Controllable photon bunching by atomic superpositions in a driven cavity

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We propose a feasible approach to generate the desired light with controllable photon bunchings by adjusting the atomic superpositions in a driven cavity. Under the large detuning limit, i.e., the cavity is far resonance with the inside atom(s), we show that the photons in the cavity are always bunchings. Typically, when the effective dispersive interaction equals the detuning between the driving and cavity fields, we find that the value of second-order correlation $g^{(2)}(0)$ inverses to the probability of the superposed atomic state. This suggests that such a value could be arbitrarily large, and thus the bunchings of the photons could be significantly enhanced.

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I. INTRODUCTION

It is well known that the discovery of two-photon bunching in thermal source by Hanbury Brown and Twiss [1] is one of the milestones in quantum optics. Indeed, the attempt to explain this experiment leads to many important developments in optics. For example, in a semiclassical framework Purcell [2] explained that the Hanbury Brown and Twiss (HBT) effect was due to the interference of wave packets, although Mandel [3] claimed that such an effect could be explained by a classical interference theory. The basic development in this topic is due to Glauber's [4] quantum coherence theory, and the nonclassical two-photon interference was verified [5]. Since then, the temporal distinguishability of photons in quantum coherence theory has been successfully utilized to interpret various multiphoton interference phenomena [6,7]. Indeed, quantum coherence of light provides a powerful platform to conceptually understand quantum physics and implement various novel quantum technologies, such as quantum information processing, quantum lithography, and "ghost" images, etc. [8].

Mathematically, the second-order correlation function $g^{(2)}(\tau)$ of the radiation field is usually utilized to describe the photon bunching feature, i.e., the probability of detecting a photon at the time t and then the second one at the time $t+\tau$. In quantum coherence theory [4], this normalized function can be defined as

$$g^{(2)}(\tau)=\frac{\langle\hat{E}^{(-)}(t+\tau)\hat{E}^{(-)}(t)\hat{E}^{(+)}(t)\hat{E}^{(+)}(t+\tau)\rangle}{\langle\hat{E}^{(-)}(t)\hat{E}^{(+)}(t)\rangle^2},\qquad(1)$$

where $\hat{E}^{(+)}(t)$ and $\hat{E}^{(-)}(t)$ indicates the complex electric field operators and $\langle\hat{X}\rangle$ refers to the relevant expected value of the field operator $\hat{X}$ for the quantum optical state. Typically, $g^{(2)}(\tau)=2$ marks the usual chaotic field, and $g^{(2)}(\tau)<1$ describes various nonclassical light sources (e.g., the single-photon one) wherein the photon is antibunching [9]. In recent years, due to the requirement of ideal quantum secure communication, various high-quality single-photon sources with the significant photon-antibunching effects have been paid much attention [10–13]. Relatively, the bunching lights [with the stronger second-order correlations, i.e., $g^{(2)}(\tau)\gg 1$] have hardly been discussed. Typically, for the thermal lights the nth order correlation is the factorial of n [14]. The stronger bunching effect of the photons is revealed in the weak squeezed vacuum field [15], even for the zero-delay correlation functions. Particularly, the recent experiments demonstrated that by using the electromagnetically induced transparencies [16], the photons emitted from an atomic ensemble into a single-mode cavity could be controlled effectively from antibunching to bunching [17], and their second-order correlation can reach to a significantly high value. In this paper we focus our attention on how to generate the desired strong bunching photons with a driven cavity QED system, wherein a few two-level atoms dispersively coupled to a single-mode field (with the frequency ω_r). It assumed that the cavity is driven by a sufficiently weak coherent field, and thus all the incoherent effects are robustly neglected. Basically, by exactly solving the relevant master equations beyond the usual mean-field approximation, all the statistical properties of the intercavity field can be derived. Without loss of the generality and for simplicity, we exactly calculate the zero-delay second-order correlation function [18]

$$g^{(2)}(0)=\frac{\langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2},\qquad(2)$$

for various coherent atomic states. Here, $a^\dagger$ (a) is the creation (annihilation) operator of the intercavity field. We found that the value of this function is strongly related to the atomic superposed states. Specifically, near the modified cavity frequencies, the value of the $g^{(2)}(0)$ is inversely related to the superposed probability of the atomic eigenstate. This provides an effective approach to generate the significantly strong bunching photons by engineering the atomic coherence.

II. BUNCHING PHOTONS BY ENGINEERING THE COHERENCE OF ATOMS IN A DRIVEN CAVITY

Under the usual Born-Markov approximation, the dynamics of the driven cavity QED system is described by the following master equation [18]:

$$\dot{\rho}=-i[H_N+H_d,\rho]-\frac{\kappa}{2}(-2\hat{a}\rho\hat{a}^\dagger+\hat{a}^\dagger\hat{a}\rho+\rho\hat{a}^\dagger\hat{a}),\quad(3)$$

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with ρ being the density matrix of the system and κ the decay rate of the intercavity photons. Above,

$$H_N = \hbar\omega_r \hat{a}^\dagger \hat{a} + \sum_{j=1}^{N} \left[\frac{\hbar\omega_j}{2}\sigma_{zj} + \hbar g_j(\sigma_{+j}\hat{a} + \sigma_{-j}\hat{a}^\dagger) \right] \quad (4)$$

describes a single-mode cavity interacting with N two-level atoms (i.e., qubits) inside the cavity [19]. Here, ω_j is the jth atom transition frequency, and g_j the coupling strength between atom and cavity. The above atomic operators are defined as $\sigma_{+j} = |1\rangle_j\langle 0|$, $\sigma_{-j} = |0\rangle_j\langle 1|$, and $\sigma_{zj} = |0\rangle_j\langle 0| - |1\rangle_j\langle 1|$. Simply, the coherent driving of the cavity H_d in Eq. (3) reads

$$H_d = \hbar\epsilon(\hat{a}^\dagger e^{-i\omega_d t} + \hat{a}e^{i\omega_d t}), \quad (5)$$

where ϵ is time-independent real amplitude and ω_d the frequency of the drivings. For simplicity, we assume that the following dispersive condition [20–22]

$$0 < \frac{g_j}{\Delta_j}, \frac{g_j g_{j'}}{\Delta_j \Delta_{jj'}}, \frac{g_j g_{j'}}{\Delta_{j'}\Delta_{jj'}} \ll 1, \quad j \neq j' = 1,2,\ldots,N \quad (6)$$

are satisfied robustly, such that the energy exchanges between the different atoms and also the energy exchange between the atom and the cavity could be effectively neglected. Here, $\Delta_j = \omega_j - \omega_r$ is the detuning between the jth qubit and the cavity, and $\Delta_{jj'} = \omega_j - \omega_{j'}$ the detuning between the jth and j'th atoms.

The central task of this work is to calculate the correlation functions of the intercavity photons, which should be related to the coherence of the atoms in the cavity. As a consequence, the quantum statistical behaviors of the output field could be determined. Specifically, we discuss how to control the correlation of photons by engineering the coherence of a single and two dispersively coupled atom(s). The demonstrated arguments should also hold for more atoms.

A. Bunching photons by engineering a single atom coherence

For the simplest case, i.e., the cavity contains only one two-level atom, we have ($\hbar = 1$ throughout the paper)

$$H_1 + H_d = \frac{\tilde{\omega}_1}{2}\sigma_{z1} + (\Delta_r + \Gamma_1\sigma_{z1})\hat{a}^\dagger\hat{a} + \epsilon(\hat{a}^\dagger + \hat{a}), \quad (7)$$

in a frame rotating at the drive field frequency ω_d. Here, $\Delta_r = \omega_r - \omega_d$ is the detuning between the cavity and driving frequencies, and $\tilde{\omega}_1 = \omega_1 + \Gamma_1$, $\Gamma_1 = g_1^2/\Delta_1$.

Substituting Eq. (7) into Eq. (3), we have the following coupled differential equations related to the time-dependent average photon number:

$$\frac{d\langle\hat{a}^\dagger\hat{a}\rangle}{dt} = -i\epsilon(\langle\hat{a}^\dagger\rangle - \langle\hat{a}\rangle) - \kappa\langle\hat{a}^\dagger\hat{a}\rangle, \quad (8)$$

with

$$\begin{aligned} \frac{d\langle\hat{a}\rangle}{dt} &= -i\Delta_r\langle\hat{a}\rangle - i\Gamma_1\langle\hat{a}\sigma_{z1}\rangle - i\epsilon - \frac{\kappa}{2}\langle\hat{a}\rangle, \\ \frac{d\langle\hat{a}\sigma_{z1}\rangle}{dt} &= -i\Delta_r\langle\hat{a}\sigma_{z1}\rangle - i\Gamma_1\langle\hat{a}\rangle - \frac{\kappa}{2}\langle\hat{a}\sigma_{z1}\rangle - i\epsilon\langle\sigma_{z1}\rangle, \end{aligned} \quad (9)$$

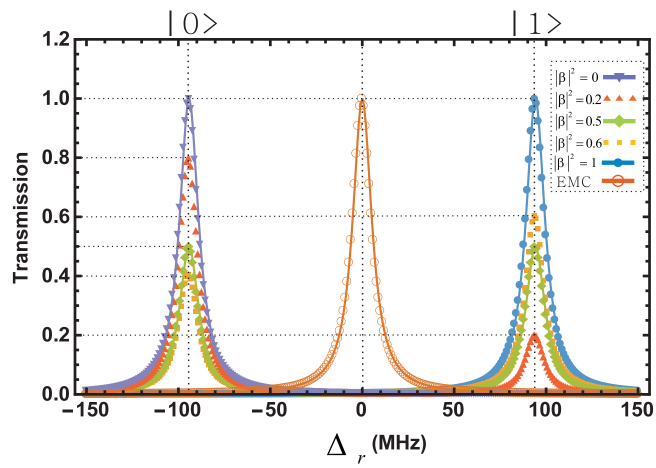

FIG. 1. Steady-state transmission spectra of the driven cavity (with a two-level atom) versus the detuning $\Delta_r = \omega_r - \omega_d$ between the cavity and driving for typical selected atomic states with $|\beta|^2 = 0,0.2,0.5,0.6,1$, respectively. As a comparison the empty-cavity (EMC) transmission is also plotted. The relevant parameters are chosen as $(\Gamma_1,\gamma) = 2\pi \times (15,1)$ MHz.

$$\begin{aligned} \frac{d\langle\hat{a}^\dagger\rangle}{dt} &= i\Delta_r\langle\hat{a}^\dagger\rangle + i\Gamma_1\langle\hat{a}^\dagger\sigma_{z1}\rangle + i\epsilon - \frac{\kappa}{2}\langle\hat{a}^\dagger\rangle, \\ \frac{d\langle\hat{a}^\dagger\sigma_{z1}\rangle}{dt} &= i\Delta_r\langle\hat{a}^\dagger\sigma_{z1}\rangle + i\Gamma_1\langle\hat{a}^\dagger\rangle + i\epsilon\langle\sigma_{z1}\rangle - \frac{\kappa}{2}\langle\hat{a}^\dagger\sigma_{z1}\rangle, \end{aligned} \quad (10)$$

$$\frac{d\langle\sigma_{z1}\rangle}{dt} = 0. \quad (11)$$

The steady-state photon number S transmitting through the cavity can be calculated as

$$S \propto \frac{\langle\hat{a}^\dagger\hat{a}\rangle_{ss}}{\epsilon^2} = \frac{\gamma^2 + \Gamma_1^2 - 2\langle\sigma_{z1}\rangle\Gamma_1\Delta_r + \Delta_r^2}{(\gamma^2 + \Gamma_1^2 - \Delta_r^2)^2 + 4\gamma^2\Delta_r^2}. \quad (12)$$

Here $\gamma = \kappa/2$. The steady-state transmission spectra of the driven cavity is schematized in Fig. 1. Certainly, if there is no atom in the cavity [i.e., for the empty-cavity (EMC) case] then the photons with the same frequency as the eigenfrequency ω_r of the cavity can transmit ideally through the cavity. The presence of the atom modifies significantly the transmission spectra of the driven photons, although it is dispersively coupled to the cavity. Specially, if the atom is prepared at one of its eigenstates $|0\rangle$ or $|1\rangle$, then the frequency of the ideal transmitted photons is shifted to $\omega_r - \Gamma_1$, or $\omega_r + \Gamma_1$. This agrees well with the experimental observations [23]. Therefore, the shifts of the transmitted peaks of the driven photons can be used to mark the eigenstates of the atom. Generally, if the atom is prepared at a generic coherence superposed state of its eigenstates, i.e., $|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$, then the two transmitted peaks, marking respectively the states $|0\rangle$ and $|1\rangle$, are revealed simultaneously. Interestingly, Fig. 1 shows that the relative heights of the two peaks are exactly equivalent to the relevant superposed probabilities $|\alpha|^2$ and $|\beta|^2$, respectively. This implies that the atomic coherence influences the transmission properties of the driven photon through the cavity.

In order to investigate how the quantum coherence of the dispersively coupled atom influences the quantum statistical properties of the intercavity field, we now calculate the relevant second-order correlation function $g^{(2)}(0)$ at the steady state. First, we find

$$\frac{d\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle}{dt} = -i2\epsilon(\langle\hat{a}^{\dagger 2}\hat{a}\rangle - \langle\hat{a}^{\dagger}\hat{a}^2\rangle) - 2\kappa\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle, \quad (13)$$

and

$$\frac{d\langle\hat{a}^{\dagger 2}\hat{a}\rangle}{dt} = i\Delta_r\langle\hat{a}^{\dagger 2}\hat{a}\rangle + i\Gamma_1\langle\hat{a}^{\dagger 2}\hat{a}\sigma_{z1}\rangle - i\epsilon(\langle\hat{a}^{\dagger 2}\rangle - 2\langle\hat{a}^{\dagger}\hat{a}\rangle) - \frac{3\kappa}{2}\langle\hat{a}^{\dagger 2}\hat{a}\rangle, \quad (14)$$

$$\frac{d\langle\hat{a}^{\dagger}\hat{a}^2\rangle}{dt} = -i\Delta_r\langle\hat{a}^{\dagger}\hat{a}^2\rangle - i\Gamma_1\langle\hat{a}^{\dagger}\hat{a}^2\sigma_{z1}\rangle - i\epsilon(2\langle\hat{a}^{\dagger}\hat{a}\rangle - \langle\hat{a}^2\rangle) - \frac{3\kappa}{2}\langle\hat{a}^{\dagger}\hat{a}^2\rangle. \quad (15)$$

It is emphasized that, under the usual mean-field approximation by neglecting any atom-field correlation [i.e., $\langle\hat{X}\sigma_\alpha\rangle \approx \langle\hat{X}\rangle\langle\sigma_\alpha\rangle$ holds for any field operator $\hat{X}$ and the atomic operator σ_α ($\alpha = z, +, -$)], one can easily check that the steady-state zero-delay second-order correlation function $g^{(2)}(0)$ of the intercavity field is always equivalent to 1. The detailed derivation is given in the Appendix. This implies that the intercavity field is also at the usual coherent state, without any bunching or antibunching effect. However, the atom-field correlation should be considered generically and the statistical properties of the intercavity field should deviate from those in the coherent states. To verify this argument, we need to deliver the exact dynamical equations for the expectable values of the atom-field operators:

$$\frac{d\langle\hat{a}^{\dagger 2}\hat{a}\sigma_{z1}\rangle}{dt} = i\Delta_r\langle\hat{a}^{\dagger 2}\hat{a}\sigma_{z1}\rangle + i\Gamma_1\langle\hat{a}^{\dagger 2}\hat{a}\rangle - i\epsilon(\langle\hat{a}^{\dagger 2}\sigma_{z1}\rangle - 2\langle\hat{a}^{\dagger}\hat{a}\sigma_{z1}\rangle) - \frac{3\kappa}{2}\langle\hat{a}^{\dagger 2}\hat{a}\sigma_{z1}\rangle, \quad (16)$$

$$\frac{d\langle\hat{a}^{\dagger}\hat{a}\sigma_{z1}\rangle}{dt} = -i\epsilon(\langle\hat{a}^{\dagger}\sigma_{z1}\rangle - \langle\hat{a}\sigma_{z1}\rangle) - \kappa\langle\hat{a}^{\dagger}\hat{a}\sigma_{z1}\rangle, \quad (17)$$

$$\frac{d\langle\hat{a}^{\dagger 2}\sigma_{z1}\rangle}{dt} = 2i\Delta_r\langle\hat{a}^{\dagger 2}\sigma_{z1}\rangle + 2i\Gamma_1\langle\hat{a}^{\dagger 2}\rangle + 2i\epsilon\langle\hat{a}^{\dagger}\sigma_{z1}\rangle - \kappa\langle\hat{a}^{\dagger 2}\sigma_{z1}\rangle, \quad (18)$$

$$\frac{d\langle\hat{a}^{\dagger}\hat{a}^2\sigma_{z1}\rangle}{dt} = -i\Delta_r\langle\hat{a}^{\dagger}\hat{a}^2\sigma_{z1}\rangle - i\Gamma_1\langle\hat{a}^{\dagger}\hat{a}^2\rangle - i\epsilon(2\langle\hat{a}^{\dagger}\hat{a}\sigma_{z1}\rangle - \langle\hat{a}^2\sigma_{z1}\rangle) - \frac{3\kappa}{2}\langle\hat{a}^{\dagger}\hat{a}^2\sigma_{z1}\rangle, \quad (19)$$

$$\frac{d\langle\hat{a}^2\sigma_{z1}\rangle}{dt} = -2i\Delta_r\langle\hat{a}^2\sigma_{z1}\rangle - 2i\Gamma_1\langle\hat{a}^2\rangle - 2i\epsilon\langle\hat{a}\sigma_{z1}\rangle - \kappa\langle\hat{a}^2\sigma_{z1}\rangle, \quad (20)$$

and also

$$\frac{d\langle\hat{a}^{\dagger 2}\rangle}{dt} = 2i\Delta_r\langle\hat{a}^{\dagger 2}\rangle + 2i\Gamma_1\langle\hat{a}^{\dagger 2}\sigma_{z1}\rangle + 2i\epsilon\langle\hat{a}^{\dagger}\rangle - \kappa\langle\hat{a}^{\dagger 2}\rangle, \quad (21)$$

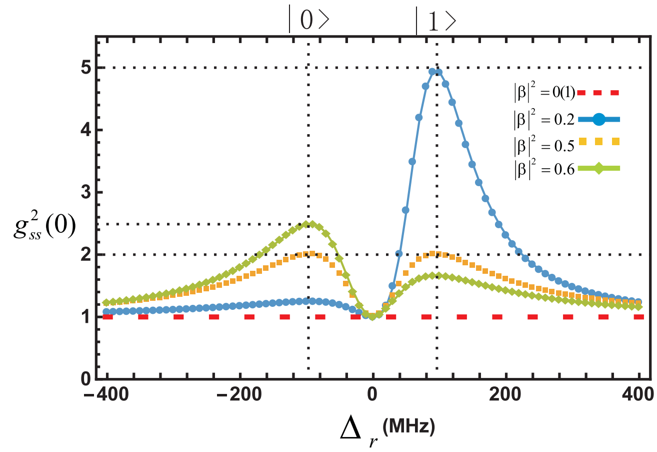

FIG. 2. The steady-state zero-delay second-order correlation function $g^{(2)}_{ss}(0)$ of the intercavity field versus the detuning Δ_r for single atomic states $|\beta|^2 = 0, 0.2, 0.5, 0.6, 1$, respectively. The other parameters are the same as in Fig. 1.

$$\frac{d\langle\hat{a}^2\rangle}{dt} = -2i\Delta_r\langle\hat{a}^2\rangle - 2i\Gamma_1\langle\hat{a}^2\sigma_{z1}\rangle - 2i\epsilon\langle\hat{a}\rangle - \kappa\langle\hat{a}^2\rangle. \quad (22)$$

The steady-state expectable values of $\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle$ and $\langle\hat{a}^{\dagger}\hat{a}\rangle$ can be obtained by solving the above coupled equations under the usual steady-state conditions. As a consequence, the zero-delay second-order correlation function $g^{(2)}(0)$ of the intercavity field can be expressed as

$$g^{(2)}_{ss}(0) = \frac{\gamma^4 + A\gamma^2 + \Gamma_1^4 + B\langle\sigma_{z1}\rangle + 6\Gamma_1^2\Delta_r^2 + \Delta_r^4}{\left(-\gamma^2 - \Gamma_1^2 + 2\langle\sigma_{z1}\rangle\Gamma_1\Delta_r - \Delta_r^2\right)^2}, \quad (23)$$

where $A = 2\Gamma_1^2 - 4\langle\sigma_{z1}\rangle\Gamma_1\Delta_r + 2\Delta_r^2$ and $B = -4\Gamma_1\Delta_r(\Gamma_1^2 - \Delta_r^2)$. It is easily seen from Eq. (23) that, the zero-delay second-order correlation function of the coherent driving field does not change after it transmits through the cavity, i.e., $g^{(2)}_{ss}(0) \equiv 1$ for the output field, if (i) $\Gamma_1 = 0$, i.e., there is no atom in the cavity or the atom in the cavity decoupled completely from the cavity; or (ii) $\langle\sigma_{z1}\rangle = \mp 1$, i.e., the dispersively coupled atom is prepared at one of its eigenstates. This indicates that, for these cases the photons through the cavity are still in the coherent states without being bunching. However, the situation is very different if the atom in the cavity is prepared at the superposition of its eigenstates, e.g., $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $\alpha, \beta \neq 0$. Indeed, Fig. 2 shows clearly that the value of the calculated zero-delay second-order correlation function $g^{(2)}_{ss}(0)$ is strongly related to the atomic coherence, i.e., $g^{(2)}(0) > 1$ is always satisfied, except at the zero-detuning point (but in this case the photons with the same frequency as the cavity cannot transmit through the cavity practically). Particularly, for the photons with the frequency shifts Γ_1 and $-\Gamma_1$, which mark, respectively, the eigenstates of the atom (see Fig. 1), one can find that the value of the above second correlation function directly depends on the superposed probability of the atomic states in the cavity. For example, if $\Delta_r = \Gamma_1$, then

$$g^{(2)}_{ss}(0) = \frac{\gamma^4 + 4\gamma^2\Gamma_1^2 - 4\langle\sigma_{z1}\rangle\gamma^2\Gamma_1^2 + 8\Gamma_1^4 - 8\langle\sigma_{z1}\rangle\Gamma_1^4}{\left(-\gamma^2 - 2\Gamma_1^2 + 2\langle\sigma_{z1}\rangle\Gamma_1^2\right)^2}. \quad (24)$$

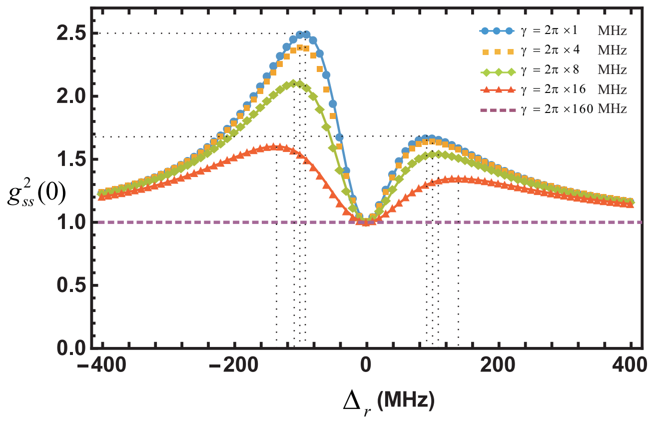

FIG. 3. The zero-delay second-order correlation function $g^{(2)}(0)$ for the single-qubit states vs the probe detuning with $|\beta|^2$ selected as 0.6. The parameter Γ_1 is chosen as $2\pi \times 15$ MHz and γ is chosen as $2\pi(1,4,8,16,160)$ MHz.

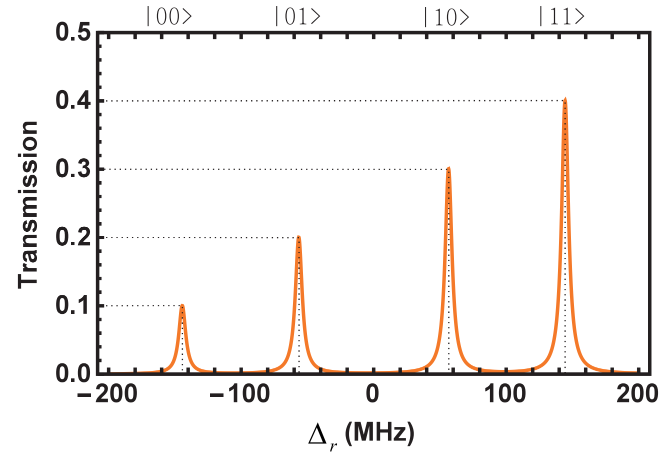

FIG. 4. The steady-state transmission spectra of the driven cavity with the two-atom superposed state: $|\psi_2\rangle = \sqrt{0.1}|00\rangle + \sqrt{0.2}|01\rangle + \sqrt{0.3}|10\rangle + \sqrt{0.4}|11\rangle$, versus the detuning Δ_r. The other parameters are chosen as $(\Gamma_1,\Gamma_2,\gamma) = 2\pi(7,16,0.5)$ MHz.

Similarly, for $\Delta_r = -\Gamma_1$, we have

$$g_{ss}^{(2)}(0) = \frac{\gamma^4 + 4\gamma^2\Gamma_1^2 + 4\langle\sigma_{z1}\rangle\gamma^2\Gamma_1^2 + 8\Gamma_1^4 + 8\langle\sigma_{z1}\rangle\Gamma_1^4}{\left(-\gamma^2 - 2\Gamma_1^2 - 2\langle\sigma_{z1}\rangle\Gamma_1^2\right)^2}. \tag{25}$$

From Eqs. (24) and (25) we find that, if the conditions

$$\begin{aligned}&(1-\langle\sigma_{z1}\rangle)^2\Gamma_1^4 \gg (1-\langle\sigma_{z1}\rangle)\gamma^2\Gamma_1^2,\ \gamma^4,\\ &(1+\langle\sigma_{z1}\rangle)^2\Gamma_1^4 \gg (1+\langle\sigma_{z1}\rangle)\gamma^2\Gamma_1^2,\ \gamma^4\end{aligned} \tag{26}$$

are satisfied, the zero-delay second-order correlation function can be simplified as

$$g_{\Gamma_1}^{(2)}(0) \sim \frac{2}{1-\langle\sigma_{z1}\rangle} = \frac{1}{|\beta|^2} \tag{27}$$

for $\Delta_r = \Gamma_1$ and

$$g_{-\Gamma_1}^{(2)}(0) \sim \frac{2}{1+\langle\sigma_{z1}\rangle} = \frac{1}{|\alpha|^2} \tag{28}$$

for $\Delta_r = -\Gamma_1$. Specifically, the conditions (26) can be simplified as

$$|\Gamma_1| \gg \gamma, \tag{29}$$

for most superposition states with the value of $\langle\sigma_{z1}\rangle$ being not close to ±1. This indicates that the value of $g_{ss}^{(2)}(0)$ could be engineered to be significantly large, and thus the strong bunching effects of the intercavity photons can be delivered by properly setting the atomic coherence in the driven cavity.

Certainly, the above arguments will be deviated, if condition (29) cannot be satisfied well. Indeed, Fig. 3 shows specifically that, for the superposition state $|\psi_1'\rangle = \sqrt{0.4}|0\rangle + \sqrt{0.6}|1\rangle$, the value of the steady-state correlation function $g_{ss}^{(2)}(0)$ decreases and the locations of the peaks deviate from those of the maximal transmissions as the decay rate γ increases.

B. Two-atom coherence and photon bunchings

The above demonstrations on single-atom coherence engineering the photon bunchings can be easily extended to the two-atom case. The transition frequencies of the two individual atoms in the driven cavity are represented as ω_1 and ω_2, respectively. The effective Hamiltonian of two individual atoms dispersively interacting with the cavity is described as

$$H_2 = \sum_{j=1}^{2}\frac{\tilde{\omega}_j}{2}\sigma_{zj} + \left(\Delta_r + \sum_{j=1}^{2}\Gamma_j\sigma_{zj}\right)\hat{a}^\dagger\hat{a}, \tag{30}$$

where $\tilde{\omega}_j = \omega_j + \Gamma_j$ and $\Gamma_j = g_j^2/\Delta_j$.

Again, the second-order correlation function $g^{(2)}(0)$ can be calculated by solving the system's master equation for the driven cavity with H_2 beyond the usual mean-field approximation again. Obviously, in this case the state-dependent frequency shift of the cavity should be jointly determined by the states of the two atoms. For example, the frequency shifts of the cavity are $-\Gamma_1 - \Gamma_2$, $-\Gamma_1 + \Gamma_2$, $\Gamma_1 - \Gamma_2$, and $\Gamma_1 + \Gamma_2$, corresponding to the two-atom computational basis states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, respectively. In order to get good resolution of the transmission peaks, we assume the condition $|\Gamma_2| - |\Gamma_1| \gg 0$. Consequently, the central frequencies of the steady-state transmission spectra mark the two-atom computational basic states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$; and their relative heights present, respectively, their superposed probabilities [24]: $|\alpha_1|^2$, $|\alpha_2|^2$, $|\alpha_3|^2$, and $|\alpha_4|^2$ in the two-atom state $|\psi\rangle = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle$, respectively. Specifically, Fig. 4 shows the steady-state transmission spectrum for a two-atom state:

$$|\psi_2\rangle = \sqrt{0.1}|00\rangle + \sqrt{0.2}|01\rangle + \sqrt{0.3}|10\rangle + \sqrt{0.4}|11\rangle. \tag{31}$$

The zero-delay second-order correlation function $g^{(2)}(0)$ can be similarly determined by calculating the steady-state expected values of the relevant cavity operators. For example, the dynamical equation of the expected value of $\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle$ reads

$$\frac{d\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle}{dt} = -2i\epsilon(\langle\hat{a}^{\dagger 2}\hat{a}\rangle - \langle\hat{a}^\dagger\hat{a}^2\rangle) - 2\kappa\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle. \tag{32}$$

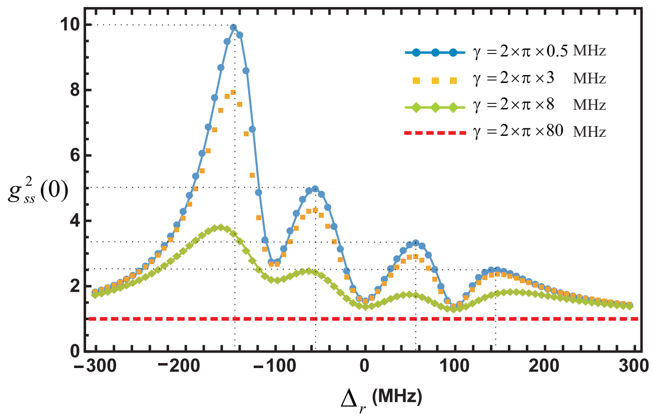

FIG. 5. The steady-state second-order correlation function of the intercavity for the two-atom state $|\psi_2\rangle$ versus the detuning Δ_r. The relevant parameters are chosen as $(\Gamma_1,\Gamma_2) = 2\pi(7,16)$ MHz, $(\gamma) = 2\pi(0.5,3,8,80)$ MHz.

This is further related to the following dynamical equations ($i = 0,1,2,3$):

$$\frac{d\langle \hat{a}B_i\rangle}{dt} = -i\Delta_r\langle \hat{a}B_i\rangle - i\sum_{j=1}^{2}\Gamma_j\langle \hat{a}B_i\sigma_{zj}\rangle + i\epsilon\langle B_i\rangle - \frac{\kappa}{2}\langle \hat{a}B_i\rangle, \quad (33)$$

$$\frac{d\langle \hat{a}^\dagger B_i\rangle}{dt} = i\Delta_r\langle \hat{a}^\dagger B_i\rangle + i\sum_{j=1}^{2}\Gamma_j\langle \hat{a}^\dagger B_i\sigma_{zj}\rangle + i\epsilon\langle B_i\rangle - \frac{\kappa}{2}\langle \hat{a}^\dagger B_i\rangle, \quad (34)$$

$$\frac{d\langle \hat{a}^2 B_i\rangle}{dt} = -2i\Delta_r\langle \hat{a}^2 B_i\rangle - 2i\sum_{j=1}^{2}\Gamma_j\langle \hat{a}^2 B_i\sigma_{zj}\rangle - 2i\epsilon\langle \hat{a}B_i\rangle - \kappa\langle \hat{a}^2 B_i\rangle, \quad (35)$$

$$\frac{d\langle \hat{a}^{\dagger 2} B_i\rangle}{dt} = 2i\Delta_r\langle \hat{a}^{\dagger 2} B_i\rangle + 2i\sum_{j=1}^{2}\Gamma_j\langle \hat{a}^{\dagger 2} B_i\sigma_{zj}\rangle + 2i\epsilon\langle \hat{a}^\dagger B_i\rangle - \kappa\langle \hat{a}^{\dagger 2} B_i\rangle, \quad (36)$$

$$\frac{d\langle \hat{a}^\dagger \hat{a} B_i\rangle}{dt} = -i\epsilon(\langle \hat{a}^\dagger B_i\rangle - \langle \hat{a}B_i\rangle) - \kappa\langle \hat{a}^\dagger \hat{a} B_i\rangle, \quad (37)$$

and

$$\frac{d\langle A_1 B_i\rangle}{dt} = i\Delta_r\langle A_1 B_i\rangle + i\sum_{j=1}^{2}\Gamma_j\langle A_1 B_i\sigma_{zj}\rangle - i\epsilon(\langle \hat{a}^{\dagger 2} B_i\rangle - 2\langle \hat{a}^\dagger \hat{a} B_i\rangle) - \frac{3\kappa}{2}\langle A_1 B_i\rangle, \quad (38)$$

$$\frac{d\langle A_2 B_i\rangle}{dt} = -i\Delta_r\langle A_2 B_i\rangle - i\sum_{j=1}^{2}\Gamma_j\langle A_2 B_i\sigma_{zj}\rangle - i\epsilon(2\langle \hat{a}^\dagger \hat{a} B_i\rangle - \langle \hat{a}^2 B_i\rangle) - \frac{3\kappa}{2}\langle A_2 B_i\rangle, \quad (39)$$

$$\frac{d\langle B_i\rangle}{dt} = 0. \quad (40)$$

Above, we have defined $A_1 = \hat{a}^{\dagger 2}\hat{a}$, $A_2 = \hat{a}^\dagger \hat{a}^2$, $B_0 = 1$, $B_1 = \sigma_{z1}$, $B_2 = \sigma_{z2}$, and $B_3 = \sigma_{z1}\sigma_{z2}$, respectively. Solving the above dynamical equations at the steady state, the second-order correlation function $g^{(2)}_{ss}(0)$ can be determined and then shown schematically in Fig. 5. These numerical results imply again that the intercavity photons still show the bunching behaviors: $g^{(2)}(0) > 1$. Also, if condition (29) is satisfied, then the inverse relationship between $g^{(2)}_{ss}(0)$ and the superposed probabilities is revealed again at the frequencies of the cavity transmission peaks.

III. DISCUSSIONS AND CONCLUSIONS

Beyond the usual mean-field approximation, in this paper we exactly calculated the steady-state second-order correlation function of the photon transmitting through the driven cavity, dispersively coupled single- and two-atom states. The results showed that the zero-delay second-order correlation function of the photons is always larger than 1, if the atom(s) in the cavity is prepared at the superposition of its computational base states. Otherwise, we have $g^{(2)}_{ss}(0) \sim 1$. Interestingly, if the condition $|\Gamma_1| \gg \gamma$ is satisfied, we found that an inverse relationship between $g^{(2)}_{ss}(0)$ and the superposed probabilities of the computational base states exists. The proposal presented here provides an effective approach to generate the significantly strong bunching photons by engineering the atomic coherence. Note that in our derivations the atomic decay had been neglected for analytical simplicity. In fact, one can check straightforwardly that the main arguments delivered above are still robust, even when the relevant atomic decays are considered. Indeed, from the master equation including the atomic decay

$$\dot{\rho} = -i[H_N + H_d,\rho] - \frac{\kappa}{2}(-2\hat{a}\rho\hat{a}^\dagger + \hat{a}^\dagger\hat{a}\rho + \rho\hat{a}^\dagger\hat{a}) - \frac{\gamma_1}{2}(-2\sigma_-\rho\sigma_+ + \sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-), \quad (41)$$

with $\sigma_\pm$ being the atomic operators and γ_1 the decay of the atom, the relevant differential equations for various expectable values on the field and atomic operators, similarly to Eqs. (8)–(11) and (13)–(22), can also be delivered. Here, it is unnecessary to write all of them. Of course, Eq. (11) for the atomic operator σ_{z1} without atomic decay should be replaced as

$$\frac{d\langle \sigma_{z1}\rangle}{dt} = -\gamma_1(\langle \sigma_{z1}\rangle + 1), \quad (42)$$

for the atomic decay γ_1. This formally results in the decay of the expected value of atomic operator σ_{z1}, i.e., $\langle \sigma_{z1}(t)\rangle = \exp(-\gamma_1 t)[\langle \sigma_{z1}(0)\rangle + 1] - 1$. However, the time to finish a single detection of the photon through the cavity is about $T = 40$ ns [25], which is significantly shorter than the decoherence

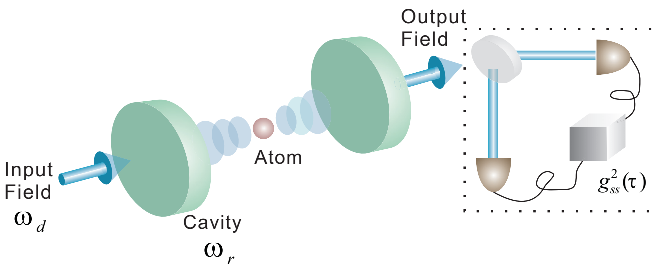

FIG. 6. A feasible scheme to generate and verify high bunching photons; the detectable steady-state second-order correlation function of the output field can be effectively enhanced by controlling the atomic superposed states.

time $T_1 \sim 7.3\ \mu$s [26] of the atom. This implies that the decay of the atom can be neglected really during the detection of the transmitted photon. Indeed, the decay time $T_1 \sim 7.3\ \mu$s [26] of the atom indicates the atomic decay rate $\gamma_1/2\pi \sim 0.02$ MHz [27], which is significantly less than the cavity's decay rate $\kappa/2\pi = 1.69$ MHz [25]. Therefore, even the decay of the atom is considered; all the numerical results demonstrated previously without the atomic decay should not be influenced manifestly. Our numerical results with the experimental atomic decay [27] really verify such an argument, which means that all the derivations in the main text are robust even when the atomic decay is considered.

Experimentally, the predictions delivered in the present work without performing the usual mean-field approximation could be immediately tested by measuring the $g^{(2)}_{ss}(\tau)$ parameter by the usual HBT setup (see Fig. 6). Probably, one of the potential challenges for the experiment is that the atom-cavity interaction is required to work in the dispersive regime. This is solvable, as the frequency of either the cavity or the artificial atom (e.g., superconducting qubits) is adjustable in principle. Therefore, the present proposal to generate the photons with significantly strong bunching effects is feasible.

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APPENDIX

In this Appendix, we verify, in detail, that the predicted photon bunchings cannot be delivered by performing the usual mean-field approximation. In fact, under the usual mean-field approximation by neglecting any atom-field correlation, e.g., $\langle \hat{X}\sigma_\alpha\rangle \approx \langle \hat{X}\rangle\langle\sigma_\alpha\rangle$ [25] for field operator $\hat{X}$ and single-atomic operator σ_α, $(\alpha = +,-,z)$, we have the following exact dynamical equations for the driven cavity with a single dispersively coupled two-level atom:

$$\frac{d\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle}{dt} = -i2\epsilon(\langle \hat{a}^{\dagger 2}\hat{a}\rangle - \langle \hat{a}^{\dagger}\hat{a}^2\rangle) - 2\kappa\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle, \tag{A1}$$

$$\frac{d\langle \hat{a}^{\dagger 2}\hat{a}\rangle}{dt} = i\Delta_r\langle \hat{a}^{\dagger 2}\hat{a}\rangle + i\Gamma_1\langle \hat{a}^{\dagger 2}\hat{a}\rangle\langle\sigma_{z1}\rangle - i\epsilon(\langle \hat{a}^{\dagger 2}\rangle - 2\langle \hat{a}^{\dagger}\hat{a}\rangle) - \frac{3\kappa}{2}\langle \hat{a}^{\dagger 2}\hat{a}\rangle, \tag{A2}$$

$$\frac{d\langle \hat{a}^{\dagger}\hat{a}^2\rangle}{dt} = -i\Delta_r\langle \hat{a}^{\dagger}\hat{a}^2\rangle - i\Gamma_1\langle \hat{a}^{\dagger}\hat{a}^2\rangle\langle\sigma_{z1}\rangle - i\epsilon(2\langle \hat{a}^{\dagger}\hat{a}\rangle - \langle \hat{a}^2\rangle) - \frac{3\kappa}{2}\langle \hat{a}^{\dagger}\hat{a}^2\rangle. \tag{A3}$$

$$\frac{d\langle \hat{a}^{\dagger 2}\rangle}{dt} = 2i\Delta_r\langle \hat{a}^{\dagger 2}\rangle + 2i\Gamma_1\langle \hat{a}^{\dagger 2}\rangle\langle\sigma_{z1}\rangle + 2i\epsilon\langle \hat{a}^{\dagger}\rangle - \kappa\langle \hat{a}^{\dagger 2}\rangle, \tag{A4}$$

$$\frac{d\langle \hat{a}^2\rangle}{dt} = -2i\Delta_r\langle \hat{a}^2\rangle - 2i\Gamma_1\langle \hat{a}^2\rangle\langle\sigma_{z1}\rangle - 2i\epsilon\langle \hat{a}\rangle - \kappa\langle \hat{a}^2\rangle. \tag{A5}$$

$$\frac{d\langle \hat{a}^{\dagger}\hat{a}\rangle}{dt} = -i\epsilon(\langle \hat{a}^{\dagger}\rangle - \langle \hat{a}\rangle) - \kappa\langle \hat{a}^{\dagger}\hat{a}\rangle, \tag{A6}$$

$$\frac{d\langle \hat{a}\rangle}{dt} = -i\Delta_r\langle \hat{a}\rangle - i\Gamma_1\langle \hat{a}\rangle\langle\sigma_{z1}\rangle - i\epsilon - \frac{\kappa}{2}\langle \hat{a}\rangle, \tag{A7}$$

$$\frac{d\langle \hat{a}^{\dagger}\rangle}{dt} = i\Delta_r\langle \hat{a}^{\dagger}\rangle + i\Gamma_1\langle \hat{a}^{\dagger}\rangle\langle\sigma_{z1}\rangle + i\epsilon - \frac{\kappa}{2}\langle \hat{a}^{\dagger}\rangle, \tag{A8}$$

respectively. The steady-state solutions to these equations deliver that

$$\langle \hat{a}^{\dagger}\hat{a}\rangle = \frac{\epsilon^2}{(\gamma - i\langle\sigma_{z1}\rangle\Gamma_1 - i\Delta_r)(\gamma + i\langle\sigma_{z1}\rangle\Gamma_1 + i\Delta_r)}, \tag{A9}$$

$$\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle = \frac{\epsilon^4}{(\gamma - i\langle\sigma_{z1}\rangle\Gamma_1 - i\Delta_r)^2(\gamma + i\langle\sigma_{z1}\rangle\Gamma_1 + i\Delta_r)^2}. \tag{A10}$$

Obviously, under such an approximation the steady-state zero-delay second-order correlation function of the intercavity filed reads

$$g^{(2)}(0) = \frac{\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle}{\langle \hat{a}^{\dagger}\hat{a}\rangle^2} \equiv 1, \tag{A11}$$

which implies that the field in the cavity should be still at the coherent state, without any bunching or antibunching behavior. This argument can also be delivered for the driven cavity with two dispersively coupled two-level atoms. Therefore, the predicted bunching effect, in the main text, of the intercavity photons could not explained by the usual mean-field approximation. Instead, it is due to the atom-field correlations beyond the mean-field approximation.

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