Measurement-device-independent quantum key distribution with pairs of vector vortex beams

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The vector vortex (VV) beam, originally introduced to exhibit a form of single-particle quantum entanglement between different degrees of freedom, has specific applications for quantum-information protocols. In this paper, by combining measurement-device-independent quantum key distribution (MDIQKD) with a spontaneous parametric-downconversion source (SPDCS), we present a modified MDIQKD scheme with pairs of VV beams, which shows a structure of hybrid entangled entanglement corresponding to intrasystem entanglement and intersystem entanglement. The former entanglement, which is entangled between polarization and orbit angular momentum within each VV beam, is adopted to overcome the polarization misalignment associated with random rotations in quantum key distribution. The latter entanglement, which is entangled between the two VV beams, is used to perform the MDIQKD protocol with SPDCS to inherit the merit of the heralded process. The numerical simulations show that our modified scheme has apparent advances both in transmission distance and key-generation rate compared to the original MDIQKD. Furthermore, our modified protocol only needs to insert q plates in a practical experiment.

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I. INTRODUCTION

Quantum key distribution [1] (QKD) has long been a promising area for application of quantum effects toward solving secure communication problems and its unconditional security can be ensured by the theory of quantum mechanics [2–4]. Despite these developments, there is still a large gap between theory and practice, in the sense that the security is based on assumptions that are not met by experimental implementations. Subtle details in implementations may introduce some laws that could potentially open side channel loopholes to attack, such as photon number splitting attack (PNS) [5], partially random phase attack [6], fake state attack [7], time shift attack [8], and blinding attack [9]. Recently, a novel protocol called measurement-device-independent quantum key distribution (MDIQKD) has been proposed by Lo *et al.* [10] and by Braunstein *et al.* [11]. Lo's paper introduces the decoy state method of MDIQKD and Braunstein's paper deals with general quantum systems and quantum measurements in MDIQKD. Since the measurement process is only used to postselect entanglement between Alice and Bob, this protocol is immune to all the detector side channel attacks. Once MDIQKD has been put forward, some modified schemes have been proposed [12–14] and several experimental demonstrations have been performed [15–18].

To our knowledge, spontaneous photometric downconversion (SPDC) processes are widely used as the sources in QKD, such as the entanglement source [19,20], and the triggered or heralded single-photon source [21,22]. Ref. [23] derived a formula for estimating the single-photon contribution for the MDIQKD with Poisson distributed heralded single-photon source (HSPS). Reference [24] proposed MDIQKD with a thermal distributed spontaneous photometric downconversion source (SPDCS) using polarization encoding to increase the fraction of the yield of single photons. However, an implementation of MDIQKD involves several error sources that affect the performance of MDIQKD. Ref. [25] analyzed various error sources and pointed out that polarization misalignment error is the primary factor contributing to the quantum bit error rate. Ref. [26] proposed a modified MDIQKD protocol using uncharacterized encoding systems as qubit sources. Ref. [27] illustrated a mismatched basis in the MDIQKD protocol. Recently, Refs. [28–30] analyzed the reference-frame-independent quantum key distribution and Refs. [31–33] exploited orbital angular momentum (OAM) in combination with polarization to encode the information in a rotation invariant photonic state in order to implement alignment-free QKD. Reference [34] realized an experiment of reference-frame-independent MDIQKD. Ref. [35] demonstrated the new form of quantum entangled entanglement between two vector vortex (VV) beams.

In this paper, by combining MDIQKD with SPDCS, we present a modified MDIQKD scheme with a photonic system composed of two VV beams, which shows a structure of hybrid entangled entanglement corresponding to intrasystem entanglement and intersystem entanglement. The former entanglement, which is entangled between polarization and orbit angular momentum within each VV beam, is adopted to overcome the polarization misalignment associated with random rotations in the transmittance process of MDIQKD. The latter entanglement, which is entangled between the two VV beams, is used to perform the MDIQKD protocol with SPDCS. Our modified scheme only needs to insert q plates in the transmittance procedure, which in turn means that the initial encoding and final decoding of information in our MDIQKD implementation protocol can be conveniently performed in the polarization space, while the transmission is done in the rotation invariant hybrid space. In particular, comparing to the original MDIQKD protocol, our scheme can effectively increase the key rate owing to maintaining the merit of MDIQKD with SPDCS by using the

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intersystem entanglement of pairs of VV beams. Moreover, comparing to the MDIQKD with SPDCS, our scheme can decrease the error rate caused by random rotation due to the nature of the intrasystem entanglement of each VV beam.

II. MODEL

In this section, we propose the schematic of the modified MDIQKD with VV beams and characterize the properties of the hybrid entangled entanglement of intrasystem and intersystem. The key rate of MDIQKD with heralded process is obtained using entangled VV beams and the quantum bit error rate is analyzed under the specific case that each VV beam is invariant under azimuthal rotations. The schematic of our modified MDIQKD model is shown in Fig. 1 and the modified MDIQKD protocol runs as follows:

A VV beam of order of m is defined in the two-dimensional Hilbert space spanned by $\{|R,\mathrm{m}\rangle,|L,-m\rangle\}$, where $|R,\mathrm{m}\rangle(|L,-m\rangle)$ is the state of a photon with uniform right (left) circular polarization carrying m of orbital angular momentum (OAM). The VV modes are generated by a specialized optical component, the q plate, a birefringent patterned slab that can couple polarization and OAM of single photons. A q plate with topological charge q maps a photon with input state $\alpha|R,0\rangle+\beta|L,0\rangle$ into the output state $\alpha|R,2q\rangle+\beta|L,-2q\rangle$ [33,34], which is exploited as an interface between VV and polarization spaces. In particular,

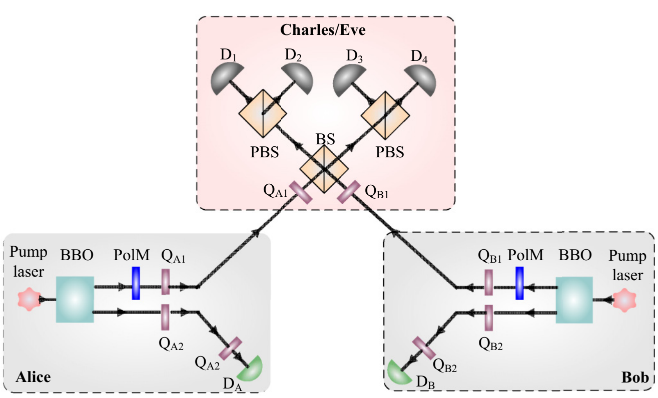

FIG. 1. The schematic diagram of the MDIQKD protocol using VV beams. Alice and Bob generated a polarization entangled photon pair by exploiting spontaneous parametric downconversion in a β-barium borate crystal (BBO). They use single-photon detectors (D_A/D_B) to detect the idler light and randomly prepare the signal light into a BB84 polarization state with a polarization modulator (PolM). Then the state pass through the q plates (Q_{A1}/Q_{B1}) and (Q_{A2}/Q_{B2}); the polarization entanglement is mapped into the intersystem entanglement of VV beams, which is contributed to perform the MDIQKD with SPDCS scheme. Before the rotation invariant signal photon is transmitted to the receiving station, another q plate with same q value transforms the four rotation invariant hybrid states back into the original polarization states. Charlie performs a partial BSM when the signal pulses from Alice and Bob arrive at a 50:50 beam splitter (BS). Four single-photon detectors (D_1-D_4) are employed to detect the results.

we consider the two balanced superpositions:

$$|\widehat{r_m}\rangle=\frac{1}{\sqrt{2}}(|R,m\rangle+|L,-m\rangle),$$
$$|\widehat{\vartheta_m}\rangle=\frac{1}{\sqrt{2}}(|R,m\rangle-|L,-m\rangle).$$

Similar to the original polarization encoding MDIQKD protocol with SPDCS, Alice randomly chooses a polarization basis from $\{H,V\}$ or $\{L,R\}$, then couples the two states into one of the qubit bases and encodes the bit values into the mode. In fact, Alice and Bob randomly encode their qubits into one of the four quantum states:

$$|H\rangle,|V\rangle,|L\rangle=1/\sqrt{2}(|H\rangle+i|V\rangle),$$
$$|R\rangle=1/\sqrt{2}(|H\rangle-i|V\rangle). \tag{1}$$

$\{L,R\}$ can be obtained by the combination of the laser's polarization and a quarter-wave plate. The q plate maps a polarization encoding qubit into hybrid polarization OAM states. By passing through the q plate with $q=1/2$, polarization states $|H\rangle,|V\rangle,|L\rangle,|R\rangle$ are transformed into rotation invariant hybrid states:

$$|H\rangle\otimes|0\rangle\overset{q\text{plate}}{\rightarrow}\frac{1}{\sqrt{2}}(|L\rangle\otimes|r\rangle)+\frac{1}{\sqrt{2}}(|R\rangle\otimes|l\rangle), \tag{2a}$$
$$|V\rangle\otimes|0\rangle\overset{q\text{plate}}{\rightarrow}\frac{i}{\sqrt{2}}(|L\rangle\otimes|r\rangle)-\frac{i}{\sqrt{2}}(|R\rangle\otimes|l\rangle), \tag{2b}$$
$$|R\rangle\otimes|0\rangle\overset{q\text{plate}}{\rightarrow}|L\rangle\otimes|r\rangle, \tag{2c}$$
$$|L\rangle\otimes|0\rangle\overset{q\text{plate}}{\rightarrow}|R\rangle\otimes|l\rangle, \tag{2d}$$

where l,r denote left- and right-handed first-order Laguerre-Gauss mode. Then, the Z basis and X basis in our hybrid qubits encoding system are transformed to

$$\{0,1\}^Z=\{|L\rangle\otimes|r\rangle,|R\rangle\otimes|l\rangle\}, \tag{3a}$$
$$\{0,1\}^X=\frac{1}{\sqrt{2}}\{|L\rangle\otimes|r\rangle\pm|R\rangle\otimes|l\rangle\}. \tag{3b}$$

Bob applies the same encoding procedure symmetrically. In our modified MDIQKD protocol, a polarization entangled photon pair is produced by exploiting spontaneous parametric downconversion in a β-barium borate crystal (BBO) and passes through two q plates, respectively. The output states will undergo the following transformation:

$$(\alpha|R\rangle+\beta|L\rangle)\otimes|0\rangle\overset{q\text{ plate}}{\rightarrow}\alpha(|L\rangle\otimes|-2q\rangle)+\beta(|R\rangle\otimes|+2q\rangle). \tag{4}$$

Then the Z basis and X basis in our modified MDIQKD with VV beams system are transformed to

$$\{0,1\}^X=\{|\widehat{r_m}\rangle,|\widehat{\vartheta_m}\rangle\}, \tag{5a}$$
$$\{0,1\}^Z=\left\{\frac{1}{\sqrt{2}}(|\widehat{r_m}\rangle+|\widehat{\vartheta_m}\rangle),\quad\frac{1}{\sqrt{2}}(|\widehat{r_m}\rangle-|\widehat{\vartheta_m}\rangle)\right\}, \tag{5b}$$

where $|\widehat{r_m}\rangle=\frac{1}{\sqrt{2}}(|R,\mathrm{m}\rangle+|L,-m\rangle)$, $|\widehat{\vartheta_m}\rangle=\frac{1}{\sqrt{2}}(|R,\mathrm{m}\rangle-|L,-m\rangle)$.

The VV beams can be easily produced by using a linear horizontal and vertical input polarization, respectively. Two

q plates with topological charge q_1 and q_2 transform the two polarization photon states into VV modes of order $m_1 = 2q_1$ and $m_2 = 2q_2$, respectively. The resulting state emerging from the source is the following:

$$\begin{aligned}
|\phi^+\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle) \overset{q\text{plate}}{\to} |\phi^+_{m_1,m_2}\rangle \\
&= \frac{1}{\sqrt{2}}\left(|\widehat{r_{m_1}}\rangle|\widehat{r_{m_2}}\rangle + |\widehat{\vartheta_{m_1}}\rangle|\widehat{\vartheta_{m_2}}\rangle\right), \\
|\phi^-\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|H\rangle - |V\rangle|V\rangle) \overset{q\text{plate}}{\to} |\phi^+_{m_1,m_2}\rangle \\
&= \frac{1}{\sqrt{2}}\left(|\widehat{r_{m_1}}\rangle|\widehat{r_{m_2}}\rangle - |\widehat{\vartheta_{m_1}}\rangle|\widehat{\vartheta_{m_2}}\rangle\right), \\
|\psi^+\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle) \overset{q\text{plate}}{\to} |\psi^+_{m_1,m_2}\rangle \\
&= \frac{1}{\sqrt{2}}\left(|\widehat{r_{m_1}}\rangle|\widehat{\vartheta_{m_2}}\rangle + |\widehat{\vartheta_{m_1}}\rangle|\widehat{r_{m_2}}\rangle\right), \\
|\psi^-\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle) \overset{q\text{plate}}{\to} |\psi^-_{m_1,m_2}\rangle \\
&= \frac{1}{\sqrt{2}}\left(|\widehat{r_{m_1}}\rangle|\widehat{\vartheta_{m_2}}\rangle - |\widehat{\vartheta_{m_1}}\rangle|\widehat{r_{m_2}}\rangle\right). \qquad (6)
\end{aligned}$$

The one-to-one mapping can be established between polarization entanglement and intersystem entanglement of VV beams, which is contributed to perform the MDIQKD with SPDCS scheme using entanglement VV beams. Alice and Bob monitor one of pair of VV beams with a practical threshold detection described by detection efficiency η_x and dark count rate d_x to perform the heralded process. The photon number distribution of heralded mixed state is

$$P_n(\mu_x) = \begin{cases} 1 - P^{\text{cor}}_{\mu_x} + \frac{d_x P^{\text{cor}}_{\mu_x}}{(1+\mu_x)P^{\text{post}}_{\mu_x}}, & n = 0 \\ \frac{\mu_x^n\{[1-(1-\mu_x)^n + d_x]\}P^{\text{cor}}_{\mu_x}}{(1+\mu_x)^n P^{\text{post}}_{\mu_x}}, & n \geqslant 1 \end{cases}, \qquad (7)$$

where $P^{\text{post}}_{\mu_x} = 1 + d_x - \frac{1}{1+\mu_x\eta_x}$ denotes the postselection probability; $P^{\text{cor}}_{\mu_x}$ is the probability that one can predict the existence of a heralded photon given a trigger one. Before Eve performs a partial Bell state measurement (BSM) on the received pulses, a second q plate with the same value transforms the VV beams back into the original polarization states. Therefore, the BSM in our MDIQKD implementation protocol can be conveniently performed in the polarization space. Considering the imperfections of the q plate, we can obtain the yield Y^Z_{11} and the error rate e^X_{11} under the situation of the transmission efficiency of about $\eta_q = 0.85$ [35–38]:

$$\begin{aligned}
Y^Z_{11} &= P_1(\mu_A)P_1(\mu_B)(1 - P_d)^2\left[4P_d^2(1 - \eta_A\eta_q)(1 - \eta_B\eta_q)\right. \\
&\quad \left. + 2P_d\left(\eta_A\eta_q + \eta_B\eta_q - \frac{3\eta_A\eta_B\eta_q^2}{2}\right) + \frac{\eta_A\eta_B\eta_q^2}{2}\right], \qquad (8)
\end{aligned}$$

$$e^X_{11} = \frac{1}{2} - \frac{\eta_A\eta_B\eta_q^2(1 - e_d)^2}{4Y^X_{11}}, \qquad (9)$$

TABLE I. List of experimental parameters used in the simulations.

	e_0	e_d	p_d	f
Reference [24]	0.5	0.015	3×10^{-6}	1.16

where $Y^X_{11} = Y^Z_{11}$. The secret key rate is given by

$$\begin{aligned}
R &= P_1(\mu_A)P_1(\mu_B)Y^Z_{11}\left[1 - H_2\left(e^X_{11}\right)\right] \\
&\quad - Q^Z_{\mu_A\mu_B} f\left(E^Z_{\mu_A\mu_B}\right)H_2\left(E^Z_{\mu_A\mu_B}\right). \qquad (10)
\end{aligned}$$

More interestingly, by passing through the q plate with $q = 1/2$, each VV beam shows intrasystem entanglement between polarization and OAM, which shows rotation invariant hybrid states under azimuthal rotations. Let us consider the effect of the random rotation on intrasystem entanglement. Each VV beam will undergo the following transformation:

$$\begin{aligned}
R[\theta](|L\rangle \otimes |r\rangle) &= R[\theta]|L\rangle \otimes R[\theta]|r\rangle \\
&= e^{i\theta}|L\rangle \otimes e^{-i\theta}|r\rangle = |L\rangle \otimes |r\rangle, \qquad (11a) \\
R[\theta](|R\rangle \otimes |l\rangle) &= R[\theta]|R\rangle \otimes R[\theta]|l\rangle \\
&= e^{-i\theta}|R\rangle \otimes e^{i\theta}|l\rangle = |R\rangle \otimes |l\rangle, \qquad (11b)
\end{aligned}$$

where l,r denotes left- and right-handed first-order Laguerre-Gauss mode. Any rotation $R[\theta]$ will leave such states unaffected since the phase shift related to the polarization state will be exactly canceled out by the phase shift of the OAM eigenstate. In the state transmission procedure, we consider a simplified model with a two-dimensional unitary matrix to describe the polarization misalignment of channel transmission [39]:

$$U_k = \begin{pmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{pmatrix}, \qquad (12)$$

where θ_k is the polarization rotation angle and $k = A,B,C$ represent the location, respectively. For each value of k, we define the polarization misalignment error $e_k = \sin^2\theta_k$ and the total error $e_d = e_A + e_B + e_C$. $e_A(e_B)$ represents the misalignment of Alice's (Bob's) channel transmission, while e_C models the misalignment of the Bell state measurement procedure. In our simulation, we adopt a practical assumption for the polarization misalignment; that is, the probability distribution of e_k is selected as $e_A = e_B = 0.475e_d$ and $e_C = 0.05e_d$. It is important to notice that the channel transmission misalignment can be removed when the original polarization state is transformed to the VV beam by using a q plate.

III. SIMULATION

With the analysis introduced in the last section, the simulations are presented to compare our results with the existing results. For simplicity, we ignore mode mismatch in our

TABLE II. List of experimental parameters used to characterize SPDCS and VV beams.

	$d_{A/B}$	$\eta_{A/B}$	$P^{\text{cor}}_{\mu_{A/B}}$	η_q
Reference [23]	5×10^{-5}	0.4	0.405844	0.85

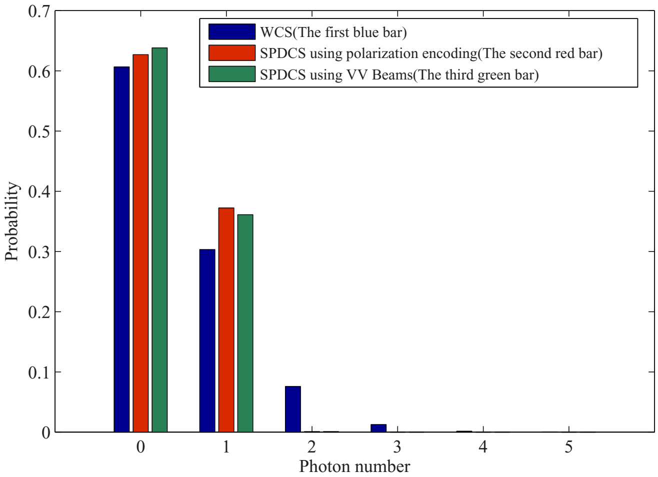

FIG. 2. A comparison for the probability of emitting a different number of photons between WCS, heralded SPDCS, and VV beams. Here, $P_{\mu_x}^{\text{cor}}$=0.405 844 and μ_x=1.425 × 10^{-3} for SPDCS while the intensity is 0.5 for WCS. The efficiency of each q plate η_{q} is 0.85 for VV beams.

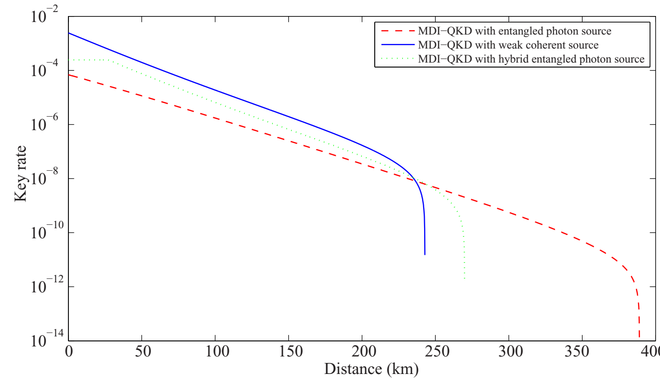

FIG. 3. Key rate versus the maximal secure transmittance distance L with different protocols. Blue solid line: Asymptotic MDIQKD protocol using weak coherent source. Red dashed line: Asymptotic MDIQKD protocol with SPDCS using polarization encoding. Green dot-dashed line: Our modified MDIQKD protocol using VV beams.

simulation and we assume that the system is operating under the normal condition, which means all other parameters are the same as the original MDIQKD. The numerical parameters used are listed in Table I, which are the same as in Ref. [25] for comparison. The parameters for SPDCS and VV beams are listed in Table II, and are borrowed from the experiment by Wang *et al.* [24].

As shown in Fig. 2, the single-photon fraction for the heralded SPDCS is bigger than that for the Weak coherent source (WCS). Though the efficiency of the q plate may bring photon loss, the single-photon fraction of VV beams is still higher than WCS. For example, the single-photon probability for SPDCS is 0.3784 when $P_{\mu_x}^{\text{cor}}$=0.405 844 and $\mu_x = 1.425 \times 10^{-3}$, while the single-photon probability for VV beams is 0.3611 when the efficiency of each q plate η_q is 0.85 for VV beams. However, that the probability for the Poisson distribution is 0.3033 when the intensity is 0.5 for WCS.

In Fig. 3, the performances of MDIQKD using WCS, SPDCS, and VV beams are compared. Our results show that the modified protocol using VV beams actually offers a higher key rate than the polarization encoding scheme owing to the higher single-photon yield rate and the maximum secure transmittance distance of our MDIQKD is longer than the original MDIQKD owing to the lower quantum bit error rate which is decreased by using a rotation invariant VV beam.

IV. CONCLUSION

In this paper, we present a modified MDIQKD protocol with pairs of VV beams. A hybrid entangled entanglement of VV beams is adopted to overcome the polarization misalignment associated with random rotations in MDIQKD with SPDCS scheme. The initial encoding and final decoding of information in our MDIQKD implementation protocol can be conveniently performed in the polarization space, while the transmission is done in the rotation invariant hybrid space. Comparing to the original MDIQKD protocol, our scheme can effectively increase the key rate owing to maintaining the merit of MDIQKD with SPDCS by using the intersystem entanglement of pairs of VV beams. Moreover, comparing to the MDIQKD with SPDCS, our scheme can decrease the error rate caused by random rotation due to the nature of the intrasystem entanglement of each VV beam. Furthermore, our modified protocol only needs to insert q plates in a practical experiment.

We noticed that though QKD systems using hybrid encoding have the potential to provide high key-generation rates and a higher tolerance to eavesdropping than is possible with polarization, the ability to transmit photons carrying OAM over large distances remains an open problem [40]. Conventional fiber does not preserve OAM mode information due to spatial-mode mixing. But in our protocol, we adopted $q = 1/2$ for all the q plates which means the VV beam only uses the first-order Laguerre-Gauss mode of OAM, which is proved to be feasible to transmit up to three different OAM modes with a specialized fiber [41]. Moreover, our protocol can extend to the free-space case easily by considering the behavior of OAM light beams in a turbulent atmosphere.

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