Teleportation of a ququart system using hyperentangled photons assisted by atomic-ensemble memories

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A single photon encoded in both the spin and the orbital angular momentum has recently been experimentally demonstrated [X.-L. Wang *et al.*, Nature **518**, 516 (2015)] with linear optics using the hyperentangled state, which can be viewed as a bipartite four-dimensional (ququart) entanglement. Here, we investigate this process from a general point of view. By exploring a controlled phase flip induced by atomic ensembles in one-side optical microcavities, we propose teleportations of general ququart systems including a two-atomic-ensemble system, a two-polarized-photon system, one photon with the polarization and spatial degrees of freedom (DOFs), and a hybrid photon-ensemble system using two hyperentangled photons. The output information may also be encoded by different physical systems up to the special requirements of a receiver. These schemes are also adapted to teleportation of a ququart system with only phases or real probability amplitudes, which is beyond previous superdense teleportation [Nature Commun. **6**, 7185 (2015)]. With these restrictions, half of the classical communication cost may be saved and experimental complexities are also reduced. Our theoretical schemes are feasible in modern physics and show the possibilities of transferring complex quantum systems for scalable quantum applications.

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I. INTRODUCTION

The transfer of quantum information between different remote memories has long been a goal of quantum communication [1–4]. One way is to directly transmit among different network nodes such as the coherent exchange of a single photon [5]. Unfortunately, inevitable losses in long-distance quantum channels will decrease transmission efficiencies. Although this problem may be easily addressed by repeated transmission of copied bits in the classical communication scenario, the distinctive quantum noncloning theorem has impeded the use of a similar transmission strategy in quantum networks. Hence, as a primitive subroutine of quantum communication, reliable long-distance quantum channels should be built and have been realized using quantum repeater schemes [2,3,6–9] and entanglement purification schemes [10,11]. Moreover, camouflaged information in the direct transmission scheme is also fragile to powerful attackers. Thus quantum entanglements can be used as key resources for secure transfer of information [12].

In this secure transmission scheme, known as quantum teleportation [12], Alice may faithfully swap unknown transmitted information to Bob's state of their shared entanglement. This famous scheme costs 4 bits + ebit (one Einstein-Podolsky-Rosen pair) per qubit. If Alice is fortunate to transfer known information, a simpler method may be used to prepare the unknown state at Bob's site [13,14]; it is known as remote state preparation (RSP). In this improved scheme, the transmitted information will first be prepared in Alice's state of their shared entanglement and then swapped to Bob's shared state using proper disentangling operations by Alice and proper feed-forward corrections by Bob. Without a transmitted quantum state involved, quantum measurement is only performed on Alice's shared state. Thus the classical communication cost and the experimental complexities of disentangling operations are reduced. However, this kind of scheme is generally restricted to transmission of special information such as phases [15] or real probability amplitudes in a subspace of the general quantum-state space [12,16].

Quantum teleportation and RSP provide useful methods to transfer quantum states from one site to another at a remote location, assisted by the previously shared entanglement and classical communication channel. Teleportation protocols have been widely implemented with different physical systems [17–30] and RSP protocols have been implemented with photon and ion qubit states [31–36]. Almost all of the previous experiments have been limited to encoding of information by only one degree of freedom (DOF) of the system even if the experimental system possesses various independent DOFs simultaneously. In fact, different DOFs of the physical system may be very useful in various quantum applications. A recent experiment shows that quantum information on a polarization photonic state can be transferred to the orbital angular momentum of another photon [37] using photonic entanglement in the spatial mode. Moreover, using hyperentangled photons, a photon with spin angular momentum and orbital angular momentum DOFs [38] or a specific class of single-photon four-dimensional states (ququarts) with only phase information [39] can be teleported with linear optics.

These experiments [37–39] have presented possibilities for transferring complex quantum systems with scalable quantum technologies. Generally, a four-dimensional system may consist of different physical systems beyond one photon with two

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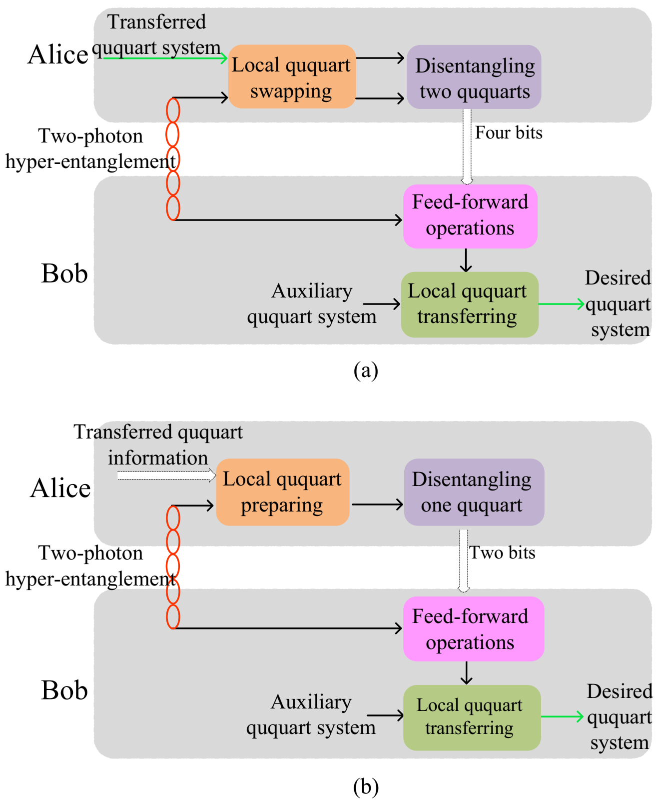

FIG. 1. Schematic of the transfer of a ququart system to another system using the two-photon hyperentanglement shown in Eq. (11). (a) An unknown ququart system. (b) A known ququart system. Here, the transferred information denotes phases or real probability amplitudes.

DOFs when a special task such as quantum computation or quantum communication is considered. Hence, it is important to show how to transfer these systems into different systems [27–29] for large-scale quantum applications. Motivated by hybrid teleportations [27–29] and teleportations of multiple DOFs of photons [38,39], in this paper we consider the teleportation of general four-dimensional systems $|\psi\rangle = \alpha_0|0\rangle + \alpha_1 e^{i\theta_1}|1\rangle + \alpha_2 e^{i\theta_2}|2\rangle + \alpha_3 e^{i\theta_3}|3\rangle$ using a two-photon hyperentanglement, as shown in Fig. 1(a). Differently from traditional teleportation [12], the input physical system may be an arbitrary combination of two qubits such as a photon and an atomic ensemble and the received physical system is also adaptable up to the special requirements of a receiver. These are generalizations of previous teleportations between light and matter [27,28] and teleportations of photons with two DOFs [38]. To complete our schemes, a primitive controlled phase flip in a hybrid system of an atomic ensemble and a photon is explored to develop a quantum teleportation architecture based on a simple module comprising an optical cavity containing a number of atoms [40]. This platform features a number of desirable properties [8,9,41–49]. With this platform in this paper, a ququart state can be faithfully teleported to a remote ququart system. The final state may be one photon with two DOFs, two atomic ensembles, two photons, or a hybrid system of a photon and an atomic ensemble. If the transformed information is restricted to phases $\{e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}\}$ [39] or real probability amplitudes $\{\alpha_0,\alpha_1,\alpha_2,\alpha_3\}$, improved schemes with less classical communication and reduced quantum measurement are presented as shown in Fig. 1(b). All of these schemes are feasible using modern techniques and may be useful for quantum communication or quantum computation with hybrid systems [9,47,49].

The paper is organized as follows. In Sec. II, an atomic-ensemble system is first presented to realize a controlled phase flip in a hybrid system of a photon and an atomic ensemble. With this controlled operation, a general ququart system of two atomic ensembles will be teleported to a remote system consisting of two atomic ensembles, one photon with two DOFs, or two photons with only polarization DOF or a hybrid system of a photon and an atomic ensemble. Thus, one ququart system may be teleported to different systems, which is very useful for large-scale quantum information processing with hybrid systems. Then, in Sec. III, a general system of one photon with two DOFs or two polarized photons or a hybrid system of a photon and an atomic ensemble can also be teleported to remote systems with different choices. All of these schemes can be simplified in Sec. IV if the transmitted information is restricted to phases or real probability amplitudes. The experimental feasibilities and fidelities of these schemes are reported in Sec. V, where some discussion is also presented.

II. TELEPORTATION OF A QUQUART SYSTEM OF TWO ATOMIC ENSEMBLES

A. An atomic-ensemble system

Figure 2(a) presents a schematic atomic ensemble composed of N cold atoms trapped in a one-sided optical cavity [8,9,40–48], in which one mirror is perfectly reflective and the other is of small transmission allowing for in-coupling and out-coupling to light. Each atom has a four-level internal structure, shown in Fig. 2(b). The two hyperfine ground states of each cold atom are denoted $|s\rangle$ and $|f\rangle$, respectively. The excited state $|e\rangle$ and high-lying Rydberg state $|r\rangle$ are two auxiliary states [46]. The atomic transition between $|f\rangle$ and $|e\rangle$ is resonantly coupled to the cavity mode at coupling rate g, which

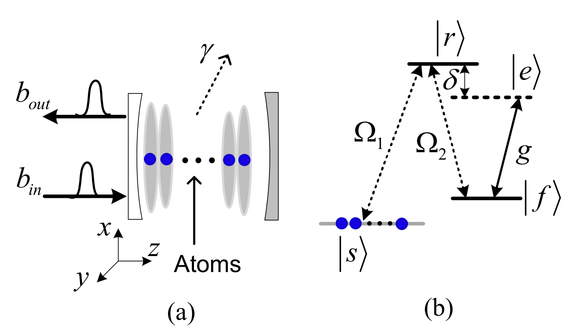

FIG. 2. (a) Building block with the atomic ensemble confined in a microcavity. γ shows fluorescence photons emitted outside the cavity. (b) Atomic level structure for emission of Stokes photons in the far-off-resonant Raman configuration. The atomic transition $|s\rangle(|f\rangle) \to |r\rangle$ is dipole forbidden using a classical laser with Rabi frequency $\Omega_1(\Omega_2)$. $|f\rangle \to |e\rangle$ is resonantly coupled to the cavity mode at coupling rate g and with an input photon in polarization $|H\rangle$.

is nearly resonantly driven by an input photon in polarization $|H\rangle$ with frequency ω. The transition $|s\rangle(|f\rangle) \to |r\rangle$ is dipole forbidden using a classical laser with Rabi frequency $\Omega_1(\Omega_2)$.

Assuming that all the atoms have been cooled to microkelvins and prepared in the ground state $|s\rangle$ in a far-of-resonant optical trap, our atomic qubit is encoded by an atomic- in the ground state $|\bar{0}\rangle$ and single collective spin-wave excitation state $|\bar{1}\rangle$, i.e.,

$$|\bar{0}\rangle = |s_1, \ldots, s_N\rangle,$$
$$|\bar{1}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |s_1, \ldots, s_{j-1}, f_j, s_{j+1}, \ldots, s_N\rangle. \quad (1)$$

An arbitrary unitary operation between $|\bar{0}\rangle$ and $|\bar{1}\rangle$ can be performed efficiently with the Rydberg blockade effect of state $|r\rangle$ by collective laser manipulations on the atomic ensemble [49]. When the Rydberg blockade shift is $\delta = 2\pi \times 100$ MHz, the transition between $|\bar{0}\rangle$ and $|\bar{1}\rangle$ can be completed with effective coupling strength $\Omega = 2\pi \times 1$ MHz [50].

In the frame rotating at the cavity frequency ω_c, the Hamiltonian of an input photon and an atomic ensemble inside a one-sided cavity can be expressed as [40,44,47]

$$\begin{aligned}\mathbf{H} = \hbar \sum_{j=1}^{N} \bigg[\left(\Delta_{\omega_c} - i\frac{\gamma_{e_j}}{2} \right) |e_j\rangle\langle e_j| \\ + i g_j (a|e_j\rangle\langle f_j| - a^+|f_j\rangle\langle e_j|) \bigg] \\ + i\hbar \frac{\kappa}{2\pi} \int [b^+(\Delta)a - b(\Delta)a^+] d(\Delta) \\ + \hbar \int b^+(\Delta) b(\Delta) d(\Delta). \end{aligned} \quad (2)$$

Here, a and b are the respective annihilation operators of the cavity mode and input photon mode, respectively, and satisfy $[a,a^+] = 1$ and $[a(x_1),b^+(x_2)] = \delta_{x_1 - x_2}$. $\Delta_{\omega_c} = \omega_0 - \omega_c$ denotes the detuning between the dipole transition frequency ω_0 and the cavity mode frequency ω_c. $\Delta = \omega - \omega_c$ denotes the detuning between the frequency of the input photon and that of the cavity mode. γ_{e_j} denotes the spontaneous emission rate of excited state $|e_j\rangle$, and g_j denotes the coupling strength between the jth atom and the cavity mode, $j = 1, \ldots, N$. For simplicity, we assume that $\gamma_{e_j} = \gamma$ and $g_j = g$ for all $j = 1, \ldots, N$ in the following [51].

For an atomic ensemble in state $|\bar{1}\rangle$ and an input photon in state $|H\rangle$, from the Hamiltonian in Eq. (2), the evolution of a joint system may be restricted to the first-order excitation subspace [40]. Thus, assume that t_0 and t_1 correspond to the moments when the photonic pulse goes in and comes out of the cavity, respectively. For cavity-field operator $\hat{a}$, input pulse field $\hat{b}$, and dipole operator σ_-, its Schrödinger equation is described as [8,9,40,44–48]

$$i\frac{da}{dt} = -ig\sigma_- - i\frac{\kappa}{2\pi} \int d\Delta b(\Delta,t), \quad (3)$$

$$i\frac{\partial b(\Delta,t)}{\partial t} = i\frac{\kappa}{2\pi} a + \Delta b(\Delta,t), \quad (4)$$

$$i\frac{d\sigma_-}{dt} = \left(\Delta_{\omega_c} - i\frac{\gamma}{2} \right) \sigma_- + iga, \quad (5)$$

where $b_{\text{in}}(t) = \frac{1}{\sqrt{2\pi}} \int e^{-i\Delta(t-t_0)} b(\Delta,t_0) d\Delta$ and $b_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}} \int e^{-i\Delta(t-t_1)} b(\Delta,t_1) d\Delta$ are the input and output pulse fields, respectively, and $b(\Delta,t_0)$ and $b(\Delta,t_1)$ are the probability amplitudes of the input photon with the frequency $\omega = \omega_c + \Delta$ at moments t_0 and t_1, respectively. The cavity output b_{out} is connected to the input by the standard input-output relation

$$b_{\text{out}} = b_{\text{in}} + \sqrt{\kappa} a. \quad (6)$$

If atoms stay in the ground states most of the time [40] [$\langle \sigma_- \rangle = -1$], from Eqs. (3)–(6), the output and input fields $b_{\text{out}}(t)$ and $b_{\text{in}}(t)$ are related by a reflection coefficient,

$$r(\Delta) \approx \frac{(\Delta - i\kappa/2)(\Delta_{\omega_c} + i\gamma/2) + g^2}{(\Delta + i\kappa/2)(\Delta_{\omega_c} + i\gamma/2) + g^2}. \quad (7)$$

The probability of an input photon's being reflected by an optical cavity module with cooperativity $C = 2g^2/(\gamma\kappa)$ is given by [51]

$$P = 1 - \frac{1 + 4C + (\Delta/\gamma)^2}{1 + 4C + 4C^2 + (\Delta/\gamma)^2}. \quad (8)$$

Note that in the case of large detuning, $(\Delta/\gamma)^2 \ll (C^2, C, 1)$, the cavity is effectively empty and the reflection probability approaches $P \to 0$.

For an atomic ensemble in state $|\bar{0}\rangle$, the atomic ensemble will be decoupled to the cavity mode. Thus, the input photon in the polarization $|H\rangle$ will be reflected by an empty cavity [40,43,49], i.e., $g = 0$. The corresponding reflection coefficient $r(\Delta)$ will be reduced to $r_0(\Delta)$, given by

$$r_0(\Delta) = \frac{\Delta - i\kappa/2}{\Delta + i\kappa/2}. \quad (9)$$

Note that when the detuning $|\Delta| \ll \gamma$ and the cooperativity $C \gg 1$, one can get a unit reflection photon with $r_0 \approx -1$ and $r \approx 1$, respectively. The interaction can be summarized to an ideal controlled phase-flip operator CZ as follows:

$$\text{CZ} = |H\rangle\langle H|(-|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|). \quad (10)$$

This ideal Λ-system unit [40,43,49] is used to realize the following teleportations of general ququart systems in Secs. II, III, and IV.

B. Teleportation of a ququart system of two atomic ensembles to one photon with two DOFs

Recently, Bao *et al.* [42] have experimentally realized teleportation between two remote atomic-ensemble quantum nodes with an average fidelity of 88(7)%. Their quantum resource is a two-photon entanglement. To extend their teleportation, in this subsection, we consider a two-photon hyperentanglement,

$$|\text{EPR}_h\rangle = |\text{EPR}_p\rangle \otimes |\text{EPR}_s\rangle, \quad (11)$$

with $|\text{EPR}_p\rangle := \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)_{AB}$ and $|\text{EPR}_s\rangle := \frac{1}{\sqrt{2}}(|d_1d_1\rangle + |d_2d_2\rangle)_{AB}$, where $\{|H\rangle,|V\rangle\}$ denotes the basis of the polarization DOF of photons, while $\{|d_1\rangle,|d_2\rangle\}$ denotes

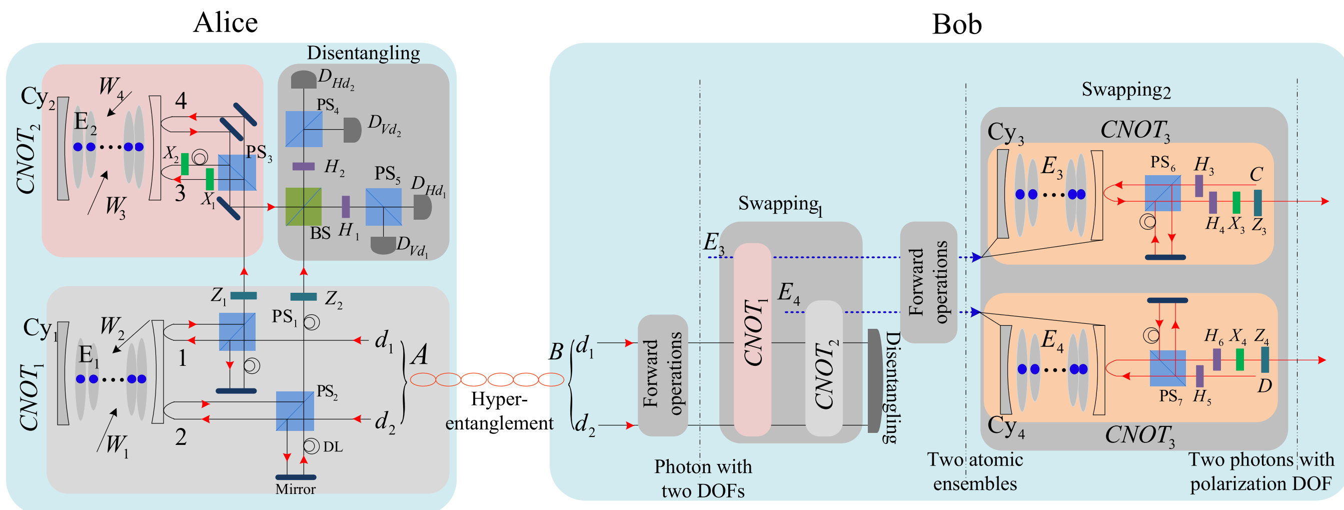

FIG. 3. Schematic of the teleportation of two atom ensembles with a two-photon hyperentanglement. H_j denotes half-wave plates to perform the Hadamard transformation on the polarization DOF of photons. W_j denotes the Hadamard transformations on the atomic ensemble. X_j denotes wave plates to perform the Pauli flip of the polarization DOF of photons. Z_j denotes wave plates to perform the Pauli phase flip of the polarization DOF of photons. PS_j represents polarizing beam splitters that transmit $|H\rangle$ and reflect $|V\rangle$. BS represents a 50%:50% polarizing beam splitter to perform the Hadamard operation on the spatial DOF of photons. Mirrors are used to reflect photons. Delays are used to postpone photonic pluses.

the basis of the spatial DOF [i.e., represents the paraxial spatial modes (Laguerre-Gauss) carrying $-\hbar$ and $+\hbar$ orbital angular momentum, respectively] of photons. Hyperentangled systems, where two-component quantum systems are entangled in every DOF [52], enable 100% efficient Bell-state analysis with only linear elements [53] and state purification [54]. In addition, hyperentanglement can also offer significant advantages in quantum secure superdense coding [55] and quantum cryptography [56,57]. With a similar two-photon hyperentanglement, Wang *et al.* [38] have studied the first teleportation of a single photon with spin and orbital angular momentum DOFs. Differently from Refs. [38] and [42], we teleport a ququart system of two atomic ensembles in this section. Moreover, a receiver may recover different ququart systems [27,28].

The schematic circuit for teleporting a ququart system of two atomic ensembles, say E_1 and E_2, is shown in Fig. 3. Suppose the transferred ququart system is in the state

$$|\psi\rangle_{E_1E_2} = (\alpha_0|\overline{00}\rangle + \alpha_1|\overline{01}\rangle + \alpha_2|\overline{10}\rangle + \alpha_3|\overline{11}\rangle)_{E_1E_2}. \quad (12)$$

Alice and Bob share the hyperentanglement $|\mathrm{EPR}_h\rangle$ in Eq. (11), where photon A belongs to Alice, while photon B belongs to Bob. First, after a Hadamard operation W_1 was performed on atomic ensemble E_1, Alice let her photon A from spatial mode d_1 (d_2) pass through PS_1 (PS_2). The transmitted component $|H\rangle$ of each spatial mode d_j is led to a cavity Cy_1 with atomic ensemble E_1 from path j. The output pulses are recombined with the corresponding reflected pulses at PS_1 (PS_2) simultaneously by accurate adjustments with delays. In detail, the joint system of photons A and B and two atomic ensembles E_1 and E_2 evolve as

$$\begin{aligned}
&|\mathrm{EPR}_h\rangle|\psi\rangle_{E_1E_2}\\
&\xrightarrow{W_1} |\mathrm{EPR}_h\rangle(W_1|\psi\rangle_{E_1E_2})\\
&\xrightarrow[\text{path 1}]{\mathrm{PS}_1,\mathrm{Cy}_1,\mathrm{PS}_1} \mathrm{CZ}_{E_1}^{Ad_1}[|\mathrm{EPR}_h\rangle(W_1|\psi\rangle_{E_1E_2})]\\
&\xrightarrow[\text{path 2}]{\mathrm{PS}_2,\mathrm{Cy}_1,\mathrm{PS}_2} \mathrm{CZ}_{E_1}^{Ad_2}\{\mathrm{CZ}_{E_1}^{Ad_1}[|\mathrm{EPR}_h\rangle(W_1|\psi\rangle_{E_1E_2})]\}\\
&\xrightarrow{W_2,Z_1,Z_2} \mathrm{CNOT}_1(|\mathrm{EPR}_h\rangle|\psi\rangle_{E_1E_2}),
\end{aligned} \quad (13)$$

where $\mathrm{CZ}_{E_1}^{Ad_j} = |H\rangle|d_j\rangle\langle d_j|\langle H|(-|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|) + |V\rangle|d_j\rangle \langle d_j|\langle V|(|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|)$ denotes a controlled phase flip of the polarization DOF of photon A from spatial mode d_j and atomic ensemble E_1 with the controlling term $|H\rangle_A$, and $\mathrm{CNOT}_1 = |H\rangle\langle H|(|\bar{0}\rangle\langle\bar{1}| + |\bar{1}\rangle\langle\bar{0}|) + |V\rangle\langle V|(|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|)$ denotes a controlled-NOT (CNOT) gate on the polarization DOF of photon A and atomic ensemble E_1 with the controlling term $|H\rangle_A$.

Second, after a Hadamard operation W_3 is performed on atomic ensemble E_2, Alice lets her photon A from spatial mode d_1 pass through PS_3. The reflected and transmitted components are led into a cavity Cy_2 with atomic ensemble E_2 from paths 3 and 4, respectively. These pulses are recombined at PS_3 simultaneously. The joint system of photons A and B and two atomic ensembles E_1 and E_2 evolves as follows:

$$\begin{aligned}
&\mathrm{CNOT}_1(|\mathrm{EPR}_h\rangle|\psi\rangle_{E_1E_2})\\
&\xrightarrow{W_3} \mathrm{CNOT}_1[|\mathrm{EPR}_h\rangle(W_3|\psi\rangle_{E_1E_2})]\\
&\xrightarrow[Cy_2,X_2,\mathrm{PS}_3]{\mathrm{PS}_3,Cy_2,X_1} \mathrm{CZ}_{E_2}^{A,d}\{\mathrm{CNOT}_1[|\mathrm{EPR}_h\rangle(W_3|\psi\rangle_{E_1E_2})]\}\\
&\xrightarrow{W_4,\bar{Z}_1} \mathrm{CNOT}_2\mathrm{CNOT}_1(|\mathrm{EPR}_h\rangle|\psi\rangle_{E_1E_2})\\
&:= |\Theta_1\rangle.
\end{aligned} \quad (14)$$

Here, $\mathrm{CZ}_{E_2}^{A,d} = |d_1\rangle\langle d_1|(-|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|) + |d_2\rangle\langle d_2|(|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|)$ denotes a controlled phase flip of the spatial DOF of photon A and atomic ensemble E_2 with the controlling term $|d_1\rangle_A$, $\mathrm{CNOT}_2 = |d_1\rangle\langle d_1|(|\bar{0}\rangle\langle\bar{1}|$

$+ |\bar{1}\rangle\langle\bar{0}|) + |d_2\rangle\langle d_2|\, (|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|)$ denotes a CNOT gate on the spatial DOF of photon A and atomic ensemble E_2 with the controlling term $|d_1\rangle$, and $|\Theta_1\rangle$ is defined by

$$|\Theta_1\rangle = \frac{1}{2}\big[|Hd_1\rangle_A\big(\sigma^e_{x,E_1}\sigma^e_{x,E_2}|\psi\rangle\big)|Hd_1\rangle_B + |Hd_2\rangle_A\big(\sigma^e_{x,E_1}|\psi\rangle\big)|Hd_2\rangle_B + |Vd_1\rangle_A\big(\sigma^e_{x,E_2}|\psi\rangle\big)|Vd_1\rangle_B + |Vd_2\rangle_A|\psi\rangle|Vd_2\rangle_B\big], \quad (15)$$

where $\sigma^e_{x,E_j} = |\bar{0}\rangle\langle\bar{1}| + |\bar{1}\rangle\langle\bar{0}|$ denotes a Pauli flip of atomic ensemble E_j. By measuring two atomic ensembles E_1 and E_2 under the basis $\{|\bar{0}\rangle,|\bar{1}\rangle\}$, the entanglement $|\Theta_1\rangle$ in Eq. (15) collapses into one of the states

$$(\alpha_3|Hd_1\rangle|Hd_1\rangle + \alpha_2|Hd_2\rangle|Hd_2\rangle + \alpha_1|Vd_1\rangle|Vd_1\rangle + \alpha_0|Vd_2\rangle|Vd_2\rangle)_{AB}, \quad (16)$$

$$(\alpha_2|Hd_1\rangle|Hd_1\rangle + \alpha_3|Hd_2\rangle|Hd_2\rangle + \alpha_0|Vd_1\rangle|Vd_1\rangle + \alpha_1|Vd_2\rangle|Vd_2\rangle)_{AB}, \quad (17)$$

$$(\alpha_1|Hd_1\rangle|Hd_1\rangle + \alpha_0|Hd_2\rangle|Hd_2\rangle + \alpha_3|Vd_1\rangle|Vd_1\rangle + \alpha_2|Vd_2\rangle|Vd_2\rangle)_{AB}, \quad (18)$$

$$(\alpha_0|Hd_1\rangle|Hd_1\rangle + \alpha_1|Hd_2\rangle|Hd_2\rangle + \alpha_2|Vd_1\rangle|Vd_1\rangle + \alpha_3|Vd_2\rangle|Vd_2\rangle)_{AB} \quad (19)$$

for the measurement outcomes $|\overline{00}\rangle$, $|\overline{01}\rangle$, $|\overline{10}\rangle$, and $|\overline{11}\rangle$ of atomic ensembles E_1 and E_2, respectively. All of these states may be useful for faithful teleportation. In fact, take the first collapsed state in Eq. (16) as an example; it may collapse into

$$|\Theta_2\rangle = (\alpha_0|Hd_1\rangle + \alpha_1|Hd_2\rangle + \alpha_2|Vd_1\rangle + \alpha_3|Vd_2\rangle)_B \quad (20)$$

by measuring photon A under the basis $\{(|H\rangle \pm |V\rangle)(|d_1\rangle \pm |d_2\rangle)/2\}$ using one BS, two PSs (PS_4 and PS_5), two half-wave plates (H_1 and H_2), and four single-photon detectors (D_{Hd_1}, D_{Hd_2}, D_{Vd_1}, and D_{Vd_2}). The recovery operations are defined in Table I. Thus Bob can recover a ququart photon B with two DOFs [38]. This means that Alice has teleported two atomic ensembles to one photon B with two DOFs. The total classical cost is four bits, where two bits are used to encode the measurement outcomes of atomic ensembles E_1 and E_2, and the other two bits are used to represent the detection results for photon A.

TABLE I. Relations between the detecting result (DR) of photon A and the feed-forward operation R on photon B for teleportation of two atomic ensembles. $\sigma_x^{p,p}$ and $\sigma_x^{p,s}$ denote Pauli flips on the polarization DOF and spatial DOF of one photon, respectively. $\sigma_z^{p,p}$ and $\sigma_z^{p,s}$ denote Pauli phase flips on the polarization DOF and spatial DOF of one photon, respectively.

DR	R performed on photon B
D_{Hd_1}	$\sigma_x^{p,p}\sigma_x^{p,s}$
D_{Hd_2}	$\sigma_z^{p,s}\sigma_x^{p,p}\sigma_x^{p,s}$
D_{Vd_1}	$\sigma_z^{p,p}\sigma_x^{p,p}\sigma_x^{p,s}$
D_{Vd_2}	$\sigma_z^{p,p}\sigma_z^{p,s}\sigma_x^{p,p}\sigma_x^{p,s}$

TABLE II. Relations between the detecting results (DR) for photon B and the feed-forward operations on two atomic ensembles, E_3 and E_4, for recovering two atomic ensembles. σ_x^e and σ_z^e denote Pauli flip and Pauli phase flip on the atomic ensembles, respectively.

	Feed-forward operation	
DR	Atomic ensemble E_3	Atomic ensemble E_4
D_{Hd_1}	σ_x^e	σ_x^e
D_{Hd_2}	σ_x^e	$\sigma_x^e\sigma_z^e$
D_{Vd_1}	$\sigma_x^e\sigma_z^e$	σ_x^e
D_{Vd_2}	$\sigma_x^e\sigma_z^e$	$\sigma_x^e\sigma_z^e$

C. Teleportation of a ququart system of two atomic ensembles to a similar system

Suppose that Alice wants to teleport a ququart system of two atomic ensembles to another two atomic ensembles for quantum computation or storage of quantum information [21–26]. After the teleportation circuit in Sec. II B, Bob should perform a local swapping circuit using two auxiliary atomic ensembles in state $|\bar{0}\rangle_{E_3}|\bar{0}\rangle_{E_4}$, as shown in Fig. 3. By implementing a CNOT gate CNOT_1 on photon B and atomic ensemble E_3 and a CNOT gate CNOT_2 on photon B and atomic ensemble E_4, from the joint system of $|\Theta_2\rangle_B|\overline{00}\rangle_{E_3E_4}$, Bob can get the following joint state:

$$|\Theta_3\rangle = (\alpha_0|Hd_1\rangle|\overline{11}\rangle + \alpha_1|Hd_2\rangle|\overline{10}\rangle + \alpha_2|Vd_1\rangle|\overline{01}\rangle + \alpha_3|Vd_2\rangle|\overline{00}\rangle)_{B,E_3E_4}. \quad (21)$$

Now, Bob should disentangle photon B using one BS, two PSs, two half-wave plates, and four single-photon detectors (as for photon A shown in Fig. 3). This may result in a new ququart system defined by

$$|\Theta_4\rangle = (\alpha_0|\overline{00}\rangle + \alpha_1|\overline{01}\rangle + \alpha_2|\overline{10}\rangle + \alpha_3|\overline{11}\rangle)_{E_3E_4}. \quad (22)$$

The recovery operations are defined in Table II. Thus they can complete the teleportation of a ququart system of two atomic ensembles to another two-atomic-ensemble system.

Of course, Bob's local swapping may be performed before the teleportation in Sec. II B. Thus Bob will change their hyperentanglement $|\text{EPR}_h\rangle$ into $\frac{1}{2}(|Hd_1\rangle|\overline{00}\rangle + |Hd_2\rangle|\overline{01}\rangle + |Vd_1\rangle|\overline{10}\rangle + |Vd_2\rangle|\overline{11}\rangle)_{A,E_3E_4}$. The followed teleportation is similar to the one in Sec. II B with a hyperentanglement $|\text{EPR}_h\rangle$. The only difference is that the feed-forward operations R in Table I should be performed on two atomic ensembles E_3 and E_4 by Bob.

D. Teleportation of a ququart system of two atomic ensembles to a two-photon system

Suppose that Alice wants to teleport a ququart system of two atomic ensembles to a two-polarized-photon system, which is an inverse of the teleportation of a photonic system to an atomic system [27,28]. Two equivalent methods can be used. One is to recover two polarized photons from one photon shown in Eq. (20) or two atomic ensembles shown in Eq. (22) by using two auxiliary photons in state $|H\rangle_{B_1}|H\rangle_{B_2}$. The other is to change their hyperentanglement $|\mathrm{EPR}_h\rangle$ in Eq. (11) into $\frac{1}{2}(|Hd_1\rangle|HH\rangle+|Hd_2\rangle|HV\rangle+|Vd_1\rangle|VH\rangle+|Vd_2\rangle|VV\rangle)_{A,B_1B_2}$. Both of these make use of a local swapping circuit between different systems. Take the first method as an example; if the photon state $|\Theta_2\rangle$ in Eq. (20) is considered, a local swapping gate should be faithfully implemented on the spatial DOF of photon B and the polarization DOF of the other auxiliary photon. This may be very difficult with only linear optics [38]. As a substitution, we consider the ququart system $|\Theta_4\rangle$ in Eq. (22) as Bob's setup.

Bob performs a CNOT gate $\mathrm{CNOT}_3=|\bar{0}\rangle\langle\bar{0}|(|H\rangle\langle V|+|V\rangle\langle H|)+|\bar{1}\rangle\langle\bar{1}|(|H\rangle\langle H|+|V\rangle\langle V|)$ on atomic ensemble E_3 and photon B_1 and a CNOT gate CNOT_3 on atomic ensemble E_4 and photon B_2, as shown in Fig. 3. Each subcircuit consists of two half-wave plates Hs, one PS, one wave plate X, one wave plate Z, one delay, and one minor. Thus Bob can change the joint system $|\Theta_4\rangle|HH\rangle_{B_1B_2}$ into

$$|\Theta_5\rangle=(\alpha_0|\overline{00}\rangle|HH\rangle+\alpha_1|\overline{01}\rangle|HV\rangle+\alpha_2|\overline{10}\rangle|VH\rangle+\alpha_3|\overline{11}\rangle|VV\rangle)_{E_3E_4,B_1B_2}. \quad (23)$$

By disentangling two atomic ensembles E_3 and E_4 under the basis $\{|\pm^e\rangle:=\frac{1}{\sqrt{2}}(|\bar{0}\rangle\pm|\bar{1}\rangle)\}$, Bob will recover a state of photons B_1 and B_2 defined by

$$|\Theta_6\rangle=\alpha_0|HH\rangle+\alpha_1|HV\rangle+\alpha_2|VH\rangle+\alpha_3|VV\rangle. \quad (24)$$

The recovery operations are defined by $\sigma_{z,B_1}^{i_1}\sigma_{z,B_2}^{i_2}$ dependent on the encoding bits i_1i_2 of all measurement outcomes, $|+^e+^e\rangle$, $|+^e-^e\rangle$, $|-^e+^e\rangle$, and $|-^e-^e\rangle$, of atomic ensembles E_3 and E_4, where σ_{z,B_j} denotes a Pauli phase flip of photon B_j.

III. TELEPORTATION OF A QUQUART PHOTONIC SYSTEM TO ANOTHER SYSTEM

A. Teleportation of two photons with only polarization DOF

With the two-photon hyperentanglement $|\mathrm{EPR}_h\rangle$ shown in Eq. (10), Alice can teleport a ququart system of two photons A_1 and A_2 in state

$$|\phi\rangle=\alpha_0|HH\rangle+\alpha_1|HV\rangle+\alpha_2|VH\rangle+\alpha_3|VV\rangle \quad (25)$$

to Bob's remote ququart system. Figure 4 presents a detailed circuit for Alice to transfer two polarized photons to two atomic ensembles. E_1 is an auxiliary atomic ensemble in state $|\bar{0}\rangle$. The subcircuit $\mathrm{CNOT}_4=|H\rangle\langle H|(|\bar{0}\rangle\langle\bar{1}|+|\bar{1}\rangle\langle\bar{0}|)+|V\rangle\langle V|(|\bar{0}\rangle\langle\bar{0}|+|\bar{1}\rangle\langle\bar{1}|)$, consisting of W_1, W_2, PS_1, delay, and minor, is used to implement a controlled flip atomic ensemble E_1 conditional on photon A_1 (the controlling term is $|H\rangle$). Here, after W_1 is performed on atomic ensemble E_1, Alice lets photon A_1 pass through PS_1 and its transmitted pulse go through cavity Cy_1 with atomic ensemble E_1; its output pulse is recombined with the corresponding reflected pulse at PS_1 simultaneously. Then W_2 is performed on atomic ensemble E_1. Thus Alice obtains $(\mathrm{CNOT}_4|\phi\rangle_{12}|\bar{0}\rangle_{E_1})$.

Similarly, another subcircuit, CNOT_4, is used to implement a CNOT gate on photon A_2 and auxiliary atomic ensemble E_2

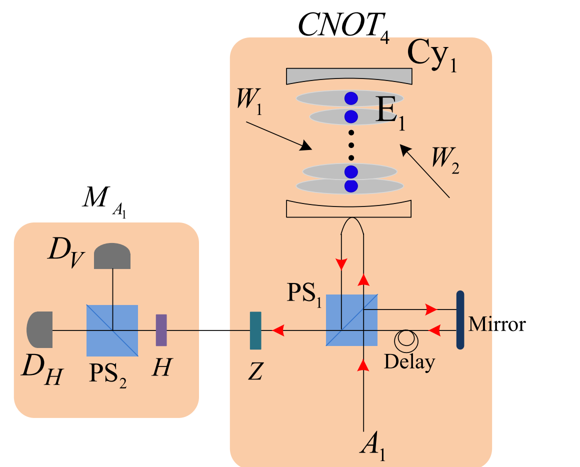

FIG. 4. Schematic teleportation of a two-photon system with a two-photon hyperentanglement. H, Z, PS_j, and W_j are defined in the Fig. 3.

in state $|\bar{0}\rangle$. Thus Alice obtains

$$\mathrm{CNOT}_4(A_2,E_2)[\mathrm{CNOT}_4(A_1,E_1)(|\phi\rangle_{A_1A_2}|\bar{0}\rangle_{E_1}|\bar{0}\rangle_{E_2})]=(\alpha_0|HH\rangle|\overline{11}\rangle+\alpha_1|HV\rangle|\overline{10}\rangle+\alpha_2|VH\rangle|\overline{01}\rangle+\alpha_3|VV\rangle|\overline{00}\rangle)_{A_1A_2,E_1E_2}. \quad (26)$$

By measuring photons A_1 and A_2 under the basis $\{|\pm^p\rangle:=(|H\rangle\pm|V\rangle)/\sqrt{2}\}$, Alice can swap a ququart system of two photons for a ququart system of two atomic ensembles defined in Eq. (22). Here, the recovery operations are defined by $(\sigma_{z,E_1}^e)^{i_1}(\sigma_{z,E_2}^e)^{i_2}\sigma_{x,E_1}^e\sigma_{x,E_2}^e$ dependent on the encoding bits i_1i_2 of all measurement outcomes, $|+^p+^p\rangle_{A_1A_2}$, $|+^p-^p\rangle_{A_1A_2}$, $|-^p+^p\rangle_{A_1A_2}$, and $|-^p-^p\rangle_{A_1A_2}$, where σ_{z,E_j} and σ_{x,E_j} denote Pauli phase flip and Pauli flip of atomic ensemble E_j, respectively. The followed quantum teleportations are the same as the circuits shown in the Fig. 3. Thus, Alice can teleport a ququart system of two photons to a ququart system of two atomic ensembles, two polarized photons, or one photon with two DOFs or a hybrid system of photons and ensemble up to special requirements of Bob.

B. Teleportation of one photon with two DOFs

Similarly to the teleportation of one photon with two DOFs [38], suppose that Alice wants to teleport a four-dimensional system of photon A' in state

$$|\phi\rangle_{A'}=(\alpha_0|Hd_1\rangle+\alpha_1|Hd_2\rangle+\alpha_2|Vd_1\rangle+\alpha_3|Vd_2\rangle)_{A'}. \quad (27)$$

She may use the swapping circuit, $\mathrm{Swapping}_1$ (consisting of CNOT gates CNOT_1 and CNOT_2), shown in Fig. 3 to swap her photonic system to a two-atomic-ensemble system defined in Eq. (22). Then the followed teleportations are the same as the circuits shown in Fig. 3. This means that Alice can teleport a four-dimensional photonic system with two DOFs to a two-atomic-ensemble system, a two-polarized-photon system, another photon with two DOFs, or a hybrid system of photons and ensemble up to Bob's requirements.

C. Teleportation of a hybrid two-qubit system

Consider a four-dimensional system of one photon A_1 and one atomic ensemble E_1 in state

$$|\phi\rangle_{A_1E_1}=(\alpha_0|H\bar{0}\rangle+\alpha_1|H\bar{1}\rangle+\alpha_2|V\bar{0}\rangle+\alpha_3|V\bar{1}\rangle)_{A_1E_1}. \quad (28)$$

With the photonic entanglement $|\text{EPR}_h\rangle$ in Eq. (11), this hybrid system can be teleported to a remote ququart system. In detail, Alice first performs a subcircuit CNOT_2 shown in Fig. 3 on photon A_1 and an auxiliary ensemble E_2 in state $|\bar{0}\rangle$. She gets $\text{CNOT}_2(A_1,E_2)(|\phi\rangle_{A_1}|\bar{0}\rangle_{E_2})$. By disentangling photon A_1 under the basis $\{|\pm^p\rangle\}$, Alice can faithfully get a four-dimensional system of two atomic ensembles in Eq. (22).

Then, from Fig. 3, Alice can teleport a ququart system of two atomic ensembles to a remote ququart system. Especially, Bob may get another hybrid system, i.e., a polarized photon is teleported to a polarized photon, while an atomic ensemble is teleported to an atomic ensemble. This can be realized from state $|\Theta_4\rangle$ shown in Eq. (22) by swapping atomic ensemble E_1 with polarized photon B' in state $|H\rangle$. This special scheme is useful for quantum computation based on large-scale quantum networks, where photons and atomic ensembles are used as different units up to their superiorities [9,47].

IV. TELEPORTATION OF A RESTRICTED QUQUART SYSTEM

If Alice knows the transferred state, the economical RSP scheme may be used to complete the same task [13,14]. The new scheme costs only half the classical communication cost of teleportation and requires a reduced local quantum measurement [see Fig. 1(b)]. Its primitive operation is Alice's local preparation of the transfer information in her subsystem of the shared entanglement. Thus the implementation complexity may be reduced because quantum joint measurements of multiple particles are difficult to perform faithfully in physical experiments. In this section, motivated by the superteleportation [39], a restricted four-dimensional system with two types of information (phases and real probability amplitudes) is teleported using the hyperentanglement in Eq. (11).

A. Teleportation of a ququart system with phase information

Assume that Alice wants to teleport phases $(1,e^{i\theta_1},e^{i\theta_2},e^{i\theta_3})$ of a four-dimensional system to Bob. Differently from the quantum teleportation in Secs. II and III, Alice does not need to make use of an input system of atomic ensembles or photons. These phases are prepared on their shared hyperentanglement $|\text{EPR}_h\rangle$ by local operations, as shown in Fig. 1(b). The followed measurement is performed on a ququart system, not a four-qubit system of general teleportation [12]. Hence, Alice only needs two bits to help Bob prepare one system with these phases. In detail, Alice first gets a new hyperentangled state,

$$|\overline{\text{EPR}}_h\rangle = \frac{1}{2}(|Hd_1\rangle|Hd_1\rangle + e^{i\theta_1}|Hd_2\rangle|Hd_2\rangle + e^{i\theta_2}|Vd_1\rangle|Vd_1\rangle + e^{i\theta_3}|Vd_2\rangle|Vd_2\rangle)_{AB}; \quad (29)$$

i.e., the transferred phase information should be faithfully swapped to the shared entanglement $|\text{EPR}_h\rangle$ by local operations. This can be realized by Alice, who knows these phases.

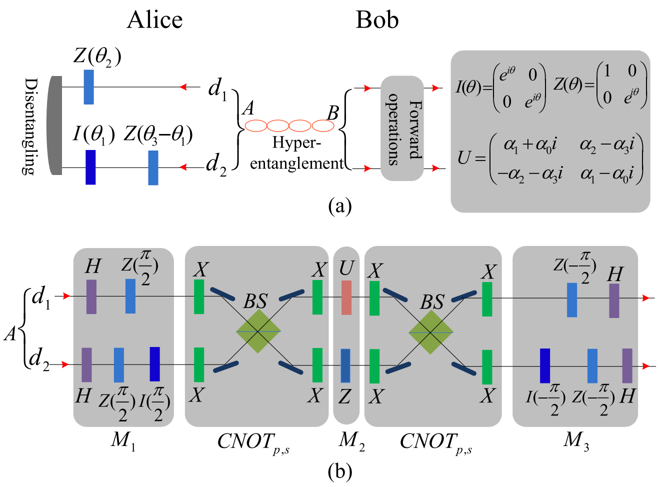

FIG. 5. (a) Schematic of the teleportation of phase information. (b) Schematic of the local swapping of real probability amplitudes with a two-photon hyperentanglement. H, X, Z, and BS are defined in the caption to Fig. 3. PS is used to transform $|H\rangle$ and reflect $|V\rangle$. U is a general unitary matrix which may be realized with rotations along the z axis and y axis of the Pauli sphere. The disentangling operation for photon A is shown in Fig. 3 and corresponding feed-forward operations on photon B are reported in Table I.

The detailed circuit is shown in Fig. 5(a). The evolution is defined as follows:

$$\begin{aligned}|\text{EPR}_p\rangle_{AB} &\xrightarrow{I(\theta_1)} \tfrac{1}{2}(|Hd_1\rangle|Hd_1\rangle + e^{i\theta_1}|Hd_2\rangle|Hd_2\rangle \\ &\quad + |Vd_1\rangle|Vd_1\rangle + e^{i\theta_1}|Vd_2\rangle|Vd_2\rangle)_{AB} \\ &\xrightarrow{Z(\theta_2)} \tfrac{1}{2}(|Hd_1\rangle|Hd_1\rangle + e^{i\theta_1}|Hd_2\rangle|Hd_2\rangle \\ &\quad + e^{i\theta_2}|Vd_1\rangle|Vd_1\rangle + e^{i\theta_1}|Vd_2\rangle|Vd_2\rangle)_{AB} \\ &\xrightarrow{Z(\theta_3-\theta_1)} \tfrac{1}{2}(|Hd_1\rangle|Hd_1\rangle + e^{i\theta_1}|Hd_2\rangle|Hd_2\rangle \\ &\quad + e^{i\theta_2}|Vd_1\rangle|Vd_1\rangle + e^{i\theta_3}|Vd_2\rangle|Vd_2\rangle)_{AB}.\end{aligned} \quad (30)$$

Now, by using the disentangling operation of photon A shown in Fig. 3, Alice can collapse this joint system $|\overline{\text{EPR}}_h\rangle$ into

$$|\phi\rangle_B = \tfrac{1}{2}(|Hd_1\rangle + e^{i\theta_1}|Hd_2\rangle + e^{i\theta_2}|Vd_1\rangle + e^{i\theta_3}|Vd_2\rangle) \quad (31)$$

after performing proper feed-forward operations listed in Table I by Bob, where two bits are used to encode her detecting results and then transmitted to Bob through the classical channel. The following local swapping may be completed by Bob as those shown in Fig. 3. Hence, Alice can teleport a four-dimensional system with phase information to Bob's different systems using two bits.

B. Teleportation of a ququart system with probability amplitudes

Assume that Alice wants to teleport real probability amplitudes $(\alpha_0,\alpha_1,\alpha_2,\alpha_3)$ of a ququart system to Bob. From Figs. 1(b) and 5(b), Alice and Bob can obtain a new hyperentangled state defined by

$$|\widetilde{\mathrm{EPR}}_h\rangle_{AB} = \alpha_0|Hd_1\rangle|Hd_1\rangle + \alpha_1|Vd_1\rangle|Vd_1\rangle + \alpha_2|Hd_2\rangle|Hd_2\rangle + \alpha_3|Vd_2\rangle|Vd_2\rangle. \quad (32)$$

Here, the subcircuit consisted of two wave plates Xs, one BS, and two other wave plates Xs used to realize a CNOT gate $\mathrm{CNOT}_{p,s}$ on the polarization DOF and spatial DOF of photon A; i.e, $\mathrm{CNOT}_{p,s} = |H\rangle\langle H|(|d_1\rangle\langle d_2| + |d_2\rangle\langle d_1|) + |V\rangle\langle V|(|d_1\rangle\langle d_1| + |d_2\rangle\langle d_2|)$. The detailed evolution is defined as follows:

$$\begin{aligned}|\mathrm{EPR}\rangle_{AB} &\xrightarrow{M_1} \tfrac{1}{2}(|Hd_1\rangle|+^p d_1\rangle + i|Vd_1\rangle|-^p d_1\rangle \\ &\quad + i|Hd_2\rangle|+^p d_2\rangle + |Vd_2\rangle|-^p d_2\rangle)_{AB} \\ &\xrightarrow[\text{eachmode}]{X,\mathrm{PS},X} \tfrac{1}{2}(i|Hd_1\rangle|-^p d_1\rangle + |Vd_1\rangle|+^p d_1\rangle \\ &\quad + |Hd_2\rangle|-^p d_2\rangle + i|Vd_2\rangle|+^p d_2\rangle)_{AB} \\ &\xrightarrow{M_2} \tfrac{1}{2}(i\beta_0|Hd_1\rangle|-^p d_1\rangle + i\beta_1'|Vd_1\rangle|-^p d_1\rangle \\ &\quad - \beta_1|Hd_1\rangle|+^p d_1\rangle + \beta_0'|Vd_1\rangle|+^p d_1\rangle \\ &\quad + i|Hd_2\rangle|-^p d_1\rangle - i|Vd_2\rangle|+^p d_2\rangle)_{AB},\end{aligned} \quad (33)$$

where $\beta_0 = \alpha_1 + \alpha_0 i, \beta_0' = \alpha_1 - \alpha_0 i$, $\beta_1 = \alpha_2 + \alpha_3 i$, and $\beta_1' = \alpha_2 - \alpha_3 i$.

Then, by using the other CNOT gate $\mathrm{CNOT}_{p,s}$ on photon A, the shared entanglement in Eq. (33) will be changed to

$$\begin{aligned}&\frac{1}{2}(i\beta_0|Hd_1\rangle|-^p d_1\rangle + i\beta_1'|Vd_2\rangle|-^p d_1\rangle \\ &\quad - \beta_1|Hd_1\rangle|+^p d_1\rangle + \beta_1'|Vd_2\rangle|+^p d_1\rangle \\ &\quad + i|Hd_2\rangle|-^p d_1\rangle - i|Vd_1\rangle|+^p d_2\rangle)_{AB} \\ &\xrightarrow{M_3} i(\alpha_0|Hd_1\rangle|Hd_1\rangle + \alpha_1|Vd_1\rangle|Vd_1\rangle \\ &\quad \alpha_2|Hd_2\rangle|Hd_2\rangle + \alpha_3|Vd_2\rangle|Vd_2\rangle)_{AB} \\ &= |\widetilde{\mathrm{EPR}}_h\rangle.\end{aligned} \quad (34)$$

Now, by disentangling photon A as shown in Fig. 3, Alice can collapse the joint system $|\widetilde{\mathrm{EPR}}_h\rangle$ into

$$|\phi'\rangle_B = \alpha_0|Hd_1\rangle + \alpha_1|Vd_1\rangle + \alpha_2|Hd_2\rangle + \alpha_3|Vd_2\rangle \quad (35)$$

using only two bits to encode her detecting results. The following local swapping may be completed by Bob as those shown in Fig. 3. Hence, Alice can teleport a four-dimensional system with real probability amplitudes to Bob's different systems using two bits.

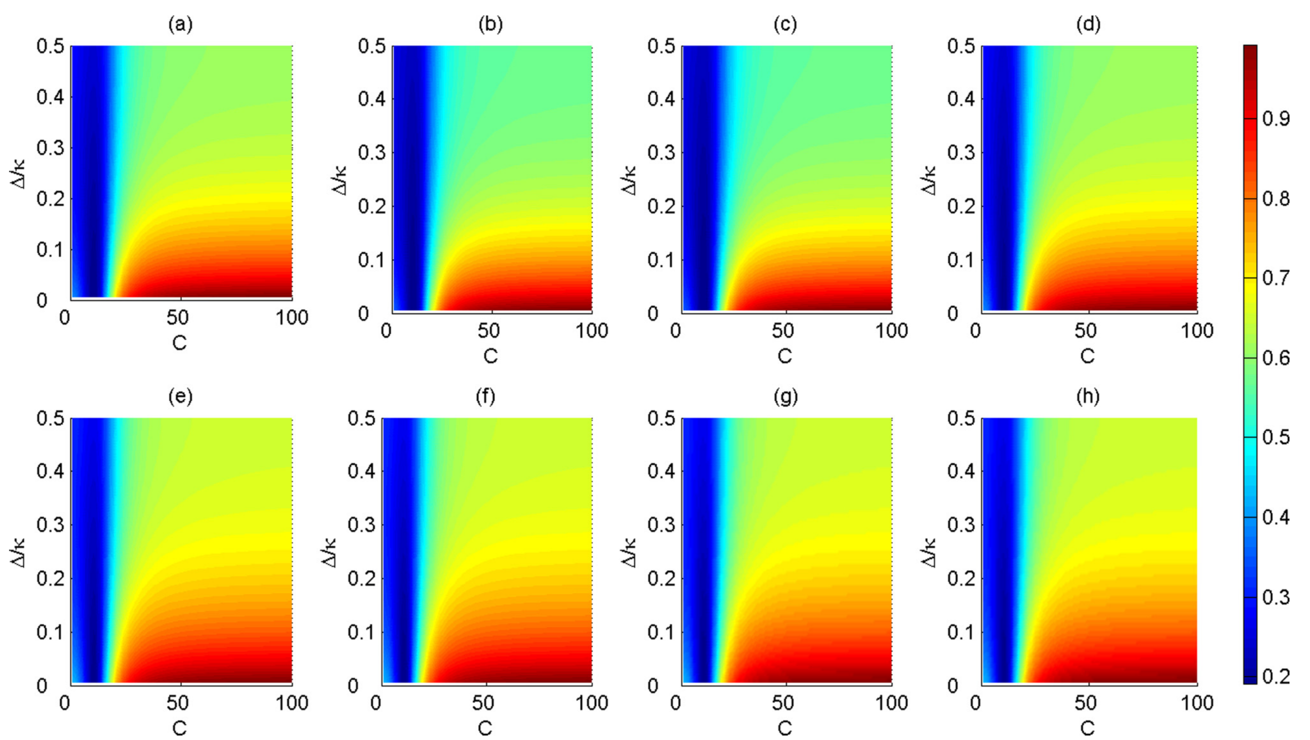

FIG. 6. Fidelities of the present eight teleportation schemes via the cooperativity C and relative detuning Δ/κ. (a) Teleportation of a two-atomic-ensemble system to another two-atomic-ensemble system. (b) Teleportation of a two-polarized-photon system to another two-polarized-photon system. (c) Teleportation of a two-polarized-photon system to one photon with two DOFs. (d) Teleportation of a hybrid system of one polarized photon and one atomic ensemble to another hybrid system. (e) Teleportation of a two-atomic-ensemble system with only phase information to another similar system. (f) Teleportation of a two-polarized-photon system with only phase information to a similar system. (g) Teleportation of a two-atomic-ensemble system with only real probability amplitudes to a similar system. (h) Teleportation of a two-polarized-photon system with only real probability amplitudes to a similar system.

V. FIDELITIES AND EFFICIENCIES OF OUR TELEPORTATIONS

The selection rules of an input pulse interacting with an atomic ensemble shown in Eq. (10) play core roles in the present teleportations. Under the resonance conditions $\Delta = \Delta_{\omega_c} = 0$, $r_0 \rightarrow -1$ and $r \rightarrow 1$ may be easily followed when the cooperativity parameter C is large enough. In this case, all of our teleportations are deterministic and faithful. The ideal coupling strength between the atomic ensemble and the optical resonator may be diminished due to experimental effects [9], which may reduce the fidelities and efficiencies. Fortunately, in recent experiments [8,9,43,45–49] on composite systems, it is possible to retain the excellent properties of atomic ensembles. The reduced efficiencies and fidelities are evaluated via the cooperativity C and the relative detuning Δ/κ. The fidelity is defined by $F = \int \langle \Psi_i | \rho_e | \Psi_i \rangle$, where ρ_e is the density matrix of the experimental state, whereas $|\Psi_i\rangle$ denotes the ideal state of the quantum system after teleportation. The efficiency E is defined by the probability of the photons being detected after the teleportation derived from the probabilistic reflection of the photons from the cavity [51]. From an experimental controlled-phase gate

$$\overline{\mathrm{CZ}} = |H\rangle\langle H|(r_0|\bar{0}\rangle\langle\bar{0}| + r|\bar{1}\rangle\langle\bar{1}|), \quad (36)$$

the fidelities and efficiencies or our teleportations are evaluated in Figs. 6 and 7, respectively. In general, the cooperativity and relative detunings may greatly affect the efficiency and fidelity of our teleportations. As those shown in Figs. 6 and 7, a high efficiency and fidelity may be achieved even in a relatively weak cooperativity if the relative detuning approaches the resonance. From a recent experiment [58] which provided the datum $(\kappa,\gamma)/2\pi \approx (53,3.0)$ MHz and the cooperativity $C > 22$, our average fidelities are greater than 94.5%, whereas the efficiencies are greater than 75.4% for the relative detuning $\Delta/\kappa = 0.2$ or greater than 97.5% and 87.6%, respectively, for $\Delta/\kappa = 0.1$. This shows that our teleportations are feasible with current technologies.

To complete our teleportations, although the perfect one-sided cavity, consisting of an ideal mirror with complete reflection and a partially reflective mirror, remains challenging, an approximate one-sided cavity may be used [47,48,59]. In actual implementations, some experimental factors, such as the detector's efficiency, the decay of the radiation to noncavity modes, and the impurities of single-photon sources, may induce various errors which will affect the efficiencies and fidelities of our teleportations. If a heralded single-photon source with probabilistic correlated photon generation is applied, the pair production levels should be averaged to avoid producing multiple pairs, for improving the impurity [60]. The experimental efficiency of the input-output process is also dependent on the photon loss deriving from the cavity mirror scattering and absorption and the nonunit efficiency of detectors. Fortunately, the photon loss can be picked out according to the response of the detectors. An additional photon filtering mechanism may be used to make up for the

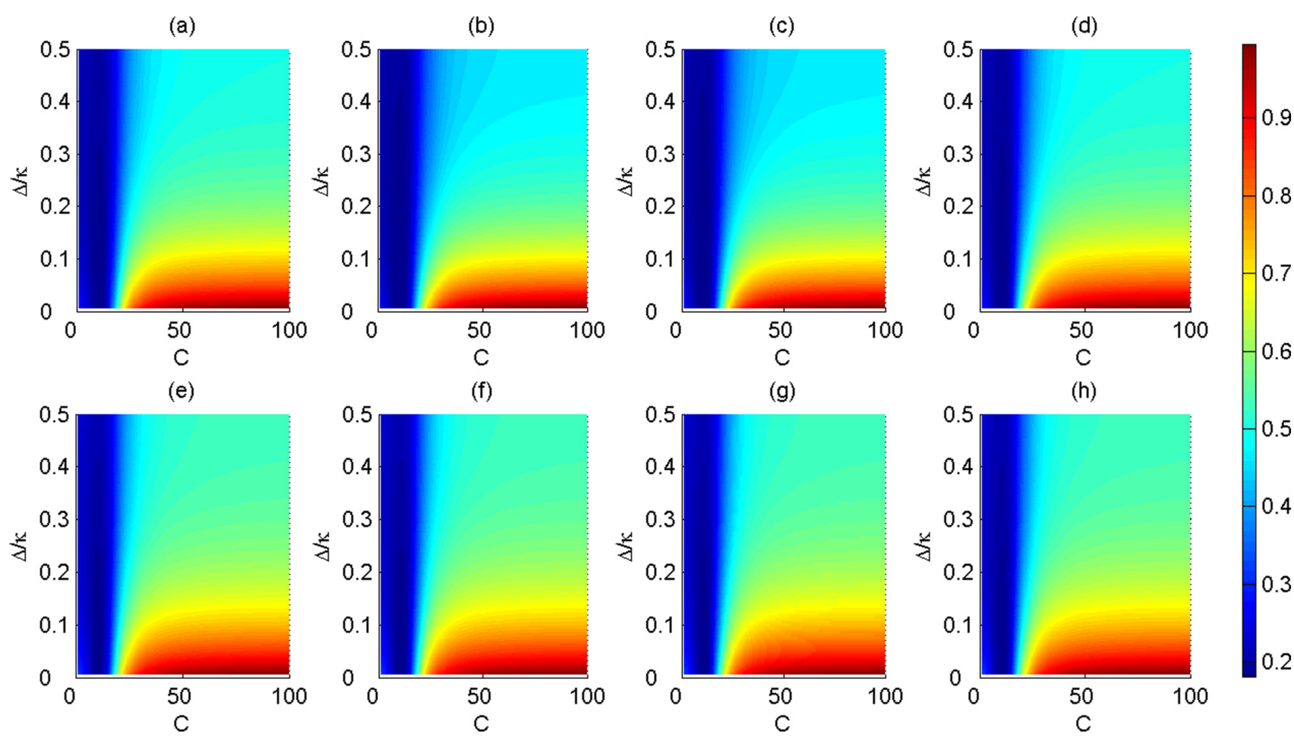

FIG. 7. Efficiencies of the present eight schemes via the cooperativity C and the relative detuning Δ/κ. Here, the scaled detuning $\gamma = 0.057\kappa$ [58]. (a) Teleportation of a two-atomic-ensemble system to another two-atomic-ensemble system. (b) Teleportation of a two-polarized-photon system to another two-polarized-photon system. (c) Teleportation of a two-polarized-photon system to one photon with two DOFs. (d) Teleportation of a hybrid system of one polarized photon and one atomic ensemble to another hybrid system. (e) Teleportation of a two-atomic-ensemble system with only phase information to another similar system. (f) Teleportation of a two-polarized-photon system with only phase information to a similar system. (g) Teleportation of a two-atomic-ensemble system with only real probability amplitudes to a similar system. (h) Teleportation of a two-polarized-photon system with only real probability amplitudes to a similar system.

photon loss, for faithful optical selection [61]. The input-output process is also affected by the photon pulse duration, which is larger than the scale of $1/\kappa$. The temporal mode of the output pulse is the same as that of the input pulse, which will lead to a faithful input-output process if the photon pulse duration T satisfies $T \gg \kappa/1$ [44]. Of course, experimental linear-optical elements also affect the success probability and fidelity [38,39].

VI. DISCUSSION AND CONCLUSION

Efficient long-distance quantum teleportation is crucial for quantum communication and quantum networking schemes [2]. The present teleportations of ququart or two-qubit systems can be easily extended to hybrid teleportations of general states. In fact, with tedious computations, by making use of n copies of two-photon hyperentanglement $|\text{EPR}_h\rangle$, all combinations of $2n$ number of photons, atomic ensembles, or a hybrid system of photons and atomic ensembles can be faithfully teleported to different systems up to the special requirements of e receiver. This means that photons and atomic ensembles may be arbitrarily hybrid for various applications. Thus our present teleportations of ququarts or two-qubit systems are primitive architectures for hybrid system applications based on a simple module comprising an optical cavity containing a number of atoms [40]. As one example, by easy combination with other photons, the bipartite hyperentanglement $|\text{EPR}_h\rangle$ can be extended to the multipartite GHZ-type hyperentanglement $|\text{GHZ}_p\rangle \otimes |\text{GHZ}_s\rangle$ assisted by the controlled phase flip in Eq. (10), where $|\text{GHZ}_p\rangle := \frac{1}{\sqrt{2}}(|H\dots H\rangle + |V\dots V\rangle)_{A_1\dots A_n}$ and $|\text{GHZ}_s\rangle := \frac{1}{\sqrt{2}}(|d_1\dots d_1\rangle + |d_2\dots d_2\rangle)_{A_1\dots A_n}$. These multipartite hyperentanglements may be used to construct a quantum teleportation network [62,63] and quantum secret sharing [64]. As another example, consider a scenario in which remote quantum attackers want to collaboratively ruin a public-key cryptosystem based on the practical difficulty of factoring the product of two large prime numbers p and q [65]. Each attacker can initialize its own register as $\sum_{i=0}^{N_j} |\bar{i}\rangle|\bar{0}\rangle$ up to its quantum ability. From the traditional Shor's decomposing algorithm [66], the followed exponential evolutions with module pq and quantum Fourier transformations can be collaboratively implemented using the teleportation-based quantum distribution computation [67–71]. Here, hybrid systems can be applied to satisfy different physical requirements of remote attackers. Thus the joint decomposition can be realized remotely and faithfully if hybrid teleportations are used. Moreover, the possible block implementation of quantum Fourier transformation may reduce the experimental complexity of this distributive attack; this will be investigated in future.

Compared with the recent experimental teleportation of a single photon with spin and orbital angular momentum DOFs [38] to another, our schemes can teleport a single photon with polarization and spatial DOFs to another system. The main difference is that the final system in our schemes may be adapted to all combinations of photons or atomic ensembles if Bob has special goals. Especially, a hybrid system can be teleported to another hybrid system, which may be useful for hybrid quantum information processing [2,47–49] by making use of different properties of photons and atomic ensembles. This is also different from the previous hybrid teleportation [37] of a polarized photon to the angular momentum of another photon. If the transferred information of a general ququart system is restricted, a recent superdense teleportation [39] has teleported a ququart photon with phases to another incurring only half the traditional transferring cost [12]. This scheme has been extended to teleportation of general ququart systems including all combinations of photons and atomic ensembles to different systems in this paper. Furthermore, similar economical schemes are proposed to transfer general ququart systems with real probability amplitudes to different systems. Combined with single-qubit operations, these teleportations can be used to realize remote CNOT gates using hyper-entanglement teleportation [67–71], in order to complete remote quantum tasks. Of course, all the schemes are completed without the joint measurement or Bell-state measurement [72]. Different from teleporting a single-photon with multiple degrees of freedom with the help of hyperentanglement Bell state analysis [73], our consideration in this paper is to show nontrivial applications of hyperentanglement in hybrid teleportations or remote quantum tasks. And the present teleportations can be easily extended to hybrid teleportations of general states with tedious computations.

In conclusion, we have investigated the possibility of quantum teleportation of a ququart system assisted by an atomic ensemble. All combinations of photons or atomic ensembles can be faithfully transferred to different ququart systems. If the transferred information is restricted, we have also teleported general ququart systems with only phase information to different systems. With this restriction, the classical communication costs are maintained and the quantum operation complexity may be reduced. Furthermore, similar economical schemes are proposed to transfer general ququart systems with real probability amplitudes to different systems. Our results are expected to be suitable for large-scale quantum application in complex quantum systems.

ACKNOWLEDGMENTS

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[1] S. J. van Enk, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **78**, 4293 (1997).

[2] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **81**, 5932 (1998).

[3] L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature **414**, 413 (2001).
[4] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).
[5] S. Ritter, C. Nölleke, C. Hahn, A. Reiserer, A. Neuzner, M. Uphoff, M. Mücke, E. Figueroa, J. Bochmann, and G. Rempe, Nature **484**, 195 (2012).
[6] D. Aghamalyan and Y. Malakyan, Phys. Rev. A **84**, 042305 (2011).
[7] Z.-B. Chen, B. Zhao, Y.-A. Chen, J. Schmiedmayer, and J.-W. Pan, Phys. Rev. A **76**, 022329 (2007).
[8] C. Simon, H. de Riedmatten, M. Afzelius, N. Sangouard, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **98**, 190503 (2007).
[9] N. Sangouard, C. Simon, H. de Riedmatten, and N. Gisin, Rev. Mod. Phys. **83**, 33 (2011).
[10] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996).
[11] J.-W. Pan, C. Simon, Č. Brukner, and A. Zeilinger, Nature **410**, 1067 (2001).
[12] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[13] C. H. Bennett, D. P. DiVincenzo, P. W. Shor, J. A. Smolin, B. M. Terhal, and W. K. Wootters, Phys. Rev. Lett. **87**, 077902 (2001).
[14] D. W. Berry and B. C. Sanders, Phys. Rev. Lett. **90**, 057901 (2003).
[15] B. Zeng and P. Zhang, Phys. Rev. A **65**, 022316 (2002).
[16] M.-X. Luo, Y. Deng, X. B. Chen, and Y. X. Yang, Quant. Info. Proc. **12**, 279 (2013).
[17] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature **390**, 575 (1997).
[18] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[19] I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, and N. Gisin, Nature **421**, 509 (2003).
[20] S. Takeda, T. Mizuta, M. Fuwa, P. van Loock, and A. Furusawa, Nature **500**, 315 (2013).
[21] C. Nölleke, A. Neuzner, A. Reiserer, C. Hahn, G. Rempe, and S. Ritter, Phys. Rev. Lett. **110**, 140403 (2013).
[22] M. Riebe, H. Häfner, C. F. Roos, W. Hänsel, J. Benhelm, G. P. T. Lancaster, T. W. Körber, C. Becher, F. Schmidt-Kaler, D. F. V. James, and R. Blatt, Nature **429**, 734 (2004).
[23] M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, and D. J. Wineland, Nature **429**, 737 (2004).
[24] L. Steffen, Y. Salathe, M. Oppliger, P. Kurpiers, M. Baur, C. Lang, C. Eichler, G. Puebla-Hellmann, A. Fedorov, and A. Wallraff, Nature **500**, 319 (2013).
[25] H. Krauter, D. Salart, C. A. Muschik, J. M. Petersen, H. Shen, T. Fernholz, and E. S. Polzik, Nat. Phys. **9**, 400 (2013).
[26] W. Pfaff, B. J. Hensen, H. Bernien, S. B. van Dam, M. S. Blok, T. H. Taminiau, M. J. Tiggelman, R. N. Schouten, M. Markham, D. J. Twitchen, and R. Hanson, Science **345**, 532 (2014).
[27] J. F. Sherson, H. Krauter, R. K. Olsson, B. Julsgaard, K. Hammerer, I. Cirac, and E. S. Polzik, Nature **443**, 557 (2006).
[28] F. Bussiéres, C. Clausen, A. Tiranov, B. Korzh, V. B. Verma, S. W. Nam, F. Marsili, A. Ferrier, P. Goldner, H. Herrmann, C. Silberhorn, W. Sohler, M. Afzelius, and N. Gisin, Nature Photon. **8**, 775 (2014).
[29] Q. Zhang, A. Goebel, C. Wagenknecht, Y.-A. Chen, B. Zhao, T. Yang, A. Mair, J. Schmiedmayer, and J.-W. Pan, Nat. Phys. **2**, 678 (2006).
[30] X.-S. Ma, T. Herbst, T. Scheidl, D. Wang, S. Kropatschek, W. Naylor, B. Wittmann, A. Mech, J. Kofler, E. Anisimova, V. Makarov, T. Jennewein, R. Ursin, and A. Zeilinger, Nature **489**, 269 (2012).
[31] N. A. Peters, J. T. Barreiro, M. E. Goggin, T.-C. Wei, and P. G. Kwiat, Phys. Rev. Lett. **94**, 150502 (2005).
[32] J. T. Barreiro, T.-C. Wei, and P. G. Kwiat, Phys. Rev. Lett. **105**, 030407 (2010).
[33] D. W. Leung and P. W. Shor, Phys. Rev. Lett. **90**, 127905 (2003).
[34] W.-T. Liu, W. Wu, B.-Q. Ou, P.-X. Chen, C.-Z. Li, and J.-M. Yuan, Phys. Rev. A **76**, 022308 (2007).
[35] W. Rosenfeld, S. Berner, J. Volz, M. Weber, and H. Weinfurter, Phys. Rev. Lett. **98**, 050504 (2007).
[36] B. Dakic, Y. O. Lipp, X. Ma, M. Ringbauer, S. Kropatschek, S. Barz, T. Paterek, V. Vedral, A. Zeilinger, C. Brukner, and P. Walther, Nat. Phys. **8**, 666 (2012).
[37] M. Erhard, H. Qassim, H. Mand, E. Karimi, and R. W. Boyd, Phys. Rev. A **92**, 022321 (2015).
[38] X.-L. Wang, X.-D. Cai, Z.-E. Su, M.-C. Chen, D. Wu, L. Li, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, Nature **518**, 516 (2015).
[39] T. M. Graham, H. J. Bernstein, T.-C. Wei, M. Junge, and P. G. Kwiat, Nature Commun. **6**, 7185 (2015).
[40] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).
[41] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, Phys. Rev. A **81**, 021804(R) (2010).
[42] X.-H. Bao, X.-F. Xuc, C.-M. Li, Z.-S. Yuan, C.-Y. Lu, and J.-W. Pan, Proc. Natl. Acad. Sci. USA **109**, 20169 (2012).
[43] B. He, A. V. Sharypov, J. Sheng, C. Simon, and M. Xiao, Phys. Rev. Lett. **112**, 133606 (2014).
[44] T. Li, G.-J. Yang, and F.-G. Deng, Opt. Express **22**, 23897 (2014).
[45] H. J. Kimble, Nature **453**, 1023 (2008).
[46] F. Mei, M. Feng, Y. F. Yu, and Z. M. Zhang, Phys. Rev. A **80**, 042319 (2009).
[47] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Rev. Mod. Phys. **85**, 553 (2013).
[48] T. G. Tiecke, J. D. Thompson, N. P. de Leon, L. R. Liu, V. Vuletić, and M. D. Lukin, Nature **508**, 241 (2014).
[49] M. Saffman, T. G. Walker, and K. Mølmer, Rev. Mod. Phys. **82**, 2313 (2010).
[50] E. Brion, L. H. Pedersen, M. Saffman, and K. Mølmer, Phys. Rev. Lett. **100**, 110506 (2008).
[51] H. J. Kimble, Phys. Scripta T **76**, 127 (1998).
[52] P. G. Kwiat, J. Mod. Opt. **44**, 2173 (1997).
[53] P. G. Kwiat and H. Weinfurter, Phys. Rev. A **58**, R2623 (1998).
[54] C. Simon and J.-W. Pan, Phys. Rev. Lett. **89**, 257901 (2002).
[55] C. Wang, F. G. Deng, Y. S. Li, X. S. Liu, and G. L. Long, Phys. Rev. A **71**, 044305 (2005).
[56] D. Bruss and C. Macchiavello, Phys. Rev. Lett. **88**, 127901 (2002).
[57] N. J. Cerf, M. Bourennane, A. Karlsson, and N. Gisin, Phys. Rev. Lett. **88**, 127902 (2002).
[58] Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, Nature **450**, 272 (2007).
[59] A. Reiserer, N. Kalb, G. Rempe, and S. Ritter, Nature **508**, 237 (2014).
[60] M. D. Eisaman, J. Fan, A. Migdall, and S. V. Polyakov, Rev. Sci. Instrum. **82**, 071101 (2011).

[61] Y. Li, L. Aolita, D. E. Chang, and L. C. Kwek, Phys. Rev. Lett. **109**, 160504 (2012).

[62] P. van Loock and S. L. Braunstein, Phys. Rev. Lett. **84**, 3482 (2000).

[63] H. Yonezawa, A. Takao, and A. Furusawa, Nature **431**, 430 (2004).

[64] S. Bandyopadhyay, Phys. Rev. A **62**, 012308 (2000).

[65] R. Rivest, A. Shamir, and L. Adleman, Commun. ACM **21**, 120 (1978).

[66] P. W. Shor, SIAM J. Comput. **26**, 1484 (1997).

[67] D. Gottesman and I. L. Chuang, Nature **402**, 390 (1999).

[68] Y. F. Huang, X. F. Ren, Y. S. Zhang, L. M. Duan, and G. C. Guo, Phys. Rev. Lett. **93**, 240501 (2004).

[69] H. F. Wang, A. D. Zhu, S. Zhang, and K. H. Yeon, Phys. Rev. A **87**, 062337 (2013).

[70] M. X. Luo, H. R. Li, and X. Wang, Quantum Inf. Comput. **15**, 1397 (2015).

[71] M.-X. Luo and X. Wang, Proc. R. Soc. A **471**, 0274 (2015).

[72] L. Ye and G. C. Guo, Phys. Rev. A **70**, 054303 (2004).

[73] Y.-B. Sheng, F.-G. Deng, and G. L. Long, Phys. Rev. A **82**, 032318 (2010).