Strong mechanical squeezing and its detection

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We report an efficient mechanism to generate a squeezed state of a mechanical mirror in an optomechanical system. We use an especially tuned parametric amplifier (PA) inside the cavity and the parametric photon phonon processes to transfer quantum squeezing from photons to phonons with almost 100% efficiency. We get 50% squeezing of the mechanical mirror which is limited by the PA. We present analytical results for the mechanical squeezing thus enabling one to understand the dependence of squeezing on system parameters like gain of PA, cooperativity, and temperature. As in cooling experiments the detrimental effects of mirror's Brownian and zero point noises are strongly suppressed by the pumping power. By judicious choice of the phases, the cavity output is squeezed only if the mirror is squeezed thus providing us a direct measure of the mirror's squeezing. Further considerable larger squeezing of the mirror can be obtained by adding the known feedback techniques.

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I. INTRODUCTION

Cavity optomechanics is based on the radiation pressure interaction between light and mechanical resonators at macroscopic scales [1]. Currently, with the rapid progress of practical technologies in cavity optomechanics, the mechanical resonator can be cooled down close to the quantum ground state [2–4]. Thus it is possible to explore quantum effects in macroscopic systems, including the superposition state [5,6], entanglement [7–9], squeezing of light [10–16], squeezing of the mechanical resonator [17–38], etc.

The quantum squeezing of mechanical modes is important as it can be used to improve the precision of quantum measurements [39]. There have been many theoretical proposals for generating squeezing of the mechanical mode [17–25,29–34]. Several experiments have reported squeezing of the mirror to different degrees. Since mechanical motion is represented by an oscillator, the most direct way to produce squeezing is via the well-known methods used to squeeze the oscillator motion. One of the early proposals was to modulate the frequency of the oscillator [17–19]. While this is the simplest, it is not easy to adopt for many different kinds of mechanical systems currently in use. Alternate methods to overcome this limitation have been suggested. These include modulation of the external laser [20–22]; use of a two-tone drives one red detuned and the other blue detuned [23]. One can use a broad band squeezed optical field and couple it into an optomechanical cavity to transfer optical squeezing into mechanical squeezing [24,25]. This method works very well and more than 50% squeezing of the mirror can be obtained [24,25]. This requires efficient coupling and a highly squeezed broad band field and thus has its own limitations. A more direct way is to have a parametric amplifier placed inside the optomechanical cavity so that the squeezing of the cavity field is generated inside the cavity. These squeezed cavity photons can interact directly with the red-detuned pump laser to produce squeezing of the mechanical mode. This is the main theme of the present work. The degree of the mechanical squeezing will be limited by the squeezing produced by the PA. However, one can use the previously used methods like the single quadrature feedback scheme [26] or the weak measurement [27,28] to substantially increase the mirror's squeezing.

While we concentrate on optomechanical couplings linear in mirror displacement, the squeezing of the mirror in quadratically coupled OMS has been investigated. In this case one can use a bang-bang technique to kick the mirror mode [29,30], use the Duffing nonlinearity [31] or use two-tone driving [32].

In this paper, we propose a scheme to generate the momentum squeezing of the movable mirror by placing a degenerate PA inside a cavity with one moving mirror. The PA is pumped at twice the frequency of the anti-Stokes sideband of the driving laser interacting the movable mirror. It is shown that the squeezing of the cavity field induced by the PA can be transferred to the movable mirror. The achieved momentum squeezing of the mirror depends on the parametric gain, the parametric phase, the power of the input laser, and the temperature of the environment. The results presented are applicable to mechanical systems in both optical and microwave cavities.

The paper is organized as follows. In Sec. II, we describe the model, give the quantum Langevin equations, and the steady-state mean values. In Sec. III, we linearize the quantum Langevin equations, derive the stability conditions, and calculate the square fluctuations in the position and momentum of the movable mirror. In Sec. IV, we discuss how the momentum squeezing of the movable mirror can be realized by using the PA inside the cavity. In Sec. V, we derive the analytical expression of the mean square fluctuation in the momentum of the movable mirror. In Sec. VI, we show how the mechanical squeezing can be measured by the output field. Our conclusions are given in Sec. VII.

II. MODEL

We consider a degenerate PA contained in a Fabry-Perot cavity with one fixed mirror and one movable mirror, as shown in Fig. 1. A degenerate parametric amplifier (PA) is generally used to produce a squeezed light [40,41]. We have shown earlier that a PA inside an optomechanical system can improve the cooling of the movable mirror [42]. It can also make the observation of the normal-mode splitting [43,44] of the movable mirror and the output field more accessible [45]. The fixed mirror is partially transmitting, while the movable

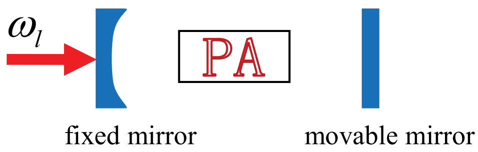

FIG. 1. Sketch of the optomechanical system to prepare the movable mirror in a squeezed state. A PA is placed inside the cavity, and the pump of the PA is not shown. Here the movable mirror is coupled to an optical cavity. This scheme also applies to a mechanical resonator capacitively coupled to a microwave cavity.

mirror is totally reflecting. The separation between the two mirrors is L. A cavity field with resonance frequency ω_c is driven by an external laser with frequency ω_l and amplitude ε_l. Our analysis and results are equally applicable to microwave systems. The intracavity photons exert a radiation pressure force on the movable mirror, causing the optomechanical interaction between the cavity field and the movable mirror. Meanwhile, the movable mirror is in contact with a thermal bath in equilibrium at temperature T, which induces a thermal Langevin force acting on the movable mirror. Under the action of these two forces, the mirror makes small oscillations around its equilibrium position. The movable mirror is treated as a quantum-mechanical harmonic oscillator with effective mass m, frequency ω_m, and energy decay rate γ_m. In the degenerate PA, we assume that a pump field at frequency $2(\omega_l+\omega_m)$ interacts with a second-order nonlinear optical crystal, thus the signal and the idler have the same frequency $\omega_l+\omega_m$. We assume that the gain of the PA is G, depending on the power of the pump driving the PA, and the phase of the pump driving the PA is θ. The Hamiltonian of the system in the rotating frame at the laser frequency ω_l is given by

$$H = \hbar(\omega_c-\omega_l)c^\dagger c - \hbar g_0 c^\dagger c(b+b^\dagger) + \hbar\omega_m\left(b^\dagger b+\frac{1}{2}\right) + i\hbar\varepsilon_l(c^\dagger - c) + i\hbar G(e^{i\theta}c^{\dagger 2}e^{-2i\omega_m t} - e^{-i\theta}c^2 e^{2i\omega_m t}), \quad (1)$$

where c and $c^\dagger$ are the annihilation and creation operators of the cavity mode, satisfying the commutator relation $[c,c^\dagger]=1$, b and $b^\dagger$ are the annihilation and creation operators of the mechanical mode, satisfying $[b,b^\dagger]=1$. The optomechanical interaction strength is $g_0 = \frac{\omega_c}{L}\sqrt{\frac{\hbar}{2m\omega_m}}$ in units of Hz, where $\sqrt{\frac{\hbar}{2m\omega_m}}$ is the zero point motion of the movable mirror. The ε_l is related to the power $\wp$ of the laser by $\varepsilon_l = \sqrt{\frac{2\kappa\wp}{\hbar\omega_l}}$ with κ being the cavity decay rate due to the leakage of photons through the partially transmitting mirror. In Eq. (1), the first and third terms describe the energies of the optical mode and the mechanical mode, respectively, the second term describes the linear optomechanical coupling between the cavity field and the movable mirror, depending on the photon number $c^\dagger c$ in the cavity, the fourth term gives the driving of the input laser, and the last term represents the coupling between the cavity field and the PA. The physical process can be illustrated in Fig. 2. Figure 2(a) shows the frequency relation among the pump photon at frequency ω_l, the cavity photon at frequency ω_c, the squeezed photon at frequency ω_c from the PA, and the

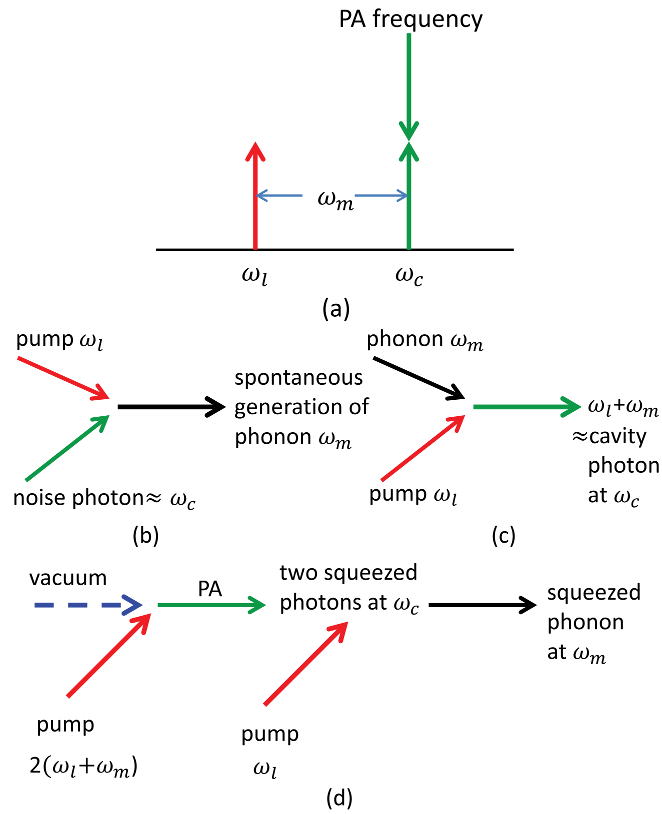

FIG. 2. Sketch of the physical processes leading to the squeezed phonons. The pump ω_l is red detuned with $\Delta=\omega_m$. The system is working in the resolved sideband limit. (a) Shows the tuning of the different frequencies. (b) and (c) The usual optomechanical processes leading to the generation of phonons and then conversion back to photons. (d) Shows how the PA leads to squeezed phonons.

phonon at frequency ω_m. Figure 2(b) shows that a phonon at frequency ω_m is spontaneously created by a red-detuned pump photon at frequency ω_l interacting with an input noise photon at frequency ω_c. Figure 2(c) shows that a cavity photon at frequency ω_c is produced when a red-detuned pump photon at frequency ω_l interacts with a phonon at frequency ω_m. Figure 2(d) shows that a squeezed phonon at frequency ω_m is generated when a red-detuned pump photon at frequency ω_l interacts with a squeezed photon at frequency $\omega_l+\omega_m \approx \omega_c$ from the PA.

According to the Heisenberg motion equation and considering the quantum and thermal noises, we obtain the quantum Langevin equations,

$$\dot{b} = ig_0c^\dagger c - i\omega_m b - \frac{\gamma_m}{2}b + \sqrt{\gamma_m}b_{in},$$
$$\dot{c} = -i(\omega_c-\omega_l)c + ig_0c(b+b^\dagger) + \varepsilon_l + 2Ge^{i\theta}c^\dagger e^{-2i\omega_m t} - \kappa c + \sqrt{2\kappa}c_{in}. \quad (2)$$

Here b_{in} is the boson annihilation operator of the thermal noise with zero mean value; its nonzero correlation functions are

$$\langle b_{in}^\dagger(t)b_{in}(t')\rangle = n_m^{th}\delta(t-t'),$$
$$\langle b_{in}(t)b_{in}^\dagger(t')\rangle = (n_m^{th}+1)\delta(t-t'), \quad (3)$$

where $n_m^{th} = \{\exp[\hbar\omega_m/(K_B T)] - 1\}^{-1}$ is the initial mean thermal excitation number in the movable mirror, and K_B is the Boltzmann constant. Moreover, c_{in} is the input quantum noise operator with zero mean value, and its nonzero correlation function is

$$\langle c_{in}^\dagger(t)c_{in}(t')\rangle = n_c^{\rm th}\delta(t-t'),$$
$$\langle c_{in}(t)c_{in}^\dagger(t')\rangle = \left(n_c^{\rm th}+1\right)\delta(t-t'), \tag{4}$$

where $n_c^{\rm th} = \{\exp[\hbar\omega_c/(K_B T)] - 1\}^{-1}$ is the initial mean thermal excitation number in the optical mode. The steady-state mean values of the system operators are

$$c_s = \frac{\varepsilon_l}{\kappa + i\Delta},$$
$$b_s = \frac{i g_0 |c_s|^2}{\frac{\gamma_m}{2} + i\omega_m}, \tag{5}$$

where $\Delta = \omega_c - \omega_l - g_0(b_s + b_s^*)$ is the effective cavity detuning from the frequency of the input laser in the presence of the radiation pressure, depending on the mechanical motion. The c_s is the steady-state amplitude of the cavity field, and b_s determines the steady-state displacement of the movable mirror. The mean numbers of the cavity photons and the mechanical phonons are given by $|c_s|^2$ and $|b_s|^2$, respectively.

III. RADIATION PRESSURE AND QUANTUM FLUCTUATIONS

In order to show the movable mirror in a squeezed state, we need to calculate the position and momentum fluctuations of the movable mirror. Here we are interested in the strong-driving regime so that the intracavity photon number $|c_s|^2$ satisfies $|c_s|^2 \gg 1$. Let $b = b_s + \delta b$ and $c = c_s + \delta c$, where δb and δc are the small fluctuation operators around the steady-state mean values, thus Eq. (2) can be linearized by neglecting higher than first-order terms in the fluctuations [46]. Introducing the slow varying fluctuation operators by $\delta b = \delta\tilde{b}e^{-i\omega_m t}$, $\delta c = \delta\tilde{c}e^{-i\Delta t}$, $b_{in} = \tilde{b}_{in}e^{-i\omega_m t}$, $c_{in} = \tilde{c}_{in}e^{-i\Delta t}$, we obtain the linearized quantum Langevin equations,

$$\delta\dot{\tilde{b}} = i[g^*\delta\tilde{c}e^{-i(\Delta-\omega_m)t} + g\delta\tilde{c}^\dagger e^{i(\Delta+\omega_m)t}] - \frac{\gamma_m}{2}\delta\tilde{b} + \sqrt{\gamma_m}\tilde{b}_{in},$$
$$\delta\dot{\tilde{c}} = -\kappa\delta\tilde{c} + ig[\delta\tilde{b}e^{-i(\omega_m-\Delta)t} + \delta\tilde{b}^\dagger e^{i(\omega_m+\Delta)t}] + 2Ge^{i\theta}\delta\tilde{c}^\dagger e^{2i(\Delta-\omega_m)t} + \sqrt{2\kappa}\tilde{c}_{in}, \tag{6}$$

where $g = g_0 c_s$ is the effective optomechanical coupling rate, depending on the power $\wp$ of the input laser. We assume that the driving field is red-detuned from the cavity resonance ($\Delta = \omega_m$), thus the anti-Stokes scattered light is nearly resonant with the cavity field. And we assume that the system is working in the resolved sideband limit $\omega_m \gg \kappa$, the mechanical quality factor is high $\omega_m \gg \gamma_m$, and the mechanical frequency ω_m is much larger than $|g|$ and $2G$. Under these conditions, the rotating wave approximation can be made, the fast oscillating term $e^{2i\omega_m t}$ in Eq. (6) can be ignored, and Eq. (6) can be simplified to

$$\delta\dot{\tilde{b}} = ig^*\delta\tilde{c} - \frac{\gamma_m}{2}\delta\tilde{b} + \sqrt{\gamma_m}\tilde{b}_{in},$$
$$\delta\dot{\tilde{c}} = -\kappa\delta\tilde{c} + ig\delta\tilde{b} + 2Ge^{i\theta}\delta\tilde{c}^\dagger + \sqrt{2\kappa}\tilde{c}_{in}. \tag{7}$$

Introducing the position and momentum fluctuations of the mechanical oscillator as $\delta Q = \frac{1}{\sqrt{2}}(\delta\tilde{b} + \delta\tilde{b}^\dagger)$ and $\delta P = \frac{1}{\sqrt{2}i}(\delta\tilde{b} - \delta\tilde{b}^\dagger)$, and the amplitude and phase fluctuations of the cavity field as $\delta x = \frac{1}{\sqrt{2}}(\delta\tilde{c} + \delta\tilde{c}^\dagger)$ and $\delta y = \frac{1}{\sqrt{2}i}(\delta\tilde{c} - \delta\tilde{c}^\dagger)$, the amplitude and phase fluctuations of the input quantum noise as $x_{in} = \frac{1}{\sqrt{2}}(\tilde{c}_{in} + \tilde{c}_{in}^\dagger)$ and $y_{in} = \frac{1}{\sqrt{2}i}(\tilde{c}_{in} - \tilde{c}_{in}^\dagger)$, and the position and momentum fluctuations of the thermal noise as $Q_{in} = \frac{1}{\sqrt{2}}(\tilde{b}_{in} + \tilde{b}_{in}^\dagger)$ and $P_{in} = \frac{1}{\sqrt{2}i}(\tilde{b}_{in} - \tilde{b}_{in}^\dagger)$, Eq. (7) can be written as the matrix form,

$$\dot{f}(t) = Mf(t) + n(t), \tag{8}$$

where $f(t)$ is the column vector of the fluctuations, and $n(t)$ is the column vector of the noise sources. Their transposes are

$$f(t)^T = (\delta Q, \delta P, \delta x, \delta y),$$
$$n(t)^T = (\sqrt{\gamma_m}Q_{in}, \sqrt{\gamma_m}P_{in}, \sqrt{2\kappa}x_{in}, \sqrt{2\kappa}y_{in}); \tag{9}$$

and the matrix M is given by

$$M = \begin{pmatrix} -\frac{\gamma_m}{2} & 0 & \frac{i}{2}(g^*-g) & -\frac{1}{2}(g+g^*) \\ 0 & -\frac{\gamma_m}{2} & \frac{1}{2}(g+g^*) & \frac{i}{2}(g^*-g) \\ \frac{i}{2}(g-g^*) & -\frac{1}{2}(g+g^*) & -(\kappa-2G\cos\theta) & 2G\sin\theta \\ \frac{1}{2}(g+g^*) & \frac{i}{2}(g-g^*) & 2G\sin\theta & -(\kappa+2G\cos\theta) \end{pmatrix}. \tag{10}$$

The stability conditions of the system can be obtained by requiring that all the eigenvalues of the matrix M have negative real parts. Applying the Routh-Hurwitz criterion [48,49], we find the stability conditions,

$$\frac{1}{4}\gamma_m^3 + 2\kappa(\kappa^2 - 4G^2) + (2\kappa + \gamma_m)(|g|^2 + 2\kappa\gamma_m) > 0,$$
$$2\kappa\gamma_m(\kappa^2-4G^2)^2 + \left[(2\kappa+\gamma_m)^2|g|^2 + (4\kappa+\gamma_m)\kappa\gamma_m^2\right](\kappa^2-4G^2) + \frac{\gamma_m^3}{4}\left[\frac{\kappa\gamma_m^2}{2} + (2\kappa+\gamma_m)|g|^2\right]$$
$$+\kappa\gamma_m(2\kappa+\gamma_m)[\kappa\gamma_m^2 + \left(2\kappa + \frac{3}{2}\gamma_m\right)|g|^2] > 0, \quad \frac{1}{4}\gamma_m^2(\kappa^2-4G^2) + |g|^2(|g|^2+\kappa\gamma_m) > 0. \tag{11}$$

Note that the stability conditions are independent of the parametric phase θ. The system stays in the stable regime only if $G < 0.5\kappa$.

By taking the Fourier transform of Eq. (8) and solving it in the frequency domain, we obtain the expressions for the position and momentum fluctuations of the movable mirror,

$$\begin{aligned}\delta Q(\omega) = {} & A_1(\omega)x_{in}(\omega) + B_1(\omega)y_{in}(\omega) + E_1(\omega)Q_{in}(\omega)\\ & + F_1(\omega)P_{in}(\omega),\\ \delta P(\omega) = {} & A_2(\omega)x_{in}(\omega) + B_2(\omega)y_{in}(\omega) + E_2(\omega)Q_{in}(\omega)\\ & + F_2(\omega)P_{in}(\omega),\end{aligned} \tag{12}$$

where

$$\begin{aligned}A_1(\omega) &= \frac{\sqrt{2\kappa}i}{d(\omega)}\{v(\omega)[G\alpha - iu(\omega)\mathrm{Im}(g)] - i|g|^2\mathrm{Im}(g)\},\\ B_1(\omega) &= \frac{\sqrt{2\kappa}}{d(\omega)}\{v(\omega)[G\beta - u(\omega)\mathrm{Re}(g)] - |g|^2\mathrm{Re}(g)\},\\ E_1(\omega) &= \frac{\sqrt{\gamma_m}}{d(\omega)}\{[u(\omega)^2 - 4G^2]v(\omega) + |g|^2u(\omega) + G\Gamma\},\\ F_1(\omega) &= \frac{\sqrt{\gamma_m}}{d(\omega)}iG(g^2e^{-i\theta} - g^{*2}e^{i\theta}),\\ A_2(\omega) &= \frac{\sqrt{2\kappa}}{d(\omega)}\{v(\omega)[G\beta + u(\omega)\mathrm{Re}(g)] + |g|^2\mathrm{Re}(g)\},\\ B_2(\omega) &= -\frac{\sqrt{2\kappa}i}{d(\omega)}\{v(\omega)[G\alpha + iu(\omega)\mathrm{Im}(g)] + i|g|^2\mathrm{Im}(g)\},\\ E_2(\omega) &= \frac{\sqrt{\gamma_m}}{d(\omega)}iG(g^2e^{-i\theta} - g^{*2}e^{i\theta}),\\ F_2(\omega) &= \frac{\sqrt{\gamma_m}}{d(\omega)}\{[u(\omega)^2 - 4G^2]v(\omega) + |g|^2u(\omega) - G\Gamma\},\end{aligned} \tag{13}$$

with $\alpha = e^{i\theta}g^* - e^{-i\theta}g$, $\beta = e^{i\theta}g^* + e^{-i\theta}g$, $\Gamma = g^2e^{-i\theta} + g^{*2}e^{i\theta}$, $v(\omega) = \frac{\gamma_m}{2} - i\omega$, $u(\omega) = \kappa - i\omega$, and

$$d(\omega) = [u(\omega)v(\omega) + |g|^2]^2 - 4G^2v(\omega)^2. \tag{14}$$

In Eq. (12), the first two terms in $\delta Q(\omega)$ and $\delta P(\omega)$ are from the radiation pressure contribution; the last two terms are from the thermal noise contribution. In the absence of the optomechanical coupling ($g = 0$), the movable mirror makes quantum Brownian motion because of the coupling to the environment, $\delta Q(\omega) = \frac{\sqrt{\gamma_m}}{\frac{\gamma_m}{2} - i\omega}Q_{in}$, $\delta P(\omega) = \frac{\sqrt{\gamma_m}}{\frac{\gamma_m}{2} - i\omega}P_{in}$. The spectra of fluctuations in the position and momentum of the movable mirror are defined by

$$\begin{aligned}S_Z(\omega) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} d\Omega\, e^{-i(\omega+\Omega)t}[\langle\delta Z(\omega)\delta Z(\Omega)\rangle\\ + \langle\delta Z(\Omega)\delta Z(\omega)\rangle], \qquad Z = Q,P.\end{aligned} \tag{15}$$

By the aid of the nonzero correlation functions of the noise sources in the frequency domain,

$$\begin{aligned}\langle Q_{in}(\omega)Q_{in}(\Omega)\rangle &= \langle P_{in}(\omega)P_{in}(\Omega)\rangle\\ &= \left(n_m^{\mathrm{th}} + \frac{1}{2}\right)2\pi\delta(\omega+\Omega),\\ \langle Q_{in}(\omega)P_{in}(\Omega)\rangle &= -\langle P_{in}(\omega)Q_{in}(\Omega)\rangle = \frac{i}{2}2\pi\delta(\omega+\Omega),\\ \langle x_{in}(\omega)x_{in}(\Omega)\rangle &= \langle y_{in}(\omega)y_{in}(\Omega)\rangle\\ &= \left(n_c^{\mathrm{th}} + \frac{1}{2}\right)2\pi\delta(\omega+\Omega),\\ \langle x_{in}(\omega)y_{in}(\Omega)\rangle &= -\langle y_{in}(\omega)x_{in}(\Omega)\rangle = \frac{i}{2}2\pi\delta(\omega+\Omega),\end{aligned} \tag{16}$$

we obtain the spectra of fluctuations in the position and momentum of the movable mirror,

$$\begin{aligned}S_Q(\omega) = {} & [A_1(\omega)A_1(-\omega) + B_1(\omega)B_1(-\omega)]\left(n_c^{\mathrm{th}} + \frac{1}{2}\right)\\ & + [E_1(\omega)E_1(-\omega) + F_1(\omega)F_1(-\omega)]\left(n_m^{\mathrm{th}} + \frac{1}{2}\right),\\ S_P(\omega) = {} & [A_2(\omega)A_2(-\omega) + B_2(\omega)B_2(-\omega)]\left(n_c^{\mathrm{th}} + \frac{1}{2}\right)\\ & + [E_2(\omega)E_2(-\omega) + F_2(\omega)F_2(-\omega)]\left(n_m^{\mathrm{th}} + \frac{1}{2}\right),\end{aligned} \tag{17}$$

where the first term proportional to $n_c^{\mathrm{th}} + \frac{1}{2}$ in $S_Q(\omega)$ and $S_P(\omega)$ is from the radiation pressure contribution, while the second term proportional to $n_m^{\mathrm{th}} + \frac{1}{2}$ is from the thermal noise contribution. In the absence of the cavity field, the spectra of fluctuations in position and momentum of the movable mirror are given by $S_Q(\omega) = S_P(\omega) = \frac{\gamma_m}{\frac{\gamma_m^2}{4} + \omega^2}(n_m^{\mathrm{th}} + \frac{1}{2})$, whose peaks are located at frequency zero with full width γ_m at half maximum. The mean square fluctuations $\langle\delta Q(t)^2\rangle$ and $\langle\delta P(t)^2\rangle$ in the position and momentum of the movable mirror are determined by

$$\langle\delta Z(t)^2\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, S_Z(\omega), \quad Z = Q,P. \tag{18}$$

Without the optomechanical coupling, we find $\langle\delta Q(t)^2\rangle = \langle\delta P(t)^2\rangle = n_m^{\mathrm{th}} + \frac{1}{2}$. For $T = 0$ K, the movable mirror is in the ground state ($n_m^{\mathrm{th}} = 0$), $\langle\delta Q(t)^2\rangle = \langle\delta P(t)^2\rangle = \frac{1}{2}$. According to the Heisenberg uncertainty principle, the product of the mean square fluctuations $\langle\delta Q(t)^2\rangle$ and $\langle\delta P(t)^2\rangle$ satisfies the following inequality:

$$\langle\delta Q(t)^2\rangle\langle\delta P(t)^2\rangle \geqslant |\frac{1}{2}[Q,P]|^2, \tag{19}$$

where $[Q,P] = i$. If either $\langle\delta Q(t)^2\rangle$ or $\langle\delta P(t)^2\rangle$ is below $\frac{1}{2}$, the state of the movable mirror exhibits quadrature squeezing. The degree of the squeezing can be expressed in the dB unit, which can be calculated by $-10\log_{10}\frac{\langle\delta P(t)^2\rangle}{\langle\delta P(t)^2\rangle_{\mathrm{vac}}}$ with $\langle\delta P(t)^2\rangle_{\mathrm{vac}}$ being the momentum variance of the vacuum state and $\langle\delta P(t)^2\rangle_{\mathrm{vac}} = \frac{1}{2}$.

IV. THE MECHANICAL SQUEEZING

In this section, we numerically evaluate the mean square fluctuations in the position and momentum of the movable mirror given by Eq. (18) to show quadrature squeezing of

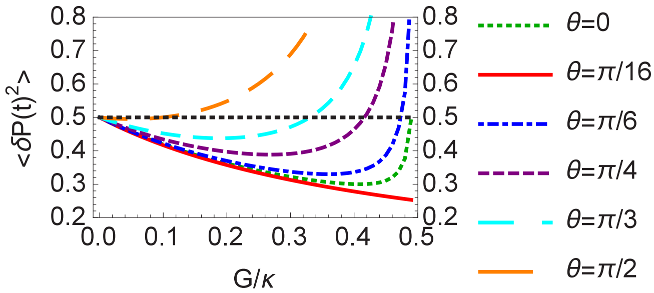

FIG. 3. The mean square fluctuation $\langle\delta P(t)^2\rangle$ versus the parametric gain G for different parametric phases $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$ when $C = 400$ and $T = 0$ K. The flat dotted line represents the momentum variance of the vacuum state $\langle\delta P(t)^2\rangle_{\mathrm{vac}} = 0.5$. The green dotted, red solid, blue dot-dashed, purple short dashed, cyan middle dashed, and orange long-dashed curves correspond to $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$, respectively.

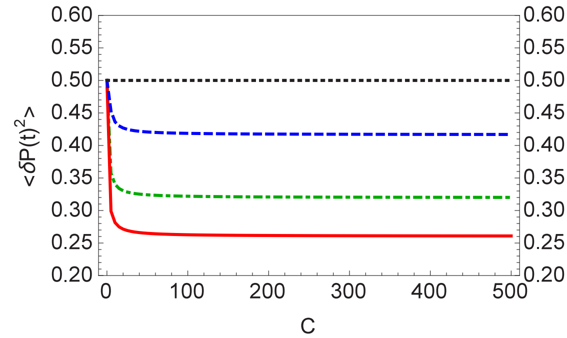

FIG. 4. The mean square fluctuation $\langle\delta P(t)^2\rangle$ versus the cooperativity parameter C for different parametric phases $\theta = 0$ (dot-dashed), $\pi/16$ (solid), $\pi/6$ (dashed) when $G = 0.46\kappa$ and $T = 0$ K. The flat dotted line represents the momentum variance of the vacuum state $\langle\delta P(t)^2\rangle_{\mathrm{vac}} = 0.5$.

the movable mirror under the action of the PA. Since our results are equally applicable to both microwave and optical systems, we choose parameters accordingly. We first choose the parameters similar to those in the experiment demonstrating mechanical squeezing in the microwave setup with two pumps [35]: $\omega_m/\kappa = 10$, $\gamma_m/\kappa = 10^{-5}$, the mechanical quality factor $Q' = \omega_m/\gamma_m = 10^6$. For convenience, we define the optomechanical cooperativity parameter $C = |g|^2/(\kappa\gamma_m)$, which is proportional to the power $\wp$ of the microwave pump. From the numerical results, it is found that $\langle\delta Q(t)^2\rangle$ cannot be less than $\frac{1}{2}$, but $\langle\delta P(t)^2\rangle$ can be less than $\frac{1}{2}$. Therefore we focus on discussing $\langle\delta P(t)^2\rangle$ here.

The mean square fluctuation $\langle\delta P(t)^2\rangle$ as a function of the parametric gain G for different parametric phases $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$ when $C = 400$ and $T = 0$ K is shown in Fig. 3. When $C = 400$, $\kappa = 5\sqrt{10}|g|$, the system is in the weak-coupling regime, and the conditions for the rotating wave approximation are satisfied. From Fig. 3, it is seen that $\langle\delta P(t)^2\rangle$=0.5 in the absence of the PA ($G = 0$), thus there is no squeezing in the momentum fluctuation of the movable mirror. In the presence of the PA ($G \neq 0$), $\langle\delta P(t)^2\rangle$ can be less than 0.5 except $\theta = \frac{\pi}{2}$. Hence the addition of the PA in the optomechanical system can realize the momentum squeezing of the movable mirror. Furthermore, it is observed that the minimum value of $\langle\delta P(t)^2\rangle$ is the smallest when $\theta = \pi/16$, which is $\langle\delta P(t)^2\rangle \approx 0.253$ at $G = 0.49\kappa$, the corresponding amount of the maximum momentum squeezing is about 49.4%; the degree of the squeezing is about 2.96 dB. The squeezing of the cavity field in the absence of the optomechanical coupling is given in the appendix. The maximum phase squeezing of the cavity field is about 2.96 dB when $G = 0.49\kappa$ and $\theta = 0$. Note that the maximum momentum squeezing of the movable mirror is equal to the maximum phase squeezing of the cavity field, but they happen at different parametric phases θ. The phase difference $\pi/16$ is related to the phase of g^2 and $\arctan[-g^2] = \pi/16$. Thus the squeezing of the cavity field is totally transferred into the movable mirror. This is because driving the system by the red-detuned laser $\Delta = \omega_m$ in the resolved sideband limit makes the optomechanical interaction between the movable mirror and the cavity field like a beamsplitter interaction.

The mean square fluctuation $\langle\delta P(t)^2\rangle$ as a function of the cooperativity parameter C for different parametric phases $\theta = 0,\pi/16,\pi/6$ when $G = 0.46\kappa$ and $T = 0$ K is shown in Fig. 4. In the absence of the optomechanical coupling ($C = 0$) between the cavity field and the movable mirror, it is seen that $\langle\delta P(t)^2\rangle = 0.5$. In the presence of the optomechanical coupling ($C \neq 0$), $\langle\delta P(t)^2\rangle$ drops to about 0.320, 0.261, 0.417 for $\theta = 0,\pi/16,\pi/6$, respectively, thus the optomechanical coupling can lead to the momentum squeezing of the movable mirror. The corresponding degrees of the squeezing are about 1.94 dB, 2.82 dB, 0.79 dB for $\theta = 0,\pi/16,\pi/6$, respectively. It is noted that the momentum squeezing of the movable mirror almost keeps constant when the cooperativity parameter C is larger than a certain value and it persists over a very wide range.

In this paragraph, we discuss previous results on mechanical squeezing. The mechanical squeezing is not larger than 3 dB in [20–22] as in this work. In the current work it is limited by the squeezing that a parametric device can produce. A relatively large mechanical squeezing can be achieved by feeding in squeezed light [24,25]. Here one gets about 6 dB squeezing by feeding in light with about 9 dB squeezing. The two-tone driving as discussed in detail in Ref. [23] can also produce large squeezing (more than 3 dB). For this, the intensity of the blue-detuned drive has to be close but smaller than the intensity of the red-detuned drive and the cooperativity parameter C has to be large. The latter requirement should not be in conflict with the dropping of the nonresonant terms in the case of two-tone driving. In addition, the mechanical squeezing beyond 3 dB can be created by quantum measurement and feedback to remove the effect of the quantum back action [33,34]. Several experiments have reported good mechanical squeezing. The best experimental mechanical squeezing is roughly 1.0 dB in [35–37], 6.2 dB in [38], 7.4 dB in [28], and 11.5 dB in [26], respectively. It is clear that additional methods are to be used to go beyond 3 dB squeezing. This is briefly discussed at the end of Sec. V.

We find that the amount of squeezing of the mechanical mirror is not very sensitive to the parameters. We next choose parameters corresponding to a typical optical cavity. We take $\omega_m/\kappa = 10$, $\gamma_m/\kappa = 10^{-3}$. The mean square fluctuation

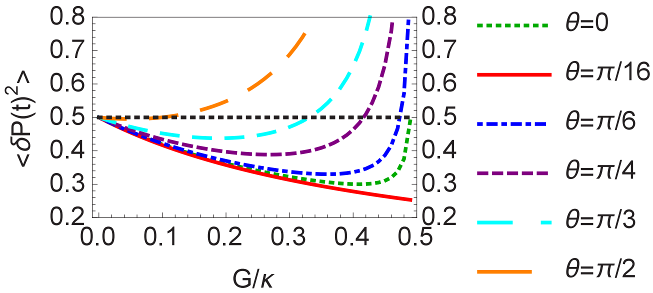

FIG. 5. The mean square fluctuation $\langle \delta P(t)^2 \rangle$ versus the parametric gain G for different parametric phases $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$ when $C = 400$ and $T = 0$ K. The flat dotted line represents the momentum variance of the vacuum state $\langle \delta P(t)^2 \rangle_{\rm vac} = 0.5$. The green dotted, red solid, blue dot-dashed, purple short dashed, cyan middle dashed, and orange long-dashed curves correspond to $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$, respectively.

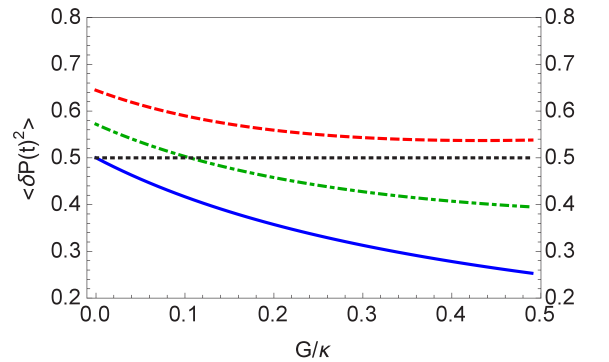

FIG. 6. The mean square fluctuation $\langle \delta P(t)^2 \rangle$ versus the parametric gain G for different temperatures of the environment $T = 0$ K (solid), 10 mK (dot-dashed), and 20 mK (dashed) when $C = 400$ and $\theta = \pi/16$. The flat dotted line represents the momentum variance of the vacuum state $\langle \delta P(t)^2 \rangle_{\rm vac} = 0.5$.

$\langle \delta P(t)^2 \rangle$ as a function of the parametric gain G for different parametric phases $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$ when $C = 400$ and $T = 0$ K is shown in Fig. 5. When $C = 400$, $\kappa = \sqrt{10}|g|/2$, the system is in the weak-coupling regime, and the conditions for the rotating wave approximation are satisfied. It is seen that $\langle \delta P(t)^2 \rangle$ takes the smallest value 0.253 when $\theta = \pi/16$ and $G = 0.49\kappa$, which is similar to that in Fig. 3.

Further for the optical case the mean square fluctuation $\langle \delta P(t)^2 \rangle$ as a function of the cooperativity parameter C for different parametric phases $\theta = 0,\pi/16,\pi/6$ when $G = 0.46\kappa$ and $T = 0$ K is similar to Fig. 4. In the presence of the optomechanical coupling, $\langle \delta P(t)^2 \rangle$ drops to about 0.320, 0.261, 0.416 for $\theta = 0,\pi/16,\pi/6$, respectively.

We next examine the effect of the Brownian noise on squeezing, i.e., the effect of the temperature of the environment. We need the values of the cavity frequency ω_c and the mechanical frequency ω_m. We consider the case of the microwave system [35]. We assume $\omega_c = 2\pi \times 6.23$ GHz and $\omega_m = 2\pi \times 3.6$ MHz. The mean square fluctuation $\langle \delta P(t)^2 \rangle$ as a function of the parametric gain G for different temperatures of the environment when $C = 400$ and $\theta = \pi/16$ is plotted in Fig. 6. For $T = 0$ K, 10 mK, and 20 mK, the corresponding initial mean thermal excitation numbers $n_c^{\rm th}$ in the optical mode are 0, 1.03×10^{-13}, and 3.22×10^{-7}, respectively; the corresponding initial mean thermal excitation numbers $n_m^{\rm th}$ in the mechanical mode are 0, 57.4, and 115.3, respectively. It is noted that increasing the temperature of the environment would decrease the momentum squeezing of the movable mirror. For example, when $G = 0.49\kappa$, $T = 0$ K and 10 mK, $\langle \delta P(t)^2 \rangle \approx 0.253$, 0.395, respectively, the corresponding degrees of the squeezing are about 2.96 dB, 1.02 dB, respectively. When the temperature of the environment is increased to $T = 20$ mK, $\langle \delta P(t)^2 \rangle$ is always larger than 0.5, thus the squeezing of the mechanical mode does not occur. We have confirmed that for the optical cavity case the results of Fig. 6 hold with almost no change. For brevity we do not present the figure for the optical cavity case.

The PA inside the OM cavity can produce a number of novel effects besides squeezing of the mirror and cooling. Some of these are generation of the genuine tripartite entangled states [50], enhancement of the precision of optomechanical position detection [51], and enhancement of the effective optomechanical interaction strength [52,53]. The latter could become important for getting closer to the single photon coupling regime.

V. ANALYTICAL APPROACH TO UNDERSTAND MECHANICAL SQUEEZING

In this section, we will present an analytical approach to understand the result of Sec. IV. In the weakly optomechanical coupling regime $\kappa \gg |g|$, in which the photons leak out of the cavity much faster than the optomechanical interaction, the cavity field follows the mechanical motion adiabatically. The adiabatical approximation can be made, thus $\delta\dot{\tilde{c}} = 0$. We obtain

$$\delta\tilde{c} = \frac{1}{\kappa^2 - 4G^2}(i\kappa g \delta\tilde{b} - i2Ge^{i\theta} g^* \delta\tilde{b}^\dagger + 2Ge^{i\theta}\sqrt{2\kappa}\tilde{c}_{in}^\dagger + \kappa\sqrt{2\kappa}\tilde{c}_{in}). \quad (20)$$

Substituting $\delta\tilde{c}$ into Eq. (7), we have

$$\delta\dot{\tilde{b}} = -\left(\frac{\kappa|g|^2}{\kappa^2 - 4G^2} + \frac{\gamma_m}{2}\right)\delta\tilde{b} + \frac{2Ge^{i\theta}g^{*2}}{\kappa^2 - 4G^2}\delta\tilde{b}^\dagger + \frac{ig^*\sqrt{2\kappa}}{\kappa^2 - 4G^2}(2Ge^{i\theta}\tilde{c}_{in}^\dagger + \kappa\tilde{c}_{in}) + \sqrt{\gamma_m}\tilde{b}_{in}. \quad (21)$$

In the absence of the PA ($G = 0$) or the cavity field ($g = 0$), it is noted that $\delta\dot{\tilde{b}}$ does not depend on $\tilde{b}^\dagger$, thus the squeezing of the movable mirror does not appear. In the presence of the PA and the cavity field, $\delta\dot{\tilde{b}}$ depends on $\tilde{b}^\dagger$. This parametric coupling can lead to the squeezing of the movable mirror. Therefore, the PA in the cavity can realize the squeezing of the movable mirror.

In the parameter domain we are working the term γ_m in the coefficient of $\delta\tilde{b}$ can be ignored. Let $G_0 = 2G/\kappa$ and we choose a value of θ such that $g^{*2}e^{i\theta} = -|g|^2$ ($\theta = \pi/16$), then we write (21) as

$$\delta\dot{\tilde{b}} = -\frac{|g|^2}{(1-G_0^2)\kappa}\delta\tilde{b} - \frac{G_0|g|^2}{(1-G_0^2)\kappa}\delta\tilde{b}^\dagger + \frac{ig^*\sqrt{2\kappa}}{(1-G_0^2)\kappa}(G_0 e^{i\theta}\tilde{c}_{in}^\dagger + \tilde{c}_{in}) + \sqrt{\gamma_m}\tilde{b}_{in}. \quad (22)$$

From Eq. (22), we get the equation for the momentum fluctuation δP as

$$\delta\dot{P} = -\frac{|g|^2}{\kappa(1+G_0)}\delta P + h(t) + f(t), \quad (23)$$

where the quantum Langevin forces are given by

$$h(t) = \frac{\sqrt{\gamma_m}}{\sqrt{2}i}(\tilde{b}_{in} - \tilde{b}_{in}^\dagger), \quad (24)$$

$$f(t) = \frac{g^*\sqrt{\kappa}}{\kappa(1+G_0)}(\tilde{c}_{in} - \tilde{c}_{in}^\dagger e^{i\theta}). \quad (25)$$

Using Eq. (25) and Eq. (4), we obtain the correlation function of $f(t)$,

$$\langle f(t)f(t')\rangle = \frac{|g|^2}{\kappa(1+G_0)^2}(1+2n_c^{th})\delta(t-t'). \quad (26)$$

The correlation function of $h(t)$ can be calculated using (24) and (3),

$$\langle h(t)h(t')\rangle = \frac{\gamma_m}{2}(1+2n_m^{th})\delta(t-t'). \quad (27)$$

We now obtain the equation for $\langle\delta P^2\rangle$ using (23), (26), and (27) as

$$\frac{\partial\langle\delta P^2\rangle}{\partial t} = -\frac{2|g|^2}{\kappa(1+G_0)}\langle\delta P^2\rangle + \frac{|g|^2}{\kappa(1+G_0)^2}(1+2n_c^{th}) + \frac{\gamma_m}{2}(1+2n_m^{th}), \quad (28)$$

and therefore we get the analytical result for the squeezing of the quadrature P in the steady state as

$$\langle\delta P^2\rangle = \frac{1}{2(1+G_0)}(1+2n_c^{th}) + \frac{\gamma_m\kappa(1+G_0)}{4|g|^2}(1+2n_m^{th}). \quad (29)$$

For $G_0 \leqslant 1$, $|g|^2/(\gamma_m\kappa) = 400$, we find

$$\langle\delta P^2\rangle \approx \frac{1}{4}(1+2n_c^{th}) + \frac{1}{800}(1+2n_m^{\mathrm{th}}), \quad (30)$$

which gives values about 0.25, 0.40, and 0.55 for $T=0$ ($n_c^{\mathrm{th}} = n_m^{\mathrm{th}} = 0$), 10 mK ($n_c^{\mathrm{th}} = 1.03\times10^{-13}$, $n_m^{\mathrm{th}} = 57.4$), and 20 mK ($n_c^{\mathrm{th}} = 3.22\times10^{-7}$, $n_m^{\mathrm{th}} = 115.3$), respectively. These analytical results are in excellent agreement with the numerical results in Fig. 6 for G/κ close to but less than 0.5. A very important feature of the result (29) which is noticed is the suppression of the Brownian noise by the cooperativity parameter C. As we have mentioned earlier and as has been realized by several others [33,34], the 3-dB limit can be broken by using the feedback mechanism as in Ref. [26]. Let η be the dimensionless feedback gain parameter, then detailed calculations show that the squeezing given by Eq. (29) is reduced by a factor of $[1+\frac{1}{2C}(1+G_0)(1+\frac{\eta}{2})]$. The maximum value of η is limited by the stability of the dynamical equations. Thus in order to get 75% squeezing (6 dB), we need the condition $\eta/C\sim 2$. Still larger squeezing is achievable by increasing the feedback. Note that stability requires that η should be not larger than $4C$.

VI. THE DETECTION OF THE MECHANICAL SQUEEZING

In this section, we analyze that the mechanical squeezing can be measured by the output field. The fluctuation $\delta c(\omega)$ of the cavity field can be obtained from Eq. (8). Using the input-output relation $c_{\mathrm{out}} = \sqrt{2\kappa}c - c_{\mathrm{in}}$ [40], we can get the fluctuation $\delta c_{\mathrm{out}}(\omega)$ of the output field. Then we define the quadrature fluctuation of the output field as

$$\delta z_{\mathrm{out}}(\omega) = \frac{1}{\sqrt{2}}[\delta c_{\mathrm{out}}(\omega)e^{-i\phi} + \delta c_{\mathrm{out}}(-\omega)^\dagger e^{i\phi}], \quad (31)$$

with ϕ being the measurement phase angle determined by the local oscillator. When $\phi=0$, $\delta z_{\mathrm{out}}(\omega) = \delta x_{\mathrm{out}}(\omega)$, which is the amplitude fluctuation of the output field. When $\phi = \pi/2$, $\delta z_{\mathrm{out}}(\omega) = \delta y_{\mathrm{out}}(\omega)$, which is the phase fluctuation of the output field. Through calculations, $\delta z_{\mathrm{out}}(\omega)$ is found to be

$$\delta z_{\mathrm{out}}(\omega) = A_z(\omega)x_{\mathrm{in}}(\omega) + B_z(\omega)y_{\mathrm{in}}(\omega) + E_z(\omega)Q_{\mathrm{in}}(\omega) + F_z(\omega)P_{\mathrm{in}}(\omega), \quad (32)$$

where

$$\begin{aligned}
A_z(\omega) &= I(\omega)\cos\phi + R(\omega)\sin\phi,\\
B_z(\omega) &= R(\omega)\cos\phi + J(\omega)\sin\phi,\\
E_z(\omega) &= -\sqrt{\gamma_m}[A_1(\omega)\cos\phi + B_1(\omega)\sin\phi],\\
F_z(\omega) &= -\sqrt{\gamma_m}[A_2(\omega)\cos\phi + B_2(\omega)\sin\phi],\\
I(\omega) &= \frac{2\kappa}{d(\omega)}v(\omega)\{|g|^2 + [u(\omega) + 2G\cos\theta]v(\omega)\} - 1,\\
R(\omega) &= \frac{4\kappa}{d(\omega)}G\sin\theta\, v(\omega)^2,\\
J(\omega) &= \frac{2\kappa}{d(\omega)}v(\omega)\{|g|^2 + [u(\omega) - 2G\cos\theta]v(\omega)\} - 1. \quad (33)
\end{aligned}$$

The spectrum of the quadrature fluctuation $\delta z_{\mathrm{out}}(\omega)$ of the output field is defined by

$$S_{z\mathrm{out}}(\omega) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} d\Omega\, e^{-i(\omega+\Omega)t}[\langle\delta z_{\mathrm{out}}(\omega)\delta z_{\mathrm{out}}(\Omega)\rangle + \langle\delta z_{\mathrm{out}}(\Omega)\delta z_{\mathrm{out}}(\omega)\rangle]. \quad (34)$$

Using Eq. (16), we find the spectrum of the quadrature fluctuation $\delta z_{\mathrm{out}}(\omega)$ of the output field,

$$S_{z\mathrm{out}}(\omega) = [A_z(\omega)A_z(-\omega) + B_z(\omega)B_z(-\omega)]\left(n_c^{\mathrm{th}} + \frac{1}{2}\right) + [E_z(\omega)E_z(-\omega) + F_z(\omega)F_z(-\omega)]\left(n_m^{\mathrm{th}} + \frac{1}{2}\right). \quad (35)$$

The output field is in a squeezed state if $S_{z\mathrm{out}}(\omega)$ is smaller than that of the vacuum state, i.e., $S_{z\mathrm{out}}(\omega) < \frac{1}{2}$.

We take $\omega_m/\kappa = 10$, $\gamma_m/\kappa = 10^{-5}$, $C = |g|^2/(\kappa\gamma_m) = 400$. Figure 7 plots the spectrum $S_{y\mathrm{out}}(\omega)$ of the phase fluctuation of the output field versus the frequency ω when

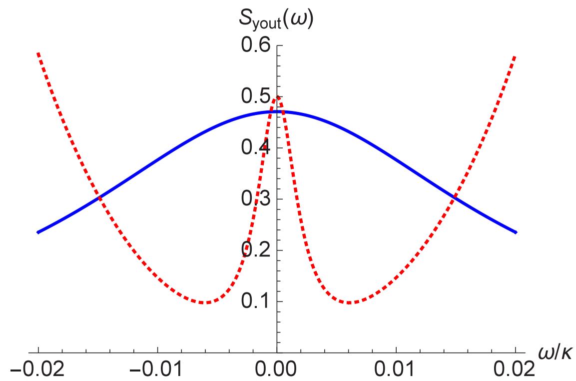

FIG. 7. The spectrum $S_{yout}(\omega)$ of the phase fluctuation of the output field versus the frequency ω when $G = 0.49\kappa$, $\theta = \pi/16$, and $T = 0$ K in the absence of the optomechanical coupling ($g = 0$) (blue solid) and in the presence of the optomechanical coupling ($g \neq 0$) (red dotted). Here the spectrum $S_{yout}(\omega)$ for $g = 0$ has been divided by 100.

$G = 0.49\kappa$, $\theta = \pi/16$, and $T = 0$ K without the optomechanical coupling ($g = 0$) and with the optomechanical coupling ($g \neq 0$). It is noted that the squeezing does not exist in the phase fluctuation of the output field for $g = 0$ because of $S_{yout}(\omega) \gg 0.5$. The squeezing exists in the phase fluctuation of the output field for $g \neq 0$ since $S_{yout}(\omega)$ can be less than 0.5 when $|\omega| \leqslant 0.0187\kappa$. Hence in the presence of the optomechanical coupling, the phase squeezing of the output field in $|\omega| \leqslant 0.0187\kappa$ is a signature of the mechanical squeezing. In the presence of the optomechanical coupling, the contour plot of the spectrum $S_{zout}(\omega)$ of the quadrature fluctuation of the output field versus the frequency ω and the phase ϕ when $G = 0.49\kappa$, $\theta = \pi/16$, and $T = 0$ K is shown in Fig. 8. The lower figure in Fig. 8 indicates the region in which the quadrature fluctuation $\delta z_{out}(\omega)$ of the output field is squeezed. Therefore, the mechanical squeezing can be detected by measuring the quadrature fluctuation of the output field [54].

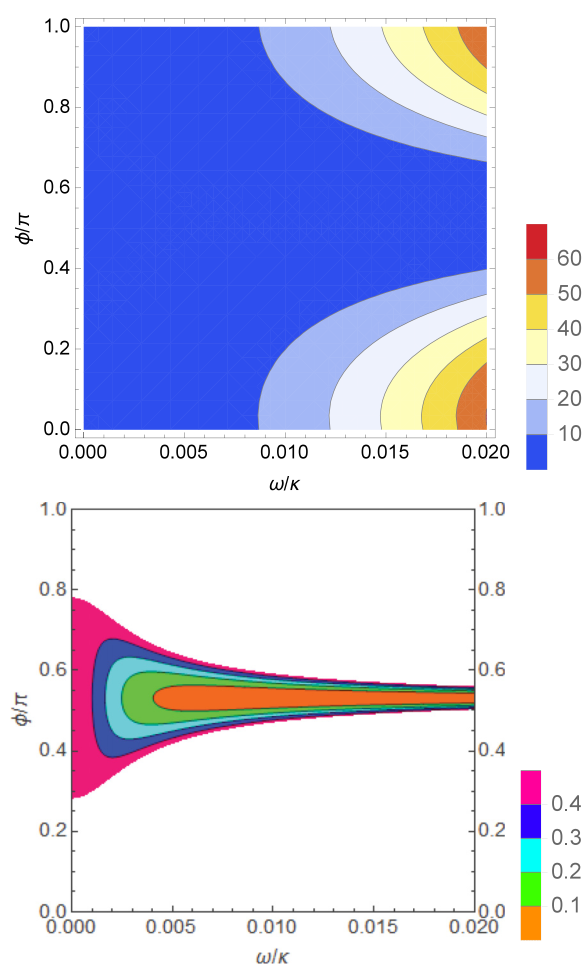

FIG. 8. The contour plot of the spectrum $S_{zout}(\omega)$ of the quadrature fluctuation of the output field versus the frequency ω and the phase ϕ when $G = 0.49\kappa$, $\theta = \pi/16$, and $T = 0$ K. The lower figure zooms the upper figure for the region around zero.

VII. CONCLUSIONS

In conclusion, we have demonstrated that the momentum fluctuation of the movable mirror can be squeezed when a PA is placed inside the cavity. It is found that the squeezing of the cavity field produced by the PA in the cavity can be fully transferred to the movable mirror in the resolved sideband limit and the thermal noise contribution is suppressed by a factor of the cooperativity parameter C. Moreover, we show that the detection of the mechanical squeezing can be realized by measuring the squeezing of the quadrature fluctuation of the output field by working in a regime of parameters when the PA does not squeeze the output field for no optomechanical coupling. In our work the degree of the mechanical squeezing will be limited by the squeezing produced by the PA. However, one can use the previously used methods like the single quadrature feedback scheme [26] or the weak measurement [27,28] to substantially increase the mirror's squeezing as explicitly discussed at the end of Sec. V. Our results and analysis are equally valid for optical and microwave systems.

APPENDIX: THE SQUEEZING OF THE CAVITY FIELD IN THE ABSENCE OF THE OPTOMECHANICAL COUPLING

For completeness and for making contact we present in this appendix what is well known for a cavity containing a PA [40,41]. In the absence of the optomechnical coupling ($g = 0$), the amplitude and phase fluctuations of the cavity field can be found from Eq. (8),

$$\delta x(\omega) = A_3(\omega)x_{in}(\omega) + B_3(\omega)y_{in}(\omega),$$
$$\delta y(\omega) = A_4(\omega)x_{in}(\omega) + B_4(\omega)y_{in}(\omega), \quad \text{(A1)}$$

where

$$A_3(\omega) = \frac{\sqrt{2\kappa}}{u(\omega)^2 - 4G^2}[u(\omega) + 2G\cos\theta],$$

$$B_3(\omega) = \frac{\sqrt{2\kappa}}{u(\omega)^2 - 4G^2}2G\sin\theta,$$

$$A_4(\omega) = \frac{\sqrt{2\kappa}}{u(\omega)^2 - 4G^2}2G\sin\theta,$$

$$B_4(\omega) = \frac{\sqrt{2\kappa}}{u(\omega)^2 - 4G^2}[u(\omega) - 2G\cos\theta]. \quad \text{(A2)}$$

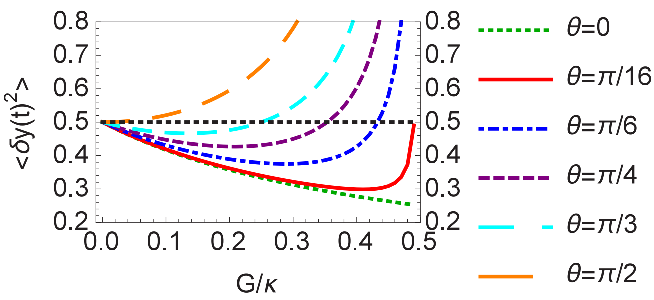

FIG. 9. The mean square fluctuation $\langle \delta y(t)^2 \rangle$ versus the parametric gain G for different parametric phases $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$ when $T = 0$ K. The flat dotted line represents the phase variance of the vacuum state $\langle \delta y(t)^2 \rangle_{\text{vac}} = 0.5$. The green dotted, red solid, blue dot-dashed, purple short dashed, cyan middle dashed, and orange long-dashed curves correspond to $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$, respectively.

Without the PA in the cavity, $G = 0$, we obtain $\delta x(\omega) = \frac{\sqrt{2\kappa}}{\kappa - i\omega} x_{in}$, $\delta y(\omega) = \frac{\sqrt{2\kappa}}{\kappa - i\omega} y_{in}$. The spectra of fluctuations in the quadratures of the cavity field are defined by

$$S_Z(\omega) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} d\Omega\, e^{-i(\omega+\Omega)t}[\langle \delta Z(\omega)\delta Z(\Omega)\rangle + \langle \delta Z(\Omega)\delta Z(\omega)\rangle], \qquad Z = x,y. \tag{A3}$$

With the help of the nonzero correlation functions of the noise sources in the frequency domain in Eq. (16), we obtain the spectra of fluctuations in the quadratures of the cavity field,

$$S_x(\omega) = [A_3(\omega)A_3(-\omega) + B_3(\omega)B_3(-\omega)]\left(n_c^{\text{th}} + \frac{1}{2}\right),$$
$$S_y(\omega) = [A_4(\omega)A_4(-\omega) + B_4(\omega)B_4(-\omega)]\left(n_c^{\text{th}} + \frac{1}{2}\right). \tag{A4}$$

In the absence of the PA in the cavity, we have $S_x(\omega) = S_y(\omega) = \frac{2\kappa}{\kappa^2+\omega^2}(n_c^{\text{th}} + \frac{1}{2})$, which have peaks located at frequency zero with full width 2κ at half maximum. The mean square fluctuations $\langle \delta x(t)^2 \rangle$ and $\langle \delta y(t)^2 \rangle$ in the quadratures of the cavity field are determined by

$$\langle \delta Z(t)^2 \rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, S_Z(\omega), \qquad Z = x,y. \tag{A5}$$

Without the PA in the cavity, we obtain $\langle \delta x(t)^2 \rangle = \langle \delta y(t)^2 \rangle = n_c^{\text{th}} + \frac{1}{2}$. For $T = 0$ K, $n_c^{\text{th}} = 0$, the cavity field is in a vacuum state, we have $\langle \delta x(t)^2 \rangle = \langle \delta y(t)^2 \rangle = \frac{1}{2}$. According to the Heisenberg uncertainty principle,

$$\langle \delta x(t)^2 \rangle \langle \delta y(t)^2 \rangle \geqslant \left| \frac{1}{2}[x,y] \right|^2, \tag{A6}$$

where $[x,y] = i$. If either $\langle \delta x(t)^2 \rangle$ or $\langle \delta y(t)^2 \rangle$ is below $\frac{1}{2}$, the cavity field is in a squeezed state. Similarly, the degree of the squeezing can be calculated by $-10\log_{10}\frac{\langle \delta y(t)^2 \rangle}{\langle \delta y(t)^2 \rangle_{\text{vac}}}$ dB, where $\langle \delta y(t)^2 \rangle_{\text{vac}}$ is the phase variance of the vacuum state and $\langle \delta y(t)^2 \rangle_{\text{vac}} = \frac{1}{2}$.

The numerical results show that $\langle \delta x(t)^2 \rangle$ cannot be less than $\frac{1}{2}$, but $\langle \delta y(t)^2 \rangle$ can be less than $\frac{1}{2}$. Thus we are interested in $\langle \delta y(t)^2 \rangle$ here. The mean square fluctuation $\langle \delta y(t)^2 \rangle$ as a function of the parametric gain G for different parametric phases $\theta = 0,\pi/16,\pi/6,\pi/4,\pi/3,\pi/2$ when $T = 0$ K is shown in Fig. 9. Note that $\langle \delta y(t)^2 \rangle = 0.5$ in the absence of the PA, thus the phase fluctuation of the cavity field is not squeezed. In the presence of the PA, it is noted that $\langle \delta y(t)^2 \rangle < 0.5$ can be obtained except $\theta = \pi/2$. Hence, the phase squeezing of the cavity field can be achieved when a PA is placed in the cavity. The best squeezing happens at $\theta = 0$ and $G = 0.49\kappa$, at which $\langle \delta y(t)^2 \rangle$ is equal to 0.253, the corresponding amount of the phase squeezing is about 49.4%, and the degree of the squeezing is $-10\log_{10}\frac{0.253}{0.5} \approx 2.96$ dB.

[1] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014).

[2] A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, and A. N. Cleland, Nature (London) **464**, 697 (2010).

[3] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature (London) **475**, 359 (2011).

[4] J. Chan, T. P. Mayer Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter, Nature (London) **478**, 89 (2011).

[5] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A **59**, 3204 (1999).

[6] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).

[7] M. Paternostro, D. Vitali, S. Gigan, M. S. Kim, C. Brukner, J. Eisert, and M. Aspelmeyer, Phys. Rev. Lett. **99**, 250401 (2007).

[8] M. J. Hartmann and M. B. Plenio, Phys. Rev. Lett. **101**, 200503 (2008).

[9] S. Huang and G. S. Agarwal, New J. Phys. **11**, 103044 (2009).

[10] C. Fabre, M. Pinard, S. Bourzeix, A. Heidmann, E. Giacobino, and S. Reynaud, Phys. Rev. A **49**, 1337 (1994).

[11] S. Mancini and P. Tombesi, Phys. Rev. A **49**, 4055 (1994).

[12] T. P. Purdy, P.-L. Yu, R. W. Peterson, N. S. Kampel, and C. A. Regal, Phys. Rev. X **3**, 031012 (2013).

[13] A. Kronwald, F. Marquardt, and A. A. Clerk, New J. Phys. **16**, 063058 (2014).

[14] K. Qu and G. S. Agarwal, New J. Phys. **16**, 113004 (2014).

[15] K. Qu and G. S. Agarwal, Phys. Rev. A **91**, 063815 (2015).

[16] D. Kilda and A. Nunnenkamp, J. Opt. **18**, 014007 (2016).

[17] G. S. Agarwal and S. A. Kumar, Phys. Rev. Lett. **67**, 3665 (1991).

[18] V. V. Dodonov and V. I. Man'ko, in *Invariants and the Evolution of Nonstationary Quantum Systems*, edited by M. A. Markov (Lebedev Institute, Moscow, 1990).

[19] V. B. Braginsky, Y. I. Vorontsov, and K. S. Thorne, Science **209**, 547 (1980).

[20] A. Mari and J. Eisert, Phys. Rev. Lett. **103**, 213603 (2009); Here the maximum position squeezing is about 50%.

[21] W.-J. Gu and G.-X. Li, Opt. Express **21**, 020423 (2013); These authors consider the dissipative OMS and the reported maximal momentum squeezing is 20%.

[22] J.-Q. Liao and C. K. Law, Phys. Rev. A **83**, 033820 (2011); The maximum reported squeezing is 40%.

[23] A. Kronwald, F. Marquardt, and A. A. Clerk, Phys. Rev. A **88**, 063833 (2013).

[24] K. Jähne, C. Genes, K. Hammerer, M. Wallquist, E. S. Polzik, and P. Zoller, Phys. Rev. A **79**, 063819 (2009).

[25] S. Huang and G. S. Agarwal, Phys. Rev. A **82**, 033811 (2010).

[26] A. Vinante and P. Falferi, Phys. Rev. Lett. **111**, 207203 (2013).

[27] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, Phys. Rev. Lett. **107**, 213603 (2011).

[28] A. Pontin, M. Bonaldi, A. Borrielli, F. S. Cataliotti, F. Marino, G. A. Prodi, E. Serra, and F. Marin, Phys. Rev. Lett. **112**, 023601 (2014).

[29] M. Asjad, G. S. Agarwal, M. S. Kim, P. Tombesi, G. Di Giuseppe, and D. Vitali, Phys. Rev. A **89**, 023849 (2014).

[30] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, C. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, Proc. Natl. Acad. Sci. USA **108**, 16182 (2011); Pulsed optomechanics was used to cool the mirror from a temperature 1100 K to 16 K; M. R. Vanner, J. Hofer, G. D. Cole, and M. Aspelmeyer, Nature Communications **4**, 2295 (2013).

[31] X.-Y. Lü, J.-Q. Liao, L. Tian, and F. Nori, Phys. Rev. A **91**, 013834 (2015).

[32] M. Benito, C. Sanchez Muñoz, and C. Navarrete-Benlloch, Phys. Rev. A **93**, 023846 (2016).

[33] R. Ruskov, K. C. Schwab, and A. N. Korotkov, Phys. Rev. B **71**, 235407 (2005).

[34] A. A. Clerk, F. Marquardt, and K. Jacobs, New J. Phys. **10**, 095010 (2008).

[35] E. E. Wollman, C. U. Lei, A. J. Weinstein, J. Suh, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, Science **347**, 53 (2015).

[36] J.-M. Pirkkalainen, E. Damskägg, M. Brandt, F. Massel, and M. A. Sillanpää, Phys. Rev. Lett. **115**, 243601 (2015).

[37] F. Lecocq, J. B. Clark, R. W. Simmonds, J. Aumentado, and J. D. Teufel, Phys. Rev. X **5**, 041037 (2015).

[38] A. Szorkovszky, G. A. Brawley, A. C. Doherty, and W. P. Bowen, Phys. Rev. Lett. **110**, 184301 (2013).

[39] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, Rev. Mod. Phys. **52**, 341 (1980).

[40] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).

[41] L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).

[42] S. Huang and G. S. Agarwal, Phys. Rev. A **79**, 013821 (2009).

[43] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. **99**, 093902 (2007).

[44] S. Gröblacher, K. Hammerer, M. Vanner, and M. Aspelmeyer, Nature (London) **460**, 724 (2009).

[45] S. Huang and G. S. Agarwal, Phys. Rev. A **80**, 033807 (2009).

[46] While for our purpose linearization approximation holds very well, a nonlinear description of the PA in an OMS can be developed using the previously developed methods in the context of optical parametric oscillators [47]. For recent advances see S. Pina-Otey, F. Jiménez, P. Degenfeld-Schonburg, and C. Navarrete-Benlloch, Phys. Rev. A **93**, 033835 (2016); P. Degenfeld-Schonburg, M. Abdi, M. J. Hartmann, and C. Navarrete-Benlloch, *ibid.* **93**, 023819 (2016).

[47] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994), Chap. 8.

[48] A. Hurwitz, in *Selected Papers on Mathematical Trends in Control Theory*, edited by R. Bellman and R. Kalaba (Dover, New York, 1964).

[49] E. X. DeJesus and C. Kaufman, Phys. Rev. A **35**, 5288 (1987).

[50] A. Xuereb, M. Barbieri, and M. Paternostro, Phys. Rev. A **86**, 013809 (2012).

[51] V. Peano, H. G. L. Schwefel, Ch. Marquardt, and F. Marquardt, Phys. Rev. Lett. **115**, 243603 (2015).

[52] X.-Y. Lü, Y. Wu, J. R. Johansson, H. Jing, J. Zhang, and F. Nori, Phys. Rev. Lett. **114**, 093602 (2015).

[53] P. D. Nation, J. Suh, and M. P. Blencowe, Phys. Rev. A **93**, 022510 (2016).

[54] The squeezing spectrum of the output changes to our benefit if feedback is added to our model. These calculations are too long to be included here.