Method for constructing shortcuts to adiabaticity by a substitute of counterdiabatic driving terms

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We propose an efficient method to construct shortcuts to adiabaticity through designing a substitute Hamiltonian to try to avoid the defect in which the speed-up protocols' Hamiltonian may involve terms which are difficult to realize in practice. We show that as long as the counterdiabatic coupling terms—even only some of them—have been nullified by the additional Hamiltonian, the corresponding shortcuts to the adiabatic process could be constructed and the adiabatic process would be sped up. As an application example, we apply this method to the popular Landau-Zener model for the realization of fast population inversion. The results show that in both Hermitian and non-Hermitian systems, we can design different additional Hamiltonians to replace the traditional counterdiabatic driving Hamiltonian to speed up the process. This method provides many choices for designing additional terms of the Hamiltonian such that one can choose a realizable model in practice.

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I. INTRODUCTION

Since Demirplack and Rice [1] and Berry [2] proposed that the addition of a suitable "counterdiabatic" (CD) term $H_{\rm CD}$ to an original time-dependent Hamiltonian $H_0(t)$ can suppress transitions between different time-dependent instantaneous eigenbases of $H_0(t)$, an emergent field named "shortcuts to adiabaticity" (STA) [3,4] which aims at designing nonadiabatic protocols to speed up the quantum adiabatic process has been brought to our attention and has attracted much interest [4–14]. To find shortcuts to adiabatic dynamics, several formal solutions which are in fact strongly related or even potentially equivalent to each other have been proposed; for instance, "counterdiabatic driving" [3,5,6] (also known as "transitionless quantum driving") and invariant-based inverse engineering [6,7]. After years of development, the theory of shortcuts to adiabatic dynamics has gradually become consummate, and STA has been applied in a wide range of fields including "fast cold atom," "fast ion transport," "fast expansions," "fast wave-packet splitting," "fast quantum information processing," and so on [4,7–24].

Nevertheless, a problem has always haunted accelerating adiabatic protocols: the structure or the values of the shortcut-driving Hamiltonian might not exist in practice. It is well known that if the Hamiltonian is hard or even impossible to realize in practice, the protocols will be useless. In view of that, several ingenious methods that aim at amending the problematic terms of the shortcut-driving Hamiltonian to satisfy the experimental requirements have been proposed in recent years [25–31]. For example, Ibáñez *et al.* [29] examined the limitations and capabilities of superadiabatic iterations to produce a sequence of STA in 2013. They calculated the additional term by an iteration method until the additional term was realizable in practice, hence the problem could be avoided. Later, in 2014, Martínez-Garaot *et al.* [26] used the dynamical symmetry of the Hamiltonian to find, by means of Lie transforms, alternative Hamiltonians that achieved the same goals as the speed-up protocols did, without directly using the CD Hamiltonian. These ideas [29–31] inspire us that finding some substitute Hamiltonians for the shortcut-driving Hamiltonian could be an efficient way to overcome the problem that the speed-up protocols' Hamiltonian may involve terms which are difficult to realize in practice. Therefore, in this paper, by using reverse thinking, we have come up with an idea to design an additional Hamiltonian which can also nullify the nonadiabatic coupling term to achieve the same goals as the shortcut-driving Hamiltonian does. Different from the previous works in which the additional term is calculated from the original Hamiltonian, we aim at finding different ways to nullify the nonadiabatic coupling and ensure the shortcut-driving Hamiltonian can be realized in practice.

II. THE SUBSTITUTE OF COUNTERDIABATIC DRIVING TERMS

The starting point is a time-dependent Hamiltonian $H_0(t)$ with N eigenstates $\{|\phi_n(t)\rangle\}$,

$$H_0(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle. \tag{1}$$

The instantaneous eigenstates satisfy

$$\langle\phi_n(t)|\phi_m(t)\rangle = \delta_{nm}, \tag{2}$$

and the closure relation

$$\sum_n |\phi_n(t)\rangle\langle\phi_n(t)| = I. \tag{3}$$

The dynamics of a system governed by Hamiltonian $H_0(t)$ is described by the Schrödinger equation

$$i\hbar\partial_t|\psi(t)\rangle = H_0(t)|\psi(t)\rangle. \tag{4}$$

In general, $|\psi(t)\rangle$ is a column vector, and we can express it as $|\psi(t)\rangle = \sum_n a_n(t)|\mu_n\rangle = [a_1(t), a_2(t), \ldots, a_n(t)]^t$, where the superscript t denotes the transpose, $\{a_n(t)\}$ are the probability amplitudes of all the bare (diabatic) states of the system, and $\{|\mu_n\rangle\}$ are the basis vectors satisfying

$$\sum_n |\mu_n\rangle\langle\mu_n| = 1, \quad \langle\mu_m|\mu_n\rangle = \delta_{mn}, \quad |\mu_m\rangle\langle\mu_n| = \sigma_{mn}, \tag{5}$$

where σ_{mn} is a matrix in which the matrix elements are all zero, except the mth line and the nth column are 1. To study adiabatic passage, we can transform the system into another picture whose bare states are the adiabatic basis (the instantaneous eigenstates of H_0) with the rotation matrix $R(t)$ which will be

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introduced in the following. In this picture, the dynamics of the system is also described by Schrödinger equation

$$i\hbar\partial_t|\psi^e(t)\rangle = H_0^e(t)|\psi^e(t)\rangle, \tag{6}$$

where the superscript e denotes the system is in the "eigen picture," and $|\psi^e(t)\rangle = [c_1(t),c_2(t),\ldots,c_n(t)]^t$.

To transform the quantum system from the Schrödinger picture to the eigen picture, the transformation equation is expressed as $|\psi^e(t)\rangle = R^\dagger|\psi(t)\rangle$, or in the form of a matrix,

$$\begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \cdots \\ S_{21} & S_{22} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}, \tag{7}$$

where $S_{mn} = \langle\phi_m|\mu_n\rangle$ and

$$R^\dagger(t) = \begin{pmatrix} S_{11} & S_{12} & \cdots \\ S_{21} & S_{22} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix}. \tag{8}$$

We can also express the rotation matrix $R^\dagger(t)$ as

$$\begin{aligned} R^\dagger &= \sum_{m,n} \sigma_{mn}\langle\phi_m|\mu_n\rangle \\ &= \sum_{m,n} |\mu_m\rangle\langle\mu_n|\langle\phi_m|\mu_n\rangle = \sum_m |\mu_m\rangle\langle\phi_m|. \end{aligned} \tag{9}$$

Adding this relationship to Eqs. (4) and (6), we obtain

$$H_0^e(t) = R^\dagger H_0 R - i\hbar R^\dagger\dot{R}, \tag{10}$$

where the overdot means time derivative and

$$R^\dagger H_0 R = \sum_n \sigma_{nn} E_n \tag{11}$$

is the diagonalization matrix for Hamiltonian $H_0(t)$, and

$$\begin{aligned} i\hbar R^\dagger\dot{R} = {} & i\hbar\sum_n \sigma_{nn}\langle\phi_n(t)|\dot{\phi}_n(t)\rangle \\ & + i\hbar\sum_{n\neq m}\sigma_{nm}\langle\phi_n(t)|\dot{\phi}_m(t)\rangle. \end{aligned} \tag{12}$$

As can be seen, the integral of the first term in Eq. (12) is just the adiabatic phase, and the second term is the nonadiabatic coupling. If $|\hbar\langle\phi_n(t)|\dot{\phi}_m(t)\rangle| \ll |E_n - E_m|$, then the transitions in the instantaneous eigenbasis are suppressed and the evolution is adiabatic. That is what is called the adiabatic condition, which limits the speed. To construct shortcuts to speed up the dynamics, a convenient way is by adding a Hamiltonian $H_1^e = i\hbar R^\dagger\dot{R}$ to counteract the nonadiabatic coupling. Returning to the Schrödinger picture,

$$H_1 = RH_1^eR^\dagger = i\hbar\dot{R}R^\dagger = i\hbar\sum_n |\dot{\phi}_n(t)\rangle\langle\phi_n(t)|. \tag{13}$$

That is, we calculate the CD term through a different way from Berry's transitionless tracking algorithm. In general, shortcuts can be constructed just by directly adding a CD term in the original Hamiltonian $H_0(t)$. However, as we mentioned above, such a CD term always makes trouble in practice. In this paper, we try to use reverse thinking to find other ways to nullify the nonadiabatic coupling. In order to obtain a general result, we further assume that the instantaneous eigenstate is $|\phi_n(t)\rangle = [\phi_{n1},\phi_{n2},\phi_{n3},\ldots]^t$, where the time-dependent ϕ_{nm} denotes the mth element of the column vector $|\phi_n(t)\rangle$. Then, we assume that there exists a Hamiltonian $H_{\rm add} = \sum_{k,l}\sigma_{kl}A_{kl}$. It should be noted that, to make sure adding a Hamiltonian is practicable in practice, it is better to choose the coefficients A_{kl} to satisfy the condition $A_{nm}^* = A_{mn}$ $(n \neq m)$ [3,7,22,26,29,31,32]. By adding this Hamiltonian into Eq. (10), we obtain

$$H^e = H_0^e + R^\dagger H_{\rm add} R, \tag{14}$$

in which

$$R^\dagger H_{\rm add} R = \sum_{n,m,k,l}\sigma_{nm}\phi_{nk}^*\phi_{ml}A_{kl}. \tag{15}$$

The term $R^\dagger H_{\rm add}R$ does not necessarily equal $i\hbar R^\dagger\dot{R}$. So long as $R^\dagger H_{\rm add}R$ can nullify the nonadiabatic coupling term $i\hbar\sum_{n\neq m}\sigma_{nm}\langle\phi_n(t)|\dot{\phi}_m(t)\rangle$, the shortcuts would be constructed. In other words, the shortcuts will be constructed as long as $\sum_{k,l}\phi_{nk}^*\phi_{ml}A_{kl} = i\hbar\langle\phi_n|\dot{\phi}_m\rangle$ $(n \neq m)$. In fact, the shortcuts are still constructible even when only some of the terms in the matrix $i\hbar\sum_{n\neq m}\sigma_{nm}\langle\phi_n(t)|\dot{\phi}_m(t)\rangle$ can be nullified. For example, if the terms $\sigma_{n1}\langle\phi_n|\dot{\phi}_1\rangle$ are nullified, the transition $|\phi_1(t)\rangle \to |\phi_{n\neq1}(t)\rangle$ will be suppressed though the transition $|\phi_{n\neq1}(t)\rangle \to |\phi_1(t)\rangle$ is allowed. In this way, the most important thing is to make sure the initial state is perfectly in the eigenstate $|\phi_1(t)\rangle$.

III. EXAMPLES OF TWO-LEVEL SYSTEMS

In the following, we take the two-level system as an example to display the feasibility of the idea proposed above. We assume a two-level Hermitian system has a ground level $|1\rangle = [1,0]^t$ and an excited level $|2\rangle = [0,1]^t$; its Hamiltonian in the interaction picture is given as

$$H_0(t) = \frac{\hbar}{2}\begin{pmatrix} -\Delta(t) & \Omega(t)e^{-i\varphi(t)} \\ \Omega(t)e^{i\varphi(t)} & \Delta(t) \end{pmatrix}, \tag{16}$$

where $\Omega(t)$ is the Rabi frequency, assumed real, and $\Delta(t)$ is the detuning. The instantaneous eigenvectors for this system are $|\phi_1\rangle = \cos\theta e^{-i\varphi}|1\rangle - \sin\theta|2\rangle$ and $|\phi_2\rangle = \sin\theta|1\rangle + \cos\theta e^{i\varphi}|2\rangle$, where $\theta = \frac{1}{2}\arctan\frac{\Omega}{\Delta}$. The corresponding eigenvalues are $E_1 = \frac{\hbar}{2}\sqrt{\Omega^2+\Delta^2}$ and $E_2 = -\frac{\hbar}{2}\sqrt{\Omega^2+\Delta^2}$. Then, the R matrix can be given as

$$\begin{aligned} R(\theta) &= \begin{pmatrix} \cos\theta e^{-i\varphi} & \sin\theta \\ -\sin\theta & \cos\theta e^{i\varphi} \end{pmatrix}, \\ R^\dagger(\theta) &= \begin{pmatrix} \cos\theta e^{i\varphi} & -\sin\theta \\ \sin\theta & \cos\theta e^{-i\varphi} \end{pmatrix}, \end{aligned} \tag{17}$$

and

$$i\hbar R^\dagger\dot{R} = \hbar\begin{pmatrix} \dot{\varphi}\cos^2\theta & (i\dot{\theta}+\frac{\dot{\varphi}}{2}\sin2\theta)e^{i\varphi} \\ (-i\dot{\theta}+\frac{\dot{\varphi}}{2}\sin2\theta)e^{-i\varphi} & -\dot{\varphi}\cos^2\theta \end{pmatrix}, \tag{18}$$

where

$$\dot{\theta} = \frac{\dot{\Omega}\Delta - \Omega\dot{\Delta}}{2(\Delta^2+\Omega^2)}. \tag{19}$$

According to the transitionless tracking algorithm, the additional Hamiltonian (the CD Hamiltonian) is

$$\begin{aligned} H_{\rm CD} &= i\hbar\sum_n |\dot{\phi}_n\rangle\langle\phi_n| \\ &= \hbar\begin{pmatrix} \dot{\varphi}\cos^2\theta & (i\dot{\theta}-\frac{\dot{\varphi}}{2}\sin2\theta)e^{-i\varphi} \\ (-i\dot{\theta}-\frac{\dot{\varphi}}{2}\sin2\theta)e^{i\varphi} & -\dot{\varphi}\cos^2\theta \end{pmatrix}, \end{aligned} \tag{20}$$

which has been well known and might cause trouble in practice (especially in multilevel and multiqubit systems). In order to tackle the problem, it might be wise to find another Hamiltonian which can also nullify the nonadiabatic coupling term and play the same role as the CD Hamiltonian. We start by assuming an additional Hamiltonian $H_{\rm add}$ which is given with unknown parameters (we have not made any hypothesis to the Hamiltonian here):

$$H_{\rm add}=\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}. \tag{21}$$

Note that, although there are many choices for the coefficients A_{mn}, the premise should be that $H_{\rm add}$ is realizable in practice. So when the coefficients are deduced, we should go back and check whether the additional Hamiltonian is realizable or not. For example, in a two-level atomic system, it is better to set $A_{12}=A_{21}^{*}$, and the boundary conditions (the phases are considered as 0 for convenience)

$$\mathrm{Re}A_{12}=\text{const}\quad\text{or}\quad \mathrm{Re}A_{12}(\tau)=\mathrm{Re}A_{12}(t_f)=0 \tag{22}$$

and

$$\mathrm{Im}A_{12}=\text{const}\quad\text{or}\quad \mathrm{Im}A_{12}(\tau)=\mathrm{Im}A_{12}(t_f)=0, \tag{23}$$

where τ is the initial time and t_f is the final time.

Then, according to Eq. (15), we obtain

$$\begin{aligned}R^{\dagger}H_{\rm add}R=&\sigma_{11}\left[A_{11}\cos^2\theta+A_{22}\sin^2\theta-(A_{12}e^{i\varphi}+A_{21}e^{-i\varphi})\frac{\sin 2\theta}{2}\right]+\sigma_{12}\left[(A_{11}-A_{22})e^{i\varphi}\frac{\sin 2\theta}{2}+A_{12}e^{2i\varphi}\cos^2\theta-A_{21}\sin^2\theta\right]\\&+\sigma_{21}\left[(A_{11}-A_{22})e^{-i\varphi}\frac{\sin 2\theta}{2}-A_{12}\sin^2\theta+A_{21}e^{-2i\varphi}\cos^2\theta\right]+\sigma_{22}\left[A_{11}\sin^2\theta+A_{22}\cos^2\theta+(A_{12}e^{i\varphi}+A_{21}e^{-i\varphi})\frac{\sin 2\theta}{2}\right].\end{aligned} \tag{24}$$

It is obvious that, as long as

$$(A_{11}-A_{22})e^{i\varphi}\frac{\sin 2\theta}{2}+A_{12}e^{2i\varphi}\cos^2\theta-A_{21}\sin^2\theta=\hbar\left(i\dot{\theta}+\frac{\dot{\varphi}}{2}\sin 2\theta\right)e^{i\varphi}, \tag{25}$$

or

$$(A_{11}-A_{22})e^{-i\varphi}\frac{\sin 2\theta}{2}-A_{12}\sin^2\theta+A_{21}e^{-2i\varphi}\cos^2\theta=\hbar\left(-i\dot{\theta}+\frac{\dot{\varphi}}{2}\sin 2\theta\right)e^{-i\varphi}, \tag{26}$$

the transition $|\phi_2(t)\rangle\rightarrow|\phi_1(t)\rangle$ or $|\phi_1(t)\rangle\rightarrow|\phi_2(t)\rangle$ is suppressed, and the shortcut is constructed. Equations (25) and (26) are the key points to realize the accelerating adiabatic protocol. They determine the condition to be satisfied to nullify the counterdiabatic coupling terms. According to Eqs. (25) and (26), we can pick out the corresponding parameters to design $H_{\rm add}$. A simple choice is to set

$$\begin{aligned}&A_{11}=-A_{22}=\hbar\eta,\\&A_{12}=A_{21}^{*}=\hbar(\alpha+i\beta)e^{-i\varphi},\end{aligned} \tag{27}$$

where α,β,η are real, to ensure $H_{\rm add}$ is Hermitian. Adding $\{A_{nm}\}$ into Eqs. (25) and (26), we obtain $\beta=\dot{\theta}$ and $\alpha\cot(2\theta)+\eta=\dot{\varphi}/2$. Then, we have

$$H^{e}=H_0^{e}+R^{\dagger}H_{\rm add}R=\hbar\begin{pmatrix}E_1/\hbar+\chi(t) & 0\\ 0 & E_2/\hbar-\chi(t)\end{pmatrix}, \tag{28}$$

where $\chi(t)=\eta\cos 2\theta-\alpha\sin 2\theta-\dot{\varphi}\cos^2\theta$. Hence, if the system's initial state is $|\psi(\tau)\rangle=[a_1(\tau),a_2(\tau)]^t$, then

$$\begin{aligned}&c_1(\tau)=a_1(\tau)\cos\theta(\tau)e^{i\varphi(\tau)}-a_2(\tau)\sin\theta(\tau),\\&c_2(\tau)=a_1(\tau)\sin\theta(\tau)+a_2(\tau)\cos\theta(\tau)e^{-i\varphi(\tau)}.\end{aligned} \tag{29}$$

By using the Schrödinger equation (6), we obtain

$$i\hbar\partial_t\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}=H^{e}\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}\Rightarrow\begin{pmatrix}c_1(t)=c_1(\tau)e^{-i\int_\tau^t E_1(t')/\hbar+\chi(t')dt'}\\ c_2(t)=c_2(\tau)e^{-i\int_\tau^t E_2(t')/\hbar-\chi(t')dt'}\end{pmatrix}. \tag{30}$$

That means the probability amplitudes $c_1(t)$ and $c_2(t)$ at time t remain the same as that at time τ with only phase difference. Returning to the interaction picture, the final state is

$$|\psi(t)\rangle=\begin{pmatrix}c_1(\tau)\cos\theta e^{-i\int_\tau^t E_1(t')/\hbar+\chi(t')dt'}e^{-i\varphi(t)}+c_2(\tau)\sin\theta e^{-i\int_\tau^t E_2(t')/\hbar-\chi(t')dt'}\\ c_2(\tau)\cos\theta e^{-i\int_\tau^t E_2(t')/\hbar-\chi(t')dt'}e^{i\varphi(t)}-c_1(\tau)\sin\theta e^{-i\int_\tau^t E_1(t')/\hbar+\chi(t')dt'}\end{pmatrix}. \tag{31}$$

It is worth noting that when $\alpha=-\dot{\varphi}/2\sin 2\theta$, $H_{\rm add}=H_{cd}$. In other words, the CD Hamiltonian calculated by the transitionless tracking algorithm is one of the cases of the present method.

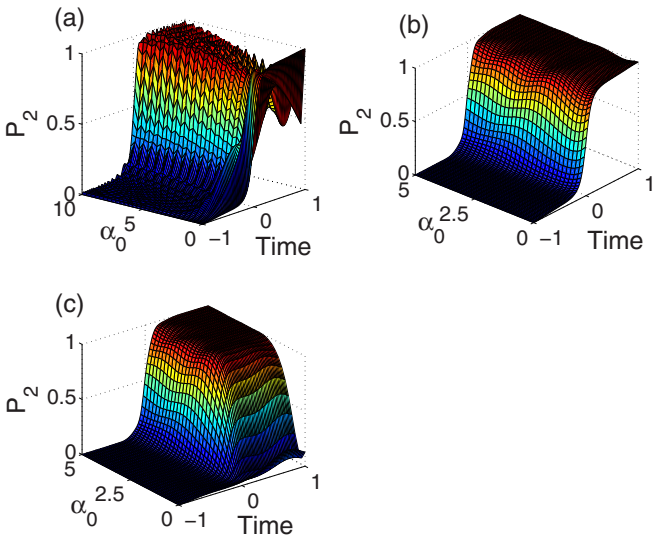

FIG. 1. The time-dependent P_2 versus α_0 (units of Ω_0) when $\{\varphi = 0,\ \zeta = 3\Omega_0,\ t_f = 1/\Omega_0\}$: (a) $\alpha = \alpha_0$ is constant; (b) $\alpha = \alpha_0\dot{\theta}$ is time dependent; (c) $\alpha = \alpha_0\dot{\theta}$ is time dependent and $\beta = 0$. The evolution time in the figure is in units of $1/\Omega_0$.

The idea can also be extended to the non-Hermitian systems. Assuming $H_{\rm add}$ is a non-Hermitian Hamiltonian, for example, the parameters are set as

$$\begin{aligned} A_{11} &= -A_{22} = \hbar(\eta + i\gamma), \\ A_{12} &= A_{21}^{*} = \hbar(\alpha + i\beta)e^{-i\varphi}, \end{aligned} \tag{32}$$

where α, β, η, and γ are all real. The choice of A_{11} and A_{22} here is just a relatively suitable example; we can also choose them as $\{A_{11} = \hbar(\eta + i\gamma),\ A_{22} = \hbar(\eta - i\gamma)\}$, $\{A_{11} = 2\hbar(\eta + i\gamma),\ A_{22} = 0\}$, or others as long as $\mathrm{Im}(A_{11} - A_{22}) \neq 0$. Then, by solving Eq. (25), we obtain $\beta + \gamma\ \sin 2\theta = \dot{\theta}$ and $\alpha\ \cot 2\theta + \eta = \frac{\dot{\varphi}}{2}$; whereas by solving Eq. (26), the result is quite different: $\beta - \gamma\ \sin 2\theta = \dot{\theta}$ and $\alpha\ \cot 2\theta + \eta = \frac{\dot{\varphi}}{2}$. That means, if the additional Hamiltonian is non-Hermitian, we cannot ideally offset all the nondiagonal terms in Eq. (18). Only one of the two transition directions between the instantaneous eigenbases $|\phi_1\rangle$ and $|\phi_2\rangle$ can be forbidden. That is to say, for the non-Hermitian system, the initial state of the system should be ideally in one of the eigenstates, i.e., $|\psi(\tau)\rangle = \cos\theta(\tau)e^{-i\varphi(\tau)}|1\rangle - \sin\theta(\tau)|2\rangle$, hence $c_1 = 1$ and $c_2 = 0$. Then, the evolution of the system is described as

$$i\hbar\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = H^{e}\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} \Rightarrow \begin{pmatrix} c_1(t) = \exp\left[-i\int_{\tau}^{t} E_1/\hbar + (\eta + i\gamma)\cos 2\theta - \alpha\ \sin 2\theta - \dot{\varphi}\ \cos^2\theta dt'\right] \\ c_2(t) = 0 \end{pmatrix}. \tag{33}$$

We find that there is a real part in the exponential term which may cause the decay. So, it would be better if we can make $\int_{\tau}^{t}\gamma\ \cos 2\theta = 0$. A simple way is by imposing $\gamma\ \cos 2\theta$ to be an odd function of time and assuming $t_f = -\tau$ (t_f is the total evolution time). The feature of this method in the non-Hermitian model is that the STA is sensitive to the initial condition of the system. The initial state should be ideally generated in the eigenstate which will not transfer to others. It should be noted here that, the imaginary part of diagonal terms usually denotes the decay of the system. In most cases, γ's form is decided by the system so that we cannot design it as desired. However, this would not affect the feasibility of the present method, because in this paper, γ would not be limited to some fixed form. It can be any arbitrary function so long as the corresponding β is realizable; for instance, if $\gamma = \mathrm{const}$, then $\beta = \dot{\theta} \pm \gamma\ \sin 2\theta$. This merit may be helpful in non-Hermitian systems which have been attracting increasing interest and have been discussed in recent years [33,34]; for example, the $\mathcal{PT}$-symmetric system [35,36].

Different adiabatic passage schemes correspond to $\Omega(t)$ and $\Delta(t)$ for the system evolution from one bare state to the other. The simplest one is the Landau-Zener scheme with constant $\Omega(t)$ and linear-in-time $\Delta(t)$:

$$\Omega(t) = \Omega_0, \quad \Delta(t) = \zeta^2 t. \tag{34}$$

In this case, $\dot{\theta} = -\Omega_0\zeta^2/[2(\Omega_0^2 + \zeta^4 t^2)]$. The additional Hamiltonian $H_{\rm add}$ is given as

$$H_{\rm add} = \hbar\begin{pmatrix} \frac{\dot{\varphi}}{2} - \alpha\ \cot 2\theta + i\gamma & (\alpha + i\dot{\theta} - i\gamma\ \sin 2\theta)e^{-i\varphi} \\ (\alpha - i\dot{\theta} + i\gamma\ \sin 2\theta)e^{i\varphi} & \alpha\ \cot 2\theta - \frac{\dot{\varphi}}{2} - i\gamma \end{pmatrix}. \tag{35}$$

First, we discuss the situation in which $\gamma = 0$ (the system is Hermitian). In the interest of the effect of α's on STA, we set $\varphi = 0$ in this part. Two kinds of α's will be discussed by numerical simulation. (1) α is time independent. Figure 1(a) shows the time-dependent population of the target state $|2\rangle$ (P_2) versus α when the initial state is $|1\rangle$ and $\{\varphi = 0,\ \zeta =$

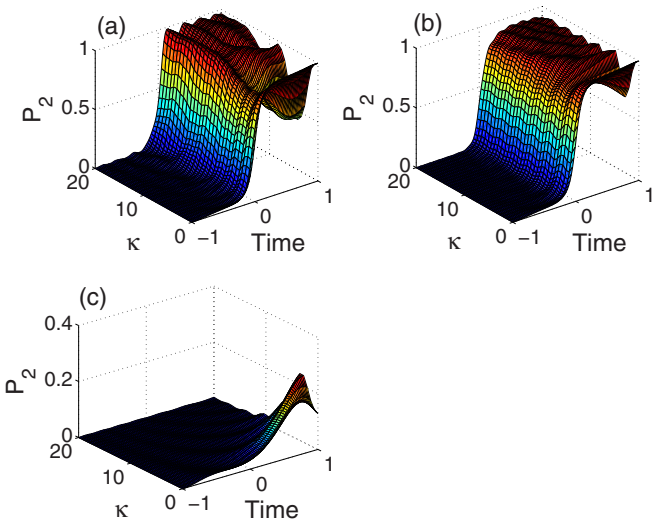

FIG. 2. The time-dependent P_2 versus κ (units of Ω_0) when $\{\varphi = \kappa t,\ \zeta = 3\Omega_0,\ t_f = 1/\Omega_0\}$: (a) based on the original transitionless tracking algorithm that $\alpha = -(\kappa/2)\sin 2\theta$; (b) based on the present method with parameter $\alpha = 0$; (c) based on H_0 without the additional term. The evolution time in the figure is in units of $1/\Omega_0$.

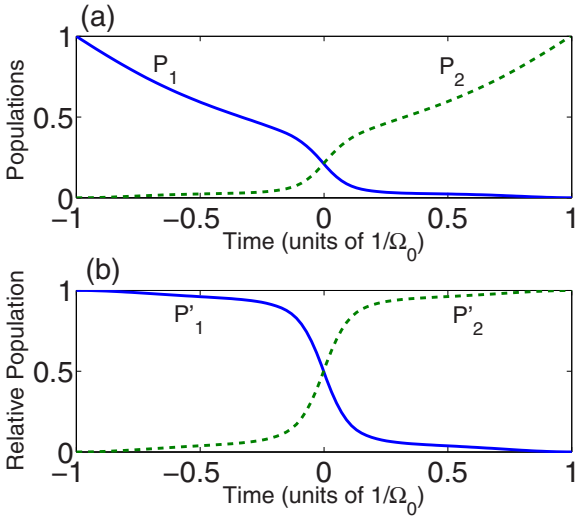

FIG. 3. (a) The populations P_1 and P_2 versus time when $\gamma = 0.5\Omega_0$. (b) The relative population P_1' and P_2' versus time when $\gamma = 0.5\Omega_0$.

$3\Omega_0,\ t_f = 1/\Omega_0\}$. The result shows that in most of the cases, the shortcut could be constructed successfully and the populations could be transferred to the target state in a very short time. The oscillation is caused by the diagonal term in Eq. (35). (2) α is time dependent. For convenience, we choose $\alpha = \alpha_0\dot{\theta}$ (α_0 is time independent). As shown in Fig. 1(b), a nearly perfect population transfer from $|1\rangle$ to $|2\rangle$ is realizable with arbitrary α_0. What is more, according to Eq. (35), it is obvious when α_0 is large enough, $\alpha_0 + i \approx \alpha_0$. This means, if we choose a relatively large α_0, we can neglect the imaginary part of A_{12} (A_{21}). This would make sense because a pulse with a form of $\alpha_0\dot{\theta}$ would be easier to realize than the form of $i\dot{\theta}$ in experiment. We plot Fig. 1(c) which shows the result when $\beta = 0$ (the other parameters are also $\{\varphi = 0,\ \zeta = 3\Omega_0,\ t_f = 1/\Omega_0\}$). From the figure, we find the population transfer would be ideally achieved as long as $\alpha_0 > 2.5$.

In the following, we will analyze the effectivity of the method when $\varphi \neq 0$. In Fig. 2(a), we give P_2 versus κ when the initial state is $|1\rangle$ and $\{\alpha = -\frac{\dot{\varphi}}{2}\sin 2\theta,\ \varphi = \kappa t,\ \zeta = 3\Omega_0\}$. As shown in the figure, when $t = t_f$, while oscillating, the fidelity of the target state $|2\rangle$ increases with increasing κ, which means if the adiabatic phase is considered, the effectivity of STA may be reduced in some situations. For comparison, in Fig. 2(b), we plot the time evolution of state $|2\rangle$ versus κ with $\{\alpha = 0,\ \varphi = \kappa t,\ \zeta = 3\Omega_0\}$. It is obvious that the second set of parameters behave better in restraining the adverse effect caused by φ than the first set. The oscillation in Figs. 2(a) and 2(b) is caused by the original Hamiltonian H_0 when Δ is large enough as shown in Fig. 2(c). In addition, it is not hard to see that using the second set of parameters to construct shortcuts can save more energy. According to Eq. (35), the eigenvalue of $H_{\rm add}$ is $E_{\pm}^{a} = \pm\hbar\sqrt{(\dot{\varphi}/2 - \alpha\ \cot 2\theta)^2 + \alpha^2 + \dot{\theta}^2}$. This means the energy cost for constructing shortcuts is the least when $\alpha = 0$.

In the following, we will briefly discuss the present method's efficiency in the non-Hermitian system. Since the system is non-Hermitian, the dynamics of the system's density operator $\rho(t)$ will be given as $\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H(t)\rho(t) - \rho(t)H^{\dagger}(t)]$, where $H(t) = H_0(t) + H_{\rm add}(t)$. First of all, we assume the population for a state $|j\rangle$ is still given as $P_j = |\langle j|\rho(t)|j\rangle|$, and display the populations P_1 and P_2 versus time in Fig. 3 with parameters $\{\alpha = 0,\ \varphi = 0,\ \zeta = 3\Omega_0,\ \gamma = 0.5\Omega_0\}$. It should be noted here that, since the Hamiltonian is non-Hermitian, if the population for a state is still given by $P_j = |\langle j|\rho(t)|j\rangle|$, the norm of the state vector given by $P_1 + P_2$ will not be conserved during the evolution. This property can be seen in Fig. 3, where the norm is not conserved during the interaction. To avoid some problems caused by $P_1 + P_2 \neq 1$, some definitions of the populations in the non-Hermitian system have been proposed [37,38]. However, since we are only concerned about the realizable possibility of a fast population inversion in the non-Hermitian system, for simplicity, we define the relative populations $P_j' = P_j/(P_j + P_k)$ $(j \neq k)$ to help with the analysis, and if not otherwise specified, $\alpha = 0$, $\varphi = 0$, and $\zeta = 3\Omega_0$ will be used throughout the discussion in this part. In Fig. 4 we display the

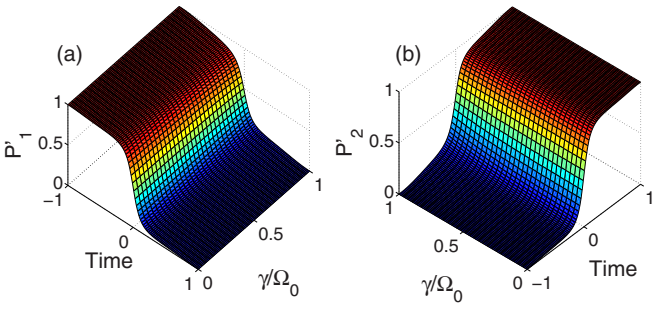

FIG. 4. (a) The time-dependent relative population P_1' versus γ. (b) The time-dependent relative population P_2' versus γ. The evolution time in the figure is in units of $1/\Omega_0$.

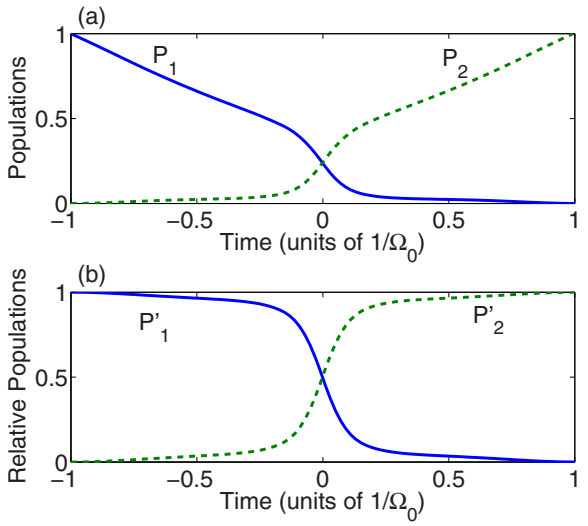

FIG. 5. (a) The populations P_1 and P_2 versus time when $\gamma = 1/(2+t^2)$. (b) The relative population P_1' and P_2' versus time when $\gamma = 1/(2+t^2)$.

time-dependent relative populations for states $|1\rangle$ [Fig. 4(a)] and $|2\rangle$ [Fig. 4(b)] versus γ, where γ is assumed to be time independent. As we can see, a fast population inversion could still be achieved even with a relatively large γ, i.e., $\gamma = \Omega_0$. As is known, in general, γ could also depend on time $\gamma = \gamma(t)$ as an effective decay rate controlled by further interactions (see, e.g., Ref. [39]). According to the form of γ in Ref. [39], we plot Fig. 5 to show that the present method can also work very well when γ is time dependent, which shows the populations versus time with the parameters mentioned above. Figure 5(b) shows the relative populations versus time, and γ is chosen as $\gamma = \frac{1}{2+t^2}$ for simplicity in plotting the figures. Moreover, if γ is controllable, or if γ could satisfy some kind of function, for example, $\gamma = \pm\dot{\theta}/\sin 2\theta$, the scheme can make the population transfer fast without increasing the coupling [35] because when $\gamma = \pm\dot{\theta}/\sin 2\theta$, the corresponding $\beta = 0$. Such an assumption can be physically realized, for instance, in two coupled optical waveguides with longitudinally varying gain and loss regions [35]. In fact, $\gamma = \pm\dot{\theta}/\sin 2\theta$ is just the result of Ref. [31] which has been analyzed and discussed in great detail.

From the analysis above, we find the real part of pulse $\mathrm{Re}A_{12}$ could be an arbitrary time-dependent function, which means the real part is obviously realizable. So, to make sure the pulses we used in the schemes are realizable, we need to confirm whether the imaginary part of the pulse is realizable or not. Figure 6 shows $\mathrm{Im}A_{12}$ versus time with different parameters when $\varphi = 0$. Shown in the figure, the shapes are all similar to Gaussian curves, which means the pulses are not hard to realize in practice. In other words, the schemes proposed in this paper are feasible in practice.

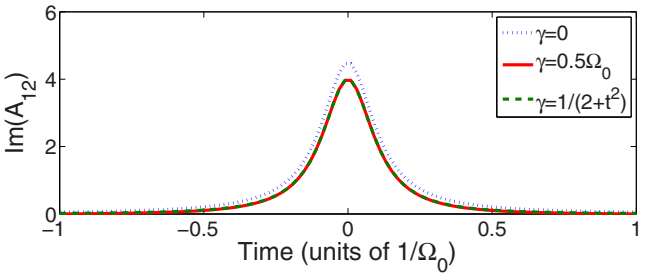

FIG. 6. The shapes of the imaginary part of the additional Hamiltonian's pulses when $\varphi = 0$. Blue dotted curve: when $\gamma = 0$; red solid curve: when $\gamma = 0.5\Omega_0$; green dashed curve: when $\gamma = 1/(2+t^2)$.

IV. CONCLUSION

In conclusion, we have proposed a different and flexible way to design the additional Hamiltonian for the original Hamiltonian to construct STA. This method may be promising as a way to avoid the defect (the speed-up protocols' Hamiltonian may involve terms which are difficult to realize in practice) because of the multiple choices of the additional Hamiltonian. We have applied this method to the Landau-Zener model as an application example, and the results show that the method works very well in two-level systems (in both Hermitian and non-Hermitian). In the Hermitian system, we find a relatively suitable α (the real part of the off-diagonal terms in the additional Hamiltonian); we can even speed up the adiabatic process without the imaginary part of the off-diagonal terms in the additional Hamiltonian. That is meaningful because amending the Rabi frequency Ω by real correction will be much easier than by imaginary correction. In the non-Hermitian system, different from Ref. [31] where γ (gain or loss of population) nullifies the counterdiabatic coupling to speed up the adiabatic evolution all alone, in this paper, γ cooperates with β (the correction of the imaginary part of the Rabi frequency) to achieve the goals. As is known, the decay γ is usually decided by the system and is uncontrollable, so a speed-up protocol with a fixed form of γ will be hard to realize and generalize. However, in our present method, the correction of Rabi frequency β cooperates with γ to construct shortcuts; hence, as long as the corresponding β is realizable in practice, the shortcuts could be constructed with arbitrary γ. Another highlight of this method is that the phase change at any time could obviously be calculated, which may have application prospects in quantum phase gates.

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