Nonreciprocal conversion between microwave and optical photons in electro-optomechanical systems

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We propose to demonstrate nonreciprocal conversion between microwave and optical photons in an electro-optomechanical system where a microwave mode and an optical mode are coupled indirectly via two nondegenerate mechanical modes. The nonreciprocal conversion is obtained in the broken time-reversal symmetry regime, where the conversion of photons from one frequency to the other is enhanced for constructive quantum interference while the conversion in the reversal direction is suppressed due to destructive quantum interference. It is interesting that the nonreciprocal response between the microwave and optical modes in the electro-optomechanical system appears at two different frequencies with opposite directions. The proposal can be used to realize nonreciprocal conversion between photons of any two distinctive modes with different frequencies. Moreover, the electro-optomechanical system can also be used to construct a three-port circulator for three optical modes with distinctively different frequencies by adding an auxiliary optical mode coupled to one of the mechanical modes.

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I. INTRODUCTION

Photons with a wide range of frequencies play an important role in the quantum information processing and quantum networks [1–4]. Microwave photons can be fast manipulated for information processing [1,2], while the optical photons are more suitable for information transfer over long distances [3,4]. However, the microwave and optical systems are not compatible with each other naturally. In order to harness the advantages of photons with different frequencies, quantum interfaces are needed to convert photons of microwave and optical modes. A hybrid quantum system should be built by combining two or more physical systems [5,6].

An optomechanical (electromechnical) system is a very good candidate to serve as a quantum interface since the mechanical resonators can be easily coupled to various electromagnetic fields with distinctively different wavelengths through radiation pressure (for reviews, see Refs. [7–10]). In recent years, enormous progress has been made in optomechanical (electromechanical) systems, such as normal-mode splitting in the strong coupling regime [11,12], ground-state cooling of mechanical resonators [13–15], and coherent state transfer between itinerant microwave (optical) fields and a mechanical oscillator [16,17]. A hybrid electro-optomechanical system wherein a mechanical resonator is coupled to both microwave and optical modes simultaneously provides us a quantum interface between microwave and optical systems [18,19]. It was proposed theoretically that high fidelity quantum state transfer between microwave and optical modes can be realized by using the mechanically dark mode, which is immune to mechanical dissipation [20–23], and this proposal was demonstrated experimentally very quickly [24–26]. The conversion between microwave and optical fields via electro-optomechanical systems has been achieved in several different experimental setups [27–29], and it was shown that the wavelength conversion process is coherent and bidirectional [28]. The electro-optomechanical systems have also been studied for strong entanglement generation between microwave photons and optical photons [30–33], and such a strong continuous-variable (CV) entanglement can be exploited for the implementation of reversible CV quantum teleportation with a fidelity exceeding the no-cloning limit [30] and microwave quantum illumination [33].

The nonreciprocal effect is the fundamental of isolators and circulators which are very important devices for information processing. Such an effect appears usually due to the broken time-reversal symmetry [34,35]. There are two main avenues to break the time-reversal symmetry for photons: (i) using magneto-optical effects (e.g., Faraday rotation) [36–45] and (ii) nonmagnetic strategies by employing optical nonlinearity [46–60] or dynamic modulation [61–79]. Nonmagnetic optical nonreciprocity based on dynamic modulation has drawn more and more attentions in recent years, and many structures have been demonstrated experimentally [61–74] or proposed theoretically [75–79].

The nonreciprocal effect has also been developed in the context of optomechanical systems. The optical nonreciprocal effect was proposed in an optomechanical system consisting of an in-line Fabry-Perot cavity with one movable mirror and one fixed mirror based on the momentum difference between forward and backward-moving light beams [80]. Nonreciprocity was also studied in a microring optomechanical system when the optomechanical coupling is enhanced in

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one direction and suppressed in the other one by optically pumping the ring resonator [81] or by resonant Brillouin scattering [82,83]. Some of us (Xu and Li) demonstrated the possibility of an optical nonreciprocal response in a three-mode optomechanical system [84] where one mechanical mode is optomechanically coupled to two linearly interacted optical modes simultaneously, and the time-reversal symmetry of the system can be broken by tuning the phase difference between the two optomechanical coupling rates [85–88]. As discussed in the theoretical outlook of a recent experiment [89], optical nonreciprocity can be achieved in distantly coupled optomechanical systems with a waveguide that can mediate a tight-binding-type coupling for both the mechanical and optical cavity modes. It is worth mentioning that the two cavity modes given in Refs. [84,89] are coupled to each other directly, so that the optical modes need to be resonant or nearly resonant. On how to obtain the nonreciprocal response between two cavity modes of distinctively different wavelengths (such as a microwave mode and an optical mode), there is still a lack of studies.

More recently, Metelmann and Clerk gave a general method for generating nonreciprocal behavior in cavity-based photonic devices by employing reservoir engineering [90]. In the spirit of the general approach of Ref. [90], here we propose an optomechanical nonreciprocal device which allows photon routing with unidirectional links combining mechanically mediated coherent and dissipative couplings. In our proposal, the links convert the signal carrier frequency from the microwave to the optical domain (or vice versa). The transmission of photons from one mode to the other is determined by the quantum interference between the two paths through the mechanically mediated coherent and dissipative couplings. Due to the broken time-reversal symmetry, the nonreciprocity is obtained when the transmission of photons from one mode to the other is enhanced for constructive quantum interference while the transmission in the reversal direction is suppressed with destructive quantum interference. It is interesting that the electro-optomechanical system shows a nonreciprocal response between the optical and microwave modes at two different frequencies with opposite directions. Moreover, after adding an auxiliary optical mode to couple to one of the mechanical modes, the electro-optomechanical system can be used as a three-port circulator for three optical modes with distinctively different frequencies.

This paper is organized as follows: In Sec. II, the Hamiltonian of an electro-optomechanical system is introduced and the spectra of the optical output fields are given. The nonreciprocal conversion between the microwave and optical photons is shown in Sec. III, and a three-port circulator for three optical modes with distinctively different frequencies is discussed in Sec. IV. Finally, we summarize the results in Sec. V.

II. MODEL

As schematically shown in Fig. 1(a), the electro-optomechanical system is composed of two cavity modes (a microwave mode and an optical mode), each of which is coupled to two nondegenerate mechanical modes. The two cavity modes cannot couple to each other directly because of the vast difference of their wavelengths. The Hamiltonian of the electro-optomechanical system is ($\hbar = 1$)

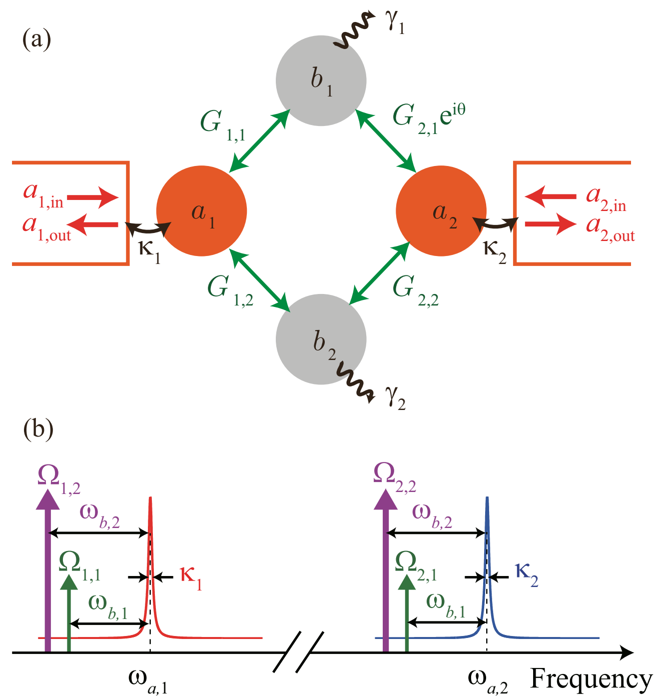

FIG. 1. (a) Schematic diagram of an electro-optomechanical system consisting of two cavity modes (a_1 and a_2) and two mechanical modes (b_1 and b_2). The cavity mode i and the mechanical mode j are coupled with effective optomechanical coupling strength $G_{i,j}$ ($i,j = 1,2$). (b) Schematic panel indicating the relevant frequencies involved in the nonreciprocal conversion process. The cavity mode i is driven by a two-tone laser at two frequencies $\omega_{a,i} - \omega_{b,1}$ and $\omega_{a,i} - \omega_{b,2}$ with amplitudes $\Omega_{i,1}$ and $\Omega_{i,2}$ in the well resolved sidebands ($\omega_{b,j} \gg \{\kappa_i,\gamma_j\}$, where the damping rate of the mechanical mode γ_j is not shown in the drawing).

$$H_{\mathrm{eom}} = \sum_{i=1,2} \omega_{a,i} a_i^\dagger a_i + \sum_{j=1,2} \omega_{b,j} b_j^\dagger b_j + \sum_{i,j} g_{i,j} a_i^\dagger a_i (b_j + b_j^\dagger) + \sum_{i,j} \Omega_{i,j} (a_i e^{i(\omega_{a,i}-\omega_{b,j})t} e^{i\phi_{i,j}} + \mathrm{H.c.}), \quad (1)$$

where a_i ($a_i^\dagger$) is the bosonic annihilation (creation) operator of the cavity mode i with resonance frequency $\omega_{a,i}$, b_j ($b_j^\dagger$) is the bosonic annihilation (creation) operator of the mechanical mode j with resonance frequency $\omega_{b,j}$, and $g_{i,j}$ is the electromechanical (optomechanical) coupling strength between the cavity mode i and the mechanical mode j ($i,j = 1,2$). The cavity mode i is driven by a two-tone laser at two frequencies $\omega_{a,i} - \omega_{b,1}$ and $\omega_{a,i} - \omega_{b,2}$ with amplitudes $\Omega_{i,1}$ and $\Omega_{i,2}$ in the well resolved sidebands ($\omega_{b,j} \gg \{\kappa_i,\gamma_j\}$) as schematically shown in Fig. 1(b), where κ_i is the decay rate of the cavity mode i and γ_j is the damping rate of the mechanical mode j. $\phi_{i,j}$ is the phase of the driving field. We can write each operator for the cavity modes as the sum of its quantum fluctuation operator and classical mean value, $a_i \to a_i + \alpha_i(t)$. In the condition that $\min[\omega_{b,j}, |\omega_{b,1} - \omega_{b,2}|] \gg \max[|g_{i,j}\alpha_i(t)|]$, the classical part $\alpha_i(t)$ can be given approximately as $\alpha_i(t) \approx \sum_{j=1,2} \alpha_{i,j} e^{i\omega_{b,j}t}$, where the classical amplitude $\alpha_{i,j}$ is determined by solving the classical equation of motion with only cavity drive $\Omega_{i,j}$ at frequency $\omega_{a,i}-\omega_{b,j}$ [31,32,91,92]. To linearize the Hamiltonian (1), we take $|\alpha_{i,j}|\gg 1$ so that we can only keep the first-order terms in the small quantum fluctuation operators; then the linearized Hamiltonian in the interaction picture with respect to $H_{\mathrm{eom},0}=\sum_{i=1,2}\omega_{a,i}a_i^\dagger a_i+\sum_{j=1,2}\omega_{b,j}b_j^\dagger b_j$ is obtained as

$$\begin{aligned}H_{\mathrm{eom,int}}=&\,G_{1,1}a_1^\dagger b_1+G_{1,1}a_1b_1^\dagger+G_{1,2}a_1^\dagger b_2+G_{1,2}a_1b_2^\dagger\\&+G_{2,1}e^{i\theta}a_2^\dagger b_1+G_{2,1}e^{-i\theta}a_2b_1^\dagger+G_{2,2}a_2^\dagger b_2\\&+G_{2,2}a_2b_2^\dagger,\end{aligned}\tag{2}$$

where $G_{i,j}=|g_{i,j}\alpha_{i,j}|$ is the effective electromechanical (optomechanical) coupling strength and the nonresonant and counter-rotating terms have been neglected. The phase of $\alpha_{i,j}$ can be controlled by tuning the phases $\phi_{i,j}$ of the driving fields. Actually, here the phases of $\alpha_{i,j}$ (three of them) have been absorbed by redefining the operators a_i and b_j, and only the total phase difference θ between them has physical effects. Without a loss of generality, θ is only kept in the terms of $a_2^\dagger b_1$ and $a_2b_1^\dagger$ in Eq. (2) and the following derivation.

By the Heisenberg equation and taking into account the damping and corresponding noise terms, we get the quantum Langevin equations (QLEs) for the operators of the optical and mechanical modes:

$$\frac{d}{dt}V(t)=-MV(t)+\sqrt{\Gamma}V_{\mathrm{in}}(t),\tag{3}$$

with the vector $V(t)=(a_1,a_2,b_1,b_2)^T$ of fluctuation operators, the vector $V_{\mathrm{in}}(t)=(a_{1,\mathrm{in}},a_{2,\mathrm{in}},b_{1,\mathrm{in}},b_{2,\mathrm{in}})^T$ of input operators, the diagonal damping matrix $\Gamma=\mathrm{diag}(\kappa_1,\kappa_2,\gamma_1,\gamma_2)$, and the coefficient matrix

$$M=\begin{pmatrix}\frac{\kappa_1}{2}&0&iG_{1,1}&iG_{1,2}\\0&\frac{\kappa_2}{2}&iG_{2,1}e^{i\theta}&iG_{2,2}\\iG_{1,1}&iG_{2,1}e^{-i\theta}&\frac{\gamma_1}{2}&0\\iG_{1,2}&iG_{2,2}&0&\frac{\gamma_2}{2}\end{pmatrix}.\tag{4}$$

$a_{i,\mathrm{in}}$ and $b_{j,\mathrm{in}}$ are the input quantum fields with zero mean values. The system is stable only if the real parts of all the eigenvalues of matrix M are positive. The stability conditions can be given explicitly by using the Routh-Hurwitz criterion [93–97]. However, they are too cumbersome to be given here. All of the parameters used in the following satisfy the stability conditions.

Let us introduce the Fourier transform for an operator o

$$\widetilde{o}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}o(t)e^{i\omega t}dt,\tag{5}$$

$$\widetilde{o^\dagger}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}o^\dagger(t)e^{i\omega t}dt;\tag{6}$$

then the solution to the QLEs (3) in the frequency domain can be given by

$$\widetilde{V}(\omega)=(M-i\omega I)^{-1}\sqrt{\Gamma}\widetilde{V}_{in}(\omega),\tag{7}$$

where I denotes the identity matrix. Using the standard input-output theory [98], the Fourier transform of the output vector $V_{\mathrm{out}}(t)=(a_{1,\mathrm{out}},a_{2,\mathrm{out}},b_{1,\mathrm{out}},b_{2,\mathrm{out}})^T$ is obtained as [99]

$$\widetilde{V}_{\mathrm{out}}(\omega)=U(\omega)\widetilde{V}_{\mathrm{in}}(\omega),\tag{8}$$

where

$$U(\omega)=\sqrt{\Gamma}(M-i\omega I)^{-1}\sqrt{\Gamma}-I.\tag{9}$$

The spectrum of the field with operator o is defined as

$$s_o(\omega)=\int_{-\infty}^{+\infty}d\omega'\langle\widetilde{o^\dagger}(\omega')\widetilde{o}(\omega)\rangle;\tag{10}$$

then the spectra of the input quantum fields, $s_{v_{\mathrm{in}}}(\omega)$, are obtained as $\langle\widetilde{v_{\mathrm{in}}^\dagger}(\omega')\widetilde{v_{\mathrm{in}}}(\omega)\rangle=s_{v_{\mathrm{in}}}(\omega)\delta(\omega+\omega')$ and $\langle\widetilde{v_{\mathrm{in}}}(\omega')\widetilde{v_{\mathrm{in}}^\dagger}(\omega)\rangle=[1+s_{v_{\mathrm{in}}}(\omega)]\delta(\omega+\omega')$, where the term "1" results from the effect of vacuum noise and $\widetilde{v_{\mathrm{in}}^\dagger}$ ($\widetilde{v_{\mathrm{in}}}$) is the Fourier transform of $v_{\mathrm{in}}^\dagger$ (v_{in}) (for $v_{\mathrm{in}}=a_{1,\mathrm{in}},a_{2,\mathrm{in}},b_{1,\mathrm{in}},b_{2,\mathrm{in}}$). The relation between the vector of the spectrum of the output fields $S_{\mathrm{out}}(\omega)$ and the vector of the spectrum of the input fields $S_{\mathrm{in}}(\omega)$ is given by

$$S_{\mathrm{out}}(\omega)=T(\omega)S_{\mathrm{in}}(\omega),\tag{11}$$

where $S_{\mathrm{in}}(\omega)=(s_{a_{1,\mathrm{in}}}(\omega),s_{a_{2,\mathrm{in}}}(\omega),s_{b_{1,\mathrm{in}}}(\omega),s_{b_{2,\mathrm{in}}}(\omega))^T$, $S_{\mathrm{out}}(\omega)=(s_{a_{1,\mathrm{out}}}(\omega),s_{a_{2,\mathrm{out}}}(\omega),s_{b_{1,\mathrm{out}}}(\omega),s_{b_{2,\mathrm{out}}}(\omega))^T$. Here $T(\omega)$ is the transmission matrix with the element $T_{v,w}(\omega)$ (for $v,w=a_1,a_2,b_1,b_2$) denoting the scattering probability from mode w to mode v. In the next section, we will focus on the photon scattering probability between the two cavity modes. For simplicity, we define $T_{12}(\omega)\equiv T_{a_1,a_2}(\omega)=|U_{12}(\omega)|^2$ and $T_{21}(\omega)\equiv T_{a_2,a_1}(\omega)=|U_{21}(\omega)|^2$, where $U_{ij}(\omega)$ represents the element at the ith row and jth column of the matrix $U(\omega)$ given in Eq. (9).

III. OPTICAL NONRECIPROCITY

We assume that the effective optomechanical coupling strengths $G_{i,j}$, the decay rates κ_i of the cavity modes, and the damping rate γ_j of the two mechanical modes satisfy the relation

$$\gamma_1\ll G_{i,j}\sim\kappa_1=\kappa_2\equiv\kappa\ll\gamma_2,\tag{12}$$

i.e., the damping of the mechanical mode 1 is much slower than the decay of the cavity modes and this is usually satisfied; the damping of the mechanical mode 2 is much faster than the decay of the cavity modes and this condition can be realized by coupling the mechanical mode 2 to an auxiliary cavity mode (more details are shown in next section). Under the assumption (12), the operators of the mechanical mode 2 can be eliminated from QLE (3) adiabatically [100,101]; then we have

$$\frac{d}{dt}V'(t)=-M'V'(t)+\sqrt{\Gamma'}V'_{\mathrm{in}}(t)-i\sqrt{\Lambda}b_{2,\mathrm{in}},\tag{13}$$

with the vector $V'(t)=(a_1,a_2,b_1)^T$ of fluctuation operators, the vector $V'_{\mathrm{in}}(t)=(a_{1,\mathrm{in}},a_{2,\mathrm{in}},b_{1,\mathrm{in}})^T$ of input operators, the diagonal damping matrices $\Gamma'=\mathrm{diag}(\kappa_1,\kappa_2,\gamma_1)$, $\Lambda=\mathrm{diag}(\gamma_{1,2},\gamma_{2,2},0)$, and the coefficient matrix

$$M'=\begin{pmatrix}\frac{\kappa_1+\gamma_{1,2}}{2}&J_2&iG_{1,1}\\J_2&\frac{\kappa_2+\gamma_{2,2}}{2}&iG_{2,1}e^{i\theta}\\iG_{1,1}&iG_{2,1}e^{-i\theta}&\frac{\gamma_1}{2}\end{pmatrix},\tag{14}$$

where the dissipative coupling strength $J_2 = 2G_{1,2}G_{2,2}/\gamma_2$, and the decay rates $\gamma_{1,2} = 4G_{1,2}^2/\gamma_2$ and $\gamma_{2,2} = 4G_{2,2}^2/\gamma_2$ are induced by the mechanical mode 2. Using the Fourier transform and the standard input-output relation, we can get the output vector $V'_{\rm out}(t) = (a_{1,\rm out}, a_{2,\rm out}, b_{1,\rm out})^T$ in the frequency domain as

$$\widetilde{V}'_{\rm out}(\omega) = U'(\omega)\widetilde{V}'_{\rm in}(\omega) - iL'(\omega)b_{2,\rm in}, \tag{15}$$

where

$$U'(\omega) = \sqrt{\Gamma'}(M' - i\omega I)^{-1}\sqrt{\Gamma'} - I, \tag{16}$$

$$L'(\omega) = \sqrt{\Gamma'}(M' - i\omega I)^{-1}\sqrt{\Lambda}. \tag{17}$$

The explicit expressions of the transmission coefficients between the two cavity modes are of the form

$$U'_{12}(\omega) = \frac{-\sqrt{\kappa_1\kappa_2}(J'_1 + J_2)}{D(\omega)}, \tag{18}$$

$$U'_{21}(\omega) = \frac{-\sqrt{\kappa_1\kappa_2}(J_1 + J_2)}{D(\omega)}, \tag{19}$$

where

$$D(\omega) = \left[\frac{\kappa_{1,\rm tot}}{2} - i(\omega - \omega_{1,1})\right]\left[\frac{\kappa_{2,\rm tot}}{2} - i(\omega - \omega_{2,1})\right] - (J_1 + J_2)(J'_1 + J_2). \tag{20}$$

Here $\kappa_{i,\rm tot}$ is the total damping rate of the cavity mode i given by

$$\kappa_{i,\rm tot} = \kappa_i + \gamma_{i,1} + \gamma_{i,2}. \tag{21}$$

The ω-dependent effective coupling strength J_1 (J'_1) (coherent coupling), the effective damping rate $\gamma_{i,1}$, and the frequency shift $\omega_{i,1}$ induced by the mechanical mode 1 are given by

$$J_1 = \frac{2G_{1,1}G_{2,1}e^{i\theta}}{\gamma_1 - i2\omega}, \tag{22}$$

$$J'_1 = \frac{2G_{1,1}G_{2,1}e^{-i\theta}}{\gamma_1 - i2\omega}, \tag{23}$$

$$\gamma_{i,1} = \frac{4G_{i,1}^2\gamma_1}{\gamma_1^2 + 4\omega^2}, \tag{24}$$

$$\omega_{i,1} = \frac{4G_{i,1}^2\omega}{\gamma_1^2 + 4\omega^2}. \tag{25}$$

We would like to note that the coherent coupling strength J_1 (J'_1) and damping rates $\gamma_{i,1}$ induced by the mechanical mode 1 are dependent on the frequency ω of the input photons, while the dissipative coupling strength J_2 and decay rates $\gamma_{i,2}$ induced by the mechanical mode 2 are independent of the frequency ω. Moreover, there are frequency shifts $\omega_{i,1}$ induced by the mechanical mode 1 but there are almost no frequency shifts induced by the mechanical mode 2.

Equations (18) and (19) imply that the transmission coefficients between the two cavity modes are determined by the quantum interference of the two paths through the mechanically mediated coherent and dissipative couplings [i.e., J_1 (J'_1) and J_2]. In constructive interference, the transmission rates will be enhanced; in contrast, the transmission rate will be suppressed with destructive interference. The nonreciprocity is obtained in the condition that one of the transmission coefficients [$U'_{12}(\omega)$ or $U'_{21}(\omega)$] is enhanced and the other one is suppressed. The nonreciprocity can be intuitively understood from the schematic diagram shown in Fig. 1(a). The input photons from one cavity mode to the other one undergo a Mach-Zehnder-type interference: one path is the hopping through the mechanical mode 1 and the other path is the hopping through the mechanical mode 2. The phase of the first path is determined by the driven fields as shown in Eq. (2). The nonreciprocal response of the electro-optomechanical system is induced by this phase, which is gauge invariant and is associated with the broken time-reversal symmetry for the system [85–87].

The perfect nonreciprocity is obtained as $|U'_{12}(\omega)| = 1, U'_{21}(\omega) = 0$ or $|U'_{21}(\omega)| = 1, U'_{12}(\omega) = 0$. In order to satisfy $U'_{12}(\omega) = 0$ or $U'_{21}(\omega) = 0$, from Eqs. (18) and (19), we should have

$$J'_1 = -J_2 \text{ or } J_1 = -J_2. \tag{26}$$

Under the assumption (12), i.e., $\gamma_1 \ll G_{i,j} \ll \gamma_2$, we have

$$|\omega| \approx \frac{G_{1,1}G_{2,1}}{G_{1,2}G_{2,2}}\frac{\gamma_2}{2}, \tag{27}$$

and

$$\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}. \tag{28}$$

After substituting Eq. (26) into Eqs. (18) and (19), we obtain the condition for $|U'_{12}(\omega)| = 1$ or $|U'_{21}(\omega)| = 1$ as

$$\frac{8J_2\sqrt{\kappa_1\kappa_2}}{[\kappa_{1,\rm tot} - i2(\omega - \omega_{1,1})][\kappa_{2,\rm tot} - i2(\omega - \omega_{2,1})]} = 1. \tag{29}$$

For simplicity we choose

$$\omega = \omega_{1,1} = \omega_{2,1}; \tag{30}$$

then the condition in Eq. (29) reduces to

$$8J_2\sqrt{\kappa_1\kappa_2} = \kappa_{1,\rm tot}\kappa_{2,\rm tot}. \tag{31}$$

Thus with the assumption (12), the nonreciprocity is obtained as the effective electromechanical (optomechanical) coupling strengths satisfy the conditions (for simplicity, we choose $G_{1,1} = G_{2,1}$ and $G_{1,2} = G_{2,2}$)

$$G_{1,1} = G_{2,1} = \frac{\kappa}{2}, \tag{32}$$

$$G_{1,2} = G_{2,2} = \frac{\sqrt{\gamma_2\kappa}}{2}, \tag{33}$$

and the perfect nonreciprocity appears around the frequencies

$$\omega = \pm\frac{\kappa}{2}. \tag{34}$$

As a specific example, under the conditions given in Eqs. (12), (32), and (33), by choosing $\theta = \pi/2$, the transmission coefficients at frequency $\omega = \kappa/2$ are given by

$$U'_{12}(\omega) \approx -1, \quad U'_{21}(\omega) \approx 0, \tag{35}$$

and the transmission coefficients at frequency $\omega = -\kappa/2$ are given by

$$U'_{12}(\omega) \approx 0, \quad U'_{21}(\omega) \approx -1. \tag{36}$$

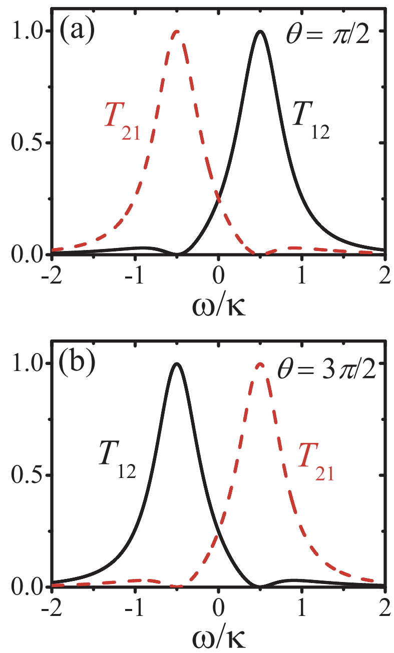

FIG. 2. Scattering probabilities $T_{12}(\omega)$ (black solid lines) and $T_{21}(\omega)$ (red dashed lines) as functions of the frequency of the incoming signal ω for different phase differences: (a) $\theta = \pi/2$ and (b) $\theta = 3\pi/2$. The other parameters are $\kappa_1 = \kappa_2 = \kappa$, $\gamma_1 = \kappa/1000$, $\gamma_2 = 16\kappa$, $G_{1,1} = G_{2,1} = \kappa/2$, and $G_{1,2} = G_{2,2} = 2\kappa$.

Under the same conditions given in Eqs. (12), (32), and (33), if we choose $\theta = 3\pi/2$, when $\omega = \kappa/2$, the transmission coefficients are given by

$$U'_{12}(\omega) \approx 0, \quad U'_{21}(\omega) \approx -1, \tag{37}$$

and when $\omega = -\kappa/2$, the transmission coefficients are given by

$$U'_{12}(\omega) \approx -1, \quad U'_{21}(\omega) \approx 0. \tag{38}$$

In Fig. 2, the scattering probabilities between the two cavity modes $T_{12}(\omega) = |U'_{12}(\omega)|^2$ and $T_{21}(\omega) = |U'_{21}(\omega)|^2$ are plotted as functions of the frequency ω of the incoming signal for different phase differences, where the parameters are given as $\kappa_1 = \kappa_2 = \kappa$, $\gamma_1 = \kappa/1000$, $\gamma_2 = 16\kappa$, $G_{1,1} = G_{2,1} = \kappa/2$, and $G_{1,2} = G_{2,2} = 2\kappa$. When $\theta \neq n\pi$ (n is an integer), the time-reversal symmetry is broken and the electro-optomechanical system exhibits a nonreciprocal response. The optimal optical nonreciprocal response is obtained when $\theta = \pi/2$ or $\theta = 3\pi/2$. As shown in Fig. 2, the electro-optomechanical system shows the nonreciprocal response between the optical and microwave modes at two different frequencies with opposite directions: when $\theta = \pi/2$ as shown in Fig. 2(a), we have $T_{21}(\omega) \approx 1$, $T_{12}(\omega) \approx 0$ at $\omega = -\kappa/2$ and $T_{12}(\omega) \approx 1$, $T_{21}(\omega) \approx 0$ at $\omega = \kappa/2$; when $\theta = 3\pi/2$ as shown in Fig. 2(b), we have $T_{12}(\omega) \approx 1$, $T_{21}(\omega) \approx 0$ at $\omega = -\kappa/2$ and $T_{21}(\omega) \approx 1$, $T_{12}(\omega) \approx 0$ at $\omega = \kappa/2$.

IV. OPTICAL CIRCULATOR

In the derivation of Sec. III, we have assumed that $\kappa_1 = \kappa_2 \ll \gamma_2$, where γ_2 should be the total damping rate of the mechanical mode 2. This assumption seems counterintuitive since usually the damping rate of the mechanical mode is smaller than the decay rate of the cavity mode. In this section, we will show that even when the intrinsic damping rate of the mechanical mode 2 (denoted by $\gamma_{2,0}$) is much smaller than the cavity decay rate κ_i, the total damping rate of the mechanical mode 2 can also satisfy the condition (12) when the mechanical resonator 2 is coupled to an auxiliary cavity mode (cavity mode 3), as shown in Fig. 3. Moreover, we will present the spectra of the output optical fields from the hybrid system which involves the electro-optomechanical system and the auxiliary cavity mode. We will show that the hybrid system can be used as a three-port circulator for three optical modes with distinctively different wavelengths at two different frequencies with opposite directions.

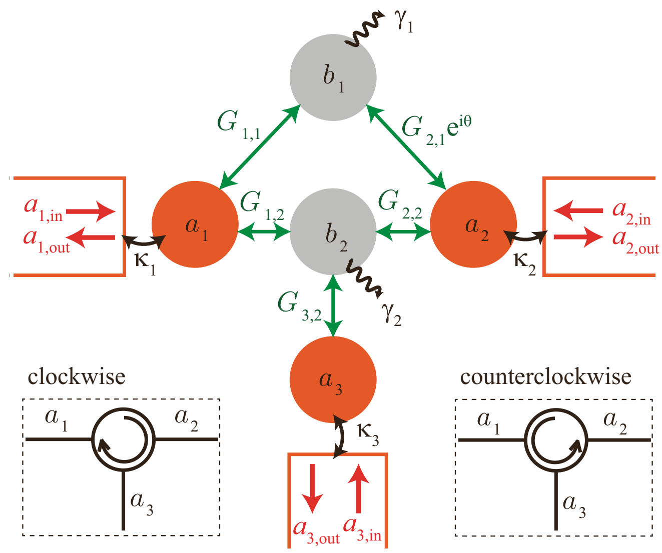

FIG. 3. Schematic diagram of a three-port (a_1, a_2, and a_3) optical circulator by an electro-optomechanical system.

The Hamiltonian of the hybrid system for the electro-optomechanical system with the auxiliary cavity mode is given by

$$H_{\mathrm{cir}} = H_{\mathrm{eom}} + H_{\mathrm{aux}}, \tag{39}$$

and

$$H_{\mathrm{aux}} = \omega_{a,3}a_3^\dagger a_3 + g_{3,2}a_3^\dagger a_3(b_2 + b_2^\dagger) + \Omega_{3,2}(a_3 e^{i(\omega_{a,3}-\omega_{b,2})t} + \mathrm{H.c.}), \tag{40}$$

where a_3 ($a_3^\dagger$) is the bosonic annihilation (creation) operator of the auxiliary cavity mode 3 with resonance frequency $\omega_{a,3}$, and $g_{3,2}$ is the electromechanical (optomechanical) coupling strength between the cavity mode 3 and the mechanical mode 2. The cavity mode 3 is driven with strength $\Omega_{3,2}$ at frequency $\omega_{a,3} - \omega_{b,2}$. In the interaction picture with respect to $H_{\mathrm{cir},0} = \sum_{i=1,2,3}\omega_{a,i}a_i^\dagger a_i + \sum_{j=1,2}\omega_{b,j}b_j^\dagger b_j$, the linearized Hamiltonian of Eq. (39) can be written as

$$H_{\mathrm{cir,int}} \approx H_{\mathrm{eom,int}} + G_{3,2}a_3^\dagger b_2 + G_{3,2}a_3 b_2^\dagger, \tag{41}$$

with the effective optomechanical coupling strength $G_{3,2} = g_{3,2}\alpha_{3,2}$. Without a loss of generality, $G_{3,2}$ is assumed to be

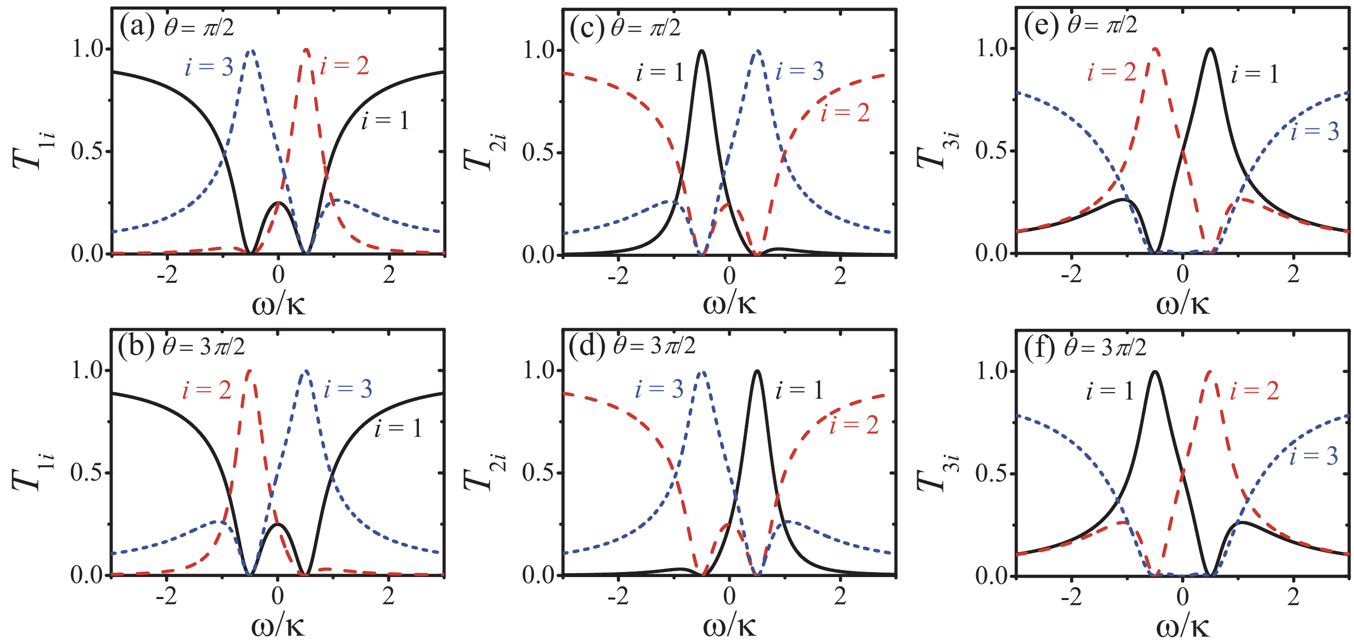

FIG. 4. Scattering probabilities (a) and (b) $T_{1i}(\omega)$, (c) and (d) $T_{2i}(\omega)$, and (e) and (f) $T_{3i}(\omega)$ ($i=1,2,3$) as functions of the frequency of the incoming signal ω for different phase differences: (a), (c), and (e) $\theta=\pi/2$; (b), (d), and (f) $\theta=3\pi/2$. The other parameters are $\kappa_1=\kappa_2=\kappa$, $\kappa_3=10\kappa$, $\gamma_1=\gamma_{2,0}=\kappa/1000$, $G_{1,1}=G_{2,1}=\kappa/2$, $G_{1,2}=G_{2,2}=2\kappa$, and $G_{3,2}=\sqrt{40}\kappa$ (thus, $\gamma_{2,\mathrm{id}}=16\kappa$).

real. The classical amplitude $\alpha_{3,2}$ is determined by solving the classical equation of motion with only the cavity drive $\Omega_{3,2}$ at frequency $\omega_{a,3}-\omega_{b,2}$.

The QLEs for the operators of the hybrid system are given as

$$\frac{d}{dt}V''(t)=-M''V''(t)+\sqrt{\Gamma''}V''_{in}(t), \tag{42}$$

with the vector $V''(t)=(a_1,a_2,a_3,b_1,b_2)^T$ of fluctuation operators, the vector $V''_{\mathrm{in}}(t)=(a_{1,\mathrm{in}},a_{2,\mathrm{in}},a_{3,\mathrm{in}},b_{1,\mathrm{in}},b_{2,\mathrm{in}})^T$ of input operators, the diagonal damping matrix $\Gamma''=\mathrm{diag}(\kappa_1,\kappa_2,\kappa_3,\gamma_1,\gamma_{2,0})$, and the coefficient matrix

$$M''=\begin{pmatrix}\frac{\kappa_1}{2} & 0 & 0 & iG_{1,1} & iG_{1,2}\\ 0 & \frac{\kappa_2}{2} & 0 & iG_{2,1}e^{i\theta} & iG_{2,2}\\ 0 & 0 & \frac{\kappa_3}{2} & 0 & iG_{3,2}\\ iG_{1,1} & iG_{2,1}e^{-i\theta} & 0 & \frac{\gamma_1}{2} & 0\\ iG_{1,2} & iG_{2,2} & iG_{3,2} & 0 & \frac{\gamma_{2,0}}{2}\end{pmatrix}. \tag{43}$$

Using the Fourier transform and the standard input-output relation, we can express the output vector $V''_{\mathrm{out}}(t)=(a_{1,\mathrm{out}},a_{2,\mathrm{out}},a_{3,\mathrm{out}},b_{1,\mathrm{out}},b_{2,\mathrm{out}})^T$ as

$$\widetilde{V}''_{\mathrm{out}}(\omega)=U''(\omega)\widetilde{V}''_{\mathrm{in}}(\omega), \tag{44}$$

where

$$U''(\omega)=\sqrt{\Gamma''}(M''-i\omega I)^{-1}\sqrt{\Gamma''}-I. \tag{45}$$

Under the assumption that the decay rate of the cavity mode 3 is much larger than the intrinsic damping rate of the mechanical mode 2 and the effective optomechanical coupling strength between the mechanical mode 2 and the cavity mode 3, i.e., $\kappa_3\gg\{\gamma_{2,0},G_{3,2}\}$, we can adiabatically eliminate the cavity mode 3; then we obtained the QLEs (3) with the replacement

$$\gamma_2\rightarrow\gamma_{2,0}+\gamma_{2,\mathrm{id}} \tag{46}$$

in the coefficient matrix, and the replacement

$$b_{2,\mathrm{in}}\rightarrow\sqrt{\gamma_{2,0}/\gamma_2}b_{2,\mathrm{in}}-i\sqrt{\gamma_{2,\mathrm{id}}/\gamma_2}a_{3,\mathrm{in}} \tag{47}$$

in the input operators vector $V_{\mathrm{in}}(t)$. Here $\gamma_{2,\mathrm{id}}$ is the effective damping rate of the mechanical mode 2 induced by the auxiliary cavity mode 3,

$$\gamma_{2,\mathrm{id}}=\frac{4G_{3,2}^2}{\kappa_3}. \tag{48}$$

$\gamma_{2,\mathrm{id}}$ can be controlled by tuning the strength of the driving field on the cavity mode 3. Even if the intrinsic damping rate of the mechanical mode 2 is much smaller than the decay rates of the cavity modes, i.e., $\gamma_{2,0}\ll\kappa_i$, the total damping rate of the mechanical mode 2 (i.e., $\gamma_2=\gamma_{2,0}+\gamma_{2,\mathrm{id}}$) still can satisfy the condition (12) when $\gamma_{2,\mathrm{id}}\gg\kappa_i$.

In the following, we will study the scattering probability between the three cavity modes. For convenience of discussion, we set $T_{ij}(\omega)\equiv T_{a_i,a_j}(\omega)=|U''_{ij}(\omega)|^2$ $(i,j=1,2,3)$. Using Eq. (45), we now show the numerical results of the scattering probabilities between the three cavity modes. As shown in Fig. 4, the electro-optomechanical system shows optical circulator behavior for the three cavity modes at two different frequencies ($\omega=\pm\kappa/2$) with opposite directions. When $\theta=\pi/2$ as shown in Figs. 4(a), 4(c), and 4(e), at frequency $\omega=-\kappa/2$, $T_{21}(\omega)\approx T_{32}(\omega)\approx T_{13}(\omega)\approx 1$ and the other scattering probabilities are equal to zero; at frequency $\omega=\kappa/2$, $T_{12}(\omega)\approx T_{23}(\omega)\approx T_{31}(\omega)\approx 1$ and the other scattering probabilities are equal to zero. When $\theta=3\pi/2$, as shown in Figs. 4(b), 4(d), and 4(f), at frequency $\omega=-\kappa/2$, $T_{12}(\omega)\approx T_{23}(\omega)\approx T_{31}(\omega)\approx 1$ and the other scattering probabilities are equal to zero; at frequency $\omega=\kappa/2$, $T_{21}(\omega)\approx T_{32}(\omega)\approx T_{13}(\omega)\approx 1$

and the other scattering probabilities are equal to zero. That is when $\theta = \pi/2$, the signal is transferred from one cavity mode to another either clockwise ($a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow a_1$) at frequency $\omega = -\kappa/2$ or counterclockwise ($a_1 \rightarrow a_3 \rightarrow a_2 \rightarrow a_1$) at frequency $\omega = \kappa/2$. In contrast to $\theta = \pi/2$, when $\theta = 3\pi/2$, the signal is transferred either counterclockwise at frequency $\omega = -\kappa/2$ or clockwise at frequency $\omega = \kappa/2$.

V. CONCLUSIONS

In summary, we have demonstrated the nonreciprocal conversion between microwave and optical photons in electro-optomechanical systems. The electro-optomechanical system shows nonreciprocal response between the microwave and optical modes at two different frequencies with opposite directions. The proposal is general and can be used to realize nonreciprocal conversion between photons of two arbitrarily different frequencies. Moreover, the electro-optomechanical system with an auxiliary optical mode can be used as a three-port circulator for three optical modes with arbitrarily different frequencies at two different frequencies with opposite directions. The electro-optomechanical system with broken time-reversal symmetry will open up a different kind of quantum interface in the quantum information processing and quantum networks.

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