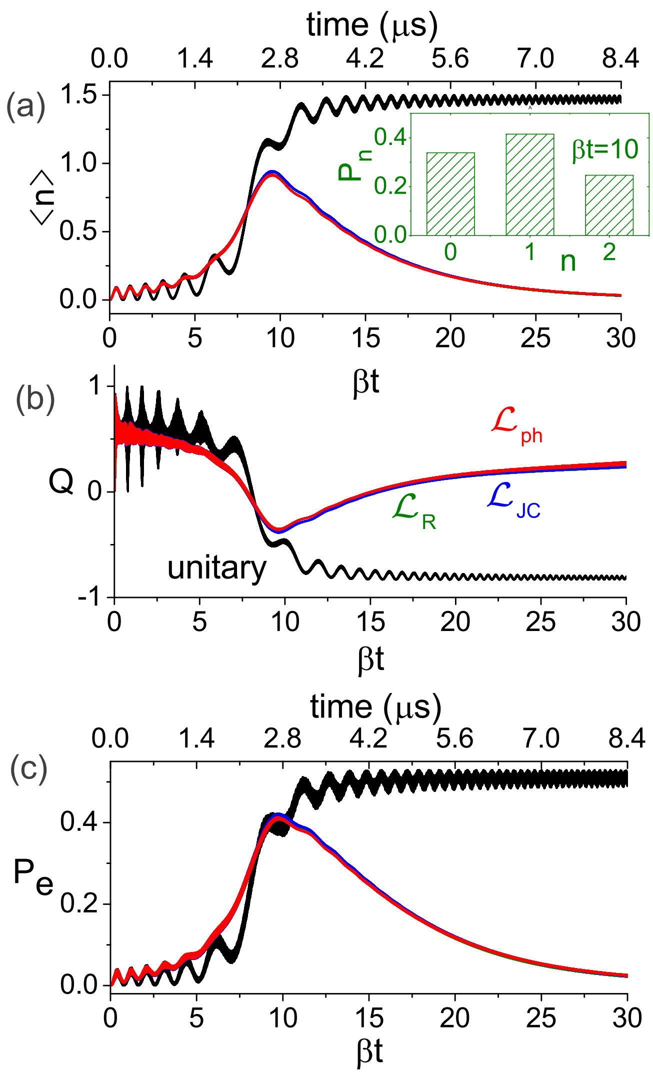

FIG. 1. Time behavior of (a) average photon number $\langle\hat{n}\rangle$, (b) Mandel's Q factor, and (c) the atomic excitation probability P_e in the resonant regime for the modulation frequency $\eta = 2\omega_0 + g_0\sqrt{2} - \nu(t)$ and initial state $|g,0\rangle$. The black lines correspond to the unitary evolutions, whereas the red, blue, and green lines correspond to the three dissipative models characterized by $\mathcal{L}_{ph}$, $\mathcal{L}_{\rm JC}$, and $\mathcal{L}_R$, respectively. The three dissipative models give almost the same results. The inset in (a) displays the photon statistics at the time instant $\beta t = 10$ under the dissipative kernel $\mathcal{L}_R$.

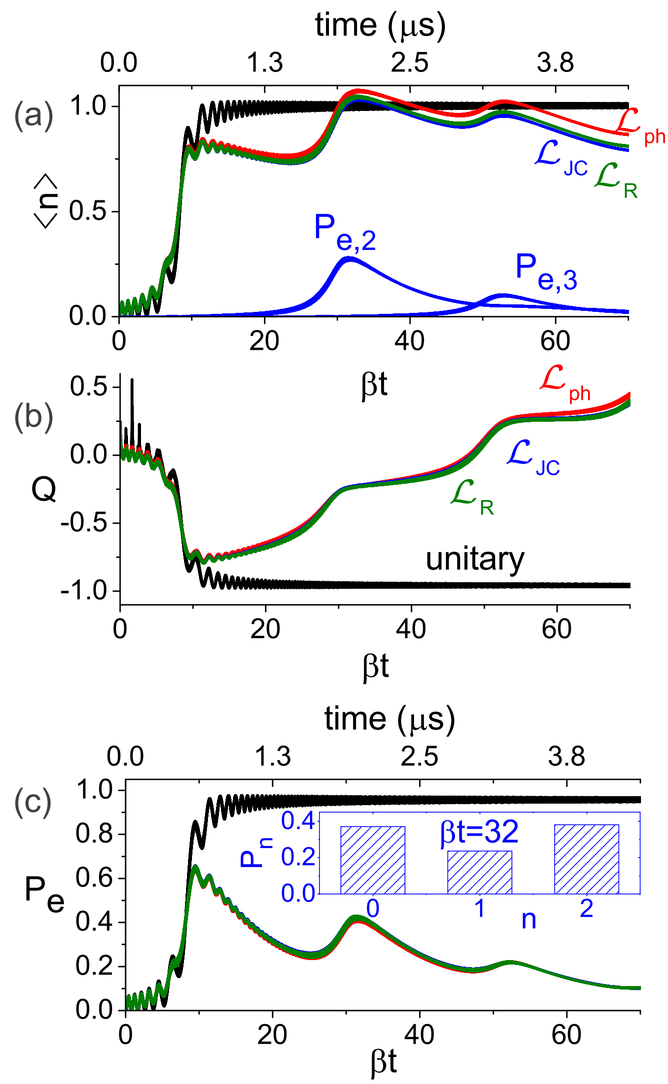

FIG. 2. Behavior of (a) $\langle\hat{n}\rangle$, (b) Q, and (c) P_e for the coupling between the states $|g,0\rangle \to |R_{2,-\mathcal{D}}\rangle$ in the dispersive regime and modulation frequency $\eta = \Delta_+ - 2(\delta_- - \delta_+) + 4\alpha - \nu(t)$. For the dissipator $\mathcal{L}_{\rm JC}$ we also show the probabilities $P_{e,n} \equiv {\rm Tr}(|e,n\rangle\langle e,n|\hat{\rho})$ (a) and the photon statistics for $\beta t = 32$ [inset in (c)].

IV. NUMERICAL RESULTS

We verified the feasibility of photon generation from vacuum via Landau-Zener sweeps of the modulation frequency in actual circuit QED setups by solving numerically the master equation (27) for the kernels $\hat{\mathcal{L}}_R$, $\hat{\mathcal{L}}_{\rm JC}$, and $\hat{\mathcal{L}}_{ph}$. We considered the standard value of $\omega_0/2\pi = 8$ GHz for the cavity frequency, the realistic qubit-field coupling strength $g_0/\omega_0 = 4\times 10^{-2}$, and the currently available dissipative rates $\kappa = 10^{-4}g_0$, $\gamma = \gamma_\phi = 7\times 10^{-4}g_0$ [63–65].

In Fig. 1 we exemplify the implementation of the transition $|g,0\rangle \to |R_{2,+}\rangle$ in the resonant regime for the parameters: $\Delta_- = 0$, $\varepsilon_\Omega = 0.01\times\Omega_0$, $\eta = 2\omega_0 + g_0\sqrt{2} - \nu(t)$, where $\nu(t) = -8\beta + (\beta^2/2)t$ and $\beta \equiv g_0\varepsilon_\Omega/(2\sqrt{2}\Delta_+)$. We plot the average photon number $\langle\hat{n}\rangle$, Mandel's Q-factor $Q = [\langle(\Delta\hat{n})^2\rangle - \langle\hat{n}\rangle]/\langle\hat{n}\rangle$ (that quantifies the spread of the photon number distribution) and the atomic excitation probability $P_e = {\rm Tr}[|e\rangle\langle e|\hat{\rho}]$. In the ideal case the system ends up approximately in the dressed state $|R_{2,+}\rangle$. Under realistic conditions the photon generation from vacuum persists for initial times, but later the system decays to the ground state associated with the kernel $\hat{\mathcal{L}}$ because the external modulation goes off-resonance. We notice that the predictions of different dissipation models are quite similar in this example so for an estimative of the time behavior one can employ the simplest phenomenological master equation. In the inset we show the photon statistics evaluated at the time interval $\beta t = 10$ for the dissipator $\mathcal{L}_R$, which confirms that one or two photons could be measured with roughly 70% probability.

In Fig. 2 we consider the transition $|g,0\rangle \to |R_{2,-\mathcal{D}}\rangle$ in the dispersive regime for the parameters: $\Delta_- = 9g_0$,

$\varepsilon_\Omega = 0.04 \times \Omega_0$, $\eta = \Delta_+ - 2(\delta_- - \delta_+) + 4\alpha - \nu(t)$, where $\nu(t) = -8\beta + (\beta^2/2)t$ and $\beta \equiv g_0\varepsilon_\Omega/2\Delta_+$. Under the unitary evolution one would generate approximately the state $|e,1\rangle$, but the dissipation alters dramatically such behavior due to successive couplings $|R_{1,\mathcal{D}}\rangle \to |R_{3,-\mathcal{D}}\rangle$, $|R_{2,\mathcal{D}}\rangle \to |R_{4,-\mathcal{D}}\rangle$ (roughly $|g,1\rangle \to |e,2\rangle$ and $|g,2\rangle \to |e,3\rangle$) induced by the modulation for long times, whereas the transitions $|R_{2,-\mathcal{D}}\rangle \to |R_{1,\mathcal{D}}\rangle$ and $|R_{3,-\mathcal{D}}\rangle \to |R_{2,\mathcal{D}}\rangle$ are caused by dissipation. These additional transitions are tested by plotting the behavior of probabilities $P_{e,n} \equiv \mathrm{Tr}[|e,n\rangle\langle e,n|\hat{\rho}]$ obtained from the kernel $\mathcal{L}_{\mathrm{JC}}$ [Fig. 2(a)]; they can be avoided by sweeping the modulation frequency in the opposite direction $\nu(t) \to -\nu(t)$ since in this case the transition $|R_{1,\mathcal{D}}\rangle \to |R_{3,-\mathcal{D}}\rangle$ becomes off-resonant for longer times. In the inset of Fig. 2(c) we show the photon statistics obtained from the kernel $\mathcal{L}_{\mathrm{JC}}$ for the time instant $\beta t = 32$, which proves that two photons could be observed experimentally. Again we notice that the predictions of the kernels $\mathcal{L}_{\mathrm{JC}}$ and $\mathcal{L}_R$ are almost indistinguishable and qualitatively identical to the predictions of $\mathcal{L}_{ph}$. So in the remainder of the paper we will only employ the dissipators $\hat{\mathcal{L}}_{\mathrm{JC}}$ and $\hat{\mathcal{L}}_{ph}$ as the evaluation of $\hat{\mathcal{L}}_R$ becomes too demanding for a large number of excitations.

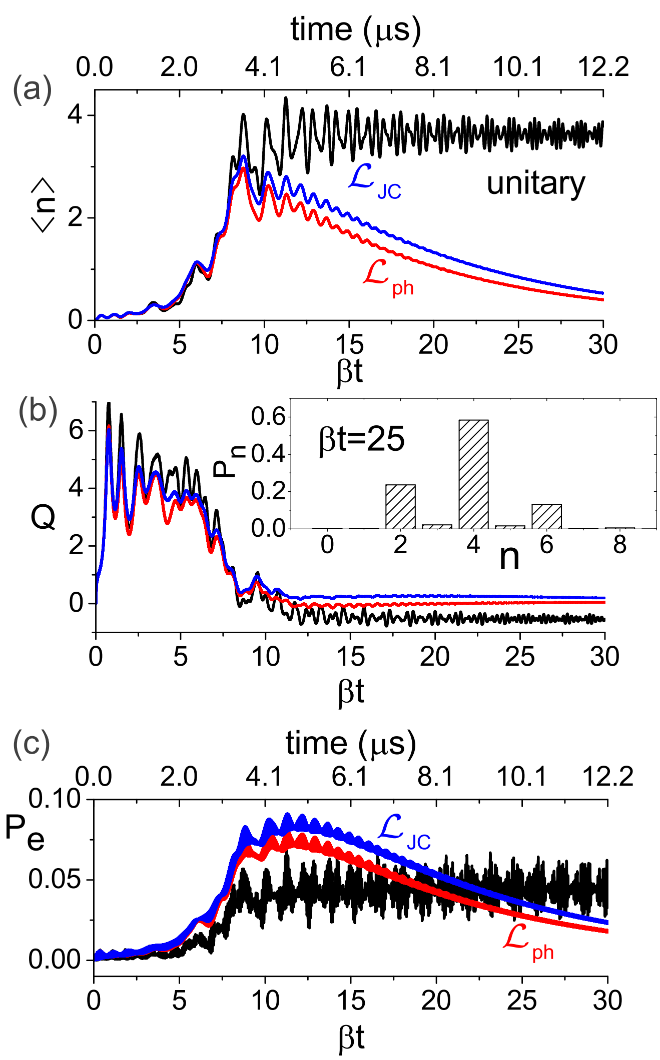

FIG. 3. Coupling of states $|g,0\rangle \to |R_{2k,\mathcal{D}}\rangle$ in the dispersive regime for the modulation frequency $\eta = 2\omega_0 + 2(\delta_- - \delta_+) - 4\alpha - \nu(t)$. The inset in (b): photon statistics under the unitary evolution for the time instant $\beta t = 25$.

In Figs. 3 and 4 we consider multiple transitions $|g,0\rangle \to |R_{2k,\mathcal{D}}\rangle$ $(k = 1,2,\ldots)$ in the dispersive regime for the parameters of Fig. 2 and the modulation frequency $\eta = 2\omega_0 + 2(\delta_- - \delta_+) - 4\alpha - \mathcal{S}\nu(t)$, where $\nu(t) = [8\beta - (\beta^2/2)t]$, $\mathcal{S} = \pm$, and $\beta \equiv \delta_-\varepsilon_\Omega/\sqrt{2}\Delta_+$. For these parameters we have $\beta/\alpha \approx 0.9$, so from the Hamiltonian (22) we expect Landau-Zener transitions among several dressed states during each avoided crossing. In Fig. 3 we set $\mathcal{S} = +$, so only the states $|R_{2k,\mathcal{D}}\rangle$ with $k \sim 1$ may become populated from the initial vacuum state as can be seen from the plots. In the ideal case the atom acquires a small probability of excitation, and up to six photons are created with non-negligible probability—this is shown in the inset of Fig. 3(b) where the photon statistics is displayed for the time instant $\beta t = 25$. Besides, the created field state is nonclassical and quite different from the usual squeezed

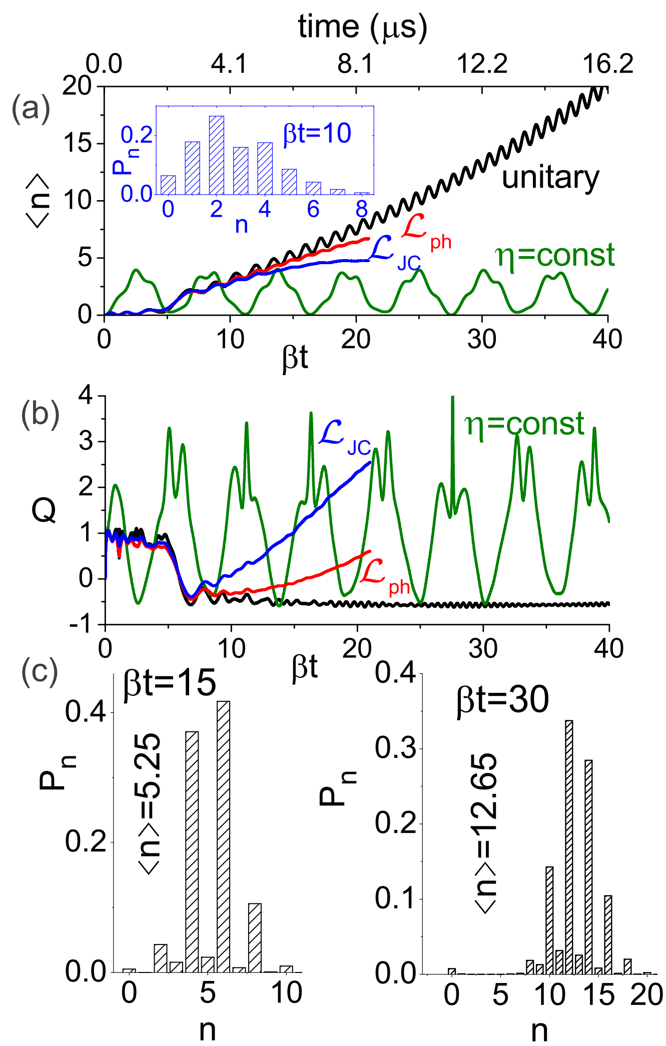

FIG. 4. Similar to Fig. 3 but for the modulation frequency $\eta = 2\omega_0 + 2(\delta_- - \delta_+) - 4\alpha + \nu(t)$. (a) and (b) Time behaviors of $\langle\hat{n}\rangle$ and Q under different dissipators where the green lines labeled "$\eta =$ const" correspond to harmonic modulation with constant frequency adjusted to maximize $\langle\hat{n}\rangle$. The inset in (a): photon statistics for $\beta t = 10$ under dissipator $\mathcal{L}_{\mathrm{JC}}$. (c) Photon statistics under unitary evolution for two time instants: $\beta t = 15$ and 30.

vacuum state created during standard DCE as corroborated by the negative values of the Q factor (recall that for the SVS one has $Q = 1 + 2\langle\hat{n}\rangle$). In the presence of dissipation the system tends to the ground state for long times since the modulation becomes off-resonant. Still it is possible to measure a meaningful average photon number $\langle\hat{n}\rangle \sim 2$ on the time scales of a few microseconds.

When $\mathcal{S} = -$ one can generate a quite large amount of photons from vacuum in the ideal case since the system undergoes successive Landau-Zener transitions towards the higher-energy dressed states. This is illustrated in Fig. 4. For adiabatic variation of η only a few photon states are populated at a time [see Fig. 4(c) for the photon statistics at the time instants $\beta t = 15$ and 30], which is reflected in the sub-Poissonian photon statistics for long times. The amount of created photons can substantially exceed the average photon number achievable for the harmonic modulation with constant frequency [27,28] [green lines in Figs. 4(a) and 4(b) for which η was found numerically to optimize the photon generation]. Our numerical simulation of dissipation is not accurate for large photon numbers, so we only show the dissipative dynamics for $\beta t < 21$. The differences between the predictions of kernels $\mathcal{L}_{JC}$ and $\mathcal{L}_{ph}$ become significant for $\langle\hat{n}\rangle \gtrsim 4$ due to the differences in the available decay channels, but the overall behavior is qualitatively similar. Our results demonstrate that several photons can be generated on the time scales of a few microseconds, but the dissipation modifies the photon statistics to super-Poissonian [see the inset in Fig. 4(a), evaluated at the time instant $\beta t = 10$ for the kernel $\mathcal{L}_{JC}$].

Finally, in Fig. 5 we study the transitions between the dressed states $|R_{n,+}\rangle \to |R_{n,-}\rangle$ in the dispersive regime for $n \leqslant M$, which roughly correspond to the transitions $|g,n\rangle \to |e,n-1\rangle$. We consider the initial state $|g,\alpha\rangle$, where $|\alpha\rangle$ denotes the coherent state with $\alpha = \sqrt{4.5}$. Other parameters are as follows: $\Delta_- = 10g_0$, $\varepsilon_\Omega/\omega_0 = 4\times 10^{-3}$, $\eta = \Delta_- + 2M(\delta_- - \delta_+) - \nu(t)$, where $\nu(t) = -7\beta + (\beta^2/2)t$, $M = 4$, and $\beta \equiv g_0\varepsilon_\Omega/\Delta_-$. As expected, the dissipation strongly affects the dynamics since excitations are lost to the environment from the very beginning and the slow modulation is unable to create additional excitations. To confirm the selective Landau-Zener sweeps between states $\{|R_{n,+}\rangle,|R_{n,-}\rangle\}$ in Figs. 5(c) and 5(d) we show the time evolution of the probabilities $P_{g,n} \equiv \mathrm{Tr}(|g,n\rangle\langle g,n|\hat{\rho})$, which change one at a time from the initial value to roughly zero ($P_{g,n}$ also undergoes fast oscillations due to the dispersive exchange of excitations between the field and the qubit). For $n > M$ the probabilities $P_{g,n}$ are not affected by the perturbation since the corresponding avoided crossings are not swept during the frequency variation [for the frequency change in the opposite directions $\nu(t) \to -\nu(t)$, only the states with $n \geqslant M$ would be affected]. Although under dissipation the occurrence of the Landau-Zener transitions is almost completely washed out in the behavior of $\langle\hat{n}\rangle$ [Fig. 5(a)], the atomic excitation probability P_e still preserves the characteristic Landau-Zener plateaus, which are transformed into peaks due to the damping [Fig. 5(b)].

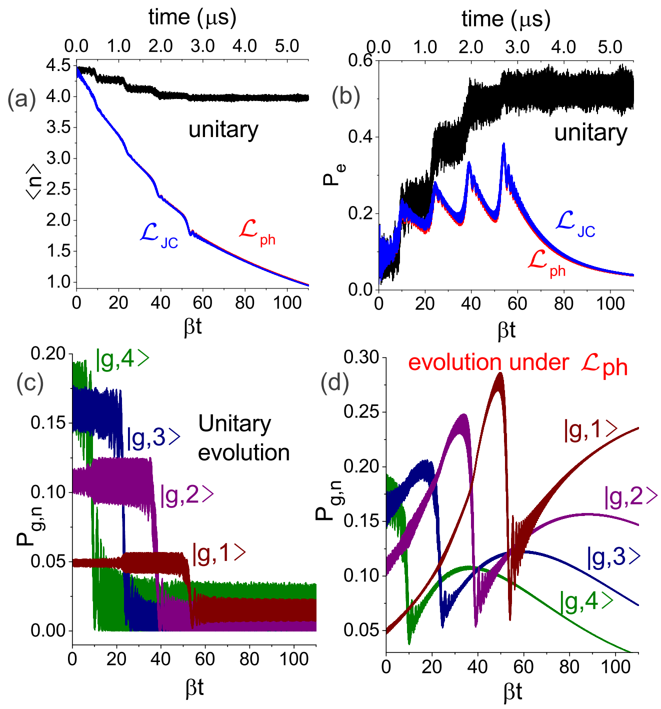

FIG. 5. Coupling of states $|R_{n,+}\rangle \to |R_{n,-}\rangle$ for $n \leqslant 4$ in the dispersive regime for the initial state $|g,\alpha\rangle$ and modulation frequency $\eta = \Delta_- + 2M(\delta_- - \delta_+) - \nu(t)$. (a) and (b) Time behavior of $\langle\hat{n}\rangle$ and P_e under different dissipators. (c) and (d) Behavior of probabilities $P_{g,n} \equiv \mathrm{Tr}(|g,n\rangle\langle g,n|\hat{\rho})$ under the unitary evolution and the kernel $\mathcal{L}_{ph}$, respectively.

V. DISCUSSION AND CONCLUSIONS

In this paper we have discussed a simple scheme to achieve photon generation from vacuum due to the counter-rotating terms in a time-dependent Rabi Hamiltonian where a suitable perturbation characterized by a sinusoidal time dependence with a slowly changing frequency is present. Because of this frequency change the perturbation intercepts several resonance frequencies of the system (described by the time-independent Rabi Hamiltonian) and can induce a single or a series of Landau-Zener processes that allow for populating the upper levels starting from the ground state. This scheme then works quite well without the need to know accurately the resonant unperturbed frequencies of the system nor accurately adjusting the shape of the modulation frequency. Besides, it is only a little sensitive to the duration of the external perturbation. The latter property is related to the fact that, provided the diabatic energies (i.e., the diagonal Rabi-basis matrix elements) in the effective Hamiltonians vary in a sufficiently wide range and with a sufficiently low speed, each Landau-Zener process essentially leads to the same final result, irrespective of the specific range and speed.

It is worth noting that although we considered the modulation of the atomic transition frequency, our method can easily be extended to the time variation of the atom-field coupling strength or both. Moreover, the effective Hamiltonians deduced here are valid for arbitrary variation of the modulation frequency. So our results can be employed to propose protocols that optimize, for instance, the photon production or generation of specific entangled states, although in these cases one would need precise knowledge of the system spectrum and to control accurately the duration of the perturbation.

We showed that the effective Landau-Zener transitions could be implemented for current parameters of dissipation provided one maintains the modulation for a time interval on the order of microseconds with relative modulation strength of a few percent. For consistent description of the experimental results the dissipation must be necessarily included because it alters significantly the unitary behavior due to the additional transitions between the system eigenstates induced by the combined action of dissipation and coherent perturbation. It is remarkable that the predictions of the phenomenological quantum optical master equation are qualitatively similar to the predictions of the microscopic dressed-picture master equation, and in many situations both predictions are almost indistinguishable (this is typical in the weak damping limit). This means that one can use the simpler phenomenological master equation to get a crude estimation about the overall behavior. The effect of random fluctuations of the modulation frequency can be incorporated into our approach as additional pure dephasing, so one does not expect major qualitative differences in this case. Therefore we hope that our protocol will facilitate the hitherto missing experimental observation of DCE due to a single qubit.

ACKNOWLEDGMENTS

A.V.D. would like to thank the Dipartimento di Fisica e Chimica of Università degli Studi di Palermo for the warm hospitality during the visit in winter of 2016. A.V.D. also acknowledges support from Brazilian agencies Conselho Nacional de Desenvolvimento Científico e Tecnológico and Fundação de Apoio à Pesquisa do Distrito Federal.

[1] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).
[2] R. J. Schoelkopf and S. M. Girvin, Nature (London) **451**, 664 (2008).
[3] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).
[4] D. I. Schuster, A. A. Houck, J. A. Schreier, A. Wallraff, J. M. Gambetta, A. Blais, L. Frunzio, J. Majer, B. Johnson, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **445**, 515 (2007).
[5] A. V. Dodonov, J. Phys.: Conf. Ser. **161**, 012029 (2009).
[6] A. V. Dodonov, R. Lo Nardo, R. Migliore, A. Messina, and V. V. Dodonov, J. Phys. B **44**, 225502 (2011).
[7] A. V. Dodonov and V. V. Dodonov, Phys. Lett. A **375**, 4261 (2011).
[8] A. V. Dodonov and V. V. Dodonov, Phys. Rev. A **85**, 015805 (2012).
[9] A. V. Dodonov and V. V. Dodonov, Phys. Rev. A **85**, 055805 (2012).
[10] A. V. Dodonov and V. V. Dodonov, Phys. Rev. A **85**, 063804 (2012).
[11] A. V. Dodonov and V. V. Dodonov, Phys. Rev. A **86**, 015801 (2012).
[12] V. V. Dodonov, in *Modern Nonlinear Optics*, Advances in Chemical Physics Series, Vol. 119, part 1, edited by M. W. Evans (Wiley, New York, 2001), pp. 309–394.
[13] V. V. Dodonov, Phys. Scr. **82**, 038105 (2010).
[14] D. A. R. Dalvit, P. A. Maia Neto, and F. D. Mazzitelli, in *Casimir Physics*, edited by D. Dalvit, P. Milonni, D. Roberts, and F. da Rosa, Lecture Notes in Physics Vol. 834 (Springer, Berlin, 2011), p. 419.
[15] P. D. Nation, J. R. Johansson, M. P. Blencowe, and F. Nori, Rev. Mod. Phys. **84**, 1 (2012).
[16] C. K. Law, Phys. Rev. A **49**, 433 (1994).
[17] C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, Nature (London) **479**, 376 (2011).
[18] P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, Proc. Natl. Acad. Sci. USA **110**, 4234 (2013).
[19] S. Felicetti, C. Sabín, I. Fuentes, L. Lamata, G. Romero, and E. Solano, Phys. Rev. B **92**, 064501 (2015).
[20] G. Benenti, A. D'Arrigo, S. Siccardi, and G. Strini, Phys. Rev. A **90**, 052313 (2014).
[21] J. R. Johansson, G. Johansson, C. M. Wilson, P. Delsing, and F. Nori, Phys. Rev. A **87**, 043804 (2013).
[22] S. Felicetti, M. Sanz, L. Lamata, G. Romero, G. Johansson, P. Delsing, and E. Solano, Phys. Rev. Lett. **113**, 093602 (2014).
[23] R. Stassi, S. De Liberato, L. Garziano, B. Spagnolo, and S. Savasta, Phys. Rev. A **92**, 013830 (2015).
[24] D. Z. Rossatto, S. Felicetti, H. Eneriz, E. Rico, M. Sanz, and E. Solano, Phys. Rev. B **93**, 094514 (2016).
[25] S. De Liberato, D. Gerace, I. Carusotto, and C. Ciuti, Phys. Rev. A **80**, 053810 (2009).
[26] A. V. Dodonov, J. Phys. A: Math, Theor. **47**, 285303 (2014).
[27] I. M. de Sousa and A. V. Dodonov, J. Phys. A: Math. Theor. **48**, 245302 (2015).
[28] D. S. Veloso and A. V. Dodonov, J. Phys. B **48**, 165503 (2015).
[29] M. Devoret, S. Girvin, and R. Schoelkopf, Ann. Phys. (NY) **16**, 767 (2007).
[30] B. Peropadre, P. Forn-Díaz, E. Solano, and J. J. García-Ripoll, Phys. Rev. Lett. **105**, 023601 (2010).
[31] J. Casanova, G. Romero, I. Lizuain, J. J. García-Ripoll, and E. Solano, Phys. Rev. Lett. **105**, 263603 (2010).
[32] R. Stassi, A. Ridolfo, O. Di Stefano, M. J. Hartmann, and S. Savasta, Phys. Rev. Lett. **110**, 243601 (2013).
[33] L. D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932).
[34] C. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932).
[35] E. Majorana, Nuovo Cimento **9**, 43 (1932).
[36] E. C. G. Stückelberg, Helv. Phys. Acta **5**, 369 (1932).
[37] N. V. Vitanov and K.-A. Suominen, Phys. Rev. A **59**, 4580 (1999).
[38] T. Chasseur, L. S. Theis, Y. R. Sanders, D. J. Egger, and F. K. Wilhelm, Phys. Rev. A **91**, 043421 (2015).
[39] A. V. Shytov, Phys. Rev. A **70**, 052708 (2004).
[40] C. E. Carroll and F. T. Hioe, J. Phys. A **19**, 2061 (1986); **19**, 1151 (1986).
[41] D. Zueco, P. Hänggi, and S. Kohler, New J. Phys. **10**, 115012 (2008).

[42] K. Saito, M. Wubs, S. Kohler, Y. Kayanuma, and P. Hänggi, Phys. Rev. B **75**, 214308 (2007).
[43] P. Ao and J. Rammer, Phys. Rev. B **43**, 5397 (1991).
[44] P. Ao and J. Rammer, Phys. Rev. Lett. **62**, 3004 (1989).
[45] J.-N. Zhang, C.-P. Sun, S. Yi, and F. Nori, Phys. Rev. A **83**, 033614 (2011).
[46] D. M. Berns, M. S. Rudner, S. O. Valenzuela, K. K. Berggren, W. D. Oliver, L. S. Levitov, and T. P. Orlando, Nature (London) **455**, 51 (2008).
[47] Guozhu Sun, Xueda Wen, Bo Mao, Jian Chen, Yang Yu, Peiheng Wu, and Siyuan Han, Nat. Commun. **1**, 51 (2010).
[48] A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Phys. Rev. A **82**, 065601 (2010).
[49] S. Fishman, K. Mullen, and E. Ben-Jacob, Phys. Rev. A **42**, 5181 (1990).
[50] D. Bouwmeester, N. H. Dekker, F. E. v. Dorsselear, C. A. Schrama, P. M. Visser, and J. P. Woerdman, Phys. Rev. A **51**, 646 (1995).
[51] K. Saito, M. Wubs, S. Kohler, P. Hänggi, and Y. Kayanuma, Europhys. Lett. **76**, 22 (2006).
[52] L. Jun-Wang, W. Chun-Wang, and D. Hong-Yi, Chin. Phys. Lett. **28**, 090302 (2011).
[53] V. V. Dodonov and A. B. Klimov, Phys. Rev. A **53**, 2664 (1996).
[54] F. Beaudoin, J. M. Gambetta, and A. Blais, Phys. Rev. A **84**, 043832 (2011).
[55] The Hamiltonian (6) may be used even in the ultra-strong-coupling regime, provided g_0 is small with respect to Δ_+.
[56] G. Vacanti, S. Pugnetti, N. Didier, M. Paternostro, G. M. Palma, R. Fazio, and V. Vedral, Phys. Rev. Lett. **108**, 093603 (2012).
[57] Although the present method is more practical to deduce the effective Hamiltonian, the approach of Ref. [26] is more suitable to account for nonlinear effects and to estimate the intrinsic frequency shifts. Such shifts are irrelevant when $\nu(t)$ undergoes a linear sweep.
[58] E. B. Davies and H. Spohn, J. Stat. Phys. **19**, 511 (1978).
[59] In fact, the relative error we make in evaluating the Hamiltonian eigenstates under such hypotheses is on the order of $\epsilon_\Omega/\max\{g_0,\Delta_-\}$ or smaller (from the expressions for $\cos\theta_n$ and $\sin\theta_n$ it is easy to find out that the relative error is on the order of $\epsilon_\Omega g_0/\Delta_-^2$ in the dispersive regime and ϵ_Ω/g_0 in the resonant regime), and the relative error in the derivation of the dissipator is on the same order.
[60] T. Werlang, A. V. Dodonov, E. I. Duzzioni, and C. J. Villas-Bôas, Phys. Rev. A **78**, 053805 (2008).
[61] A. V. Dodonov, Phys. Scr. **86**, 025405 (2012).
[62] W. Vogel and D.-G. Welsch, *Quantum Optics* (Wiley, Berlin, 2006).
[63] G. Kirchmair, B. Vlastakis, Z. Leghtas, S. E. Nigg, H. Paik, E. Ginossar, M. Mirrahimi, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **495**, 205 (2013).
[64] D. Ristè, M. Dukalski, C. A. Watson, G. de Lange, M. J. Tiggelman, Y. M. Blanter, K. W. Lehnert, R. N. Schouten, and L. DiCarlo, Natur (London) **502**, 350 (2013).
[65] L. Sun, A. Petrenko, Z. Leghtas, B. Vlastakis, G. Kirchmair, K. M. Sliwa, A. Narla, M. Hatridge, S. Shankar, J. Blumoff, L. Frunzio, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Nature (London) **511**, 444 (2014).