

Near-surface effect on interatomic resonance interactionQi-Zhang Yuan, Chun-Hua Yuan,^{*} and Weiping Zhang*State Key Laboratory of Precision Spectroscopy, Quantum Institute for Light and Atoms, Department of Physics, East China Normal University, Shanghai 200062, People's Republic of China*

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We study theoretically the resonance interaction between two atoms located near a dielectric-plate surface with quantum electrodynamics. One atom is initially excited and the other one is in the ground state. The effect of the surface exerted on the atoms can sufficiently change the interatomic resonance interaction. This near-surface effect on resonance interaction can be used to control atoms and can help to build on-chip quantum devices, new types of light sources, and quantum storage devices.

DOI: [10.1103/PhysRevA.93.032517](https://doi.org/10.1103/PhysRevA.93.032517)**I. INTRODUCTION**

In recent years, the atom chip has been a hot research domain in quantum physics [1,2]. To meet the development trends of integration and miniaturization of quantum experimental installations, a good method is to manipulate atomic ensembles near a chip. The atom chip has broad use in technologies such as optical communication, quantum storage, and quantum control. Moreover, the atom chip can reduce the cost and increase the reliability of experiments [3]. In the atom-chip experiments, the atom-chip distance can fall to the nanometer scale, and in this case the interaction between the atom and chip becomes sufficiently strong. Furthermore, the interatomic interaction will also be affected by the surface. The nature of the atom chip and interatomic interactions is one kind of mechanical effect of the vacuum electromagnetic field, which is generally called the Casimir effect, named after the Dutch physicist Hendrik Casimir. The Casimir effect is an important quantum phenomenon and has continued to attract the interest of researchers since the publication of the famous work on the attraction between two uncharged conducting plates in 1948 [4]. In recent decades, the Casimir effect lies at the core of developing nano-engineering [5–7], and research on the repulsive Casimir force is especially hot [8–22].

The vacuum electromagnetic field can lead to a mechanical force between macro subjects as well as between neutral microparticles, such as atoms or molecules [23]. By exchanging virtual photons, two neutral atoms (or one atom and one surface) can interact with each other, and the vacuum electromagnetic field acts as a medium in the interaction. Generally, the Van de Waals interaction is used to name the interaction between atoms, while the interaction between one atom and the surface of one macro subject is called the Casimir–Polder interaction. The interaction between two identical atoms with one of them initially in an excited state is called the resonance interaction (RI) [24–26]. The RI can lead to some collective behavior of the atoms, for example, collective spontaneous emission [27–31] and the collective atomic level shift [32]. Therefore, it plays an important role in laser cooling [33], producing entanglement [34,35], quantum storage [36], cold-molecule formation [37–39], quantum transmission, and building new types of quantum light sources [40].

The modifications of RI have been studied extensively over the past several decades, such as by using microcavities [41], nanofibers [42], waveguides [43], etc. Because of the broad use of RI and the fast development of atom-chip technology, we think it is significant to combine these two areas and study the RI between atoms near the surface of a chip. Because the surface is a boundary for the electromagnetic field, the density of states of the vacuum electromagnetic field near the surface must change, and the RI between the atoms located near the surface must be affected. On the other hand, by making use of this effect, we can change how atoms behave with one surface.

For these reasons, investigating the effect exerted on atoms by a nearby surface becomes important. In this paper, we study theoretically the RI between two atoms located near a dielectric-plate surface. For the atoms very close to the surface, we approximately treat the surface as infinitely large. Quantum electrodynamics is employed to describe the interaction between the vacuum electromagnetic field and the two atoms, whereas the reflection of electromagnetic waves from the dielectric plate is calculated by the classical Maxwell theory. The retardation effect of light in propagation is brought into consideration. We analytically calculated the interatomic RI characterized by the spontaneous radiation rates and the atomic level shifts. Our results show that, when the atoms are located near a surface, the RI depends on not only the interatomic displacement but also the atom-surface distance. Because the effect of the surface on interatomic RI becomes notable only when atoms are very close to the surface, we call the effect the “near-surface effect” (NSE). The NSE provides the possibility of controlling RI by using a surface.

This paper is organized as follows: The formalism of the interaction between two atoms and the electromagnetic vacuum is reviewed in Sec. II, then we introduce the mode function of the vacuum electromagnetic field in Sec. III, followed by Sec. IV in which we calculate the damping rates and level shifts of atomic excited states that characterize the interatomic RI. Some numerical examples are given, and the properties of the NSE are discussed in Sec. V. Finally, we conclude in Sec. VI.

II. DYNAMICS OF INTERATOMIC RESONANCE INTERACTION

In this section, a brief review of the dynamical analysis of dipole-reservoir interaction will be given. The system we

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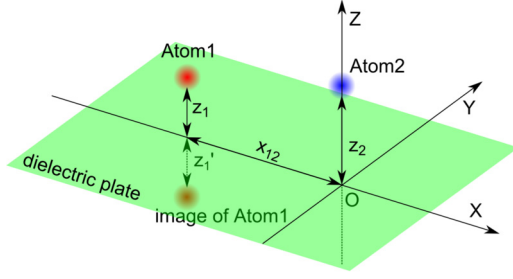


FIG. 1. The scheme investigated is illustrated. The dielectric surface is located in the x - y plane. The distance between atom 1 (atom 2) and the surface is labeled as z_1 (z_2). The distance between the drop shadows of the two atoms on surface is x_{12} .

studied is shown in Fig. 1. We use 1 and 2 to label the two atoms. z_1 (z_2) is used to represent the distance from atom 1 (2) to the dielectric-plane surface. Without loss of generality, we assume the two atoms are all located in the x - z plane, and the distance between the drop shadows of the two atoms on the surface is x_{12} .

We calculate the atom-light interaction problem in the Schrödinger picture. The two atoms are marked as atom 1 and atom 2. The quantum state of the system can be expressed by

$$|\psi\rangle = \sum_{i=1,2} C_i |i\rangle |0\rangle + \sum_{i=3,4} \sum_{\mathbf{k}\lambda} D_{i,\mathbf{k}\lambda} |i\rangle |\mathbf{k}\lambda\rangle, \quad (1)$$

in which the basis kets $|1\rangle$ – $|4\rangle$ are

$$\begin{aligned} |1\rangle &= |e_1 g_2\rangle, & |2\rangle &= |g_1 e_2\rangle, \\ |3\rangle &= |e_1 e_2\rangle, & |4\rangle &= |g_1 g_2\rangle, \end{aligned} \quad (2)$$

where $|e\rangle$ and $|g\rangle$ are the quantum states of atoms. $|g_i\rangle$ represents atom i in the ground state and $|e_j\rangle$ means atom j in the excited state. For example, the ket $|1\rangle = |e_1 g_2\rangle$ means atom 1 is excited while atom 2 is in the ground state. The electromagnetic field state $|\mathbf{k}\lambda\rangle$ means a photon with wave vector \mathbf{k} and polarization λ exists, while $|0\rangle$ means there is no photon (the electromagnetic vacuum state). C_i and $D_{i,\mathbf{k}\lambda}$ are probability amplitudes. We bring $|3\rangle = |e_1 e_2\rangle$ into consideration because we hold the counter-rotating wave terms in the interaction Hamiltonian, which make transitions $|1\rangle|0\rangle \leftrightarrow |3\rangle|\mathbf{k}\lambda\rangle \leftrightarrow |2\rangle|0\rangle$ possible. For the eigenenergy of intermediate state $|3\rangle|\mathbf{k}\lambda\rangle$ to be different from that of initial state $|1\rangle|0\rangle$, $|2\rangle|0\rangle$, and final state $|4\rangle|\mathbf{k}_0\lambda\rangle$, state $|3\rangle|\mathbf{k}\lambda\rangle$ is short lived. When the atom-light system jumps to state $|3\rangle|\mathbf{k}\lambda\rangle$, it will jump back to $|1\rangle|0\rangle$ or $|2\rangle|0\rangle$ in a very short time. For this reason, all the eigenstates with photon number larger than 1 can at best negligibly contribute to the atom-light interaction and have been dropped in Eq. (1) [44,45].

The total Hamiltonian \hat{H} can be divided into three parts: the atomic Hamiltonian \hat{H}_A , the electromagnetic Hamiltonian \hat{H}_F , and the atom-light interaction Hamiltonian \hat{H}_{int} . The atomic Hamiltonian can be written as

$$\hat{H}_A = \hbar \sum_i \omega_i \sigma_{ii}, \quad (3)$$

where $\sigma_{ij} = |i\rangle\langle j|$ are the atomic flip operators. The electromagnetic Hamiltonian is

$$\hat{H}_F(\mathbf{r}) = \sum_{\mathbf{k}\lambda} \hbar \omega_{\mathbf{k}} \left(a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} + \frac{1}{2} \right), \quad (4)$$

where $a_{\mathbf{k}\lambda}^\dagger$ and $a_{\mathbf{k}\lambda}$ are creation and annihilation operators for the photon of mode $\mathbf{k}\lambda$, respectively. \hbar is the modified Planck constant and $\omega_{\mathbf{k}}$ is the angular frequency of the photon. The interaction Hamiltonian is

$$\hat{H}_{\text{int}} = - \sum_{a=1,2} \hat{\mathbf{d}}_a \cdot \hat{\mathbf{E}}(\mathbf{r}_a), \quad (5)$$

where $\hat{\mathbf{d}}_a = \sum_{ij} \mathbf{d}_{ij}^a \sigma_{ij}$ are the electric-dipole operators with \mathbf{d}_{ij}^a being the dipole transition matrix elements, and $\hat{\mathbf{E}}(\mathbf{r})$ is the electric-field operator, which is given by

$$\hat{\mathbf{E}}(\mathbf{r}) = \sum_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda} \hat{a}_{\mathbf{k}\lambda} \mathbf{E}(\mathbf{k}\lambda, \mathbf{r}) + \text{H.c.} \quad (6)$$

The electric wave mode function $\mathbf{E}(\mathbf{k}\lambda, \mathbf{r})$ is determined by the refractive index of the surrounding medium and the boundary conditions. $A_{\mathbf{k}\lambda}$ is the normalization coefficients.

By making use of the Schrödinger equation

$$i \hbar \frac{\partial |\psi\rangle}{\partial t} = \hat{H} |\psi\rangle, \quad (7)$$

we can get the equations of motion of the probability amplitudes C_i and $D_{i,\mathbf{k}\lambda}$:

$$\dot{C}_1 = \frac{i}{\hbar} \sum_{a=1,2} \sum_{i'=3,4} \sum_{\mathbf{k}\lambda} D_{i',\mathbf{k}\lambda} A_{\mathbf{k}\lambda} \mathbf{d}_{3i'}^a \cdot \mathbf{E}(\mathbf{k}\lambda, \mathbf{r}_a), \quad (8)$$

$$\dot{C}_2 = \frac{i}{\hbar} \sum_{a=1,2} \sum_{i'=3,4} \sum_{\mathbf{k}\lambda} D_{i',\mathbf{k}\lambda} A_{\mathbf{k}\lambda} \mathbf{d}_{2i'}^a \cdot \mathbf{E}(\mathbf{k}\lambda, \mathbf{r}_a), \quad (9)$$

$$\begin{aligned} \dot{D}_{3,\mathbf{k}\lambda} &= -i(\omega_{\mathbf{k}} + \omega_0) D_{3,\mathbf{k}\lambda} \\ &+ \frac{i}{\hbar} \sum_{a=1,2} \sum_{i'=1,2} C_{i'} A_{\mathbf{k}\lambda}^* \mathbf{d}_{3i'}^a \cdot \mathbf{E}^*(\mathbf{k}\lambda, \mathbf{r}_a), \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{D}_{4,\mathbf{k}\lambda} &= -i(\omega_{\mathbf{k}} - \omega_0) D_{4,\mathbf{k}\lambda} \\ &+ \frac{i}{\hbar} \sum_{a=1,2} \sum_{i'=1,2} C_{i'} A_{\mathbf{k}\lambda}^* \mathbf{d}_{4i'}^a \cdot \mathbf{E}^*(\mathbf{k}\lambda, \mathbf{r}_a). \end{aligned} \quad (11)$$

To solve this group of equations, we first calculate the formal solutions of Eqs. (10) and (11):

$$\begin{aligned} D_{3,\mathbf{k}\lambda}(t) &= \frac{i}{\hbar} A_{\mathbf{k}\lambda}^* \sum_{a=1,2} \sum_{i'=1,2} \mathbf{d}_{3i'}^a \cdot \mathbf{E}^*(\mathbf{k}\lambda, \mathbf{r}_a) C_{i'}(t) \\ &\times \left[\pi \delta(\omega_{\mathbf{k}} + \omega_0) - \frac{i}{(\omega_{\mathbf{k}} + \omega_0)} \right], \end{aligned} \quad (12)$$

$$\begin{aligned} D_{4,\mathbf{k}\lambda}(t) &= \frac{i}{\hbar} A_{\mathbf{k}\lambda}^* \sum_{a=1,2} \sum_{i'=1,2} \mathbf{d}_{4i'}^a \cdot \mathbf{E}^*(\mathbf{k}\lambda, \mathbf{r}_a) C_{i'}(t) \\ &\times \left[\pi \delta(\omega_{\mathbf{k}} - \omega_0) - \frac{i}{(\omega_{\mathbf{k}} - \omega_0)} \right], \end{aligned} \quad (13)$$

in which the Markov approximation has been used;

$$\int_0^t C_{i'}(t') e^{-i(\omega_k - \omega_0)(t-t')} dt' \approx C_{i'}(t) \left[\pi \delta(\omega_k - \omega_0) - \frac{i}{(\omega_k - \omega_0)} \right]. \quad (14)$$

Substituting Eqs. (12) and (13) into Eqs. (8) and (9), we get the equations for C_1 and C_2 :

$$\dot{C}_1 = A_{11}C_1(t) + A_{12}C_2(t), \quad (15)$$

$$\dot{C}_2 = A_{22}C_2(t) + A_{21}C_1(t), \quad (16)$$

in which the coefficients $A_{aa(b)}$ are defined by

$$A_{aa} = -\frac{1}{\hbar^2} \sum_{k\lambda} |A_{k\lambda}|^2 \left\{ G_{aa}(\mathbf{k}\lambda) \pi \delta(\omega_k - \omega_0) - G_{aa}(\mathbf{k}\lambda) \frac{i}{(\omega_k - \omega_0)} - G_{bb}(\mathbf{k}\lambda) \frac{i}{(\omega_k + \omega_0)} \right\},$$

$$A_{ab} = -\frac{1}{\hbar^2} \sum_{k\lambda} |A_{k\lambda}|^2 \left\{ G_{ba}(\mathbf{k}\lambda) \pi \delta(\omega_k - \omega_0) - G_{ba}(\mathbf{k}\lambda) \frac{i}{(\omega_k - \omega_0)} - G_{ab}(\mathbf{k}\lambda) \frac{i}{(\omega_k + \omega_0)} \right\}, \quad (17)$$

where $a = 1, 2$ and $b = 1, 2$ are the atom labels and $a \neq b$. The functions $G_{aa(b)}(\mathbf{k}\lambda) \equiv \mathbf{d}_a \cdot \mathbf{E}^*(\mathbf{k}\lambda, \mathbf{r}_a) \mathbf{E}(\mathbf{k}\lambda, \mathbf{r}_{a(b)}) \cdot \mathbf{d}_{a(b)}$, where the dipole transition matrix elements \mathbf{d}_{ij}^a are denoted by \mathbf{d}_a for simplicity. The coefficients A_{aa} ($a = 1, 2$) describe the interaction between one single atom a ($a = 1, 2$) and the vacuum field, while A_{ab} ($a, b = 1, 2$) describe the interaction between two atoms. Equations (15) and (16) can be worked out:

$$C_1(t) = C_2(0) e^{\frac{A_{11}+A_{22}}{2}t} \frac{A_{12}}{\Theta} (e^{\frac{1}{2}\Theta t} - e^{-\frac{1}{2}\Theta t}) + C_1(0) e^{\frac{A_{11}+A_{22}}{2}t} \frac{(A_{11} - A_{22} + \Theta) e^{\frac{1}{2}\Theta t} - (A_{11} - A_{22} - \Theta) e^{-\frac{1}{2}\Theta t}}{2\Theta},$$

$$C_2(t) = C_1(0) e^{\frac{A_{11}+A_{22}}{2}t} \frac{A_{21}}{\Theta} (e^{\frac{1}{2}\Theta t} - e^{-\frac{1}{2}\Theta t}) - C_2(0) e^{\frac{A_{11}+A_{22}}{2}t} \frac{(A_{11} - A_{22} - \Theta) e^{\frac{1}{2}\Theta t} - (A_{11} - A_{22} + \Theta) e^{-\frac{1}{2}\Theta t}}{2\Theta}, \quad (18)$$

where

$$\Theta = \sqrt{(A_{11} - A_{22})^2 + 4A_{12}A_{21}}.$$

In the initial condition that $C_1(0) = 1$ and $C_2(0) = 0$, we have

$$C_1(t) = e^{\frac{A_{11}+A_{22}}{2}t} \frac{(A_{11} - A_{22} + \Theta) e^{\frac{1}{2}\Theta t} - (A_{11} - A_{22} - \Theta) e^{-\frac{1}{2}\Theta t}}{2\Theta}, \quad C_2(t) = e^{\frac{A_{11}+A_{22}}{2}t} \frac{A_{21}}{\Theta} (e^{\frac{1}{2}\Theta t} - e^{-\frac{1}{2}\Theta t}). \quad (19)$$

According to Eq. (5), the interaction energy between the atom pair and the surface can be calculated by

$$E_{\text{int}} = \langle \hat{H}_{\text{int}} \rangle = - \sum_{a=1,2} \sum_{ij} \sum_{k\lambda} A_{k\lambda} \mathbf{d}_{ij}^a \cdot \mathbf{E}(\mathbf{k}\lambda, \mathbf{r}_a) C_i^* D_{j,k\lambda} + \text{c.c.} = i\hbar(A_{11}C_1^*C_1 + A_{22}C_2^*C_2 + A_{12}C_1^*C_2 + A_{21}C_2^*C_1) + \text{c.c.} \quad (20)$$

The total interaction energy can be divided into two parts: the atom-surface resonance Casimir-Polder interaction (RCPI) energy

$$E_{\text{RCPI-}a} = i\hbar A_{aa} |C_a|^2 + \text{c.c.}, \quad (21)$$

and atom-atom RI energy

$$E_{\text{RI}} = i\hbar(A_{12}C_1^*C_2 + A_{21}C_2^*C_1) + \text{c.c.} \quad (22)$$

After calculating coefficients A_{aa} and A_{ab} ($a, b = 1, 2$) and using them in Eqs. (19)–(22), the behavior of the atom pair can be fully described. According to Eq. (21), $E_{\text{RCPI-}a}$ is proportional to the population of state $|a = 1, 2\rangle$, while the interatomic RI is proportional to the coherence between state $|1\rangle$ and $|2\rangle$. Different from the Van de Waals interaction and the Casimir-Polder interaction, the lifetime of RI and RCPI is not infinite because of spontaneous emission. Our results reflect this feature of RI and RCPI.

In next section, we define the mode function $\mathbf{E}(\mathbf{k}\lambda, \mathbf{r})$ and the normalization constant $A_{k\lambda}$ which are needed in calculating the detailed expressions of A_{aa} and A_{ab} ($a, b = 1, 2$).

III. QUANTIZATION OF ELECTROMAGNETIC FIELD NEAR DIELECTRIC SURFACE

In the previous section, we briefly reviewed the formulas describing the interatomic RI. To calculate the detailed expressions of A_{aa} and A_{ab} ($a, b = 1, 2$), we need to define the mode functions of the electromagnetic field near the surface. In this section, we introduce the method of quantization and present the mode function $\mathbf{E}(\mathbf{k}\lambda, \mathbf{r})$ as well as the normalization constant $A_{k\lambda}$. The total electric field can be expressed as

$$\mathbf{E}(\mathbf{r}) = \sum_{k\lambda} E_{k\lambda} \mathbf{E}(\mathbf{k}\lambda, \mathbf{r}) + \text{c.c.}, \quad (23)$$

in which $\mathbf{E}(\mathbf{k}\lambda, \mathbf{r})$ is the mode function. The mode function can be divided into incident wave and reflected waves:

$$\mathbf{E}(\mathbf{k}\lambda, \mathbf{r}) = \mathbf{W}(\mathbf{k}\lambda, \mathbf{r}) + \mathbf{S}(\mathbf{k}\lambda, \mathbf{r}).$$

The incident light is a plane wave,

$$\mathbf{W}(\mathbf{k}\lambda, \mathbf{r}) = \hat{\mathbf{e}}_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}},$$

and the reflected wave can be written as

$$S(\mathbf{k}\lambda, \mathbf{r}) = g_\lambda(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}'} e^{i\mathbf{k}' \cdot \mathbf{r}},$$

where $g_\lambda(\mathbf{k})$ are the reflection coefficients and λ refers to the polarization of light. \mathbf{k} is the wave vector of the incident wave, \mathbf{k}' refers to the reflected light, and i is the imaginary unit. Then we obtain the electric wave mode function

$$E(\mathbf{k}\lambda, \mathbf{r}) = \hat{\mathbf{e}}_{\mathbf{k}\lambda} e^{i\mathbf{k} \cdot \mathbf{r}} + g_\lambda(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}'\lambda} e^{i\mathbf{k}' \cdot \mathbf{r}}. \quad (24)$$

$\hat{\mathbf{e}}_{\mathbf{k}\lambda}$ are the unit vectors perpendicular to the wave vector \mathbf{k} . The expressions of $g_\lambda(\mathbf{k})$ are decided by the properties of the reflecting surface and the coordinate system we choose. To quantize the electric field, we replace the amplitude $E_{\mathbf{k}\lambda}$ in Eq. (23) with $A_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda}$ to obtain Eq. (6). According to Eq. (24), the normalization constant

$$A_{\mathbf{k}\lambda} = \frac{1}{\kappa_{\mathbf{k}\lambda}} \left(\frac{\hbar \omega_{\mathbf{k}}}{2\epsilon_0 V} \right)^{1/2}, \quad (25)$$

in which

$$\kappa_{\mathbf{k}\lambda} = \sqrt{1 + |g_\lambda(\mathbf{k})|^2}. \quad (26)$$

Having specified the normalization constant and the mode function, we can calculate the expressions of A_{aa} and A_{ab} ($a, b = 1, 2$) which characterize the behavior of the two interacting atoms.

$$\begin{aligned} \gamma_{\text{free}} &= \frac{\pi}{2\hbar\epsilon_0(2\pi)^3} \int_{-\infty}^{\infty} k^3 \delta(k - k_0) dk \int_{\Omega} d\Omega \sum_{\lambda} \frac{1}{\kappa_{\mathbf{k}\lambda}^2} \mathbf{d}_a \cdot [\hat{\mathbf{e}}_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda} + |g_\lambda(\mathbf{k})|^2 \hat{\mathbf{e}}_{\mathbf{k}'\lambda} \hat{\mathbf{e}}_{\mathbf{k}'\lambda}] \cdot \mathbf{d}_a, \\ \delta_{\text{free}} &= \frac{1}{2\hbar\epsilon_0(2\pi)^3} \int_{-\infty}^{\infty} \frac{k^3}{k - k_0} dk \int_{\Omega} d\Omega \sum_{\lambda} \frac{1}{\kappa_{\mathbf{k}\lambda}^2} \mathbf{d}_a \cdot [\hat{\mathbf{e}}_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda} + |g_\lambda(\mathbf{k})|^2 \hat{\mathbf{e}}_{\mathbf{k}'\lambda} \hat{\mathbf{e}}_{\mathbf{k}'\lambda}] \cdot \mathbf{d}_a. \end{aligned} \quad (30)$$

Before calculating the integrals, we must first choose a coordinate system and characterize the reflection coefficients. As shown in Fig. 2, we set the x - y plane on the dielectric surface, and the z direction is just the normal direction. The homogeneous, isotropic, dielectric half space is a bulk of continuum material, on which the reflection of light can be treated classically. In this coordinate system, for a plane-dielectric surface with permittivity $n(\omega_k)$, it is not difficult to calculate the Fresnel reflection coefficients by classical Maxwell theory,

$$\begin{aligned} g_{\perp}(\mathbf{k}) &= \frac{\cos \theta - n(\omega_k) \cos \theta''}{\cos \theta + n(\omega_k) \cos \theta''}, \\ g_{\parallel}(\mathbf{k}) &= \frac{n(\omega_k) \cos \theta - \cos \theta''}{n(\omega_k) \cos \theta + \cos \theta''}, \end{aligned} \quad (31)$$

where θ is the incident angle in vacuum while θ'' is the refraction angle in the dielectric material, satisfying $n(\omega_k) \sin \theta'' = \sin \theta$. ω_k is the angular frequency of the incident light. g_{\perp}

IV. DAMPING RATES AND ATOMIC LEVEL SHIFTS

According to Eqs. (21) and (22), A_{aa} describes the behavior of one single atom while A_{ab} refers to the collective properties of the two atoms. Since the mode function $E(\mathbf{k}\lambda, \mathbf{r})$ and the normalization constant $A_{\mathbf{k}\lambda}$ have been specified, their detailed expressions will be calculated in this section.

A. Behavior of one single atom

We first calculate A_{aa} in this section. Considering the atom pair to be located in a free half space, the continuum limit is

$$\sum_{\mathbf{k}} \rightarrow \frac{V}{(2\pi)^3} \int d^3k = \frac{V}{(2\pi)^3} \int_0^{\infty} k^2 dk \int_{\Omega} d\Omega,$$

where $\int_{\Omega} d\Omega$ represents angular integration. The coefficient A_{aa} can be divided into two parts,

$$A_{aa} = A_{aa, \text{free}} + A_{aa, r}, \quad (27)$$

in which

$$A_{aa, \text{free}} \equiv -\gamma_{\text{free}} + i\delta_{\text{free}}, \quad (28)$$

$$A_{aa, r} \equiv -\gamma_{s, r-a} + i\delta_{s, r-a}, \quad (29)$$

where γ_j and δ_j ($j = \text{free or } s, r$) are real parameters. We use γ and δ in Eqs. (28) and (29) because the real and imaginary part of A_{aa} are respectively related to the spontaneous radiation rate and the atomic level shift. The subscripts of the parameters are chosen according to their physical meaning. $A_{aa, \text{free}}$ refers to the interaction of a single atom with free vacuum field, while $A_{aa, r}$ describes the interaction between the atom and surface. Substituting Eqs. (24) and (25) into Eq. (17), we obtain

and g_{\parallel} are, respectively, the reflection coefficients of light with electric components polarized perpendicular and parallel

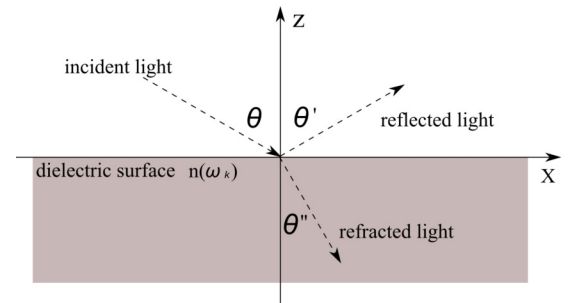


FIG. 2. The reflection of an electromagnetic wave from the dielectric surface with refraction index $n(\omega_k)$ is shown schematically. The surface is in the x - y plane and the normal direction is the z direction. θ is the incident angle, θ' is the reflection angle, and θ'' is the refraction angle.

to the incident plane. The properties of the surface are described by Eq. (31), which are decided by the dielectric permittivity of the surface $n(\omega_k)$. Making use of the reflection coefficients given by Eq. (31), the detailed expression of A_{aa} can be calculated. The angular integral can be written as $\int_{\Omega} d\Omega \rightarrow \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi$. For the surface rotationally symmetric with the z axis as the symmetry axis, the reflection coefficients $g_{\lambda}(\mathbf{k})$ are independent of ϕ . Given this, it is not difficult to prove

$$\begin{aligned} & \int_{\Omega} d\Omega \sum_{\lambda} \frac{1}{\kappa_{k\lambda}^2} (\hat{\mathbf{e}}_{k\lambda} \hat{\mathbf{e}}_{k\lambda} + |g_{\lambda}(\mathbf{k})|^2 \hat{\mathbf{e}}_{k'\lambda} \hat{\mathbf{e}}_{k'\lambda}) \\ &= \int_{\Omega} d\Omega \sum_{\lambda} \hat{\mathbf{e}}_{k\lambda} \hat{\mathbf{e}}_{k\lambda} = \frac{8}{3} \pi \mathbf{I}. \end{aligned} \quad (32)$$

By substituting Eq. (32) into Eq. (30), we obtain

$$\gamma_{\text{free}} = \frac{1}{2} \Gamma_0, \quad (33)$$

$$\delta_{\text{free}} = \frac{1}{2\pi k_0^3} \Gamma_0 \int_{-\infty}^{\infty} \frac{k^3}{k - k_0} dk, \quad (34)$$

where $\Gamma_0 = |\mathbf{d}_a|^2 k_0^3 / (3\hbar\pi\epsilon_0)$ is exactly the damping rate of a single atom in free vacuum. Equation (33) exactly agrees with the well-known result on spontaneous radiation of a single atom in free vacuum [46]. According to Eq. (33), γ_{free} is half of the single-atom free spontaneous radiation rate. It is natural that the damping rate of the probability amplitude is exactly half that of the probability. δ_{free} is the vacuum Lamb shift. The expression (34) is obviously divergent, which can be dealt with the Bethe's method [47] and is given by

$$\delta_{\text{free}} = \frac{1}{2\pi k_0^3} \Gamma_0 \int_{-\infty}^{\infty} \frac{k^3}{k - k_0} dk \rightarrow \frac{1}{2\pi} \Gamma_0 \ln\left(\frac{mc}{\hbar k_0}\right).$$

Because the free Lamb shift does not lead to any mechanical force in this problem, we ignore it in the following.

Furthermore, according to Eq. (27), $A_{aa,r}$ can be calculated by

$$\begin{aligned} \gamma_{s,r-a} &= \frac{\pi}{2\hbar\epsilon_0(2\pi)^3} \int_{-\infty}^{\infty} k^3 \delta(k - k_0) dk \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \sum_{\lambda} \frac{1}{\kappa_{k\lambda}^2} \mathbf{d}_a \cdot [g_{\lambda}(\mathbf{k}) \hat{\mathbf{e}}_{k\lambda} \hat{\mathbf{e}}_{k'\lambda} e^{-i2kz_a \cos\theta} + g_{\lambda}^*(\mathbf{k}) \hat{\mathbf{e}}_{k'\lambda} \hat{\mathbf{e}}_{k\lambda} e^{i2kz_a \cos\theta}] \cdot \mathbf{d}_a, \\ \delta_{s,r-a} &= \frac{1}{2\hbar\epsilon_0(2\pi)^3} \int_{-\infty}^{\infty} \frac{k^3}{k - k_0} dk \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \sum_{\lambda} \frac{1}{\kappa_{k\lambda}^2} \mathbf{d}_a \cdot [g_{\lambda}(\mathbf{k}) \hat{\mathbf{e}}_{k\lambda} \hat{\mathbf{e}}_{k'\lambda} e^{-i2kz_a \cos\theta} + g_{\lambda}^*(\mathbf{k}) \hat{\mathbf{e}}_{k'\lambda} \hat{\mathbf{e}}_{k\lambda} e^{i2kz_a \cos\theta}] \cdot \mathbf{d}_a. \end{aligned} \quad (35)$$

Substituting Eq. (31) into Eq. (35), we get

$$\gamma_{s,r-a} = \frac{3\Gamma_0}{8} \hat{\mathbf{d}}_a \cdot \left[\begin{aligned} & (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) \int_0^{\pi/2} \sin\theta \frac{\cos^2\theta - n^2(k_0)\cos^2\theta''}{\cos^2\theta + n^2(k_0)\cos^2\theta''} \cos(2k_0z_a \cos\theta) d\theta \\ & - (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) \int_0^{\pi/2} \sin\theta \cos^2\theta \frac{n^2(k_0)\cos^2\theta - \cos^2\theta''}{n^2(k_0)\cos^2\theta + \cos^2\theta''} \cos(2k_0z_a \cos\theta) d\theta \\ & + 2\hat{\mathbf{z}}\hat{\mathbf{z}} \int_0^{\pi/2} \sin^3\theta \frac{n^2(k_0)\cos^2\theta - \cos^2\theta''}{n^2(k_0)\cos^2\theta + \cos^2\theta''} \cos(2k_0z_a \cos\theta) d\theta \end{aligned} \right] \cdot \hat{\mathbf{d}}_a, \quad (36)$$

$$\delta_{s,r-a} = \frac{3\Gamma_0}{8} \hat{\mathbf{d}}_a \cdot \left\{ \begin{aligned} & (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) \left[-\frac{1}{2k_0z_a} - \int_0^{\pi/2} \sin\theta \frac{\cos^2\theta - n^2(k_0)\cos^2\theta''}{\cos^2\theta + n^2(k_0)\cos^2\theta''} \sin(2k_0z_a \cos\theta) d\theta \right] \\ & - (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) \left[\frac{2}{(2k_0z_a)^3} - \int_0^{\pi/2} \sin\theta \cos^2\theta \frac{n^2(k_0)\cos^2\theta - \cos^2\theta''}{n^2(k_0)\cos^2\theta + \cos^2\theta''} \sin(2k_0z_a \cos\theta) d\theta \right] \\ & + 2\hat{\mathbf{z}}\hat{\mathbf{z}} \left[-\frac{1}{2k_0z_a} - \frac{2}{(2k_0z_a)^3} - \int_0^{\pi/2} \sin^3\theta \frac{n^2(k_0)\cos^2\theta - \cos^2\theta''}{n^2(k_0)\cos^2\theta + \cos^2\theta''} \sin(2k_0z_a \cos\theta) d\theta \right] \end{aligned} \right\} \cdot \hat{\mathbf{d}}_a. \quad (37)$$

By checking the expression of $\gamma_{s,r}$ and $\delta_{s,r}$, we find that they depend on z_a . When the distance between atom a ($a = 1, 2$) and the dielectric surface changes, both the radiation rate and level shift of atom a must be modified. $\gamma_{s,r}$ and $\delta_{s,r}$ respectively represent the change of the single-atom spontaneous radiation rate and the atomic level shift due to the surface. $\gamma_{s,r}$ is the origin of the Purcell effect, and $\delta_{s,r}$ brings about the RCPI between the atom and the surface. Our results agree with the former well-known expressions [46].

B. Collective behavior of two atoms

A_{ab} describes the collective behavior of two atoms. Using notation similar to that in Eq. (27), we have

$$A_{ab} = A_{ab,d} + A_{ab,r}, \quad (38)$$

in which

$$A_{ab,d} \equiv -\gamma_{c,d} + i\delta_{c,d}, \quad (39)$$

$$A_{ab,r} \equiv -\gamma_{c,r} + i\delta_{c,r}, \quad (40)$$

where $A_{ab,d}$ describes the interaction of two atoms by directive exchange of virtual photons, while $A_{ab,r}$ refers to the exchange of virtual photons with reflection on the surface. The letter ‘‘c’’ in the subscript represents the ‘‘collective behavior’’ and ‘‘d’’ is for the ‘‘directive photons exchange between atoms.’’ Similarly, ‘‘r’’ in Eq. (40) is for the ‘‘photon exchange by reflection on the

surface." We first calculate $A_{ab,d}$, and it is not difficult to obtain

$$\gamma_{c,d} = \frac{3\Gamma_0}{4} \hat{\mathbf{d}}_b \cdot \left[\mathbf{I} \left(\frac{\sin k_0 r_{ab}}{k_0 r_{ab}} + \frac{\cos k_0 r_{ab}}{k_0^2 r_{ab}^2} - \frac{\sin k_0 r_{ab}}{k_0^3 r_{ab}^3} \right) + \hat{\mathbf{r}}_{ab} \hat{\mathbf{r}}_{ab} \left(-\frac{\sin k_0 r_{ab}}{k_0 r_{ab}} - 3 \frac{\cos k_0 r_{ab}}{k_0^2 r_{ab}^2} + 3 \frac{\sin k_0 r_{ab}}{k_0^3 r_{ab}^3} \right) \right] \cdot \hat{\mathbf{d}}_a, \quad (41)$$

$$\delta_{c,d} = \frac{3\Gamma_0}{4} \hat{\mathbf{d}}_b \cdot \left[\mathbf{I} \left(\frac{\cos k_0 r_{ab}}{k_0 r_{ab}} - \frac{\sin k_0 r_{ab}}{k_0^2 r_{ab}^2} - \frac{\cos k_0 r_{ab}}{k_0^3 r_{ab}^3} \right), + \hat{\mathbf{r}}_{ab} \hat{\mathbf{r}}_{ab} \left(-\frac{\cos k_0 r_{ab}}{k_0 r_{ab}} + 3 \frac{\sin k_0 r_{ab}}{k_0^2 r_{ab}^2} + 3 \frac{\cos k_0 r_{ab}}{k_0^3 r_{ab}^3} \right) \right] \cdot \hat{\mathbf{d}}_a, \quad (42)$$

where $\hat{\mathbf{r}}_{ab} = (\mathbf{r}_a - \mathbf{r}_b)/r_{ab}$ is the unit vector pointing from atom a to atom b ($a \neq b$). $\gamma_{c,d}$ and $\delta_{c,d}$ respectively represents the atomic collective spontaneous radiation rate and the level shift in boundaryless space. Equations (41) and (42) exactly agree with the former result obtained in the free vacuum [25].

The most important and interesting part in this paper is the change in atomic collective behavior because of the nearby surface, which is described in Eq. (40). According to Eq. (17), we have

$$\gamma_{c,r} = \frac{3\Gamma_0}{4} \hat{\mathbf{d}}_b \cdot \left\{ \begin{aligned} & (\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}}) \int_0^{\pi/2} d\theta \sin \theta \frac{\cos^2 \theta - n^2(k_0) \cos^2 \theta''}{\cos^2 \theta + n^2(k_0) \cos^2 \theta''} \frac{J_1(k_0 x_{ab} \sin \theta)}{k_0 x_{ab} \sin \theta} \cos[k_0(z_a + z_b) \cos \theta] \\ & + \hat{\mathbf{y}}\hat{\mathbf{y}} \int_0^{\pi/2} d\theta \sin \theta \frac{\cos^2 \theta - n^2(k_0) \cos^2 \theta''}{\cos^2 \theta + n^2(k_0) \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \cos[k_0(z_a + z_b) \cos \theta] \\ & + (\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}}) \int_0^{\pi/2} \sin \theta \cos^2 \theta d\theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} \frac{J_1(k_0 x_{ab} \sin \theta)}{k_0 x_{ab} \sin \theta} \cos[k_0(z_a + z_b) \cos \theta] \\ & - \hat{\mathbf{x}}\hat{\mathbf{x}} \int_0^{\pi/2} \sin \theta \cos^2 \theta d\theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \cos[k_0(z_a + z_b) \cos \theta] \\ & + \hat{\mathbf{z}}\hat{\mathbf{z}} \int_0^{\pi/2} \sin \theta \sin^2 \theta d\theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \cos[k_0(z_a + z_b) \cos \theta] \\ & + (\hat{\mathbf{z}}\hat{\mathbf{x}} - \hat{\mathbf{x}}\hat{\mathbf{z}}) \int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_1(k_0 x_{ab} \sin \theta) \sin[k_0(z_a + z_b) \cos \theta] \end{aligned} \right\} \cdot \hat{\mathbf{d}}_a, \quad (43)$$

$$\delta_{c,r} = \frac{3\Gamma_0}{4} \hat{\mathbf{d}}_b \cdot \left\{ \begin{aligned} & -(\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}}) \int_0^{\pi/2} \sin \theta \frac{\cos^2 \theta - n^2(k_0) \cos^2 \theta''}{\cos^2 \theta + n^2(k_0) \cos^2 \theta''} \frac{J_1(k_0 x_{ab} \sin \theta)}{k_0 x_{ab} \sin \theta} \sin[k_0(z_a + z_b) \cos \theta] d\theta \\ & - \hat{\mathbf{y}}\hat{\mathbf{y}} \int_0^{\pi/2} \sin \theta \frac{\cos^2 \theta - n^2(k_0) \cos^2 \theta''}{\cos^2 \theta + n^2(k_0) \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \sin[k_0(z_a + z_b) \cos \theta] d\theta \\ & - (\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}}) \int_0^{\pi/2} \sin \theta \cos^2 \theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} \frac{J_1(k_0 x_{ab} \sin \theta)}{k_0 x_{ab} \sin \theta} \sin[k_0(z_a + z_b) \cos \theta] d\theta \\ & + \hat{\mathbf{x}}\hat{\mathbf{x}} \int_0^{\pi/2} \sin \theta \cos^2 \theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \sin[k_0(z_a + z_b) \cos \theta] d\theta \\ & - \hat{\mathbf{z}}\hat{\mathbf{z}} \int_0^{\pi/2} \sin \theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \sin[k_0(z_a + z_b) \cos \theta] d\theta \\ & + \hat{\mathbf{z}}\hat{\mathbf{z}} \int_0^{\pi/2} \sin \theta \cos^2 \theta \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_0(k_0 x_{ab} \sin \theta) \sin[k_0(z_a + z_b) \cos \theta] d\theta \\ & + (\hat{\mathbf{z}}\hat{\mathbf{x}} - \hat{\mathbf{x}}\hat{\mathbf{z}}) \int_0^{\pi/2} \frac{n^2(k_0) \cos^2 \theta - \cos^2 \theta''}{n^2(k_0) \cos^2 \theta + \cos^2 \theta''} J_1(k_0 x_{ab} \sin \theta) \sin^2 \theta \cos \theta \cos[k_0(z_a + z_b) \cos \theta] d\theta \\ & - (\mathbf{I} + \hat{\mathbf{z}}\hat{\mathbf{z}}) \left[\frac{1}{2} \frac{1}{k_0(z_a + z_b)} + \frac{1}{k_0^3(z_a + z_b)^3} \right] \end{aligned} \right\} \cdot \hat{\mathbf{d}}_a, \quad (44)$$

where J_0 and J_1 are the Bessel functions of zeroth order and first order, respectively. $x_{ab} = x_a - x_b$ is the distance between the drop shadows of atoms a and b ($a \neq b$) on the surface (see Fig. 1). Without loss of generality, to get the expressions (43) and (44), we assume that atom a and b are located in the x - z plane, which means $y_a = y_b = 0$ and this assumption brings simplicity to the calculation. The origin of $\gamma_{c,r}$ and $\delta_{c,r}$ is the exchange of virtual photons between the two atoms by reflection from the surface. The value of $\gamma_{c,r}$ and $\delta_{c,r}$ depend on the distance between the drop shadows of the two atoms x_{ab} as well as the sum of the distance of the two atoms from the surface, $z_a + z_b$; in other words, the distance between one atom and the image of the other atom [$r_{a'b} = [x_{ab}^2 + (z_a + z_b)^2]^{1/2}$]. The refraction index of the surface is contained in the expressions of $\gamma_{c,r}$ and $\delta_{c,r}$, which

means the RI is affected by the properties of the dielectric surface.

When the expressions of $\gamma_{c,r}$ and $\delta_{c,r}$ depend on not only the relative displacement of the two atoms but also the atom-surface distance, we can say that they are three-body dependent. As a consequence, the interatomic RI also becomes three-body dependent. By changing the atom-surface distances, the collective damping rates and atomic level shifts of the two interacting atoms must be changed, and this effect provides one method to control the RI between atoms. If the atoms are far from the surface, $k_0(z_a + z_b) \gg 1$ and $\cos[k_0(z_a + z_b) \cos \theta]$ [or $\sin[k_0(z_a + z_b) \cos \theta]$] oscillates very rapidly with respect to θ . Because of the rapid oscillation of the integrands, the value of integrals in Eqs. (43) and (44) will be small and $\delta_{c,r}$ and $\gamma_{c,r}$ tend to zero. On the other hand, when the atoms are

close to the surface, $k_0(z_a + z_b) \ll 1$ and the integrals can be sufficiently large, then the contribution from $\gamma_{c,r}$ and $\delta_{c,r}$ must be considerable. When the effect induced by the surface can only be notable when the atoms are located close to it, we call this effect the near-surface effect (NSE).

V. NEAR-SURFACE EFFECT

We calculated the expressions of coefficients A_{aa} and A_{ab} ($a, b = 1, 2$), which characterize the behavior of two atoms near one surface. A_{aa} describes the behavior of the single atom a while A_{ab} ($a \neq b$) determines the collective properties of the two atoms. We showed that coefficients A_{aa} and A_{ab} can be divided into two parts: the free-vacuum-field part and the surface-reflection part. The latter part is traceable to the reflection of virtual photons from the surface and brings about the changes in the RCPI as well as in the RI. By using these changes, one surface can be used to control the atom-surface RCPI and the interatomic RI. Although the results given in the last section generally describe the NSE on RI and RCPI, the expressions are not directly perceived through the senses. In this section, we consider some special cases and plot the figures that can visually show the NSE exerted on the atoms.

We consider one scheme in which the interatomic axes are parallel to the surface. We change the atom-surface distance and show the effect exerted on the atoms by the surface. When the distances from the surface to the two atoms are equal, $A_{11} = A_{22}$ and $A_{12} = A_{21}$, Eq. (19) reduces to

$$\begin{aligned} C_1(t) &= \frac{e^{(A_{11}+A_{12})t} + e^{(A_{11}-A_{12})t}}{2}, \\ C_2(t) &= \frac{e^{(A_{11}+A_{12})t} - e^{(A_{11}-A_{12})t}}{2}. \end{aligned} \quad (45)$$

Moreover, we consider the atoms to be isotropically polarized ($d_x = d_y = d_z = d/\sqrt{3}$). When there is no external polarization field, it is reasonable to treat the polarization to be isotropic. The relative inductivity of the surface is set to 3.

Under the conditions discussed above, we assume that atom 1 is initially excited. The interatomic distance is fixed to be $5k_0^{-1}$, while the atom-surface distance changes. In Fig. 3 we plot P_1 ($=|C_1|^2$) and P_2 ($=|C_2|^2$) as functions of dimensionless time $\Gamma_0 t$ for different surface-atom distances. P_1 (P_2) is used to represent the probability of atom 1 (2) in the excited state. According to Fig. 3, when the atoms are in free space, P_1 decays alone and atom 2 can hardly be excited. This behavior of the population shows there is no interaction between the atoms. But if the atoms are located very closed to the surface; for example, $z_1 = z_2 = 0.2k_0^{-1}$, the populations of states $|1\rangle$ and $|2\rangle$ oscillate rapidly and damply. The oscillations of P_1 and P_2 are complementary, which indicates that the interatomic RI is strong and the energy of the electron exchanges between the two atoms. The damping of P_1 and P_2 is due to the spontaneous emission of the atoms. When the atoms are moved farther from the surface, those oscillations become slower, which means the interatomic RI is weakened. Figure 3 proves that NSE can sufficiently effect the interatomic RI. In this example, the RI is strongly enhanced when the atoms are close to the surface, and the strength of the RI can be modified by changing the atom-surface distance.

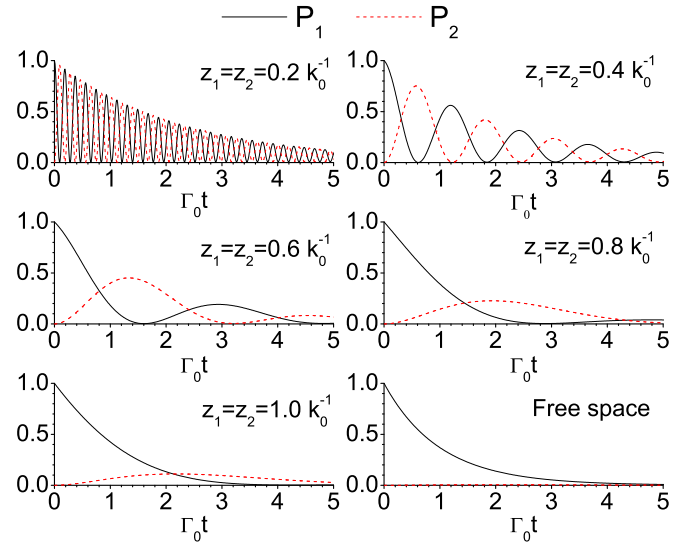


FIG. 3. The probabilities P_1 and P_2 as functions of dimensionless time $\Gamma_0 t$ plotted for different surface-atom distances. Atom 1 is initially excited. The interatomic distance is fixed at $5k_0^{-1}$. If these two atoms are in free space, atom 2 can hardly be excited and atom 1 just damps alone. But when the two atoms are very close to the surface, the populations of states $|1\rangle$ and $|2\rangle$ oscillate rapidly, which reveals a strong interatomic RI and energy transfer. Damping of P_1 and P_2 is due to the spontaneous emission of the two atoms.

This features is very useful in building quantum devices to control energy and information transfer between qubits.

After studying the populations of the atomic excited states, we consider the time evolution of the RI potential energy. In the case of $z_1 = z_2$, Eqs. (21) and (22) can be rewritten as

$$E_{\text{RI}} = -2\hbar \text{Im} A_{12}(C_1^* C_2 + C_1 C_2^*), \quad (46)$$

$$E_{\text{RCPI-}a} = -2\hbar \text{Im} A_{aa}|C_a|^2. \quad (47)$$

Using Eqs. (46) and (47), we plot the interatomic RI potential as a function of time for different atom-surface distances in Fig. 4. Initially, there is no interatomic RI because light needs time to propagate from one atom to the other. The speed of light is finite so the two atoms cannot feel each other at the very beginning. Then the RIs are built up between the two atoms by exchanging virtual photons. E_{RI} increases with time and goes to one maximal value. But the lifetime of RI is finite because of spontaneous radiation, so all curves in Fig. 4 finally decay with time and tend to zero. According to Fig. 4, when the atoms are in free space, the RI can hardly be built up between the atoms. But if the atoms are moved close to the surface ($z_1 = z_2 = 0.2k_0^{-1}$), the RI becomes much stronger than that of the case of large atom-surface distance. This result agrees with the analysis we made in the previous paragraph. Moreover, the time evolution of atom-surface RCPI potentials are plotted in Fig. 5. Similar to the RI, the RCPI is stronger when the atoms are close to the surface.

Figure 4 shows the time evolution of the interatomic RI, and we fix the time $\Gamma_0 t = 1$ and plot the RI versus the interatomic distance x_{12} in Fig. 6. We find that, when the atoms are close to the surface, the interatomic RI is sufficiently stronger than

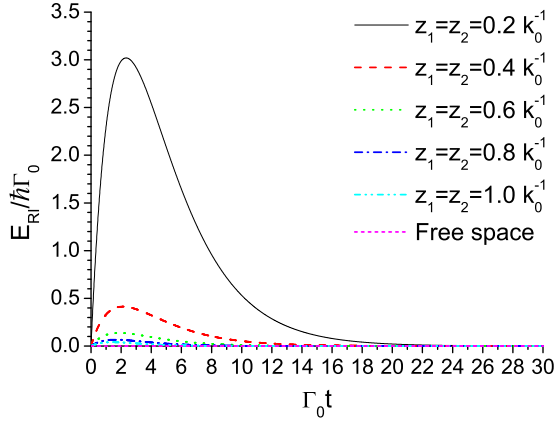


FIG. 4. The dimensionless RI energy $E_{\text{RI}}/\hbar\Gamma_0$ as a function of dimensionless time $\Gamma_0 t$ for different atom-surface distances. The curves describe the process of the buildup, maximizing and decay of interatomic RI. When the atoms are very close to the surface, the RI is much stronger than that in the case of large atom-surface distance. The RI is enhanced by the NSE.

that for large atom-surface distance. This result proves that NSE can enhance the interatomic RI. We further define the atom-exchange symmetry and antisymmetry states as

$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \quad (48)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle), \quad (49)$$

and the corresponding probability amplitudes

$$C_+ = \frac{1}{\sqrt{2}}(C_1 + C_2) = \frac{1}{\sqrt{2}}e^{(A_{11}+A_{12})t},$$

$$C_- = \frac{1}{\sqrt{2}}(C_1 - C_2) = \frac{1}{\sqrt{2}}e^{(A_{11}-A_{12})t}. \quad (50)$$

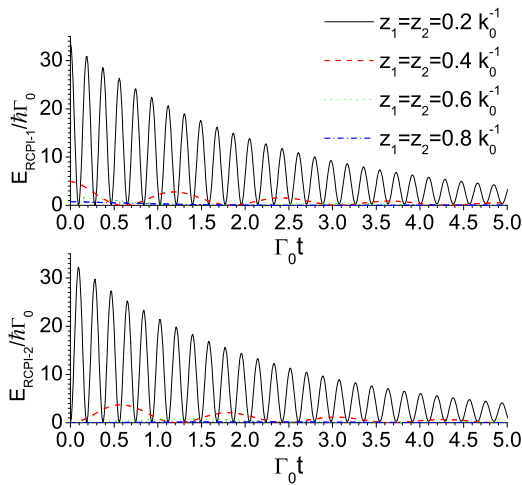


FIG. 5. The upper (lower) figure describes the dimensionless RCPI potential energy $E_{\text{RCPI-1}}/\hbar\Gamma_0$ ($E_{\text{RCPI-2}}/\hbar\Gamma_0$) between atom 1 (atom 2) and the surface as a function of dimensionless time $\Gamma_0 t$ for different atom-surface distances. Because the RCPI energy is proportional to the population of the atomic excited state, the curves in this figure are similar to those in Fig. 3.

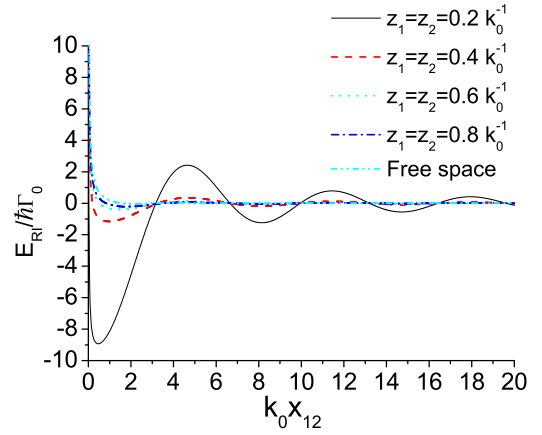


FIG. 6. The dimensionless RI potential energy $E_{\text{RI}}/\hbar\Gamma_0$ at $\Gamma_0 t = 1$ plotted as a function of dimensionless interatomic distance $k_0 x_{12}$ for different atom-surface distances. When the atoms are close to the surface, the RI energy is much stronger than that for large atom-surface distance.

According to Eq. (50), $\gamma_{\text{free}} + \gamma_{\text{s,r}} + \gamma_{\text{c,d}} + \gamma_{\text{c,r}}$ and $\gamma_{\text{free}} + \gamma_{\text{s,r}} - \gamma_{\text{c,d}} - \gamma_{\text{c,r}}$ represent the damping rates of the atom-exchange-symmetric and -antisymmetric states, respectively. By changing the atom-surface distance, the damping rates of the symmetric and -antisymmetric states can be modified, as shown in Fig. 7. When the atoms are close to the surface, the interatomic interaction is enhanced and the super-radiation is suppressed. This result is physically reasonable and not difficult to understand. The interatomic RI, as well as the resonance energy transfer, is mediated by the virtual photon exchange between the two atoms. If the radiation rate is large, the spontaneous emission is enhanced, the atoms emit a photon

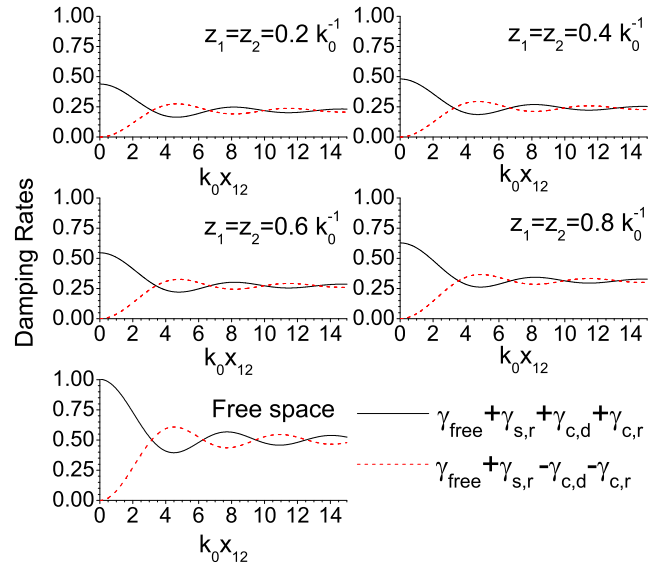


FIG. 7. The damping rates of atom-exchange-symmetric state ($\gamma_{\text{free}} + \gamma_{\text{s,r}} + \gamma_{\text{c,d}} + \gamma_{\text{c,r}}$) and -antisymmetric state ($\gamma_{\text{free}} + \gamma_{\text{s,r}} - \gamma_{\text{c,d}} - \gamma_{\text{c,r}}$) versus the dimensionless interatomic distance $k_0 x_{12}$ for different atom-surface distances. The distances from the two atoms to the surface are equal ($z_1 = z_2$).

very quickly, and then the excitation energy goes away. As a consequence, the RI between atoms just vanishes. But when the spontaneous radiation is suppressed, we can imagine that a photon (or excitation energy, in other words) is “trapped” by the atoms and can be exchanged many times. As a result, the interatomic energy transfer and the RI are enhanced. The control of the damping rate has potential applications in quantum storage, building light sources and producing entangled states.

With the examples given above, we show virtually the change of the interatomic RI because of the NSE. These results show that the effect of the surface can be notable when the atoms are nearby, which agrees with the analysis in Sec. IV.

VI. CONCLUSION

Motivated by the development of research on the RI and atom-chip technology, we investigate theoretically the NSE exerted on the interatomic RI. We find that the surface can sufficiently change the atomic damping rates and level shifts. The change values are given by $\gamma_{c,r}$ and $\delta_{c,r}$, whose values depend not only on the relative position of the two atoms but also on the atom-surface distances. Because of the contribution from $A_{ab,r}$, the interatomic RI is tunable by changing the atom-surface distance. When the atoms are

close to the surface, the interatomic RI can be sufficiently enhanced. In this case, because of the strong interaction, the energy exchange rate between the two atoms becomes much larger than the decay rate of atomic excited states, so the oscillation of the populations of atomic excited states is observable. The frequency of population oscillation is a function of atom-surface distance, which means the energy exchange between two atoms is controllable by one surface. This feature is very useful in production of the quantum transmission devices. Furthermore, the spontaneous radiation rates of atoms can be suppressed by the surface, which is beneficial to produce quantum storage and entanglement. Our results can help in understanding the dynamical evolution of atoms near a surface and contribute to the production of on-chip quantum devices.

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