

## Experimental demonstration of genuine multipartite quantum nonlocality without shared reference frames

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Multipartite quantum nonlocality is an important diagnostic tool and resource for both researches in fundamental quantum mechanics and applications in quantum information protocols. Shared reference frames among all parties are usually required for experimentally observing quantum nonlocality, which is not possible in many circumstances. Previous results have shown violations of bipartite Bell inequalities with approaching unit probability, without shared reference frames. Here we experimentally demonstrate genuine multipartite quantum nonlocality without shared reference frames, using the Svetlichny inequality. A significant violation probability of 0.58 is observed with a high-fidelity three-photon Greenberger-Horne-Zeilinger state. Furthermore, when there is one shared axis among all the parties, which is the usual case in fiber-optic or earth-satellite links, the experimental results demonstrate the genuine three-partite nonlocality with certainty. Our experiment will be helpful for applications in multipartite quantum communication protocols.

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### I. INTRODUCTION

Nonlocality, a kind of correlation between space separated parties which cannot be properly explained by any local theory, is one of the most striking features of quantum mechanics. With the development of quantum information science, nonlocality has been far beyond a helpful concept for deeper understanding of quantum mechanics, it is also recognized as important and necessary resources [1,2] to harness many quantum information protocols [3], such as quantum teleportation [4], quantum key distribution [4,5], reduction of communication complexity [6,7], certification and expansion of randomness [8,9], and especially device independent quantum information tasks [10,11].

Extensive studies have been focused on bipartite nonlocality, both theoretically [12–15] and experimentally [16–18]. When researches move from a bipartite to multipartite case, much more complex and richer structures arise. It no doubt makes the properties and characterization of multipartite nonlocal correlations more interesting. However, on the other hand, it also results in much more challenging problems, both on the theoretical [12] and experimental aspects [19].

To verify multipartite quantum nonlocality, a relevant class of Bell inequalities have been developed, such as the Mermin-Ardehali-Belinskii-Klyshko (MABK) inequality [20–22], the Werner-Wolf-Zukowski-Brukner (WWZB) inequality [23,24], the Mermin (M) inequality [20], the Mermin-Klyshko (MK) inequality [20,22], and the Svetlichny (S) inequality [25]. Among these inequalities, the S inequality has special importance because the violation of it demonstrates the nonlocality to be genuinely multipartite, and there have been some experiments [26,27] testing the Svetlichny inequality. When testing these inequalities, an important requirement is that spatially separated parties should share a common reference

frame [28,29]. But in practical experiments, aligning the reference frames of each party is resource intensive and technically demanding, especially for distant parties, which might be an obstacle for implementing various multipartite quantum communication protocols.

Several methods have been developed to circumvent this problem. One possible method is to use specially encoded multiqubit states which are invariant under collective rotations of each local reference frame [30,31]. However, it requires multiqubit entangled state preparation, which is still challenging in technique. Another way to align measurements is to establish a shared reference frame by using correlated quantum systems [32,33], which will consume an intensive resource for coherently exchanging many entangled quantum systems. Recently, Liang *et al.* show that random measurements can also lead to nonlocal correlations between measurement outcomes [34]. After that, there are several simulations and proofs working on finding optimal measurement settings to improve the violation probability of Bell inequalities [16,35–37], or even with certainty violation probability [17,18,36,38]. Up to now, there have been three experiments [16–18] seeking to demonstrate violations of Bell inequalities without shared reference frames. In Refs. [16,17] photon polarizations are used as an encoding qubit, while in Ref. [18] hybrid encoded logic qubits of spin and orbital angular momentum degrees of freedom are employed to work in a rotation invariant way. However, all these experiments are limited in the bipartite case.

In this paper, with a high fidelity three-photon GHZ state, we experimentally demonstrate genuine multipartite quantum nonlocality without shared reference frames, by choosing the tetrahedral measurement bases proposed by Senel *et al.* [36]. The violation probability of the Svetlichny inequality is 0.58. Furthermore, when one axis of the reference frames can be aligned, like the case in the fiber optics or earth-to-satellite links, we even demonstrate genuine three-partite nonlocality with certainty, using a Y shape measurement base. Numerical analysis shows that the choice of the Y shape measurement base also has a robust resistance to state

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preparation noise. We believe that such results will be helpful for multipartite alignment-free QKD protocols [37], as well as other multipartite quantum communication protocols.

## II. THEORETICAL SCHEME AND NUMERICAL ANALYSIS

For simplicity we consider a tripartite system, each subsystem is measured by one observer. For each subsystem, the measurement setting and the outcome are denoted by  $M_1, M_2, M_3$  and  $r_1, r_2, r_3$ , respectively. Then the joint outcome probabilities of the tripartite system can be written as  $P(r_1 r_2 r_3 | M_1 M_2 M_3)$ . In a local hidden variable model, the definition of the standard multipartite nonlocality means that the joint outcome probabilities cannot be written as  $P(r_1 r_2 r_3 | M_1 M_2 M_3) = \int P(r_1 | M_1, \lambda) P(r_2 | M_2, \lambda) P(r_3 | M_3, \lambda) \rho(\lambda) d\lambda$ , where  $\lambda$  is a shared local hidden variable,  $\int \rho(\lambda) d\lambda = 1$ , with  $\rho(\lambda) \geq 0$ , and  $P(r_k | M_k, \lambda) (k \in 1, 2, 3)$  is the probability of the  $k$ th observer measuring observable  $M_k$  with outcome  $r_k$  for a given local hidden variable  $\lambda$ . However, the definition of the genuine multipartite nonlocality [25] gives a stronger form of multipartite correlations than the standard multipartite nonlocality, which means that the joint outcome probabilities cannot be written as

$$\begin{aligned} & P(r_1 r_2 r_3 | M_1 M_2 M_3) \\ &= \int P(r_1 | M_1, \lambda) P(r_2 r_3 | M_2 M_3, \lambda) \rho(\lambda) d\lambda \\ &+ \int P(r_2 | M_2, \mu) P(r_1 r_3 | M_1 M_3, \mu) \rho(\mu) d\mu \\ &+ \int P(r_3 | M_3, \nu) P(r_1 r_2 | M_1 M_2, \nu) \rho(\nu) d\nu. \quad (1) \end{aligned}$$

Based on these definitions, an operational way to verify the system's nonlocal properties is to check the multipartite Bell type inequalities, especially the S inequality [25], for the genuine multipartite nonlocality. When experimentally testing these inequalities, usually we need to align the reference frames for each subsystem, which poses stringent requirements for experimental techniques. However, recently it is also found that the violation of Bell inequalities is still possible even without alignment of reference frames [34, 36, 38]. In [36], choosing a tetrahedral measurement scheme for each observer, a high violation probability of the S inequality is achieved.

In the following, we focus on the tripartite case. For this case, the S inequality can be constructed from the S polynomials as  $S_3 = \frac{1}{4}(S_2 a'_3 + S_2 a_3)$ , where  $S_2 = \frac{1}{4}(a_1 a_2 + a_1 a'_2 + a'_1 a_2 - a'_1 a'_2)$ ,  $S'_2 = \frac{1}{4}(a'_1 a'_2 + a'_1 a_2 + a_1 a'_2 - a_1 a_2)$ , and  $a_k (a'_k)$  is the measurement performed on the  $k$ th observer. When each product term in the S polynomial is replaced by its corresponding expectation values, such as  $a_1 a_2 a'_3$  being replaced by  $E(a_1 a_2 a'_3)$ , we get a corresponding S expression. And the absolute value of this S expression is the S inequality value, denoted as  $I_S$ . Now the S inequality is written as  $I_S \leq 1$ . According to the measurement scheme in Ref. [36], starting from sharing a multipartite state, each observer measures his subsystem in four bases, which define measurement directions evenly spaced over the Bloch sphere, corresponding to the four vertices of a tetrahedron in the Bloch sphere. Thus, for three observers, a total of  $4^3 = 64$  kinds of joint measurements are performed, and  $(4 \times 3)^3 = 1728$  different S inequality values can be obtained. Then we pick out the maximal value among

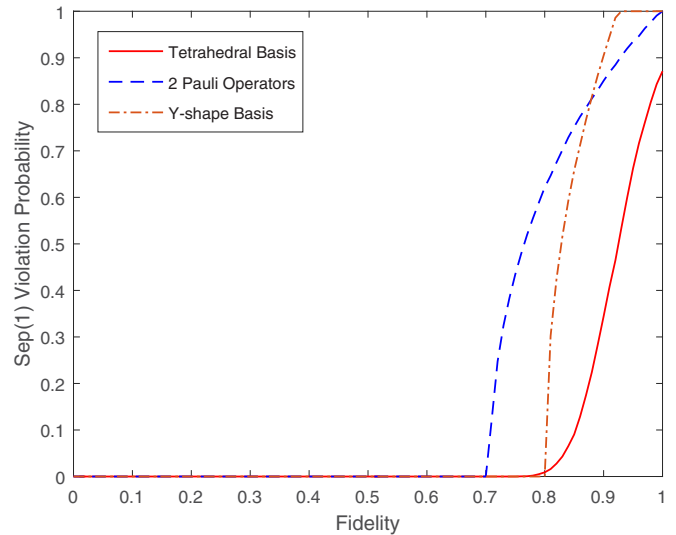


FIG. 1. Numerical simulation results of S inequality violation probability versus fidelity  $V$  with noisy GHZ states  $\rho_V$ , and without shared reference frame. For the case of sharing no common axis, the red solid line denotes the result with the tetrahedral measurement scheme and its highest violation probability 0.871 is achieved at  $V = 1$ . For the case of sharing only one common axis, the blue dashed line is the result with the two perpendicular measurement directions scheme, in which unit violation probability is achieved when  $V = 1$ . And the brown dash-dot line is the result of applying our Y shape measurement scheme, in which unit violation probability is maintained in a large area between  $V = 1$  and  $V = 0.93$ .

them to see whether it violates the S inequality. After a large number of repetitions of the above trials, we will get the violation probability  $P_S$  of the S inequality, which is the ratio of the number of trials in which the S inequality is violated over the total number of trials. Note that, to introduce the alignment-free effect, each observer should equivalently make a random local rotation  $R_k$  (for the  $k$ th observer) on his subsystem before performing his measurements. Here  $R_k$  is given as  $R_k = \cos \frac{\theta_k}{2} \mathbb{I} - i \sin \frac{\theta_k}{2} (n_k^1 \sigma_1 + n_k^2 \sigma_2 + n_k^3 \sigma_3)$ , where  $\theta_k$  and  $n_k^j$  are real,  $\sum_j n_k^j = 1$ , and  $\sigma_k (k = 1, 2, 3)$  are the Pauli operators.

Starting from a three-qubit Greenberger-Horne-Zeilinger (GHZ) state, using the tetrahedral measurement bases, numerical simulation in [36] shows a 88% high violation probability of the  $S_3$  inequality. However, for the purpose of experimental testing, we need to further analyze the situation of nonideal initial state preparation. Suppose that the noisy GHZ state is  $\rho_V = V |\text{GHZ}_3\rangle\langle\text{GHZ}_3| + (1 - V) \frac{I}{8}$ , where  $I$  is the identity,  $|\text{GHZ}_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ , and  $0 \leq V \leq 1$  ( $V$  denotes the fidelity or visibility). With the tetrahedral bases, we numerically simulate the violations of the S inequality without a shared reference frame. In Fig. 1 the red solid line shows the simulation results. The maximum violation probability 0.871 is achieved by the ideal GHZ state, and the violation probability reduced quickly as the fidelity  $V$  decreases. With a fidelity less than 0.8, there will be hardly any violation of the S inequality. Such results pose a high requirement on the GHZ state preparation fidelity.

As mentioned in the Introduction, a usually encountered practical situation is that the observers can share one reference

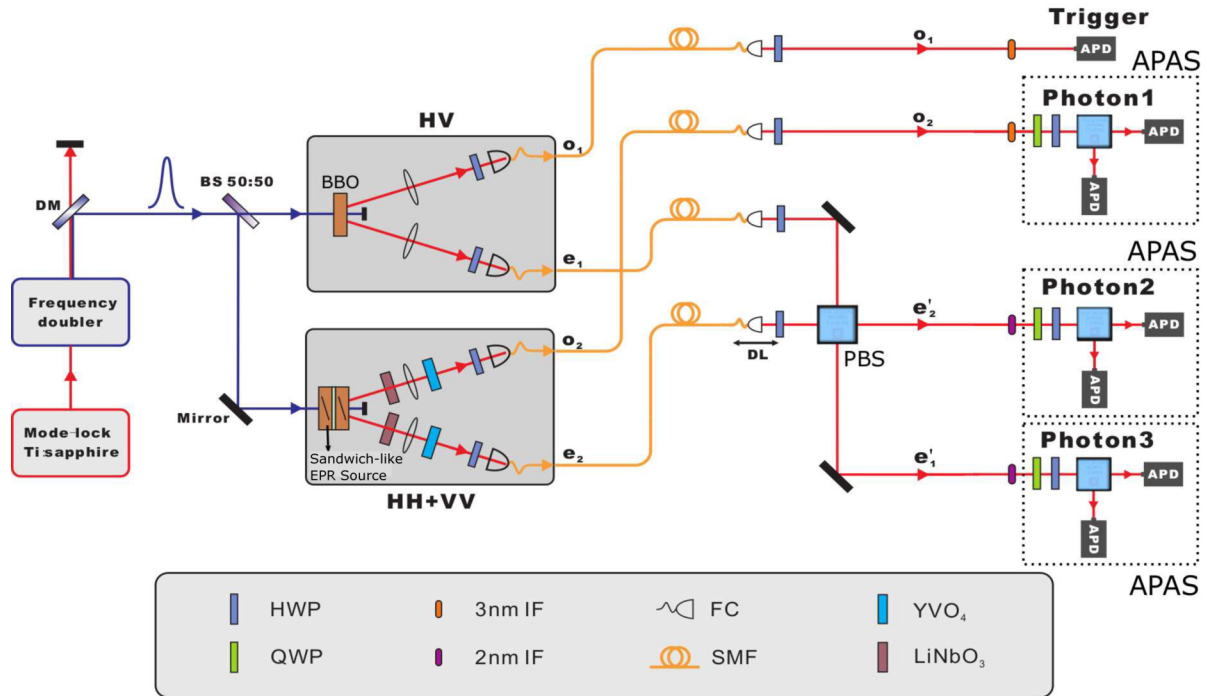


FIG. 2. Experimental setup. The abbreviations of the components are DM, dichroic mirror; HWP, half-wave plate; FC, fiber coupler; BS, beam splitter; QWP, quarter-wave plate; IF, interference filter; PBS, polarizing beam splitter; APD, avalanche photodiode; SMF, single-mode fiber; and DL, delay line. In the main figure, two photon pairs emitted from one EPR source and one product state source are coupled into single mode fibers. The DL is translation stage for tuning the arrival time of photons to interfere them at the PBS. The polarization states of the photons are analyzed by one QWP, one HWP, one PBS, and two APDs. The output signals of the APDs are sent to the coincidence unit.

frame axis, such as the cases in fiber-optic or earth-satellite communications. So we also numerically analyze the model in which the random local rotations performed before the measurement by each observer are restricted as  $R_k^z(\theta) = \cos \frac{\theta}{2} \mathbb{I} - i \sin \frac{\theta}{2} \sigma_3$  (supposing that the  $Z$  axis is shared), where  $R_k^z(\theta)$  is randomized according to  $\theta$ . In Ref. [36] it is shown that in this case the S inequality can be violated with certainty using two perpendicular measurement operators on the restricted plane. However, for the nonideal initial GHZ state, our numerical results show that the violation probability can achieve 1 only when  $V = 1$  and it drops quickly as  $V$  decreases, which means that it is very difficult to achieve the violation with certainty in real experiments. In Fig. 1 the blue dashed line shows our numerical results for this measurement scheme.

To achieve the S inequality violation with certainty in a real experiment, we propose and numerically analyze a new Y shape measurement scheme, which is more robust against the state preparation noise. The Y shape bases have three measurement directions in the restricted plane, and each is separated 120 deg with each other. With the Y shape bases, when  $V$  decreases, a unit violation probability still can be achieved in a certain region of  $V$ . The brown dash-dot line in Fig. 1 illustrates that the guaranteed violation can still be achieved even when the fidelity is as low as 0.93.

### III. EXPERIMENT SETUP AND RESULTS

In our experiment, two cases are tested to demonstrate the genuine multipartite nonlocality without shared reference

frames. One is testing the tripartite S inequality for the  $|\text{GHZ}_3\rangle$  state without any alignment among each observer, using the tetrahedral measurement scheme. The other is testing the tripartite S inequality for  $|\text{GHZ}_3\rangle$  with one shared axis (the  $Z$  axis), using the Y shape measurement scheme.

The experimental setup is shown in Fig. 2. The pulse train from a mode-locked Ti-sapphire laser (with a central wavelength 780 nm, a repetition rate of 76 MHz, and a duration of 140 fs) first passes through a second harmonic generator (SHG). Then the frequency-doubled ultraviolet pulse train emitted from the SHG is split into two beams with identical powers. One beam is sent to pump a single type-II beam like a phase-matching BBO crystal to prepare photon pairs in the product state  $|HV\rangle$ , where  $H$  and  $V$  denote the horizontal and vertical polarization states of the photons, respectively. The other beam is sent to pump a sandwich-like EPR source [39] consisting of two 1-mm-thick beta-barium borate (BBO) crystals to prepare the state of  $\frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$ . Then the two photon pairs are collected by four single mode fibers. After that, the photon in the output mode  $o_1$  is directly detected by an avalanche photodiode (APD) as a trigger. And the photon in the output mode  $e_1$  is first transformed to the state of  $|H+V\rangle$  by a half-wave plate and then directed to interfere with the photon in the output mode  $e_2$  on a polarizing beam splitter (PBS), which is set to transmit (reflect)  $H(V)$  polarization. When there is one and only one photon in each of the four spatial modes  $o_1, o_2, e'_1, e'_2$ , the three photons in  $o_2, e'_1, e'_2$  modes are successfully prepared in the state  $|\text{GHZ}_3\rangle$ .

Before testing the S inequality, we first perform the quantum state tomography on the prepared  $|\text{GHZ}_3\rangle$  state. For this

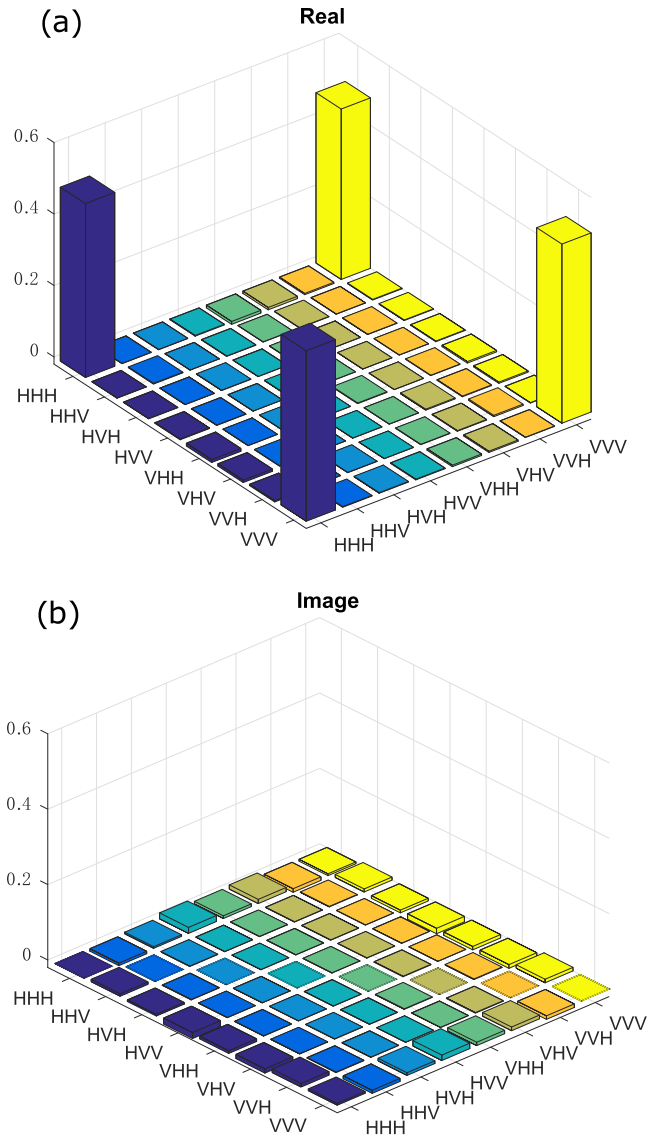


FIG. 3. The result of the tomography density matrix for  $|\text{GHZ}_3\rangle$ . Data collection time for each measurement setting is 800 s. (a) The real part of the tomography density matrix. (b) The image part of the tomography density matrix.

purpose, each photon is spectrally filtered with a interference filter (centered at 780 nm, with 2 nm bandwidth for  $e'_1$ ,  $e'_2$  modes and 3 nm bandwidth for  $o_1$ ,  $o_2$  modes). Then an automated polarization analysis system (APAS) consisting of one half-wave plate, one quarter-wave plate, one polarizing beam splitter, and two single photon detectors are employed to measure the photons in the  $o_2$ ,  $e'_1$ , and  $e'_2$  modes in appropriate polarization basis. At last, a multichannel coincidence unit is used to register all possible coincidence counts. Thus, we can reconstruct the density matrix  $\rho_{\text{expt}}$  of the prepared state from these data (see Fig. 3 for tomography results). With  $\rho_{\text{expt}}$  we find that the fidelity of the prepared three photon state with  $|\text{GHZ}_3\rangle$  is  $F = \langle \text{GHZ}_3 | \rho_{\text{expt}} | \text{GHZ}_3 \rangle = 0.971 \pm 0.003$ . And the typical four-photon coincidence rate is 1 Hz. We also directly test the S inequality with shared reference frames, using  $\rho_{\text{expt}}$ . The measured S value is  $1.37 \pm 0.05$ , which shows

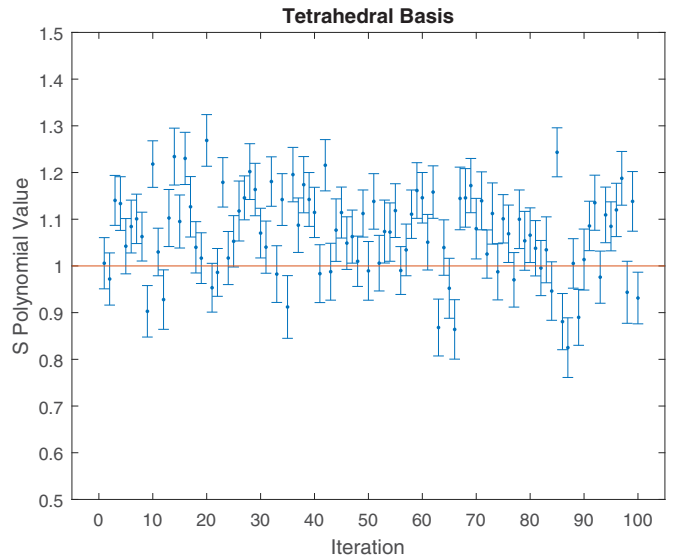


FIG. 4. The measured S inequality values for 100 experimental trials in the case of sharing no common axis, using the tetrahedral measurement scheme.

a violation of seven standard errors and is close to the maximal S value of 1.414 with ideal state  $|\text{GHZ}_3\rangle$ .

In the next step we test the case of measuring in tetrahedral bases without shared reference frames for our  $|\text{GHZ}_3\rangle$  state. To obtain the violation probability of the S inequality, we perform the experiment for 100 trials. In each trial the four measurement bases for each observer are defined by the four vertices of a randomly rotated, inscribed tetrahedron of the Bloch sphere. So there are a total of  $4^3 = 64$  joint measurement settings in each trial. For each measurement setting, the data collection time is 100 s. By recording all possible four-photon coincidence counts among the four APAS, we can calculate the total of  $(4 \times 3)^3 = 1728$  S inequality values and check whether the maximal value in them violates the S inequality. The experimental results of the 100 trials are shown in Fig. 4. We can see that among the 100 trials, there are 58 trials which violate the S inequality for more than one standard error. This gives us a rough estimation of the violation probability to be 0.58.

To study the case of violating the S inequality with one shared axis, we employ the Y shape measurement scheme, assuming that the Z axis is shared among different observers. We perform a total of 120 trials of the experiment. Each trial has the same experimental process as described above, except that the chosen measurement bases changed to the Y shape bases and the random rotations are limited in the  $\sigma_x$ - $\sigma_y$  plane. So, for each trial, we perform  $3^3 = 27$  joint measurements and get  $(3 \times 2)^3 = 216$  S inequality values. The experimental results are shown in Fig. 5. In contrast to the results in Fig. 4, we can see clearly that all 120 trials illustrate a violation of the S inequality. And the smallest S inequality value among the 120 trials is  $1.089 \pm 0.05$ .

When discussing the results in the above two cases, although in the first alignment-free case we have demonstrated genuine tripartite nonlocality with significant probability, the difference between the experimental results and the numerical simulation result of a 0.871 violation probability is still large.

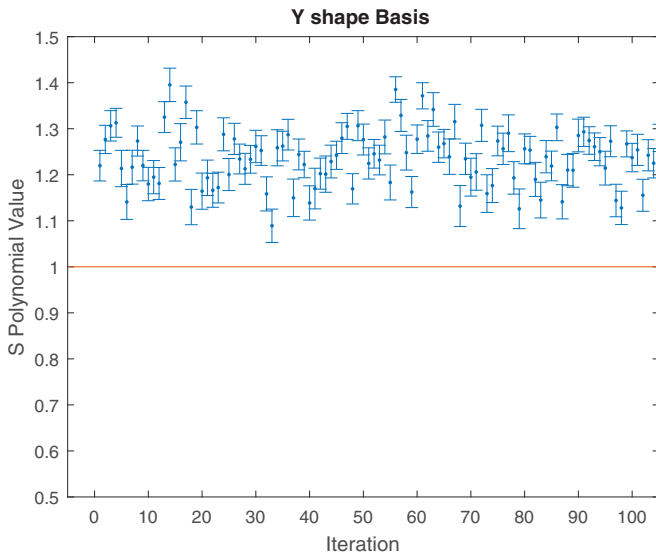


FIG. 5. The measured S inequality values for 120 experimental trials in the case of sharing only one common axis, using the Y shape measurement scheme.

This is mainly due to our nonideal state fidelity 0.97 of  $|\text{GHZ}_3\rangle$  and the limited standard error 0.05 of the S inequality values. Through numerical simulation, the experimentally reconstructed density matrix  $\rho_{\text{expt}}$  with a 0.97 fidelity will lead to a large decrease of the violation probability from 0.871 to 0.692. Furthermore, when accounting for the standard error 0.05 of the S inequality values, we cannot take the trials which only violate the S inequality with less than one standard error into account, which leads to a further decrease of the violation probability. Lastly, to estimate a probability seriously, 100 is not an adequate number of trials in fact, and this number is limited by the limited data collection time of the whole experiment. However, fortunately, a more practical situation is actually the second case, in which one common axis exists. For such a partial-alignment-free case, the experiment results give out a unit probability even with the unavoidable state preparation noise.

It is worth noting that, although our results seem to have some similarity with the device-independent entanglement witness (DIEW) [40,41], they are very different in terms of

their starting points and purposes. The DIEW is used to avoid false-positive entanglement detections when the devices are unreliable, i.e., misalignment of their devices should not make separable states to violate the DIEW, which would probably happen with the traditional entanglement witnesses [40]. However, in our work, the purpose is to see violations of the inequality with success probability as high as possible when the input states are genuine multipartite nonlocal but no reference frames are shared. So we aim to avoid some false-negative nonlocality detection. In fact, for nonlocality detection with Bell inequalities, there is no opportunity for false-positive nonlocality detection, in spite of the misaligned reference frames. This is determined by the construction of Bell inequalities: the measurement operators in a Bell inequality can take various directions, but such variations would never make a local state to violate the inequality, it only gives us a way to maximize the violation for a nonlocal state.

#### IV. CONCLUSION

In summary, we experimentally demonstrate genuine multipartite nonlocality in an alignment-free case and a partial-alignment-free case. When there is no shared reference axis, a significant violation probability of the S inequality is observed with a tetrahedral measurement scheme. Furthermore, if one reference axis is shared, the violation of the S inequality is achieved with certainty using a Y shape measurement scheme, even when the initial GHZ state is noisy. Our results illustrate that the multipartite nonlocality is more ubiquitous than people have ever thought and would be useful for studying multipartite nonlocality in the scenarios of practically realizing multipartite quantum communication tasks among distant parties.

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