Nonanalytic pulse discontinuities as carriers of information

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In this paper, we consider optical pulses encoded with two nonanalytic points, and we evaluate the *detectable* information of these pulses in media supporting slow- and fast-light propagation. It is shown that, in some configurations of slow- (subluminal) light propagation, the signal is not readily detectable, albeit the arrival of the encoded nonanalytic points at the receiver. It is thus argued that, from a practical point of view, information should not be entirely associated with the pulse discontinuities. On the other hand, it is confirmed that for propagation in vacuum or a fast-light medium, detectable information is bounded by the nonanalytic points, which create a space-time window within which detectable information cannot escape. As such, the distinction between the nonanalytic points of a signal, its energy transport, and detectable information is demonstrated.

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I. INTRODUCTION

An electromagnetic pulse can be regarded as a superposition of many monochromatic waves. In a dispersive medium, each of these waves propagates with a different phase velocity. Depending on the pulse spectrum, carrier frequency, propagation distance, and medium dispersion strength, the wave packet typically experiences reshaping during propagation. In some cleverly designed configurations, pulse peak propagation with superluminal group velocity (SGV) or negative group velocity (NGV) can be observed [1–4]. On the other hand, it is possible to slow down the pulse with an amount of delay equivalent to several orders of magnitude of the pulse temporal width [5–10]. These phenomena, referred to as *abnormal group velocities* (AGVs), are in full compliance with Einstein's causality. The fundamental rules governing AGVs are well established and can be attributed to spectral reshaping [11–18] and energy exchange between the propagating pulse and the medium [19,20].

Compliance of AGVs with Einstein's causality hinges on the fact that, unlike the pulse front, the input and output pulse peaks are noncausally connected. This has been experimentally demonstrated in Ref. [16], in which a bandpass amplifier was used as a medium supporting negative group delay. The input pulse was truncated at the position of its peak; nevertheless, the "time-advanced" output pulse possessed a well-recognized peak that maintained its shape for a finite duration after which an oscillatory (nonmeaningful) response was observed.

More specifically, it has been observed that the oscillatory response took place after the nonanalytic point (or the singularity due to the truncation of input pulse peak) arrived at the channel output. This suggests that for a superluminal pulse, the detection of meaningful information beyond the arrival of the strictly luminal pulse singularity (nonanalytic point) at the output is fundamentally prohibited [21]. A similar behavior has been demonstrated at optical frequencies in a ring resonator structure supporting superluminal propagation [22,23].

The strictly luminal speed of pulse singularities (nonanalytic points) has been experimentally confirmed in other structures supporting fast and slow light [24,25]. Furthermore, it has been accepted that the propagation of information should be associated with the propagation of these strictly luminal pulse discontinuities [26,27] since they carry the unpredictable behavior of the pulse and cannot be extrapolated using Taylor's expansion [12]. As such, pulse singularities have been widely regarded as the true information carriers.

On the other hand, a family of researchers have considered *detectable* information of an electromagnetic pulse not as a localized point but rather a quantity associated with the time instant at which the signal-to-noise ratio [SNR(t)] at the receiver side reaches a predetermined threshold, according to the following expression:

$$\mathrm{SNR}(t)=\frac{P_{\mathrm{sig}}(t)}{P_{\mathrm{med\ noise}}(t)+P_{\mathrm{det\ noise}}(t)},\qquad(1)$$

where $P_{\mathrm{sig}}(t)$ is the signal power, $P_{\mathrm{det\ noise}}(t)$ is the noise associated with the detector circuit, and $P_{\mathrm{med\ noise}}(t)$ is the total classical and quantum noise associated with the medium at a given propagation distance. This later operational definition has been adopted in Refs. [28–31], to name a few.

Although both definitions of information have been extensively used in the literature, the connection between them remained unclear. In this paper, we bridge the gap between the two definitions of information. We use fast- and slow-light media as examples to demonstrate the physical mechanism of propagation of pulse singularities as opposed to the propagation of *detectable* information. We thus show that associating information with the arrival of nonanalytic points at the receiver is not an entirely valid approach, in general. Our methodology is briefly described in the following subsection.

II. METHODOLOGY

We consider causal Gaussian pulse propagation in media that exhibit slow (and fast) group velocity (v_g). A nonanalytic point (singularity) is introduced by terminating the pulse at the position of its peak. Accordingly, a space-and-time "window" bounded by two discontinuities is generated. The first discontinuity represents the transitional turn-on of the pulse, whereas the second discontinuity arises from the sudden pulse termination (turn-off). Such an approach can be effectively interpreted as if there are two superimposed windows at the input of the medium. The first window is the aforementioned window bounded by the two discontinuities, and the second window represents the pulse envelope. Initially, the boundaries of both windows are perfectly aligned before propagating inside the medium. During pulse propagation, the two windows travel at different speeds, where the relative velocity depends on whether the medium supports fast- or slow-light propagation.

For the case of a medium supporting slow-light propagation, the "discontinuities window" arrives at the output much earlier than a significant part of the *subluminal* envelope of the input pulse. On the other hand, in a vacuum channel (or a medium supporting superluminal/negative group delay), the "discontinuities window" arrives at the output with (right after) the arrival of the pulse envelope.

To interpret the arrival of pulse discontinuities in the context of the arrival of *detectable* information, we evaluate the detectable information according to the operational definition in Eq. (1). It is confirmed that in fast-light propagation, detectable information arrives at a time instant located within the space-time "discontinuities window" from which it can never escape. On the other hand, this is not necessarily the case in media supporting slow light, where information is readily detected at a time instant within or outside the "discontinuities window." Such analysis provides physical interpretations that are useful for any signaling scheme utilizing dispersive channels.

This paper is organized as follows: in Sec. III, we introduce the slow-light medium considered in the analysis. In Sec. IV, we use the finite-difference time-domain method (FDTD) to calculate the field response at the output of the slow-light medium when the input pulse is truncated. In Sec. V, we incorporate the SNR(t) analysis to compare between the arrival of detectable information as opposed to the arrival of discontinuities in slow-light media. In Sec. VI, we describe the behavior of pulse discontinuities in the case of superluminal propagation. Finally, the conclusion is presented in Sec. VII.

III. FIELD PROPAGATION IN A CAUSAL SLOW-LIGHT MEDIUM

We chose an input Gaussian pulse centered at a carrier frequency 47.7 THz (near infrared) with pulse width ($2T$) equal to 0.24 ps. Here, the temporal width T corresponds to e^{-1} of the Gaussian pulse peak amplitude. The input pulse peak interacts with the medium at time $t_0 \sim 3T$. The causal input pulse is expressed as

$$f(t) = u(t)e^{-(\frac{t-t_0}{T})^2}\sin(\omega_c t), \quad (2)$$

where $u(t)$ is a unit step function with a zero value for all time instants $t < 0$ to enforce causality.

For a medium that exhibits slow group velocity, we consider a causal passive Lorentzian medium with a single-resonance whose index of refraction is given by

$$n(\omega) = \sqrt{1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + 2i\delta\omega}}. \quad (3)$$

Here, ω_p and ω_0 refer to the plasma and resonance frequencies, respectively, and δ is the phenomenological linewidth. The Lorentzian model expressed in Eq. (3) gives an accurate description of anomalous dispersion encountered in a vast range of media. In this analysis, we adopt numerical values that correspond to AlGaN, listed in Table I, as a case study.

TABLE I. Numerical values for the parameters of the single-resonance Lorentzian model (AlGaN).

$\omega_0/(2\pi)$	1.921×10^{14} Hz
$\omega_p/(2\pi)$	3.328×10^{14} Hz
$\delta/(2\pi)$	9.756×10^{12} Hz
L	30 μm

The group delay encountered in such a medium is expressed as $\tau_g = \frac{-d\phi}{d\omega}$, where ϕ is the phase of the term $e^{-\omega\,\mathrm{Im}[n(\omega)]L/c}$, and L is the medium length.

The group delay of AlGaN is plotted in Fig. 1. It is observed that the medium exhibits negative group delay in the frequency region around the resonance ω_0. On the other hand, the frequency range around $\omega/(2\pi) = 47.7$ THz corresponds to subluminal τ_g. Evidently, at very low (and high) frequencies, the medium exhibits strictly luminal behavior, thus enforcing causality.

The field is calculated using the FDTD method by incorporating the auxiliary differential equation (ADE) method [32]. In this computation, the discrete time steps have a granularity of $\Delta t = 4 \times 10^{-18}$ s and the space steps are $\Delta z = 4 \times 10^{-9}$ m.

The output response of the medium is plotted in Fig. 2 and compared with propagation in vacuum (with the same length $L = 30$ μm). It is shown that the subluminal pulse peak is retarded with a duration equal to $\sim$0.1133 ps, in agreement with the group delay in Fig. 1.

Thus far, we have presented a scenario in which a causal Gaussian pulse propagates in a medium with subluminal group velocity. In this analysis, we selected AlGaN as a test case; however, other causal media that support subluminal propagation, such as atomic systems with electromagnetic induced transparency (EIT) [33], can be considered in the analysis instead.

In the next section, we describe the propagation dynamics of the pulse discontinuities in the selected medium.

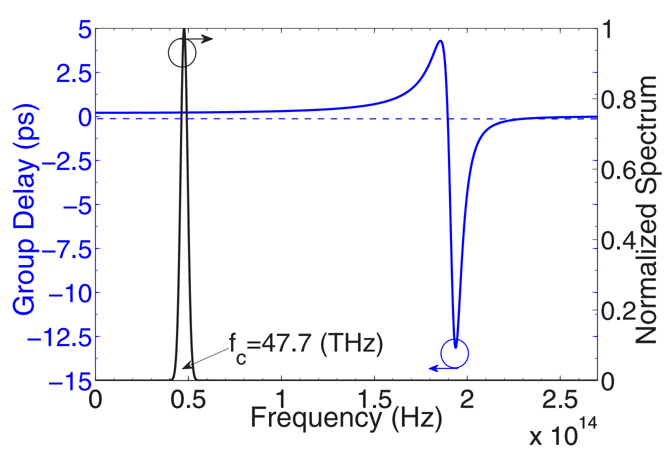

FIG. 1. Input Gaussian excitation spectrum fitted on the group delay (τ_g) of AlGaN. τ_g is calculated for a length $L = 30$ μm. The blue dashed line corresponds to in-vacuum τ_g.

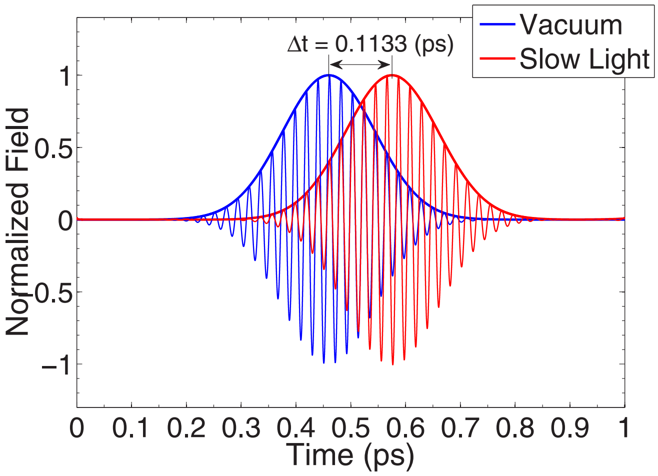

FIG. 2. Comparison between pulse propagation in a subluminal medium (AlGaN) and vacuum with $L = 30\ \mu$m. The pulse propagating in the subluminal medium is delayed by $\Delta t = 0.1133$ ps with respect to the in-vacuum pulse.

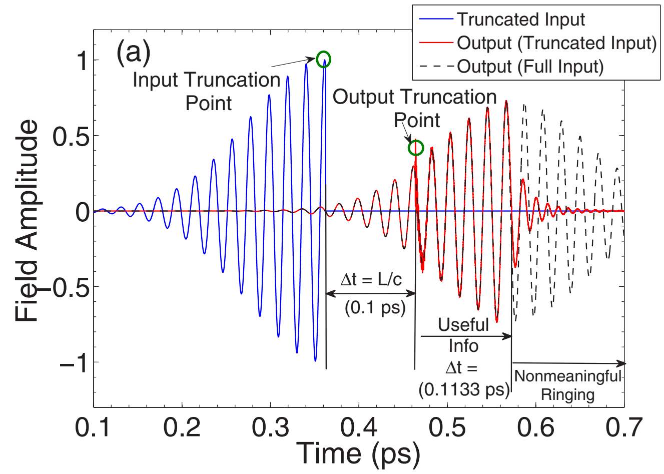

FIG. 3. Output response of the subluminal medium and vacuum when excited by an input pulse terminated at its peak. (a) After a propagation distance of 30 μm in the slow-light medium. (b) After propagation in vacuum with the same length. For both cases (a) and (b), the input singularities arrive at the output at the same time instant (~0.463 ps).

IV. PROPAGATION OF DISCONTINUITIES IN A CAUSAL SLOW-LIGHT MEDIUM

In this section, we investigate the speed of discontinuities in causal slow-light media. The input Gaussian excitation is turned on for a duration that is roughly equal to $\sim 3T$ (0.36 ps) before a sudden transitional turn-off of the signal (discontinuity) is introduced at the position of the pulse peak. The responses of both the slow-light and vacuum channels (with equal length $L = 30\ \mu$m) are plotted in Fig. 3. The output response of the slow-light medium—excited by the terminated input—is plotted and compared with the corresponding output response that would have evolved on the medium if the input pulse had not been terminated.

Some interesting behavior can be observed in Fig. 3: First, it is shown that the sharp termination (marked by a green circle)—which is mathematically represented as a point of nonanalyticity (singularity) due to the sudden turn-off of the input—propagates with a strictly luminal speed c, as expected. This point and the points immediately following it contain high-frequency components that experience the medium as if it were a vacuum channel. Accordingly, the singularities arrive at the output after a duration that is equal to $t = L/c$. This time instant, associated with the arrival of the pulse discontinuity, is the same in both the slow-light medium [depicted in Fig. 3(a)] and in the vacuum channel [Fig. 3(b)], and it is equal to 0.463 ps.

Second, although the pulse discontinuity reaches the output sooner than part of the subluminal energy content of the pulse, this singularity is followed by *meaningful* energy content. Here, a meaningful response denotes an interval over which the output response of the terminated pulse matches the response of the full input pulse (if not terminated) [21]. This interval is denoted in Fig. 3(a) as "Useful Info," and it extends for a duration that is roughly equal to the delay duration $\Delta t = \tau_g - L/c \sim 0.1133$ ps, where τ_g is the subluminal group delay. This implies that for the slow light, *the medium is capable of reproducing the input excitation at least for a period* $\Delta t \sim 0.1133$ *ps—as if the singularity did not exist.*

One might think that the medium would no longer have a meaningful response at the output after the singularity (pulse termination) has arrived, in analogy with fast light [16]. However, this is not true as the medium keeps reacting even past the arrival of the singularity. Certainly, this is a consequence of the medium reacting to the portion of the input pulse preceding the singularity at the input of the channel. The remainder of the output energy following the singularity is coherent, conserved, and can be distinguished without distortion for an amount of $\Delta t \sim 0.1133$ ps. Finally, after $\Delta t \sim 0.1133$ ps, the output pulse starts ringing and has no resemblance to the response of the unterminated input.

Third, for the case of vacuum, the output response represents a mere phase shift of the input pulse in space and time. This is evident in the truncation observed at the output as a response to the input termination.

The above observations can be interpreted as follows: a window that has two boundaries is created; the first boundary is the discontinuity at the front due to the transitional turn-on of the pulse, and the other boundary is the second discontinuity as a result of the pulse peak termination. This window maintains a fixed duration ($\sim 3T$) during propagation and propagates strictly at c. At a propagation distance equal to 30 μm, the subluminal pulse envelope is retarded (with delay $\Delta t \sim$ 0.1133 ps with respect to the discontinuity window). However, the space-time window bounded by the two discontinuities is aligned with its counterpart in the vacuum channel [depicted in Fig. 3(b)]. To study the relation between the arrival of pulse discontinuities as opposed to the arrival of detectable information content of the pulse [34], the signal-to-noise ratio [SNR(t)] is calculated at the receiver side, as discussed next.

V. ARRIVAL OF PULSE DISCONTINUITIES AND DETECTABLE INFORMATION IN SLOW-LIGHT MEDIA

In this section, we investigate the relation between the points of nonanalyticity of a causal pulse and its detectable information content. As such, we apply the approach expressed in Eq. (1). Here, the propagating pulse represents an on-off keying (OOK) communication scheme. The noise sources include the following: shot noise, detector thermal noise, and dark current. The SNR(t) is calculated for the subluminal pulse presented in Fig. 3(a) at a propagation distance $L = 30$ μm and compared to in-vacuum pulse propagation, as depicted in Fig. 4(a). The corresponding fields after propagating distance $L = 30$ μm are plotted in Fig. 4(b). The solid black rectangle marks the boundaries of the space-time "discontinuities window" (singularities due to pulse turn-on and turn-off). Evidently, the boundaries of such a window are aligned in the case of vacuum and the slow-light medium [the turn-off singularity in the slow pulse is pronounced by the sudden shot in the red pulse at $t \sim 0.463$ ps in Fig. 4(b)].

From the SNR analysis, the following observations can be made: First, the pulse front in both the slow and in-vacuum pulses occurs at a strictly luminal speed (L/c) but at a very low value of SNR (−250 dB).

Second, for the case of slow light, there is an abrupt increase in the SNR that corresponds to the arrival of the second precursor (Brillouin precursor). If the detectable information is defined by the time instant at which the SNR exceeds the 0 dB level, when the signal level is comparable to the noise level, then it is observed that detectable information is recorded at 0.38 ps, while the singularity due to the pulse truncation occurs at 0.463 ps. This implies that detectable information (marked by a green circle) is localized within the window of discontinuities (marked by a black rectangle). Such an observation is consistent with the behavior that one would expect for the case of fast-light propagation.

On the other hand, if we consider the case of longer propagation distances ($L = 70$ μm), which incurs larger group delay, the detectable information is no longer localized within the window of discontinuities (nonanalytic points); this is contrary to the aforementioned scenario and the case of fast light, as depicted in Fig. 5(a). The pulse truncation singularity (marked by black square) occurs at 0.59 ps in both pulses, while the detectable information (marked by green circle) is recorded at 0.68 ps, implying the leakage of detectable information outside the window of discontinuities. The SNR(t)

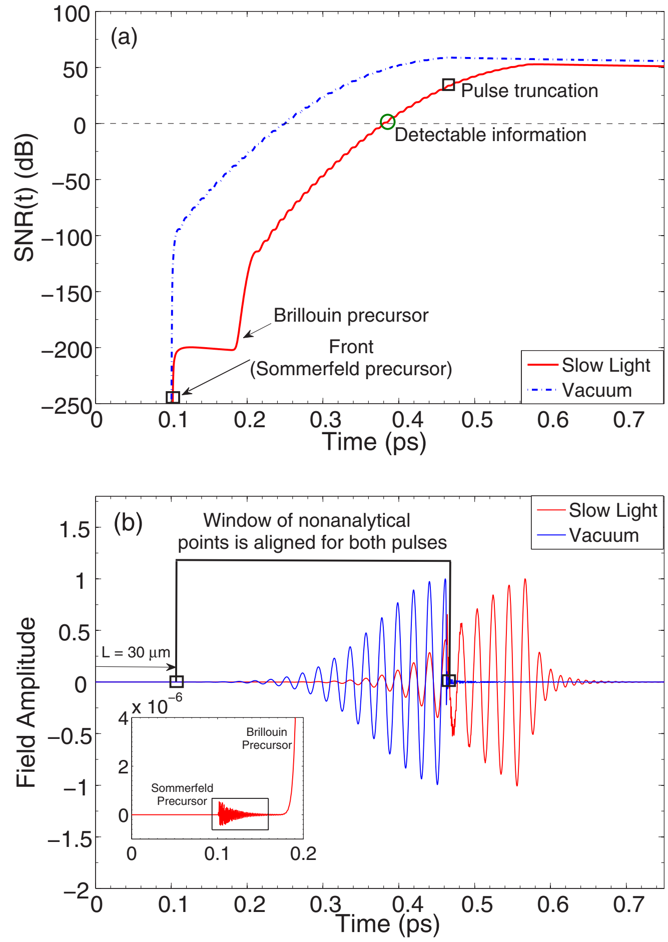

FIG. 4. (a) SNR analysis in slow and vacuum channels ($L =$ 30 μm). The detectable information, marked by 0 dB, lies within the window of discontinuities. (b) Pulse propagation in slow and vacuum channels. The two discontinuities (pulse front and truncation) are aligned in both media. The high-frequency oscillations due to the pulse termination travel at (c) and arrive, simultaneously, at 0.463 ps in both media.

analysis confirms the arrival of the front, in both vacuum and the slow-light medium, simultaneously with a very low SNR level.

It is now more pronounced that the window of discontinuities (marked by the black rectangle) is *time advanced* with respect to the center of mass (energy) of the pulse, as depicted in Fig. 5(b). This can be interpreted as if the pulse envelope has escaped outside the space-time "discontinuities window." These observations suggest that the detectable information content of subluminal pulses can still be retrieved even after the arrival of the pulse discontinuities at the receiver. Therefore, *detectable information (signal velocity) cannot be entirely associated with nonanalytic pulse singularities*.

It should be noted that while the above conclusion has been drawn for AlGaN, it can be generalized for any causal subluminal medium, as long as the high-frequency components (associated with the nonanalytic pulse discontinuities) experience a strictly luminal delay during propagation. This is valid in the current AlGaN model in addition to any causal atomic system [33]. The main difference lies mostly in the

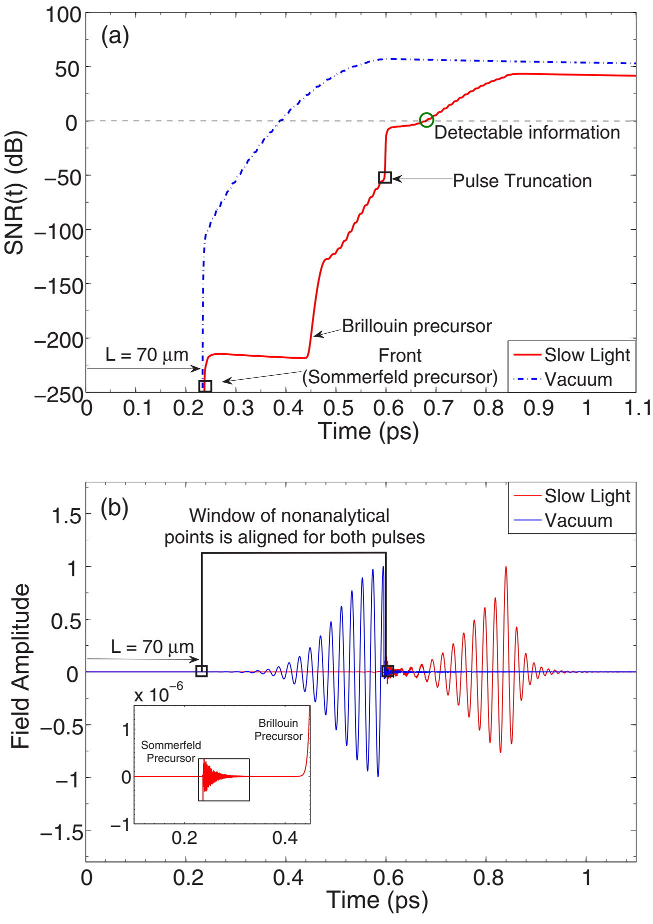

FIG. 5. (a) SNR analysis in slow and vacuum channels ($L = 70$ μm). The detectable information, marked by 0 dB, lies within the window of discontinuities. (b) Pulse propagation in slow and vacuum channels. The two discontinuities (pulse front and truncation) are aligned in both media. The high-frequency oscillations due to the pulse termination travel at (c) and arrive, simultaneously, at 0.59 ps in both media.

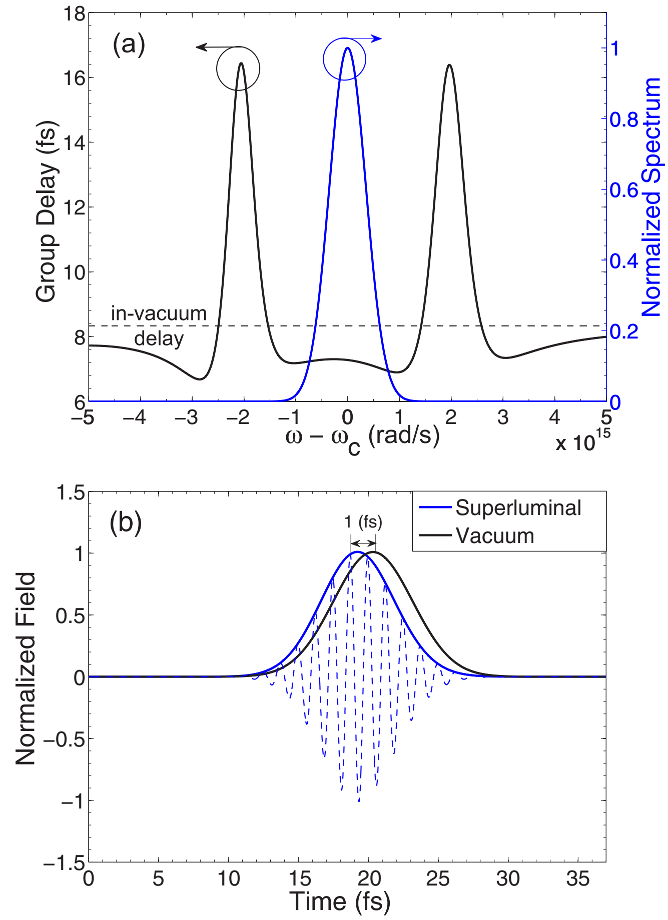

FIG. 6. Superluminal medium response in the frequency and time domain. (a) Input pulse spectrum fitted over the medium group delay. The horizontal axis denotes the frequency detuning, and the horizontal black dashed line marks the in-vacuum group delay. (b) Comparison between the time-domain responses of the superluminal medium and vacuum channel after propagating a distance $L = 2.5$ μm. The superluminal pulse experiences 1 fs time advancement in addition to a slight compression of 3.75% of the initial FWHM.

nature and quantification of the noise associated with the detectable information calculations.

In this section, the distinction between pulse discontinuities and detectable information has been illustrated in slow-light media. In the next section, we briefly discuss the behavior of pulse discontinuities in superluminal media to confirm previous predictions.

VI. PROPAGATION OF INFORMATION IN A CAUSAL SUPERLUMINAL MEDIUM

We consider a double-resonance Lorentzian medium with gain to model distortionless superluminal propagation. The refractive index of such a medium is expressed as

$$n(\omega) = \sqrt{1 + \frac{\omega_{p,1}^2}{\omega^2 - \omega_{0,1}^2 + 2i\delta\omega} + \frac{\omega_{p,2}^2}{\omega^2 - \omega_{0,2}^2 + 2i\delta\omega}}, \quad (4)$$

where $\omega_{p,j}$ and $\omega_{0,j}$ ($j = 1$ or 2) are the plasma and resonance frequencies associated with the first and second resonances, and δ is the phenomenological linewidth. The group delay of the gain medium, as a function of frequency detuning, is depicted in Fig. 6(a).

The medium supports superluminal propagation in the frequency range around $\omega/(2\pi) = 5 \times 10^{15}$ Hz [where $L/c - (\tau_g)_{\mathrm{sup}} \sim 1$ fs]. As such, we chose an input Gaussian excitation centered at $\omega_c/(2\pi) = 5 \times 10^{15}$ Hz and pulse width $2T = 8$ fs. The output response of the medium is computed using the FDTD method [32], with discrete time steps $\Delta t = 2 \times 10^{-18}$ s and space steps 1.25×10^{-9} m. This choice of parameters ensures computational stability for long propagation distances in the gain medium.

The output response of the medium, after propagating for 2.5 μm, is plotted in Fig. 6(b) and compared with the case of vacuum. The superluminal pulse experiences peak advancement $\sim$1 fs in addition to slight compression.

To investigate the causal connection between the input and output pulses, we carry out the same approach presented in the previous section. We introduce a sudden turn-off in the input pulse at the position of its peak. The output response is

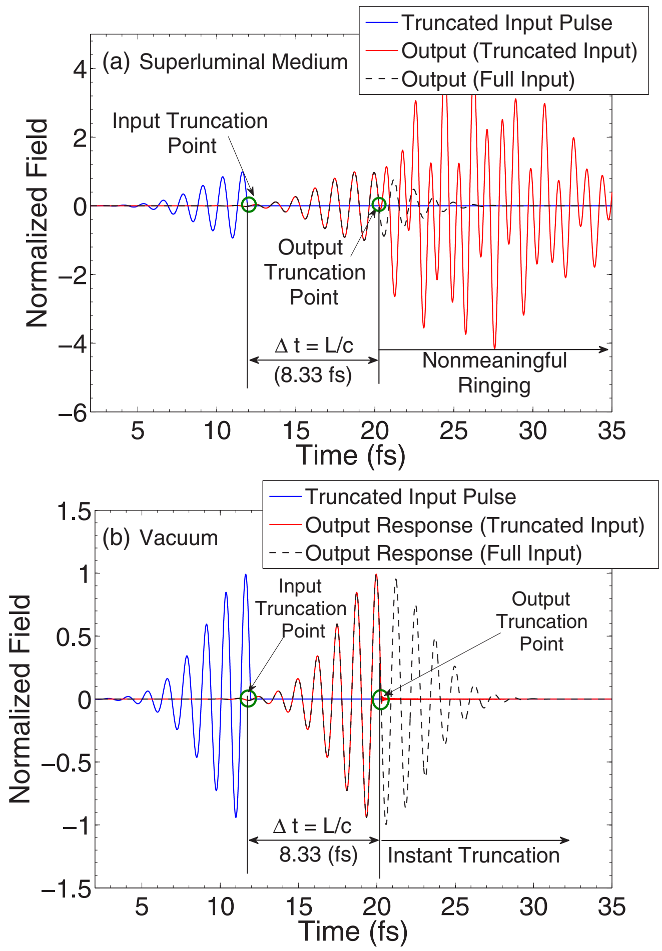

FIG. 7. (a) Propagation of pulses truncated at their peak. (a) Propagation in a superluminal medium. The pulse does not carry meaningful information beyond the arrival of the singularity (discontinuity) at ~20 fs. (b) Propagation in vacuum. The truncated pulse is a mere shifted version of the input truncated pulse.

depicted in Fig. 7(a) for the case of a superluminal pulse and in vacuum [Fig. 7(b)] for comparison. The results obtained in both figures can be summarized as follows:

First, it is observed that the discontinuity (marked by a green circle)—which corresponds to the abrupt termination of the input pulse—arrives at the output after a strictly luminal duration equal to L/c regardless of the medium type, in agreement with the behavior reported in subluminal pulse propagation.

Second, more interestingly, when the input pulse is abruptly turned off in the superluminal medium, the remainder of the energy content past the output singularity conveys nonmeaningful ringing (unlike a subluminal pulse). The energy transferred from the (gain) medium to the field beyond the pulse singularity does not convey additional useful information about the input pulse. This implies that all the input information that interacted with the medium before the termination point has already evolved at the output before the singularity reached the receiver. This behavior is generic and is in full agreement with [16,22,24–26].

The above observations imply that detectable information (in fast light) is always localized within the space-time "discontinuity window," unlike the slow-light propagation, for which it has been shown that detectable information is not necessarily localized within this window.

Finally, it is worth noting that the double-resonance Lorentzian model of Eq. (4) represents a vast class of atomic media [30,33] for which the above conclusion, the localization of detectable information within the "discontinuity window," is entirely valid.

VII. CONCLUSION

By transmitting optical truncated pulses over dispersive media, it has been shown that for pulses encoded with multiple discontinuities, the information content can remain below the detection threshold (not practically detectable), albeit the arrival of all the nonanalytic points (pulse discontinuities) at the receiver. From a practical point of view, information thus should not be entirely associated with the pulse discontinuities. Furthermore, the distinction between the evolution of pulse discontinuities and the detectable information content of the pulse has been demonstrated. This analysis provides alternative interpretations for any communication scheme that utilizes slow- or fast-light propagation.

[1] L. Brillouin, *Wave Propagation and Group Velocity*, Pure and Applied Physics Series (Academic, New York, 1960).
[2] R. Y. Chiao and A. M. Steinberg, Tunneling times and superluminality, Prog. Opt. **37**, 345 (1997).
[3] R. Landauer and Th. Martin, Barrier interaction time in tunneling, Rev. Mod. Phys. **66**, 217 (1994).
[4] H. G. Winful, Tunneling time, the Hartman effect, and superluminality: A proposed resolution of an old paradox, Phys. Rep. **436**, 1 (2006).
[5] R. W. Boyd and D. J. Gauthier, Slow and fast light, Prog. Opt. **43**, 497 (2002).
[6] Y. A. Vlasov, M. O'Boyle, H. F. Hamann, and S. J. McNab, Active control of slow light on a chip with photonic crystal waveguides, Nature (London) **438**, 65 (2005).
[7] L. Thévenaz, Slow and fast light in optical fibres, Nat. Photon. **2**, 474 (2008).
[8] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Storage of Light in Atomic Vapor, Phys. Rev. Lett. **86**, 783 (2001).
[9] R. S. Tucker, P.-C. Ku, and C. J. Chang-Hasnain, Slow-light optical buffers: Capabilities and fundamental limitations, J. Lightwave Technol. **23**, 4046 (2005).
[10] J. B. Khurgin, Optical buffers based on slow light in electromagnetically induced transparent media and coupled resonator structures: Comparative analysis, J. Opt. Soc. Am. B **22**, 1062 (2005).
[11] E. L. Bolda, R. Y. Chiao, and J. C. Garrison, Two theorems for the group velocity in dispersive media, Phys. Rev. A **48**, 3890 (1993).

[12] K. Wynne, Causality and the nature of information, Opt. Commun. **209**, 85 (2002).

[13] Z. Yang, Physical mechanism and information velocity of fast light: A time-domain analysis, Phys. Rev. A **87**, 023801 (2013).

[14] G. Diener, Superluminal group velocities and information transfer, Phys. Lett. A **223**, 327 (1996).

[15] M. Mojahedi, E. Schamiloglu, F. Hegeler, and K. J. Malloy, Time-domain detection of superluminal group velocity for single microwave pulses, Phys. Rev. E **62**, 5758 (2000).

[16] J. M. W. Mitchell and R. Y. Chiao, Causality and negative group delays in a simple bandpass amplifier, Am. J. Phys. **66**, 14 (1998).

[17] A. H. Dorrah, A. Ramakrishnan, and M. Mojahedi, Time-frequency dynamics of superluminal pulse transition to the subluminal regime, Phys. Rev. E **91**, 033206 (2015).

[18] J. Peatross, S. A. Glasgow, and M. Ware, Average Energy Flow of Optical Pulses in Dispersive Media, Phys. Rev. Lett. **84**, 2370, (2000).

[19] J. Peatross, M. Ware, and S. A. Glasgow, Role of the instantaneous spectrum on pulse propagation in causal linear dielectrics, J. Opt. Soc. Am. A **18**, 1719 (2001).

[20] M. Ware, S. A. Glasgow, and J. Peatross, Energy transport in linear dielectrics, Opt. Express **9**, 519 (2001).

[21] Meaningful information implies that the output field resembles the input in amplitude and phase over a finite duration.

[22] M. Tomita, H. Amano, S. Masegi, and A. I. Talukder, Direct Observation of a Pulse Peak Using a Peak-Removed Gaussian Optical Pulse in a Superluminal Medium, Phys. Rev. Lett. **112**, 093903 (2014).

[23] Unlike electronic circuits, a ring structure has a non-negligible length; accordingly, the output ringing happens after a strictly luminal delay $\sim L/c$, where L is the propagation distance and c is the speed of light in vacuum.

[24] R. Suzuki and M. Tomita, Causal propagation of non analytical points in fast- and slow-light media, Phys. Rev. A **88**, 053822 (2013).

[25] M. Tomita, H. Uesugi, P. Sultana, and T. Oishi, Causal information velocity in fast and slow pulse propagation in an optical ring resonator, Phys. Rev. A **84**, 043843 (2011).

[26] W. F. Silva, D. R. Solli, A. J. Corcho, D. P. Caetano, and J. M. Hickmann, Fast light, Non-analytical points and the speed of information using pulses described by functions with compact support in *Slow and Fast Light Conference, Optical Society of America, Washington, DC, 2006* (OSA Publishing, Washington DC, 2006).

[27] W. F. Silva, M. A. Alencar, D. P. Caetano, A. J. Corcho, and J. M. Hickmann, Pulses with non analytical points described by functions with compact support: The information velocity in a fast-light medium in *Slow and Fast Light Conference, Optical Society of America, Hawaii, United States, July 2009* (OSA Publishing, Washington, DC, 2009).

[28] A. Kuzmich, A. Dogariu, L. J. Wang, P. W. Milonni, and R. Y. Chiao, Signal Velocity, Causality, and Quantum Noise in Superluminal Light Pulse Propagation, Phys. Rev. Lett. **86**, 3925 (2001).

[29] M. Z. Feng, W. Sorin, and R. S. Tucker, Fast light and the speed of information transfer in the presence of detector noise, IEEE Photon. J. **1**, 213 (2009).

[30] A. H. Dorrah and M. Mojahedi, Velocity of detectable information in faster-than-light pulses, Phys. Rev. A **90**, 033822 (2014).

[31] A. H. Dorrah, L. Kayili, and M. Mojahedi, Superluminal propagation and information transfer: A statistical approach in the microwave domain, Phys. Lett. A **378**, 3218 (2014).

[32] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, 3rd ed. (Artech House, Norwood, MA, 2005), Chap. 9, pp. 361–365.

[33] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Electromagnetically induced transparency: Optics in coherent media, Rev. Mod. Phys. **77**, 633 (2005).

[34] Recall from the Introduction that in some interpretations of information, the pulse singularities (front and termination) have been associated with genuine information.