

Casimir-Polder force on a V-type three-level atom near a structure containing left-handed materialsJingping Xu,^{1,2} Shenglong Chang,² Yaping Yang,^{2,*} and M. Al-amri^{1,†}¹*The National Center for Applied Physics (NCAP), King Abdulaziz City for Science and Technology (KACST), Riyadh 11442, Saudi Arabia*²*MOE Key Laboratory of Advanced Micro-Structured Materials, School of Physics Science and Engineering, Tongji University, Shanghai 200092, People's Republic of China*

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The Casimir-Polder (CP) force acting on a V-type three-level atom which is initially prepared in two different kinds of superposition states, i.e., subradiant and superradiant states, is investigated. The influence of quantum interference on force evolution due to two-dipole transitions is analyzed in detail. It is found that the orientation of the atomic dipole moment has significant influence on the Casimir-Polder force and consequently its evolution. For the ideal degenerate V-type atom with two parallel dipoles, quantum interference leads to population trapping as well as the cancellation of the CP force when the atom is prepared initially in a subradiant state. However, the result changes when we consider the practical Zeeman V-type atom whose two dipole moments are perpendicular to each other. Since quantum interference in such an atom must occur in an anisotropic environment, it is possible to trap atomic population and enhance the CP force simultaneously by preparing the atom initially in sub-radiant states. In principle, our results can be found in an arbitrary anisotropic environment, and here we describe a structure containing left-handed materials to highlight our findings.

DOI: [10.1103/PhysRevA.93.012514](https://doi.org/10.1103/PhysRevA.93.012514)**I. INTRODUCTION**

The Casimir-Polder (CP) force refers to the force between neutral atoms and bulk materials [1]. It originates from the quantum fluctuation of both the electromagnetic field and the atomic dipole moment. With the development of cavity quantum electrodynamics, it is feasible under the present technology to trap, manipulate, and detect either few atoms or even a single atom [2]. Therefore, the CP force can be detected and no longer be ignored in research that is related to atom optics. Several applications of the CP force in atom optics have been extensively carried out in recent decades [3–7]. For example, it can be used to realize atomic Mach-Zehnder-type interferometers [3], flat quantum reflective mirrors accompanied by gravitational force [4], and evanescent-wave elements for atom guiding [5]. Unpleasantly, the CP force is a disturbing factor in nanodevices, which leads to undesired sticking of small objects to surfaces [6] and diminishes the depth of magneto-optical traps when near the surface [7]. The CP potential of an atom near a monolayer made by periodically arranged metallic and dielectric nanospheres has also been analyzed [8]. Recently, it was proposed to excite a Rydberg atom through a close-by oscillating mirror mediated by the CP force [9]. All the aforementioned applications are based on the CP force acting on atoms in the ground state. Nevertheless, the force acting on excited atoms has attracted plenty of interest in recent years [10–17]. In general, the CP force on an excited atom is much stronger than that on the ground state, and varies sinusoidally with distance from the surface [14], and even becomes repulsive [10]. The reason is that the CP force on the excited atom originates from real photon emission, and mainly relates to the electromagnetic mode at the atomic transition frequency. Therefore, it is easy to control the CP force on an excited atom by tailoring the electromagnetic properties of the

environment. For example, in a structure made of left-handed metamaterials and a metal, the CP potential of an excited atom can form a barrier away from surface to levitate particle [16]. Furthermore, when the atom is close to a surface made of left-handed metamaterials and zero-index metamaterials [17], the CP force acting on an initially excited two-level atom can be not only significant for a longer time, but also nearly independent of dipole orientation, even if the atom is far away from the surface. In addition to papers on two-level atom systems, there are several concerning the force acting on multilevel systems, including molecules [14,15,18] and multilevel atoms [15]. However, the influence of quantum interference in multilevel atoms was not much discussed before due to the discretely separated levels [14,15,18]. Quantum interference among different decay channels of the atom is an important issue of quantum optics and has attracted a great deal of attention for a long time. It is behind many fascinating phenomena in the fields of lasers and quantum information, such as coherence trapping of a population, lasing without inversion, electromagnetically induced transparency, ultranarrow spectral lines, and gain without inversion [19]. In addition, quantum interference of V-type three-level atoms has been extended to affect the evolution of entanglement between two atoms [20,21].

Here the influences of quantum interference on the CP force acting on V-type three-level atoms are studied in detail. There are two kinds of three-level atoms that we investigated and compared with each other; One is the ideal degenerate V-type atom with two parallel dipole moments, and the other is the Zeeman V-type atom whose two dipole moment are left rotated and right rotated respectively. To achieve significant quantum interference, one has to consider two kinds of initial states, i.e., one is a subradiant state and the other is a superradiant state. The atom is assumed to be placed near a structure made of left-handed metamaterials (LHMs) and a metal, which had been adopted in previous papers [16,17].

This paper is organized as follows: In Sec. II we introduce the model and theory needed to explore the Casimir-Polder force. In Sec. III, we analyze the CP force on two kinds

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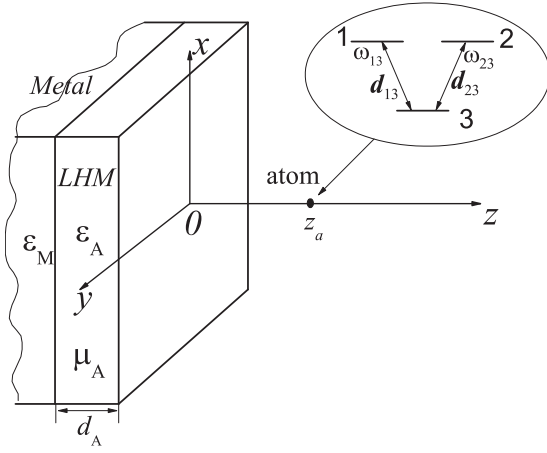


FIG. 1. Scheme of a three-level atom near a structure which is made of a left-handed material (LHM) slab mounted on a metal.

of V-type three-level atoms. In Sec. IV, we draw the conclusions.

II. MODEL AND FORMULAS

A three-level atom near a double-layer structure is considered, shown in Fig. 1. The right layer is a left-handed metamaterial slab with permittivity ϵ_A , permeability μ_A , and thickness d_A . The left substrate is metal with permittivity ϵ_M . The atom at position $\mathbf{r}_a = (0, 0, z_a)$ has two upper levels $|1\rangle$ and $|2\rangle$, and one ground level $|3\rangle$. The corresponding transition frequencies and dipole moments are ω_{i3} and \mathbf{d}_{i3} with $i = 1, 2$, respectively.

The system Hamiltonian is [22]

$$H = \sum_{\lambda=e,m} \int d^3\mathbf{r} \int_0^\infty d\omega \hbar \omega \hat{\mathbf{f}}_\lambda^+(\mathbf{r}, \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega) + \hbar\omega_{13}|1\rangle\langle 1| + \hbar\omega_{23}|2\rangle\langle 2| - \hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_a). \quad (1)$$

$\hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega)$ and $\hat{\mathbf{f}}_\lambda^+(\mathbf{r}, \omega)$ are the fundamental operators of the structure-assisted fields which satisfy the commutation relationship of $[\hat{\mathbf{f}}_\lambda^+(\mathbf{r}, \omega), \hat{\mathbf{f}}_{\lambda'}(\mathbf{r}', \omega')] = \delta_{\lambda\lambda'} \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega')$ and the rules $\hat{\mathbf{f}}_\lambda^+(\mathbf{r}, \omega)|\{0\rangle\rangle = |1_\lambda(\mathbf{r}, \omega)\rangle$ and $\hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega)|\{0\rangle\rangle = 0$. Here $|\{0\rangle\rangle$ is the vacuum state, and $\hat{\mathbf{d}} = \mathbf{d}_{13}\hat{\sigma}_{13} + \mathbf{d}_{31}\hat{\sigma}_{31} + \mathbf{d}_{23}\hat{\sigma}_{23} + \mathbf{d}_{32}\hat{\sigma}_{32}$ is the atom dipole operator, in which $\hat{\sigma}_{ij}$ is the atomic transition operator between $|i\rangle$ and $|j\rangle$. The transition between $|1\rangle$ and $|2\rangle$ is dipole forbidden.

The operator of the electric field is expressed in terms of the fundamental operators as

$$\hat{\mathbf{E}}(\mathbf{r}) = \sum_{\lambda=e,m} \int d^3\mathbf{r}' \int_0^\infty d\omega \vec{\mathbf{G}}_\lambda(\mathbf{r}, \mathbf{r}', \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}', \omega) + \text{H.c.} \quad (2)$$

In this paper, we just consider the initial state as

$$|\psi(0)\rangle = (c_1|1\rangle + c_2|2\rangle) \otimes |\{0\rangle\rangle. \quad (3)$$

It refers to the fact that the structure-assisted field is in the vacuum state $|\{0\rangle\rangle$ while the three-level atom is in the coherent superposition state between $|1\rangle$ and $|2\rangle$ initially.

The measurable CP force is the expectation value of the operator of the electromagnetic force acting on the atom, which

has the expression under the long-wavelength approximation as [12]

$$\mathbf{F} = \left\langle \nabla[\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r})] + \frac{d}{dt}[\hat{\mathbf{d}} \times \hat{\mathbf{B}}(\mathbf{r})] \right\rangle_{\mathbf{r}=\mathbf{r}_a}. \quad (4)$$

The right two terms refer to the dipole force and the Lorentz force respectively. It should be noticed that because the transition between $|1\rangle$ and $|2\rangle$ is forbidden, there are only five matrix elements $\langle \hat{\sigma}_{11}(t) \rangle$, $\langle \hat{\sigma}_{22}(t) \rangle$, $\langle \hat{\sigma}_{33}(t) \rangle$, $\langle \hat{\sigma}_{12}(t) \rangle$, and $\langle \hat{\sigma}_{21}(t) \rangle$ which take part in the dynamical evolution of force (shown later). Considering the case of $\omega_{13} \approx \omega_{23}$, the Lorentz force is nearly canceled [12]. So we ignore the Lorentz force and consider only the dipole force in this paper. In the Heisenberg picture, we reach the mean force after some deductions as [12,17]

$$\mathbf{F}(t) = i \frac{\mu_0}{\pi} \int_0^\infty d\omega \omega^2 \int_{t_0}^t d\tau e^{-i\omega(t-\tau)} \times \{ \nabla \langle \hat{\mathbf{d}}(t) \cdot \text{Im} \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, \omega) \cdot \hat{\mathbf{d}}(\tau) \rangle \}_{\mathbf{r}=\mathbf{r}_a} + \text{c.c.} \quad (5)$$

Here c.c means complex conjugation. The remaining task is to determine the dipole-dipole correlation function in Eq. (5) as

$$\langle \hat{\mathbf{d}}(t) \hat{\mathbf{d}}(\tau) \rangle = \sum_{mn} \sum_{m'n'} \mathbf{d}_{mn} \mathbf{d}_{m'n'} \langle \hat{\sigma}_{mn}(t) \hat{\sigma}_{m'n'}(\tau) \rangle, \quad (6)$$

$$m, m', n, n' = 1, 2, 3.$$

Assuming that Lamb shifts are already included in the transition frequencies, and adopting the transformations $\hat{\sigma}'_{ij}(t) = \hat{\sigma}_{ij}(t) e^{-i\omega_{ij}t}$ with $\omega_{ij} = \omega_{i3} - \omega_{j3}$, the simultaneous master equations of atomic operators are as follows:

$$\dot{\hat{\sigma}}'_{11}(t) = -\gamma_1 \hat{\sigma}'_{11}(t) - \frac{\kappa_{12}}{2} [\hat{\sigma}'_{12}(t) e^{i\omega_{12}t} + \hat{\sigma}'_{21}(t) e^{-i\omega_{12}t}], \quad (7)$$

$$\dot{\hat{\sigma}}'_{22}(t) = -\gamma_2 \hat{\sigma}'_{22}(t) - \frac{\kappa_{21}}{2} [\hat{\sigma}'_{21}(t) e^{-i\omega_{12}t} + \hat{\sigma}'_{12}(t) e^{i\omega_{12}t}], \quad (8)$$

$$\dot{\hat{\sigma}}'_{12}(t) = -\frac{1}{2}(\gamma_1 + \gamma_2) \hat{\sigma}'_{12}(t) - \frac{1}{2} \kappa_{12} \hat{\sigma}'_{22}(t) e^{-i\omega_{12}t} - \frac{1}{2} \kappa_{21} \hat{\sigma}'_{11}(t) e^{-i\omega_{12}t}, \quad (9)$$

$$\dot{\hat{\sigma}}'_{13}(t) = -\frac{1}{2} \gamma_1 \hat{\sigma}'_{13}(t) - \frac{1}{2} \kappa_{12} \hat{\sigma}'_{23}(t) e^{-i\omega_{12}t}, \quad (10)$$

$$\dot{\hat{\sigma}}'_{23}(t) = -\frac{1}{2} \gamma_2 \hat{\sigma}'_{23}(t) - \frac{1}{2} \kappa_{21} \hat{\sigma}'_{13}(t) e^{i\omega_{12}t}. \quad (11)$$

Here, γ_1 (γ_2) is the spontaneous decay rate from level $|1\rangle$ ($|2\rangle$) to ground level $|3\rangle$, and κ_{12} and κ_{21} are the collective damping rates due to quantum interference. They have the following expressions:

$$\gamma_1 = 2 \frac{\mu_0}{\hbar} \omega_{13}^2 \mathbf{d}_{13} \cdot \text{Im} \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}, \omega_{13}) \cdot \mathbf{d}_{31}, \quad (12)$$

$$\gamma_2 = 2 \frac{\mu_0}{\hbar} \omega_{23}^2 \mathbf{d}_{23} \cdot \text{Im} \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}, \omega_{23}) \cdot \mathbf{d}_{32}, \quad (13)$$

$$\kappa_{12} = 2 \frac{\mu_0}{\hbar} \omega_{23}^2 \mathbf{d}_{13} \cdot \text{Im} \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}, \omega_{23}) \cdot \mathbf{d}_{32}, \quad (14)$$

$$\kappa_{21} = 2 \frac{\mu_0}{\hbar} \omega_{13}^2 \mathbf{d}_{23} \cdot \text{Im} \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}, \omega_{13}) \cdot \mathbf{d}_{31}. \quad (15)$$

According to the simultaneous Eqs. (7)–(11), the atomic operator $\hat{\sigma}'_{ij}(t)$ can be written as the product of a slowing evolution operator $\hat{\sigma}''_{ij}(t)$ and a time-dependent exponential function, such as $\hat{\sigma}'_{ij}(t) = \hat{\sigma}''_{ij}(t)e^{-\gamma_j t}$ with $\gamma_{ij} = (\gamma_i + \gamma_j)/2$ and $\gamma_3 = 0$.

Therefore, the approximation $\hat{\sigma}_{ij}(t) \approx \hat{\sigma}_{ij}(\tau)e^{i\omega_j(t-\tau)}e^{-\gamma_j(t-\tau)}$ can be adopted. Inserting this approximation and the equal-time correlation $\langle \hat{\sigma}_{ij}(\tau)\hat{\sigma}_{nm}(\tau) \rangle = \langle \hat{\sigma}_{im}(\tau) \rangle \delta_{jn}$ into Eq. (6), we get the dipole-dipole correlation function as

$$\langle \hat{\mathbf{d}}(t)\hat{\mathbf{d}}(\tau) \rangle = \langle \hat{\sigma}_{11}(\tau) \rangle \mathbf{d}_{13}\mathbf{d}_{31}e^{(i\omega_{13}-\gamma_{13})(t-\tau)} + \langle \hat{\sigma}_{22}(\tau) \rangle \mathbf{d}_{23}\mathbf{d}_{32}e^{(i\omega_{23}-\gamma_{13})(t-\tau)} + \langle \hat{\sigma}_{12}(\tau) \rangle \mathbf{d}_{13}\mathbf{d}_{32}e^{(i\omega_{13}-\gamma_{13})(t-\tau)} \\ + \langle \hat{\sigma}_{21}(\tau) \rangle \mathbf{d}_{23}\mathbf{d}_{31}e^{(i\omega_{23}-\gamma_{13})(t-\tau)} + \langle \hat{\sigma}_{33}(\tau) \rangle \mathbf{d}_{31}\mathbf{d}_{13}e^{(-i\omega_{13}-\gamma_{13})(t-\tau)} + \langle \hat{\sigma}_{33}(\tau) \rangle \mathbf{d}_{32}\mathbf{d}_{23}e^{(-i\omega_{23}-\gamma_{13})(t-\tau)}. \quad (16)$$

Therefore the force can be deduced as

$$\mathbf{F}(t) = \sum_{mn} \langle \hat{\sigma}_{mn}(t) \rangle F_{mn}(\mathbf{r}_A), \quad m, n = 1, 2, 3. \quad (17)$$

Five density matrix elements [i.e., $\langle \hat{\sigma}_{11}(t) \rangle$, $\langle \hat{\sigma}_{22}(t) \rangle$, $\langle \hat{\sigma}_{33}(t) \rangle$, $\langle \hat{\sigma}_{12}(t) \rangle$, and $\langle \hat{\sigma}_{21}(t) \rangle$] as well as five time-independent force amplitudes [i.e., $F_{11}(\mathbf{r}_A)$, $F_{22}(\mathbf{r}_A)$, $F_{12}(\mathbf{r}_A)$, $F_{21}(\mathbf{r}_A)$, and $F_{33}(\mathbf{r}_A)$] contribute to the CP force on a V-type three-level atom. The time-independent force amplitudes have the expressions

$$F_{11}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr} \{ [\boldsymbol{\alpha}_{11}(\mathbf{r}_a, i\xi) + \boldsymbol{\alpha}_{11}(\mathbf{r}_a, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, i\xi) \} \\ + \{ \mu_0 \omega_{13}^2 [\mathbf{d}_{13} \cdot \nabla \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, \omega_{13}) \cdot \mathbf{d}_{31}] \Theta(\omega_{13}) + \text{c.c.} \}, \quad (18)$$

$$F_{22}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr} \{ [\boldsymbol{\alpha}_{22}(\mathbf{r}_a, i\xi) + \boldsymbol{\alpha}_{22}(\mathbf{r}_a, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, i\xi) \} \\ + \{ \mu_0 \omega_{23}^2 [\mathbf{d}_{23} \cdot \nabla \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, \omega_{23}) \cdot \mathbf{d}_{32}] \Theta(\omega_{23}) + \text{c.c.} \}, \quad (19)$$

$$F_{12}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr} \{ [\boldsymbol{\alpha}_{12}(\mathbf{r}_a, i\xi) + \boldsymbol{\alpha}_{12}(\mathbf{r}_a, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, i\xi) \} \\ + \{ \mu_0 \omega_{13}^2 [\mathbf{d}_{13} \cdot \nabla \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, \omega_{13}) \cdot \mathbf{d}_{32}] \Theta(\omega_{13}) + \text{c.c.} \}, \quad (20)$$

$$F_{21}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr} \{ [\boldsymbol{\alpha}_{21}(\mathbf{r}_a, i\xi) + \boldsymbol{\alpha}_{21}(\mathbf{r}_a, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, i\xi) \} \\ + \{ \mu_0 \omega_{23}^2 [\mathbf{d}_{23} \cdot \nabla \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, \omega_{23}) \cdot \mathbf{d}_{31}] \Theta(\omega_{23}) + \text{c.c.} \}, \quad (21)$$

$$F_{33}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr} \{ [\boldsymbol{\alpha}_{33}(\mathbf{r}_a, i\xi) + \boldsymbol{\alpha}_{33}(\mathbf{r}_a, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_a, i\xi) \}. \quad (22)$$

The force amplitude can be divided into two parts: the dispersion part and the resonant part. In the above five force amplitudes, there always exists the dispersion part which is characterized by the integration over ξ ($\boldsymbol{\alpha}_{ij}$ is the susceptibility as a function of frequency and position [12]). Except F_{33} the other four force amplitudes contain a resonant part which is characterized by the Θ function. The reason for $F_{33}(\mathbf{r}_A)$ containing only the dispersion part, i.e., Eq. (22), is that there is no real photon process when the atom is in the ground level [3]. However, compared with the resonant parts, the dispersion part is ignored when the atom is away from the surface. Thus, we focus on the resonant part of the CP force in this work, and find that such a resonant force is sensitive to the quantum interference.

Among these force amplitudes, $F_{12}(\mathbf{r}_A)$ and $F_{21}(\mathbf{r}_A)$ relate to the quantum interference because they originate from the cross coupling between \mathbf{d}_{13} and \mathbf{d}_{23} . We call them cross-coupling forces. Therefore, quantum interference not only affects the population evolution through κ_{12} but also contributes to the amplitude of the Casimir-Polder force due to $F_{12}(\mathbf{r}_A)$ and

$F_{21}(\mathbf{r}_A)$. The combination of these two facts can achieve a meaningful way to make Casimir-Polder force controllable.

III. ANALYSIS

Before discussing the influence of quantum interference on the CP force, we start by reviewing the case of an initially excited two-level atom for comparison. This case can be easily retrieved by setting $\mathbf{d}_{23} = \mathbf{d}_{32} = 0$. There are only two force amplitudes involved, i.e., $F_{11}(\mathbf{r}_A)$ and $F_{33}(\mathbf{r}_A)$. The evolution of the force is determined by the matrix elements $\langle \hat{\sigma}_{11}(t) \rangle$ and $\langle \hat{\sigma}_{33}(t) \rangle$, in which $\langle \hat{\sigma}_{11}(t) \rangle$ decays exponentially with rate γ_1 , as does the Casimir-Polder force.

According to previous work [17], when the atom is located at a position leading to the inhibition of spontaneous decay, the resonant part of $F_{11}(\mathbf{r}_A)$ becomes strong. In physics, inhibition of a decay rate is the result of destructive interaction between the reflected photon and the atom itself [22]. In an open environment, the decay rate is deeply inhibited when such interaction gets much stronger, which reflects significantly on

the force acting on the atom. Hence, the atom can suffer a strong resonant CP force and last for a long time. A conclusion can be inferred that if we design an environment where atomic spontaneous decay can be inhibited, the resonant force acting on the atom is more significant than in free space. An example is when the atom is located directly at the surface of an ideal metal without dissipation. It is known that the atom with parallel dipoles is inhibited totally from decay when it is located on an ideal metal surface, while the CP force at such a position has an infinite value at least theoretically [16]. Although in real experiments, the decay rate of an atom near a real metal always increases due to the additional near-field interaction between the atom and absorbing surfaces, the decay channel through the propagating wave mode is indeed inhibited.

Now a question of interest will be, if we can inhibit atomic decay through other means, does it also improve the resonant Casimir-Polder force acting on atom? The alternative method to control atomic decay is of course quantum interference between different decay channels in a multilevel atom [19,20,21]. For an ideal degenerate V-type three-level atom, the atomic decay is totally inhibited through quantum interference, if it is initially prepared in the subradiant state; meanwhile the

atomic decay can be enhanced by preparing the atom in the superradiant state [19]. Valid quantum interference has two prerequisites: First is the neglect of the detuning ω_{12} compared to the decay rate, so that the approximation $\omega_{13} \approx \omega_{23} = \omega_0$ is reasonable. Second is the equivalent value between the collective damping rate κ_{12} and the decay rates γ_1 (γ_2).

In the following we will discuss two kinds of V-type three-level atom to explore the influence of quantum interference on the CP force.

A. Quantum interference with $d_{13}||d_{32} = d_0$

In this ideal case, the two dipole momenta d_{13} and d_{32} are equal to each other, i.e., $d_{13}||d_{32} = d_0$. We call this kind of atom “the ideal V-type three-level atom.” This atomic model had been adopted in many previous works concerning quantum interference [19,20,21]. Since these two degenerate transition dipoles interact with the same electromagnetic mode, the corresponding decay rates and collective damping are the same, i.e., $\gamma_1 = \gamma_2 = \kappa_{12} = \kappa_{21} = \gamma$, despite the environmental involvement. Similarly, the time-independent force amplitudes can be simplified as

$$F_{11}(\mathbf{r}_A) = F_{22}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr}\{[\alpha_{11}(\mathbf{r}_A, i\xi) + \alpha_{11}(\mathbf{r}_A, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_A, i\xi)\} + \mu_0 \omega_0^2 2\text{Re}[d_0 \cdot \nabla \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_A, \omega_0) \cdot d_0] \Theta(\omega_0), \quad (23)$$

$$F_{12}(\mathbf{r}_A) = F_{21}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr}\{[\alpha_{12}(\mathbf{r}_A, i\xi) + \alpha_{12}(\mathbf{r}_A, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_A, i\xi)\} + \mu_0 \omega_0^2 2\text{Re}[d_0 \cdot \nabla \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_A, \Omega_0) \cdot d_0] \Theta(\omega_0), \quad (24)$$

$$F_{33}(\mathbf{r}_A) = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{Tr}\{[\alpha_{33}(\mathbf{r}_A, i\xi) + \alpha_{33}(\mathbf{r}_A, -i\xi)] \cdot \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_A, i\xi)\}. \quad (25)$$

Notice that $F_{11}(\mathbf{r}_A)$, $F_{22}(\mathbf{r}_A)$, $F_{12}(\mathbf{r}_A)$, and $F_{21}(\mathbf{r}_A)$ are the same for both the resonant parts as well as the dispersion parts because $d_{13} = d_{32} = d_0$ and $\omega_{13} = \omega_{23} = \omega_0$.

When the atom is prepared, initially, in the subradiant state as $|\psi(t=0)\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, the initial density matrix elements are

$$\begin{aligned} \langle \hat{\sigma}_{11}(0) \rangle &= \langle \hat{\sigma}_{22}(0) \rangle = 1/2, \\ \langle \hat{\sigma}_{12}(0) \rangle &= \langle \hat{\sigma}_{21}(0) \rangle = -1/2, \end{aligned} \quad (26)$$

and the others are zero.

By inserting them into the master equations (7)–(9), it is found that matrix elements do not evolve anymore due to the destructive interference. Therefore, the CP force is time independent according to Eq. (17). However, the force is also canceled due to quantum interference as follows:

$$\begin{aligned} F(t) &= \langle \hat{\sigma}_{11}(t) \rangle F_{11}(\mathbf{r}_A) + \langle \hat{\sigma}_{22}(t) \rangle F_{22}(\mathbf{r}_A) + \langle \hat{\sigma}_{12}(t) \rangle F_{12}(\mathbf{r}_A) \\ &\quad + \langle \hat{\sigma}_{21}(t) \rangle F_{21}(\mathbf{r}_A) + \langle \hat{\sigma}_{33}(t) \rangle F_{33}(\mathbf{r}_A) \\ &= [\langle \hat{\sigma}_{11}(0) \rangle + \langle \hat{\sigma}_{22}(0) \rangle + \langle \hat{\sigma}_{12}(0) \rangle + \langle \hat{\sigma}_{21}(0) \rangle] F_{11}(\mathbf{r}_A) \\ &= 0. \end{aligned} \quad (27)$$

This means that the force disappears when the atom is prepared in the subradiant state. Quantum interference leads to population trapping and force cancellation simultaneously. Therefore, an atom prepared in the subradiant state will pass through a waveguide without any perturbation. Without quantum interference, there always exist the CP force on either a multilevel atom or molecule regardless of the state preparation [12,14,15,18], even if the atom is in the ground state.

The situation switches when the atom is prepared initially in the superradiant state, i.e., $|\psi(t=0)\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$. The initial density matrix elements are

$$\langle \hat{\sigma}_{11}(0) \rangle = \langle \hat{\sigma}_{22}(0) \rangle = \langle \hat{\sigma}_{12}(0) \rangle = \langle \hat{\sigma}_{21}(0) \rangle = 1/2, \quad (28)$$

and the others are zero.

It is easy to see that matrix elements evolve in phase under the formation of $\langle \hat{\sigma}_{ij}(t) \rangle = 0.5e^{-2\gamma t}$, $i, j = 1, 2$, and the CP force acting on the atom then would be

$$F(t) = 2F_{11}(\mathbf{r}_A)e^{-2\gamma t} + F_{33}(\mathbf{r}_A)(1 - e^{-2\gamma t}). \quad (29)$$

From Eq.(29), the initial force amplitude is $2F_{11}$, while the force decays exponentially with rate 2γ if we ignore the

force on the ground state F_{33} . As a comparison, for an initially excited two-level atom, its force follows the relation $F(t) = F_{11}(\mathbf{r}_A)e^{-\gamma t} + F_{33}(\mathbf{r}_A)(1 - e^{-\gamma t})$ [17]. That means the force amplitude of an excited two-level atom is one-half of that of a three-level atom in the superradiant state. Consequently, the decay rate of the force on an excited two-level atom is also one-half of that of the three-level atom. Therefore, the three-level atom prepared in the superradiant state suffers doubled forces due to the constructive interference, but its lifetime is slashed in half when compared to the excited two-level atom.

Hence, for the ideal three-level atom with two parallel dipole momenta, quantum interference can modify both the population and the amplitude of the CP force. Furthermore, these two modifications are in phase. One needs to prepare an atom in the subradiant state in order to get population trapping but CP force cancellation, and vice versa.

Actually the ideal three-level atom with two parallel dipole moments is more of an academic model and has rarely been met in real atomic systems until now. The realistic V-type three level atom usually possesses two perpendicular dipole moments, whose quantum interference should result in an anisotropic vacuum [23]. So in the next section we will discuss such an alternative three-level atom system and find that it can achieve the more exciting scenario in which the enhancement of the CP force is accompanied by simultaneous trapping of population.

B. Quantum interference with $d_{13} \perp d_{32}$

Naturally, applying a magnetic field on a two-level atom results in Zeeman splitting which produces two nearly degenerate dipole moments with $\omega_{13} \approx \omega_{23} = \omega_0$. Such a pair of dipole moments are perpendicular to each other; one is left-rotating polarized and the other is right-rotating polarized. They are represented by $\mathbf{d}_{13} = d_0\mathbf{e}_+$ and $\mathbf{d}_{23} = d_0\mathbf{e}_-$, in which $\mathbf{e}_+ = (\mathbf{e}_z + i\mathbf{e}_x)/\sqrt{2}$ and $\mathbf{e}_- = (\mathbf{e}_z - i\mathbf{e}_x)/\sqrt{2}$ refer to right-rotating and left-rotating unit vectors, respectively. We call such a three-level atom “the Zeeman atom.” Because its two dipole moments are perpendicular to each other, there is no quantum interference in free space.

However, an anisotropic environment can revive the quantum interference of the Zeeman atom [23]. Here the anisotropy means that the diagonal elements of the Green tensor are not the same. For the structure shown in Fig. 1, $\vec{\mathbf{G}}_{xx}(\mathbf{r}_a, \mathbf{r}_a, \omega_0)$ is different from $\vec{\mathbf{G}}_{zz}(\mathbf{r}_a, \mathbf{r}_a, \omega_0)$. The corresponding decay and collective damping rates defined in Eqs. (12)–(15) now are transformed into

$$\begin{aligned} \gamma_1 = \gamma_2 &= \frac{\mu_0}{\hbar} \omega_0^2 d_0^2 [\text{Im}\vec{\mathbf{G}}_{xx}(\mathbf{r}_a, \mathbf{r}_a, \omega_0) + \text{Im}\vec{\mathbf{G}}_{zz}(\mathbf{r}_a, \mathbf{r}_a, \omega_0)] \\ &= (\Gamma_{\parallel} + \Gamma_{\perp})/2, \end{aligned} \quad (30)$$

$$\begin{aligned} \kappa_{12} = \kappa_{21} &= \frac{\mu_0}{\hbar} \omega_0^2 d_0^2 [\text{Im}\vec{\mathbf{G}}_{zz}(\mathbf{r}_a, \mathbf{r}_a, \omega_0) - \text{Im}\vec{\mathbf{G}}_{xx}(\mathbf{r}_a, \mathbf{r}_a, \omega_0)] \\ &= (\Gamma_{\perp} - \Gamma_{\parallel})/2. \end{aligned} \quad (31)$$

Here, $\Gamma_{\perp} = 2\mu_0\omega_0^2 d_0^2 \text{Im}\vec{\mathbf{G}}_{zz}(\mathbf{r}_a, \mathbf{r}_a, \omega_0)/\hbar$ is the spontaneous decay rate of the dipole moment \mathbf{d}_0 perpendicular to the interface, i.e., along the z axis, while $\Gamma_{\parallel} =$

$2\mu_0\omega_0^2 d_0^2 \text{Im}\vec{\mathbf{G}}_{xx}(\mathbf{r}_a, \mathbf{r}_a, \omega_0)/\hbar$ is the spontaneous decay rate of the dipole moment \mathbf{d}_0 parallel to the interface, i.e., along the x axis. Concerning the atomic population, quantum interference works significantly when κ_{12} approaches γ_1 . Therefore, if the atom is located in an environment such that $\Gamma_{\parallel} = 0$, we get $\gamma_1 = \gamma_2 = \kappa_1 = \kappa_2 = \Gamma_{\perp}/2$. It means that the Zeeman atom equivalently possess two parallel dipoles both perpendicular to the surface. Up to now several anisotropic environments have been designed to revive the quantum interference in Zeeman atoms [24–32]; for example, placing the atom in a photonic crystal [24,25], in left-handed materials in waveguides [26,27,28,29], near metal nanoshells [30], near single negative metamaterials [31], and near metasurfaces [32].

The resonant part of the CP force can also be divided into two components, of which one originates from the dipole component that is parallel to the surface, and the other from the component that is perpendicular to it, as follows:

$$\begin{aligned} F_{11}^r(\mathbf{r}_a) &= F_{22}^r(\mathbf{r}_a) \\ &= \mu_0\omega_0^2 d_0^2 \nabla \text{Re}[\vec{\mathbf{G}}_{xx}(\mathbf{r}, \mathbf{r}_a, \omega_0) + \vec{\mathbf{G}}_{zz}(\mathbf{r}, \mathbf{r}_a, \omega_0)]_{r=r_a} \\ &= (F_{\parallel} + F_{\perp})/2, \end{aligned} \quad (32)$$

$$\begin{aligned} F_{12}^r(\mathbf{r}_a) &= F_{21}^r(\mathbf{r}_a) \\ &= \mu_0\omega_0^2 d_0^2 \nabla \text{Re}[\vec{\mathbf{G}}_{zz}(\mathbf{r}, \mathbf{r}_a, \omega_0) - \vec{\mathbf{G}}_{xx}(\mathbf{r}, \mathbf{r}_a, \omega_0)]_{r=r_a} \\ &= (F_{\perp} - F_{\parallel})/2. \end{aligned} \quad (33)$$

When a Zeeman atom is placed in an anisotropic environment with $\vec{\mathbf{G}}_{xx} \neq \vec{\mathbf{G}}_{zz}$, the collective damping rate κ_{12} as well as the cross-coupling forces $F_{12}^r(\mathbf{r}_a)$ and $F_{21}^r(\mathbf{r}_a)$ survive. According to Eq. (27), if the cross-coupling force $F_{12}^r(\mathbf{r}_a)$ has a direction opposite to $F_{11}^r(\mathbf{r}_a)$, we can get the trapping of population and the enhancement of the CP force simultaneously.

We take the structure of Fig. 1 as an example. The permittivity and permeability of the LHM slab are

$$\begin{aligned} \varepsilon_A &= 1 + \frac{(0.8\omega_0)^2}{(0.8246\omega_0)^2 - \omega^2 - i(0.001\omega_0)\omega}, \\ \mu_A &= 1 + \frac{(0.8\omega_0)^2}{(0.8246\omega_0)^2 - \omega^2 - i(0.001\omega_0)\omega}. \end{aligned} \quad (34)$$

Therefore, at the atomic transition frequency $\omega = \omega_0$, the slab has negative indices $\varepsilon_A(\omega_0) = \mu_A(\omega_0) \approx -1.001 + i0.006$. The permittivity of the metal is chosen as

$$\varepsilon_M = 1 - \frac{(4\omega_0)^2}{\omega^2}. \quad (35)$$

Hence, its permittivity is negative and its refractive index is purely imaginary at $\omega = \omega_0$.

The thickness of the LHM slab is set to be $d_A = 2\lambda$ with $\lambda = 2\pi c/\omega_0$. The time-independent force amplitudes $F_{\parallel}(z_a)$ and $F_{\perp}(z_a)$ are plotted in Fig. 2(a). The decay rates $\Gamma_{\parallel}(z_a)$ and $\Gamma_{\perp}(z_a)$ are shown in Fig. 2(b). The unit of force is $B = \mu_0|d_0|^2\omega_0^4/4\pi^2c^2$, while $\Gamma_0 = d_0^2\omega_0^3/(3\pi\varepsilon_0\hbar c^3)$ is the decay rate in free space.

Due to the phase compensation of the LHM slab, both the force and decay rates near the position $z_a = 2\lambda$ clearly vary. We call the position $z_a = 2\lambda$ the focal position. From

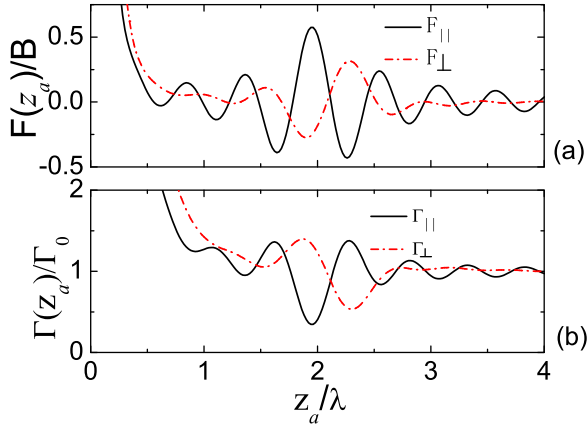


FIG. 2. (a) The forces acting on dipole components that are parallel and perpendicular to the interface, $F_{||}(z_a)$ and $F_{\perp}(z_a)$, as functions of position. (b) Spontaneous decay rate of the dipole components that are parallel and perpendicular to the interface, $\Gamma_{||}(z_a)$ and $\Gamma_{\perp}(z_a)$, as functions of position. The Zeeman atom is situated near a single LHM-metal structure with $d_A = 2\lambda$; indices are given by Eqs. (34) and (35).

Fig. 2(b), at the position $z_a = 2\lambda$, $\Gamma_{||}(z_a)$ is deeply inhibited, while $\Gamma_{\perp}(z_a)$ is enhanced. According to Eqs. (30) and (31), κ_{12} and γ_1 are both positive. On the other hand, from Fig. 2(a), at the position $z_a = 2\lambda$, $F_{||}(z_a)$ and $F_{\perp}(z_a)$ have opposite directions, and furthermore $F_{||}(z_a)$ is much stronger than $F_{\perp}(z_a)$. According to Eqs. (32) and (33), $F_{11}^r(\mathbf{r}_a)$ is positive while the cross-coupling force $F_{12}^r(\mathbf{r}_a)$ is negative. Here, quantum interference on the atomic population and the amplitude of the force are out of phase. For clarity, the amplitudes of the force $F_{ij}(z_a)$, $i, j = 1, 2$, as functions of position are plotted in Fig. 3(a), while the decay rates γ_1, γ_2 and the collective damping rates κ_{12}, κ_{21} are plotted in Fig. 3(b).

From Fig. 3, all the quantities are significant at the focal point $z_a = 2\lambda$ which means the quantum interference works at such position. In Fig. 3(a), at the focal point, the amplitudes of the cross-coupling forces F_{12} and F_{21} are attractive and have the value of $-0.38B$, while the diagonal forces F_{11} and F_{22} are

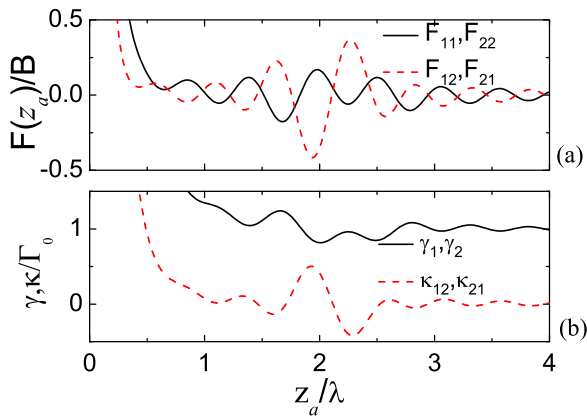


FIG. 3. (a) The time-independent force amplitude $F_{ij}(z_a)$ and (b) $\gamma_1, \gamma_2, \kappa_{12}$, and κ_{21} as functions of position. The V-type three-level atom is situated near a single LHM-metal structure with $d_A = 2\lambda$; indices are given by Eqs. (34) and (35).

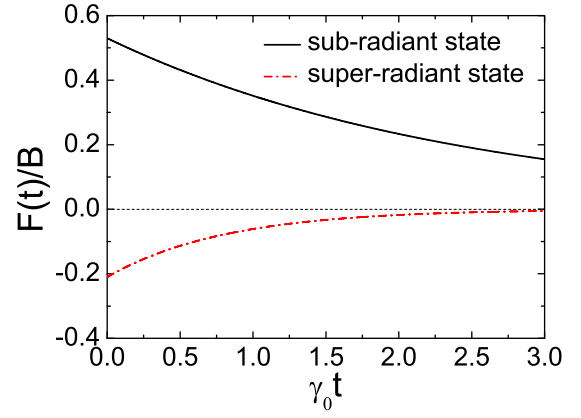


FIG. 4. The force evolution when an atom is placed at the focus point $z_a = 2\lambda$ of the structure shown in Fig. 1. The black solid curve refers to the case of an initially subradiant state, while the red dash-dotted curve refers to that of initially superradiant state.

repulsive and have the value of $0.16B$. In Fig. 3(b), at the focal point $z_a = 2\lambda$, both the decay and collective damping rates are positive and approach each other.

Now, we look into the three-level atom prepared initially in the subradiant state as $|\psi(t=0)\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, taking the initial matrix elements to be the same as Eq. (26). At the initial time, the force amplitude acting on the atom is $0.54B$. After that the force evolves and is determined by population evolution. Due to the initial subradiant state, the population will decay slowly compared to that in free space as, $\gamma = 0.82\Gamma_0$ and $\kappa = 0.45\Gamma_0$. The force shows up for a longer time compared with the free-space case. The force evolution is clearly indicated by the solid line in Fig. 4.

When the three-level atom is initially prepared in the superradiant state of $|\psi(t=0)\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$ with initial matrix elements as in Eq. (28), the force is absorptive and has the value of $-0.22B$ at the initial time. Due to the superradiant state, the atomic population decays much more quickly than in free vacuum, and so does the CP force evolution. The force evolution in this case is shown by the red dashed curve in Fig. 4.

Comparing the black solid curve with the red dashed curve in Fig. 4, it is apparent that the force with the initially subradiant state has a stronger amplitude, and lasts for a longer time, while the force of the initially superradiant state is weaker and lasts for a shorter time. This is opposite to the case of the ideal three-level atom with parallel dipoles. Therefore, considering the realistic Zeeman three-level atom, quantum interference based on an anisotropic environment is helpful for approaching a stronger resonant force with a longer action time.

To improve our result, we give an extreme example although it is not practical, and that is the LHM slab possessing ideal indices with $\epsilon_A = \mu_A = -1$ as well as an ideal metal mirror with $r^{TE} = -r^{TM} = -1$. The amplitudes of the force $F_{ij}(z_a), i, j = 1, 2$, as functions of position are plotted in Fig. 5(a), while the decay and collective damping rates are plotted in Fig. 5(b).

In this ideal case when the atom is placed at $z_a = 2\lambda$, the collective damping rate κ_{12} (κ_{21}) equals the decay rate γ_1 (γ_2)

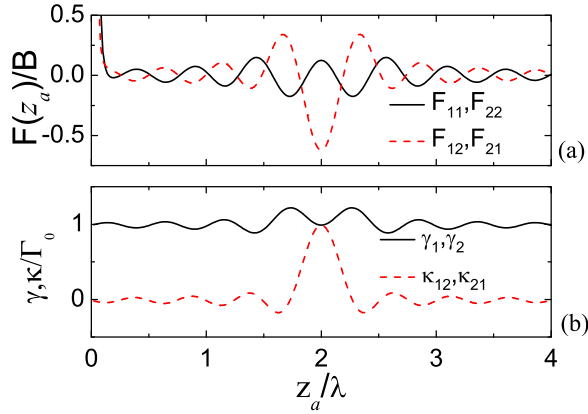


FIG. 5. (a) The time-independent force amplitude $F_{ij}(z_a)$ and (b) γ_1 , γ_2 , κ_{12} , and κ_{21} as functions of position. The Zeeman atom is situated near a single LHM-metal structure with $d_A = 2\lambda$; the index of the LHM at atomic frequency is assumed to be $\epsilon_A(\omega_0) = \mu_A(\omega_0) = -1$.

with values of Γ_0 , i.e., Fig. 5(b), and therefore, the atomic population can be trapped completely by destructive quantum interference [23]. In addition, the amplitude of the force F_{ij} is enhanced a little in Fig. 5(a) compared with the case in Fig. 3(a). The maxima of F_{12} and F_{21} reach $-0.63B$, while F_{11} and F_{22} decrease to $0.12B$ at the focus point $z_a = 2\lambda$.

If the Zeeman atom is prepared initially in the subradiant state as $|\psi(t=0)\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$ and also the force amplitude acting on the atom is $0.75B$, one will undoubtedly have population trapping, and of course the repulsive force will exist for a much longer time due to the subradiant state. This is the way to achieve atomic population trapping as well as a repulsive CP force. However, since ideal LHMs cannot yet be fabricated, we can just approach the result shown in Fig. 4.

Essentially, our result is also valid for other anisotropic environments, as shown in Refs. [24–26,30,31]. Anisotropically induced quantum interference can inhibit the decay of an atomic population. Meanwhile, the anisotropy leads to a difference between the cross-coupling force and the diagonal force defined in Eqs. (32) and (33) because neither $F_{||}$ nor F_{\perp} is null. Therefore, with the initially subradiant state, atomic

population is trapped, and meanwhile the resonant CP force will last for a long time.

IV. CONCLUSIONS

The Casimir-Polder force on a real excited three-level atom has been discussed in this paper. The role of quantum interference, between different transition channels, and its influence on the CP force is analyzed in detail.

The evolution of the CP force depends clearly on the atomic population and the amplitude of the force shown in Eq. (17). Nonetheless, quantum interference can modify both the atomic population and the amplitude of the resonant CP force. The effect of quantum interference on CP force evolution depends on the details of the atomic system.

For the ideal three-level atom with two degenerate parallel dipoles, and when the atom is initially prepared in the subradiant state, atomic population is trapped but the CP force is canceled. However, when the Zeeman atom with two circular polarized dipoles is considered, quantum interference can play an important role particularly due to an anisotropic environment. Such anisotropic-environment-induced quantum interference can lead to the trapping of atomic population. Additionally, it leads to significant amplitude of the CP force when the atom is initially prepared in the subradiant state. Therefore, this is evidence that one can possibly maintain a stronger CP force for a long time through quantum interference. In this paper, we adopted an anisotropic environment made of left-handed materials mounted on metal to perform the calculation. However, our result can be generalized to other anisotropic structures.

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