Symmetry-assisted resonance transmission of noninteracting identical particles

D. Sokolovski,[1,2,*] J. Siewert,[1,2] and L. M. Baskin[3]

[1]*Departmento de Química-Física, Universidad del País Vasco, UPV/EHU, Leioa, Spain*

[2]*IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain*

[3]*The Bonch-Bruevich State University of Telecommunications, Pr. Bolshevikov 22-1, Saint-Petersburg 193232, Russia*

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We show that a "pileup" effect occurring for a train of noninteracting identical particles incident on the same side of a resonance scatterer leads to significant interference effects, different from those observed in Hong-Ou-Mandel experiments. These include characteristic changes in the overall transmission rate and full counting statistics, as well as "bunching" and "antibunching" behavior in the all-particle transmission channel. With several resonances involved, pseudoresonant driving of the two-level system in the barrier may also result in a sharp enhancement of scattering probabilities for certain values of temporal delay between the particles.

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I. INTRODUCTION

Quantum statistical effects accompanying scattering of noninteracting identical particles are among some of the most intriguing predictions of quantum mechanics. Their studies, both theoretical and experimental, now constitute an extensive research field [1–23]. If two such particles, prepared in wave-packet states, meet head on in free space, they will eventually "pass through each other," just like their distinguishable counterparts. The situation is different if such particles coincide inside a scatterer, with the possibility of two (or more) distinct scattering outcomes for each particle. In the celebrated Hong-Ou-Mandel (HOM) experiment [2], the particles entering a scatterer from opposite sides are seen to leave the barrier predominantly from the same side (bosons) or from opposite sides (fermions). The HOM effect has found important practical applications in quality testing of single-photon sources [4], entanglement detection [5], entanglement swapping [6], and quantum metrology [7]. Its generalizations, to name a few, include observation of multiple photon bunching effects [8–10], scattering of photons by multiport beam splitters, and their use as interferometers for identical particles in spatially separated modes [11–13], Efforts to extend HOM interference experiments to bosononic or fermionized cold atoms [14] are currently under way [15,16].

Perhaps surprisingly, little studied to date remains the case complementary to that of HOM, in which the particles enter the barrier from the *same* side and "meet" there owing to a "pileup effect," if the barrier is capable of detaining the first particle long enough for the following particle(s) to catch up with it. The statistical effects are, in this case, quite different from those predicted for the HOM interference and are most pronounced in resonance tunneling, where a particle spends in the barrier roughly the lifetime of the corresponding metastable state, which can be long for sharp resonances.

In this paper, we give the general theory of the effect and demonstrate how the said pileup alters the transmission rate for initially correlated many-particle states. There are complex interference effects in the scattering statistics of the incident particles, whose wave-packet modes do not overlap prior to their arrival at the scatterer. A preliminary analysis of the case of two fermionized atoms can be found in [24]. In [25] it was demonstrated that interference effects of a similar kind will arise whenever a particle simultaneously populates several wave-packet modes. For brevity we use the term *particles* (fermions or bosons) to refer to both cold atoms and photons, equally amenable to our analysis.

The rest of the paper is organized as follows: in Sec. II we discuss the correlation between initial particles. In Sec. III we analyze the correlations acquired in scattering. In Sec. IV we introduce a generating function for the scattering statistics. In Section V we show how quantum statistical effects vanish for particles well separated initially. Sections VI and VII discuss the two- and N-particle cases, respectively. In Sec. VIII we apply our general approach to resonance transmission across a scatterer supporting one or more metastable states. Section IX reportss our conclusions.

II. INITIAL CORRELATIONS BETWEEN THE PARTICLES

Consider, in one dimension, a source sending N identical noninteracting particles of mass μ [26] in wave-packet states, $\psi_n(x_n)$, as illustrated in Fig. 1,

$$\psi_n(x_n,t) = (2\pi)^{-1/2}\int A_n(p)\exp[ipx_n - iE(p)(t+t_n)]dp,$$
$$E(p) = p^2/2\mu, \quad (1)$$

towards a finite-width potential barrier at times $0 = t_1 < t_2 < \cdots < t_N$. Particles possessing intrinsic degrees of freedom are assumed to be prepared in the same spin state, which is unchanged by interaction with the barrier. To simplify the notations, we omit the spin indices for the states, (1), as well as for the operators a_n^+ and a_n, creating and annihilating an incident particle in state ψ_n.

In the Heisenberg representation, these operators are given by

$$a_n^+(t) = (2\pi)^{-1/2}\int A_n(p)\exp[-iE(t+t_n)]a^+(p)dp,$$
$$a_n = (a_n^+)^\dagger, \quad (2)$$

*dgsokol15@gmail.com

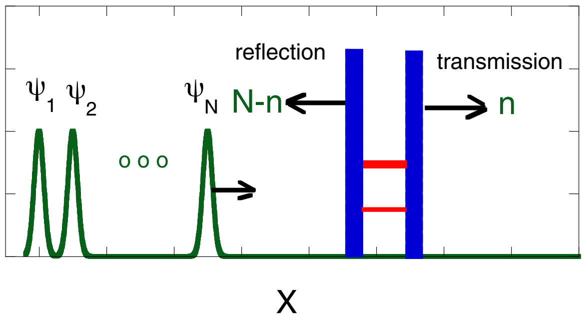

FIG. 1. Schematic showing a "train" of N wave packets incident on a double barrier which supports two resonance levels.

where the superscript dagger denotes Hermitian conjugation, and the plane-wave creation and annihilation operators $a^+(p)$ and $a(p)$ obey the usual commutation relations, $[a(p),a^+(p')]_\mp \equiv a(p)a^+(p') \mp a^+(p')a(p) = 2\pi\delta(p-p')$, with the upper and lower signs corresponding to bosons and fermions, respectively (cf. also Ref. [27]). Their (anti)commutators coincide with the overlaps between the wave packets, (1),

$$[a_m,a_n^+]_\mp = \int A_m^*(p)A_n(p)\exp[iE(p)(t_m-t_n)]dp \equiv I_{mn}, \quad (3)$$

where $I_{nn}=1$ and $I_{mn}=I_{nm}^*$. The symmetry of the incident state has no effect on the initial probability density provided $I_{nm}=\delta_{nm}$, e.g., for the delays between emissions, $|t_m-t_n|$, large enough for the rapid oscillations of the exponential in (3) to destroy the integral for all $m \neq n$. We refer to such particles as *initially uncorrelated*. Alternatively, the particles may be prepared in a *correlated* initial state, and below we consider both these cases. It is readily seen that spreading of freely moving wave packets does not alter the commutation relations, (3). Thus, the symmetrized or antisymmetrized wave function describing N incident particles is given by

$$|\Psi_{\text{in}}(t)\rangle = K^{-1/2}\prod_{n=1}^{N} a_n^+(t)|0\rangle, \quad (4)$$

where $|0\rangle$ is the vacuum state and K is the normalization constant. Evaluating the norm $\langle\Psi_{\text{in}}(t)|\Psi_{\text{in}}(t)\rangle$ and using Wick's theorem [28] to bring the operator product to the normal order, we have (the upper and lower signs are for bosons and fermions, respectively)

$$K = \sum_{\sigma(N)}(\pm 1)^{p(\sigma(N))}\prod_{i=1}^{N} I_{i\sigma_i} \equiv S^{\pm}[I_{mn}], \quad (5)$$

where $\sigma(N)$ is a permutation of the indices $(1,\ldots,N)$, σ_i is the element in position i after reordering, and $p(\sigma)$ is its parity [29].

III. CORRELATIONS BETWEEN SCATTERED PARTICLES

At long times, after all particles have left the barrier area, each wave packet ends up split into the transmitted (t) and reflected (r) parts. Thus, as $t\to\infty$, the wave function has the form

$$|\Psi_{\text{out}}(t)\rangle = K^{-1/2}\prod_{n=1}^{N}[t_n^+(t)+r_n^+(t)]|0\rangle, \quad (6)$$

where the corresponding creation and annihilation operators are given by

$$t_n^+(t) = \int T(p)A_n(p)\exp[-iE(t+t_n)]a^+(p)dp,$$
$$r_n^+(t) = \int R(p)A_n(p)\exp[-iE(t+t_n)]a^+(-p)dp,$$
$$t_n = (t_n^+)^\dagger, \qquad r_n = (r_n^+)^\dagger, \quad (7)$$

and $T(p)$ and $R(p)$ are the barrier transmission and reflection amplitudes for a particle with momentum p. Since $|T(p)|^2 + |R(p)|^2 = 1$, as $t\to\infty$ we also have

$$T_{mn} \equiv [t_m,t_n^+]_\mp = \int |T(p)|^2 A_m^*(p)A_n(p)\exp[iE(p)(t_m-t_n)]dp,$$
$$R_{mn} \equiv [r_m,r_n^+]_\mp = I_{mn} - T_{mn}, \quad (8)$$

while all remaining commutators vanish. In Eqs. (8) $T_{mn} = T_{nm}^*$ is a Hermitian matrix of the overlaps between the transmitted parts of the wave packets, and its diagonal elements T_{nn} coincide with the probabilities w_n that the nth particle will be transmitted on its own:

$$T_{nn} = \int |T(p)|^2|A_n(p)|^2 dp \equiv w_n. \quad (9)$$

We note that even initially uncorrelated particles may become correlated as a result of scattering. This will happen, for example, if each transmitted one-particle state is significantly broadened in the coordinate space (narrowed in the momentum space), so that the integrals in Eq. (8) do not vanish, even if the integrals in Eq. (3) did.

IV. THE GENERATING FUNCTION AND FULL COUNTING STATISTICS

For N identical particles, there are $N+1$ outcomes, with $n=0,1,\ldots,N$ particles crossing in the barrier whose probabilities, $W(n,N)$, we study next. It is convenient to construct a generating function [30] $G(\alpha)$,

$$G^{\pm}(\alpha) = \lim_{t\to\infty}\langle\Psi_{\text{out}}(t)|\Psi(t,\alpha)\rangle, \quad (10)$$

where $|\Psi(t,\alpha)\rangle \equiv K^{-1/2}\prod_{n=1}^{N}[\alpha t_n^+(t)+r_n^+(t)]|0\rangle$ and K is defined by Eq. (5). By Wick's theorem, we have

$$G^{\pm}(\alpha) = S^{\pm}[\Delta_{mn}^{\pm}]/S^{\pm}[I_{mn}], \quad (11)$$

where the matrix Δ is given by

$$\Delta_{mn}^{\pm} = I_{mn} + (\alpha-1)T_{mn}, \quad n,m = 1,2,\ldots,N. \quad (12)$$

For the mean number of transmissions, $\bar{n}_T \equiv \sum_{n=0}^{N} nW(n,N)$, we have

$$\bar{n}_T(N,t_1,t_2,\ldots,t_N) = \partial_\alpha G^{\pm}(\alpha)|_{\alpha=1} = \sum_{j=1}^{N} S^{\pm}[I_{mn}^{(j)}]/S^{\pm}[I_{mn}], \quad (13)$$

where $I_{mn}^{(j)}$ is the matrix obtained from I_{mn} by replacing the elements in the jth row, $I_{j1},\dots,I_{jN}$, with $T_{j1},\dots,T_{jN}$. The full counting statistics of an N-particle process are evaluated by noting that

$$W^{\pm}(n,N,t_1,t_2,\dots,t_N) = \frac{1}{n!}\partial_\alpha^n G^{\pm}|_{\alpha=0} = \sum_{j_1<j_2<\dots<j_n}^{N} S^{\pm}\big[I_{mn}^{(j_1,j_2,\dots,j_n)}\big]/S^{\pm}[I_{mn}], \quad (14)$$

where $I_{mn}^{(j_1,j_2,\dots,j_n)}$ is the matrix obtained from R_{mn} by replacing the elements in rows $j_1,j_2,\dots,j_n$, with the corresponding rows in the matrix T_{mn}. Next we briefly discuss what would happen if the particles were not identical.

V. THE DISTINGUISHABLE PARTICLES LIMIT

The appearance of correlations between initially uncorrelated particles can be explained in the following way. The particles are well separated initially, and if they leave the scatterer quickly enough, each scattering event occurs independently. If, on the other hand, the scatterer detains each particle for a significant period of time, Bose or Fermi statistical effects become important while several (or all) particles are still inside. This, in turn, may alter the measurable probabilities for various outcomes, which is the effect we seek to describe here. This is not possible if the particles are distinguishable and do not interact with each other. Such particles cannot "meet" in the scatterer (or anywhere else), they are always transmitted independently, and it does not matter whether they all arrive at the same time or their arrivals are separated by long time intervals.

Assume, for simplicity, that the individual tunneling probabilities in Eq. (9) are equal for all particles, $w_i = w_j \equiv w$. Then the probability that n of N distinguishable particles (DPs) will be transmitted is given by the binomial distribution,

$$W^{\mathrm{DP}}(n,N) = C_n^N w^n (1-w)^{N-n}, \quad (15)$$

where C_n^N is the binomial coefficient. The mean number of transmissions, also independent of the choice of $t_1,t_2,\dots,t_N$, is given by

$$\overline{n}_T^{\mathrm{DP}}(n,N) = \sum_{i=1}^{N} w_i = wN. \quad (16)$$

It is readily seen that the DP limit, (17), is reached if identical particles arrive at the scatterer after long intervals, $|t_i - t_j| \to \infty$. Indeed, in this limit all operators in Eqs. (2) and (7) commute, both I_{mn} and T_{mn} are diagonal, and for $w_i = w_j \equiv w$, Eqs. (14) yield

$$W^{\pm}(n,N,t_1,t_2,\dots,t_N) \to W^{\mathrm{DP}}(n,N). \quad (17)$$

We are, however, more interested in the case where quantum statistical effects do lead to measurable changes in in the channel probabilities, $W^{\pm}(n,N,t_1,t_2,\dots,t_N)$, and consider it next.

VI. THE TWO-PARTICLE CASE ($N = 2$)

In the simplest case of just two particles, $N = 2$, Eqs. (13) and (14) yield

$$W^{\pm}(2,2) = \frac{w_1 w_2 \pm |T_{12}|^2}{1 \pm |I_{12}|^2},$$
$$W^{\pm}(1,2) = \frac{w_1(1-w_2) + w_2(1-w_1) \pm 2\mathrm{Re}(T_{12}R_{12}^*)}{1 \pm |I_{12}|^2},$$
$$\overline{n}_T = \frac{w_1 + w_2 \pm 2\mathrm{Re}[T_{12}I_{12}^*]}{1 \pm |I_{12}|^2}, \quad (18)$$

which coincides with the results in [24] if the particles have the same momentum distribution, $A_1(p) = A_2(p)$.

The last of Eqs. (18) shows that if one sends to the scatterer initially correlated pairs of identical particles, ($I_{12} \neq 0$), the mean number of transmissions per pair may be different from the result obtained for two DPs in the same wave-packet states.

If the pairs are not initially correlated, we always have $\overline{n}_T = \overline{n}_T^{DP}$, but the bosons (fermions) are more (less) likely to exit the scatterer on the same side. This behavior will be observed if the two particles, initially well separated from each other, meet in the scatterer, so that $T_{12} \neq 0$.

VII. THE N-PARTICLE CASE

Equations (13) and (14) show that the results in the previous section also hold for an arbitrary number of particles, $N > 2$. The mean number of transmissions may be affected by the symmetry of the initial state, $\overline{n}_T \neq \sum_{j=1}^N j w_j$, if and only if the particles are correlated initially, $I_{mn} \neq \delta_{mn}$.

For initially uncorrelated particles, $I_{mn} = \delta_{mn}$, the symmetry changes the probabilities $W(n,N)$, but not $\overline{n}$, provided $T_{mn} \neq w_n \delta_{mn}$. In this case, "bunching" and "antibunching" types of behavior can be observed for bosons and fermions in the probability that all N particles will be transmitted, $W(N,N)$. Since the matrix T_{mn} is positive definite, the Hadamard inequality for determinants [31] and its analog for permanents [32] ensure that $W_{\pm}(N,N) \gtrless \prod_{i=1}^N w_i$ (cf. also Ref. [33]). Thus, N bosons (fermions) are more (less) likely to be transmitted all together than DPs in the same one-particle states. Note that this argument cannot be extended to the probabilities $W(n<N,N)$ or to initially correlated initial states, $I_{mn} \neq \delta_{mn}$.

VIII. RESONANCE TRANSMISSION

A system likely to show these effects is a resonance barrier, where, due to the long delay in traversing it, even particles that are well separated initially have a chance to "pile up" in the scatterer. The transmission coefficient of such a barrier can be written as a sum of narrow Breit-Wigner peaks (see, for example, Ref. [34]),

$$|T(p)|^2 = \sum_l \frac{\Gamma_l^2}{\left(p^2/2\mu - E_l^r\right)^2 + \Gamma_l^2}, \quad (19)$$

and even for initially uncorrelated particles, the shape of $|T(p)|^2|A(p)|^2$ may be narrow enough to ensure that T_{mn} is not diagonal even if $I_{mn} = \delta_{mn}$.

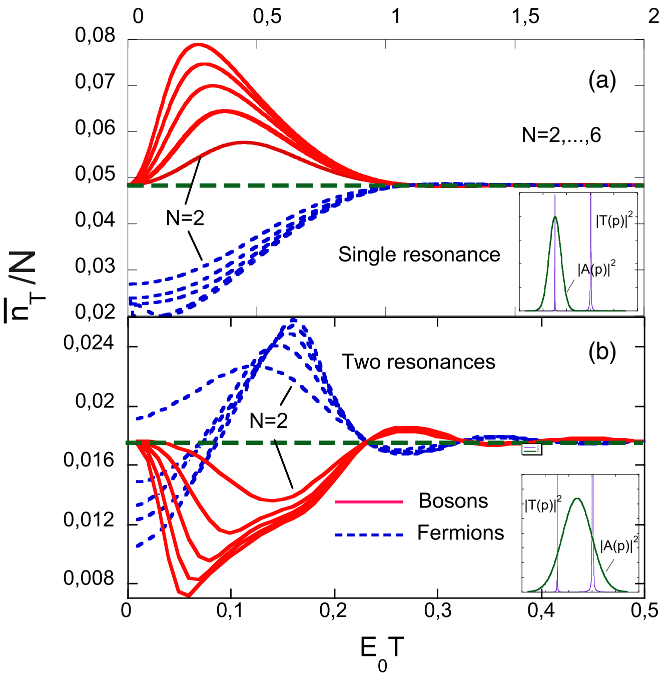

FIG. 2. Mean number of transmissions for N particles in identical Gaussian wave packets vs time T between emissions: (a) through a single resonance level $E_1^r/E_0 = 0.41$, $\Gamma_1/E_0 = 0.0087$, $p_0\sigma = 3.77$, and $E_0 \equiv p_0^2/2\mu$; (b) through two resonance levels $E_1^r/E_0 = 0.95$, $\Gamma_1/E_0 = 0.038$, $E_2^r/E_0 = 3.82$, $\Gamma_2/E_0 = 0.28$, and $p_0\sigma = 6.04$. Dashed horizontal lines correspond to the limit of distinguishable particles. Insets: Momentum distribution $|A(p)|^2$ and transmission coefficient $|T(p)|^2$.

Up to this point our treatment has been general. Throughout the rest of the paper we consider the special case where the particles are emitted after equal intervals, $t_{n+1} - t_n = T$. We examine the dependence of the quantities of interest on the time T by evaluating numerically the determinants and permanents in Eqs. (13) and (14). Figure 2 shows the mean number of transmissions for N particles emitted in identical Gaussian states of a coordinate width σ and a mean momentum p_0:

$$A(p) \equiv A_n(p) = (\sigma^2/2\pi)^{1/4} \exp[(p - p_0)^2\sigma^2/4]. \quad (20)$$

The scatterer supports two resonant metastable states with energies $E_{1,2}^r$ and widths $\Gamma_{1,2}$, of which one or both can be accessed by the incident particle, as shown in the insets in Figs. 2(a) and 2(b). With only one level involved, the mean number of transmissions $\bar{n}_T(T)$ for bosons increases to a maximum value for some correlated initial state (illustrated in Fig. 3 together with its fermionic counterpart) and then returns to the DP limit for initially uncorrelated particles [see Fig. 2(a)]. For fermions, the Pauli principle mostly reduces $\bar{n}_T$ to levels below the DP level, which, for the maximally correlated states, obtained as $T \to 0$ [35], is considerably decreased.

With two metastable states involved, interference between resonances reverses the effect: for $0.1 < p_0^2/2\mu T < 0.25$, $\bar{n}_T$ is suppressed for bosons and enhanced for fermions [see Fig. 2(b)].

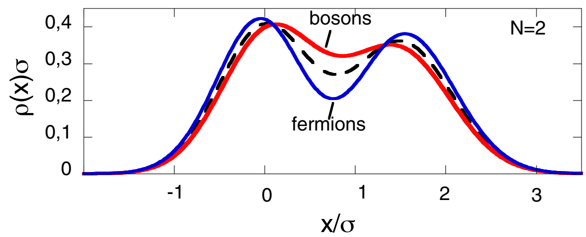

FIG. 3. One-particle density $\rho(x)$ [36] (normalized to 2) for an initially correlated two-particle Gaussian state, $A_1(p) = A_2(p) \sim \exp[-(p - p_0)\sigma^2/2]$, $p_0\sigma = 6$, $p_0 T/\mu\sigma = 1.5$, $p_0^2 t/2\mu = 4.5$, and $T \equiv t_2 - t_1$. Also shown by the dashed line is $\rho(x)$ for two distinguishable particles in the same one-particle states.

The scattering probabilities $W(n,N,T)$ for the single-resonance case, plotted in Fig. 4 for $N = 4$, show smooth deviations from the DP values in Eq. (17) before reaching these values as the time between arrivals tends to ∞. We note that the probabilities $W(N,N)$ never fall below (exceed) their DP level for bosons (fermions), as discussed in the previous section. We note also that the increase or decrease in $\bar{n}_T$ results from a similar increase or decrease in the probability of the one-particle transmission channel, $W(1,N)$.

With two resonances accessible to the particles, the picture is more interesting. For bosons, $W(n,N,T)$ exhibit maxima whenever the time between emissions coincides with a multiple of the difference in resonant energies, $T \approx T_k = 2\pi k/(E_2^r - E_1^r)$, $k = 1,2,\ldots$. The peaks are most pronounced for the (N,N) channel [cf. Fig. 5(a)] and, as shown in Fig. 6(a), become sharper as N increases. This is another consequence of the symmetrization of the initial state, which, with each

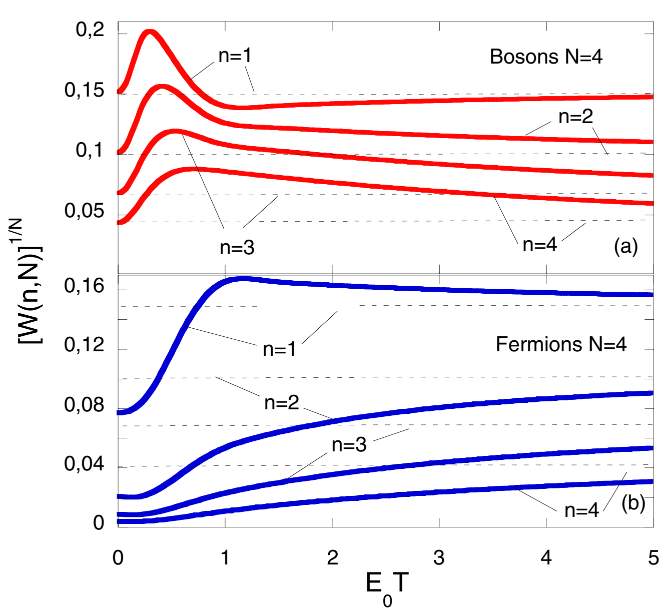

FIG. 4. (a) Probabilities that $n = 1, 2, 3, 4$ bosons will be transmitted for $N = 4$ (single resonance); (b) same as (a), but for fermions. Parameters are as in Fig. 2(a). Dashed horizontal lines indicate the corresponding values for distinguishable particles given by Eq. (17) Incident particles may be considered uncorrelated for $E_0 T \gtrsim 0.4$.

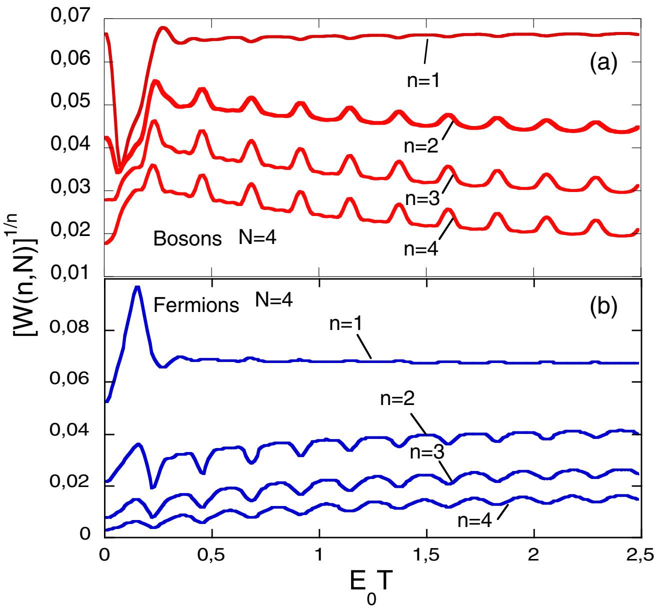

FIG. 5. (a) Probabilities that $n = 1, 2, 3, 4$ bosons will be transmitted for $N = 4$ (two resonances); (b) same as (a), but for fermions. Parameters are as in Fig. 2(b). Incident particles may be considered uncorrelated for $E_0T \gtrsim 0.4$.

particle distributed between the wave packets in Fig. 1, appears to produce quasiperiodic excitation of the metastable two-level system supported by the barrier. With an increasing number of particles, the excitation looks more periodic, and the "resonance" condition $T \approx T_k$ needs to be satisfied with ever greater accuracy. Note that similar (yet not identical) interference effects have been predicted for scattering trains of wave-packet modes representing a single particle (for details see [25]).

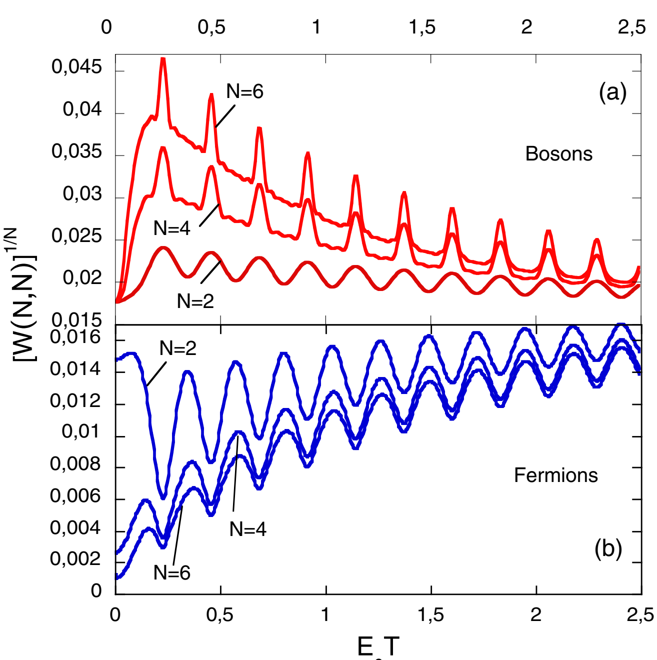

FIG. 6. (a) Probabilities that all bosons will be transmitted for $N = 2, 4, 6$ (two resonances); (b) same as (a), but for fermions. Parameters are as in Fig. 2(b).

For fermions, probing two resonance states, the peaks at $T = T_k$ are replaced by dips, which appear, for example in the probability $W(2,4)$ shown in Fig. 5(b). In contrast to the bosonic case, these dips are never seen in the (N,N) channel, where $W(N,N,T)$ undergoes sinusoidal oscillations, no matter how large the number of particles N is [see Fig. 6(b)].

Experimental observation of effects of the Pauli principle on resonance tunneling would be possible for cold atoms in the Tonks-Girardeau regime, injected into a quasi-one-dimensional trap with laser-induced barriers [37]. An optical realization of the bosonic experiment would consist in sending identically polarized photons toward a Fabry-Perot interferometer or injecting them in a waveguide with narrowing, imitating a one-dimensional barrier. If required, a correlated initial state can be produced by scattering several uncorrelated particles off a long-lived resonance and selecting the outcome in one of the n-particle transmission channels.

IX. DISCUSSION AND CONCLUSIONS

In summary, Bose or Fermi statistics can significantly change the scattering outcomes for a train of noninteracting identical particles impinged on the same side of a scatterer. Physically, the effect requires the simultaneous presence of several particles inside the scatterer. This may occur if the initial state is already correlated or if the particles, well separated initially, "pile up" inside, as a result of a scattering delay. In the latter case, the statistics of the process are affected in such a way that the mean number of transmissions per train, $\overline{n}_T$, remains unaffected. For a correlated initial state, $\overline{n}_T$ may be larger or smaller than that for a train composed of distinguishable noninteracting particles.

Mathematically, this is an interference phenomenon arising from the presence of additional terms in the (anti)symmetrized wave function, which disappears if the particles can be distinguished. Its analysis is extremely simple, owing to the commutation of the evolution and symmetrization operators: it is sufficient first to solve the corresponding one-particle problems and then to evaluate the asymptotic exchange integrals as $t \to \infty$. Interference plays the most fundamental role in quantum mechanics, and we think it unlikely that a more detailed or more "physical" explanation of the effect can be provided.

To conclude, scattering of trains of identical particles offers a variety of interference effects, very different from those observed in the HOM experiments, some of which are discussed in detail in Sec. VIII. Observation of such effects is within the capability of modern experimental techniques.

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[1] S. Aaronson and A. Arkhipov, Theory Comput. **9**, 143 (2013).
[2] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[3] D. Sokolovski, Phys. Rev. Lett. **110**, 115302 (2013).
[4] F. W. Sun and C. W. Wong, Phys. Rev. A **79**, 013824 (2009).
[5] Y. H. Shih and C. O. Alley, Phys. Rev. Lett. **61**, 2921 (1988).
[6] M. Halder, A. Beveratos, N. Gisin, V. Scarani, C. Simon, and H. Zbinden, Nat. Phys. **3**, 692 (2007).
[7] P. Walther, J.-W. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni, and A. Zeilinger, Nature **429**, 158 (2004).
[8] J. G. Rarity and P. R. Tapster, J. Opt. Soc. Am. B **6**, 1221 (1989).
[9] Z. Y. Ou, J.-K. Rhee, and L. J. Wang, Phys. Rev. Lett. **83**, 959 (1999).
[10] X.-L. Niu, Y.-X. Gong, B. Liu, Y.-F. Huang, G.-C. Guo, and Z. Y. Ou, Opt. Lett. **34**, 1297 (2009).
[11] M. C. Tichy, M. Tiersch, F. de Melo, F. Mintert, and A. Buchleitner, Phys. Rev. Lett. **104**, 220405 (2010).
[12] K. Mayer, M. C. Tichy, F. Mintert, T. Konrad, and A. Buchleitner, Phys. Rev. A **83**, 062307 (2011).
[13] N. Spagnolo, C. Vitelli, L. Sansoni, E. Maiorino, P. Mataloni *et al.*, Phys. Rev. Lett. **111**, 130503 (2013).
[14] M. Girardeau, J. Math. Phys. **1**, 516 (1960).
[15] M. Bonneau, J. Ruaudel, R. Lopes, J.-C. Jaskula, A. Aspect, D. Boiron, and C. I. Westbrook, Phys. Rev. A **87**, 061603(R) (2013).
[16] R. Lopes, A. Imanaliev, A. Aspect, M. Cheneau, D. Boiron, and C. I. Westbrook, Nature **520**, 66 (2015).
[17] M. A. Khan and M. N. Leuenberger, Phys. Rev. B **90**, 075439 (2014).
[18] D. Marian, E. Colomés, and X. Oriols, J. Phys.: Condens. Matter **27**, 245302 (2015).
[19] J.-D. Urbina, J. Kuipers, Q. Hummel, and K. Richter, arXiv:1409.1558.
[20] G. Zürn, F. Serwane, T. Lompe, A. N. Wenz, M. G. Ries, J. E. Bohn, and S. Jochim, Phys. Rev. Lett. **108**, 075303 (2012).
[21] M. Rontani, Phys. Rev. Lett. **108**, 115302 (2012).
[22] S. Hunn, K. Zimmermann, M. Hiller, and A. Buchleitner, Phys. Rev. A **87**, 043626 (2013).
[23] F. Hassler, M. V. Suslov, G. M. Graf, M. V. Lebedev, G. B. Lesovik, and G. Blatter, Phys. Rev. B **78**, 165330 (2008).
[24] D. Sokolovski and L. M. Baskin, Phys. Rev. A **90**, 024101 (2014).
[25] D. Sokolovski, Phys. Rev. A **91**, 052104 (2015).
[26] For a photon in a waveguide μ is the effective mass acquired as a result of transversal confinement.
[27] R. Loudon, Phys. Rev. A **58**, 4904 (1998).
[28] G. C. Wick, Phys. Rev. **80**, 268 (1950).
[29] Thus, $S^+[I_{mn}]$ is the permanent (per) of the matrix I_{mn}, and $S^-[I_{mn}]$ is its determinant (det).
[30] For other use of probability generating functions see, e.g., E. B. Rokower and N. B. Abraham, J. Phys. A: Math. Gen. **11**, 1879 (1978).
[31] R. A. Horn and C. R. Johnson, *Matrix Analysis* (Cambridge University Press, New York, 1986).
[32] M. Marcus, Proc. Am. Math. Soc. **15**, 967 (1964).
[33] M. Pons, D. Sokolovski, and A. del Campo, Phys. Rev. A **85**, 022107 (2012).
[34] D. Bohm, *Quantum Theory I* (Addison-Wesley, Reading, MA, 1965), p. 287.
[35] Note that for fermions, $|\Psi_{in}\rangle$ in Eq. (4), always normalized to unity, tends to a finite limit as $t_m - t_n \to 0$. For example, for two atoms prepared in Gaussian states of width σ, the one-particle density $\rho(x)$ tends to $\exp(-2x^2/\sigma^2)[1 + 4x^2/\sigma 2]$ as $t_2 \to t_1$. We study the scattering of this maximally correlated fermionic state, $I_{mn} = 1$, whenever the limit $t_m - t_n \to 0$ is taken in the rest of the paper.
[36] One-particle density is defined in the usual way, $\rho(x) = \langle\Psi|a^+(x)a(x)|\Psi\rangle$, where $a^+(x)$ creates a particle at a location x.
[37] T. P. Meyrath, F. Schreck, J. L. Hanssen, C. S. Chuu, and M. G. Raizen, Phys. Rev. A **71**, 041604(R) (2005).