Nearly deterministic preparation of the perfect W state with weak cross-Kerr nonlinearities

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Relying on weak cross-Kerr nonlinearities, we propose a nearly deterministic generation scheme of the three-photon polarization-entangled perfect W state which can be applied to the perfect teleportation of an unknown single-photon state and has robust entanglement against the loss of one photon of them. Three photons entangle together by virtue of the bus function of the coherent state serving as the intermediate among them. In the scheme, three processes are executed successively and two kinds of modules are inserted into the circuit, where the homodyne measurement and the photon number measurement are aptly performed. By means of classical feedforward techniques, single-photon unitary transformation operations are performed on the corresponding photons based on the obtained measurement outcomes, by which the generation efficiency of the perfect W state aims to nearly unity. Moreover, some currently available optical elements are applied in the generation process, which offer facilities for the practical implementation.

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I. INTRODUCTION

More charming than classical mechanics, quantum mechanics provides us with intriguing properties in the microsystems. Specially, quantum entanglement is the typical representative of attracting features embodying the mysterious quantum world different from the classical one. Quantum entanglement is on the core of quantum information, and focuses considerable attention on it and becomes an appealing issue due to its enchanting merits.

However, the classification on entangled states is still an open question. As we know, a Bell state [1,2] is the maximal class of two-qubit entangled states. Under stochastic local operations and classical communication (SLOCC), three-qubit entangled states can be classified into Greenberger-Horne-Zeilinger (GHZ) states and W states [3]. However, the study encounters troubles when researchers attempt to find the characterization and universal classification methods of multiqubit entangled states, and the multiqubit entanglement maintains its mystery thus far. Just because of this, considerable time and effort is devoted to study the properties of multiqubit entanglement, for example, Dicke states [4], decoherence-free states [5–8], cluster states [9–11], χ-type states [12,13], and other entangled states [14–18].

W states are a class of nonmaximally entangled quantum states, but they remain entanglement more robust than other entangled states, that is, two remaining qubits can be still entangled when anyone of three qubits is traced out while it is impossible for GHZ states. Exploiting it, quantum mechanics against local hidden variable theory can be tested [2,19], similarly to other entangled states. Moreover, W states are a necessary kind of physics resources for many important applications in quantum information processing, quantum teleportation and superdense coding [20–22], quantum deterministic secure communication and key distribution [23,24], quantum telecloning [25], and broadcasting [26], just to name a few.

By virtue of its characterization, teleportation employing the quantum channel of a general W state cannot be realized with the unity probability, whereas Agrawal *et al.* [27] proposed a kind of W states, by which perfect teleportation (successful probability of 100%) can be realized if the appropriate operations are performed.

The class of W states proposed by Agrawal *et al.* can be denoted as

$$|W_r\rangle = \frac{1}{\sqrt{2+2r}}(|HHV\rangle + \sqrt{r}e^{i\zeta}|HVH\rangle + \sqrt{r+1}e^{i\tau}|VHH\rangle), \tag{1}$$

where $|H\rangle$ ($|V\rangle$) denotes the horizontal (vertical) polarization state corresponding to the qubit $|0\rangle$ ($|1\rangle$), r denotes a real number, and ζ and τ refer to phases.

If we let $r = 1$ and $\zeta = \tau = 0$, we can obtain the perfect W state denoted as

$$|W\rangle = \frac{1}{2}|HHV\rangle + \frac{1}{2}|HVH\rangle + \frac{1}{\sqrt{2}}|VHH\rangle. \tag{2}$$

Utilizing this class of W states as the quantum channel, perfect teleportation can be realized [27,28]; quantum information can be split [29], shared [30], and securely transmitted [31]. Moreover, the W states can be changed to the maximally entangled W state by concentration [32,33].

Above all, the preparation of W states is the preliminary condition of these fascinating applications, so it is under intense research. Possessing the merits of higher transmission velocity, longer coherence time, easier manipulation, higher information density, and lower energy cost than electrons utilized as information carriers, the photons are the promising candidate for fulfilling the tasks of quantum information processing. Among the viable selections for quantum bit

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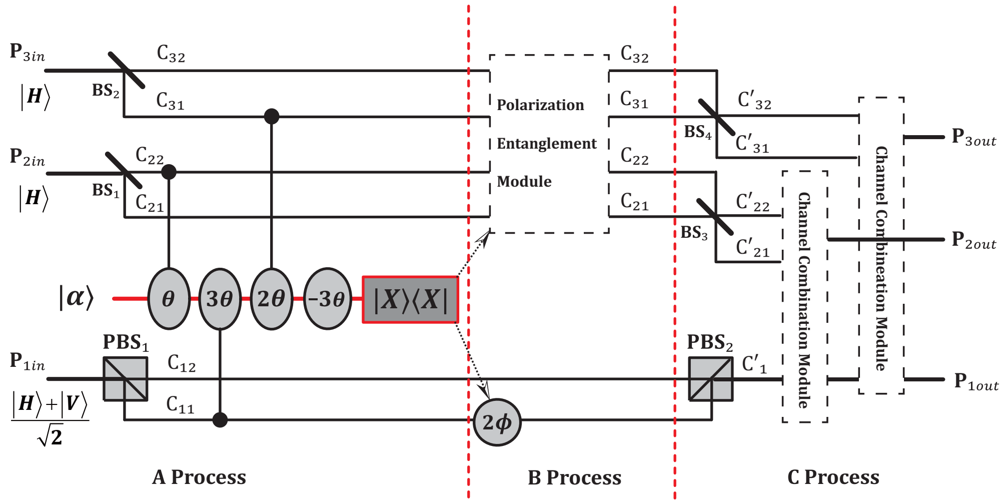

FIG. 1. Illustration plot for generating the polarization-entangled perfect W state. All the channels which the photons pass through are supposed to be equal length, and they can be modulated if not. The beam splitters (BS) have equal probabilities (50:50) of transmission and reflection. The polarization beam splitters (PBS) reflect $|V\rangle$ mode and transmit $|H\rangle$ mode. Influenced by cross-Kerr nonlinearities, the phase shift 3θ, the phase shift θ, and the phase shift 2θ of the coherent state $|\alpha\rangle$ will arise from the interaction with the photon 1, the photon 2, and the photon 3 which pass through the channel C_{11}, the channel C_{22}, and the channel C_{31}. The phase shift -3θ resulting from the phase modulation is carried out on the coherent state. Based on the measurement outcomes, a polarization entanglement operation and a phase shift $2\phi(x,k\theta)$ $(k=0,1,2,3)$ operation should be performed on the photons (2, 3) passing through the appropriate channels and the photon 1 passing through the channel C_{11}, respectively. Before three photons leave the circuit, six channels are combined into three channels by exploiting PBS_2 on the photon 1 and two channel combination operations on the photon 2 and the photon 3. The dotted line with arrow denotes classical feedforward. Three sections separated by two red dashed lines represent three processes. A: spatial entanglement process; B: polarization entanglement process; C: channel combination process.

carriers, the polarization modes of the photons are easily operated, so they are suitable for quantum bits.

Proposing the linear standard model, Knill *et al.* [34] achieved an important advance on the effective nonlinear interaction between the photons, which improves the disadvantageous situation where the meaningful and available quantum gate operations are very difficult to achieve at the level of single photon due to the very small photon-photon interaction [35,36]. However, the successful probability based on only the linear standard model is limited [37]. Fortunately, by virtue of cross-Kerr nonlinearities, the limitation on the probability for quantum information processing is broken down [38], so the high probability for fulfilling quantum information processing is expected.

In context, we consider the generation of the perfect W state with the help of weak cross-Kerr nonlinearities. Exploiting weak cross-Kerr nonlinearities, there are some schemes for generating entangled states, such as GHZ states [39,40], cluster states [41–45], Dicke states [46], χ-type states [47], and four-photon decoherence-free states [48]. Even so, the similar construction cannot be used to obtain W states with unity successful probability due to features of W states. So far, only a few probabilistic schemes for generating polarization-entangled W states are proposed [40,46,49,50]. With regard to the perfect W state, there is no optical scheme considering its generation with successful probability approaching unity.

In this paper, we limit our attention to the generation of the perfect W state denoted as Eq. (2). Based on weak cross-Kerr nonlinearities, a nearly deterministic generation scheme of the perfect W state with the polarization degree of freedom is proposed. First, we describe the generation process of the perfect W state in Sec. II. Secondly, we discuss the fidelity and feasibility on the practical implementation for preparing the perfect W state in Sec. III. Finally, we conclude our work with Summary in Sec. IV.

II. SCHEME FOR GENERATING THE THREE-PHOTON POLARIZATION-ENTANGLED PERFECT W STATE

In what follows, we present the scheme for generating the three-photon polarization-entangled perfect W state abided by the following three processes, which is also illustrated in Fig. 1.

A. Spatial entanglement process of three photons

Suppose the photons (1,2,3) to be in the state $\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)_1\otimes|H\rangle_2\otimes|H\rangle_3$, and let them enter into the circuit shown as Fig. 1 from the input ports.

The channel of the photon 1 is divided to the channels (C_{11},C_{12}) by a polarization beam splitter, PBS_1, due to the different polarization modes $(|V\rangle,|H\rangle)$. Passing through beam splitters, BS_1 and BS_2, which have the following function between two input modes (a,b) and two output modes (c,d): $a^\dagger\rightarrow(c^\dagger+d^\dagger)/\sqrt{2}, b^\dagger\rightarrow(c^\dagger-d^\dagger)/\sqrt{2}$, the photon 2 and the photon 3 enter into the channels (C_{21},C_{22}) and the channels (C_{31},C_{32}).

Undergoing cross-Kerr nonlinearities provided by Kerr media, the evolution process of three photons interacting with a coherent state $|\alpha\rangle$ can be expressed as

$$\frac{1}{2\sqrt{2}}(|H\rangle_1|C_{12}\rangle+|V\rangle_1|C_{11}\rangle)(|H\rangle_2|C_{21}\rangle+|H\rangle_2|C_{22}\rangle)(|H\rangle_3|C_{31}\rangle+|H\rangle_3|C_{32}\rangle)|\alpha\rangle \xrightarrow{\text{Kerr media}}$$
$$\frac{1}{2\sqrt{2}}|H\rangle_1|C_{12}\rangle|H\rangle_2|H\rangle_3(|C_{22}\rangle|C_{31}\rangle|\alpha\rangle+|C_{21}\rangle|C_{31}\rangle|\alpha e^{-i\theta}\rangle+|C_{22}\rangle|C_{32}\rangle|\alpha e^{-i2\theta}\rangle+|C_{21}\rangle|C_{32}\rangle|\alpha e^{-i3\theta}\rangle)$$
$$+\frac{1}{2\sqrt{2}}|V\rangle_1|C_{11}\rangle|H\rangle_2|H\rangle_3(|C_{21}\rangle|C_{32}\rangle|\alpha\rangle+|C_{22}\rangle|C_{32}\rangle|\alpha e^{i\theta}\rangle+|C_{21}\rangle|C_{31}\rangle|\alpha e^{i2\theta}\rangle+|C_{22}\rangle|C_{31}\rangle|\alpha e^{i3\theta}\rangle). \quad (3)$$

After the hybrid system leaves Kerr media, X measurement [38,51,52] is performed on the coherent state, and there presents four groups of measurement outcomes corresponding to four scenarios of phase shifts $(0,\pm\theta,\pm 2\theta,\pm 3\theta)$ denoted as Eq. (3). It needs to be noted that the phase shift $k\theta$ and the phase shift $-k\theta$ cannot be differentiated from each other when X measurement is performed, so three photons entangle together with the spatial modes in the four scenarios,

$$\xrightarrow{\text{Measurement}} \frac{1}{\sqrt{2}}(|H\rangle_1|C_{12}\rangle|H\rangle_2|H\rangle_3|C_{2m}\rangle|C_{3j}\rangle e^{i\phi(x,k\theta)}+|V\rangle_1|C_{11}\rangle|H\rangle_2|H\rangle_3|C_{2n}\rangle|C_{3i}\rangle e^{-i\phi(x,k\theta)}), \quad (4)$$

where $\phi(x,k\theta)=-\alpha\sin k\theta(x-2\alpha\cos k\theta)$, α denotes the amplitude of the coherent state $|\alpha\rangle$, and $m=2,n=1,i=2,j=1$ for $k=0$; $m=1,n=2,i=2,j=1$ for $k=1$; $m=2,n=1,i=1,j=2$ for $k=2$; $m=1,n=2,i=1,j=2$ for $k=3$.

If zero phase shift occurs, no phase modulation is necessary. According to the measurement outcomes, a phase shift $2\phi(x,k\theta),(k=1,2,3)$ operation should be performed on the photon 1 passing through the channel C_{11}, which is used to equalize the phases of the two superposition terms in the scenarios of three different nonzero phase shifts.

For simplifying description in the later processes, we take an example as the representative of four different scenarios of phase shifts. Provided that the X measurement outcome indicates zero phase shift $(k=0)$, the entangled state of three photons with spatial modes can be denoted as

$$\xrightarrow{\text{Zero phase shift}} \frac{1}{\sqrt{2}}(|H\rangle_1|C_{12}\rangle|H\rangle_2|H\rangle_3|C_{22}\rangle|C_{31}\rangle+|V\rangle_1|C_{11}\rangle|H\rangle_2|H\rangle_3|C_{21}\rangle|C_{32}\rangle). \quad (5)$$

As for the other three scenarios $(k=1,2,3)$, the lines which will enter into the polarization entanglement module need to be modulated, and the corresponding rules can be deduced from Eq. (4).

B. Polarization entanglement process of the photons (2,3)

Following the preceding example, if zero phase shift $(k=0)$ is witnessed by the X measurement, a polarization entanglement module illustrated in Fig. 2 is put into the channels $(C_{22},C_{31};C_{21},C_{32})$ of the photons (2,3) to entangle them with the polarization degree of freedom, where $m=2,n=1,i=2,j=1$.

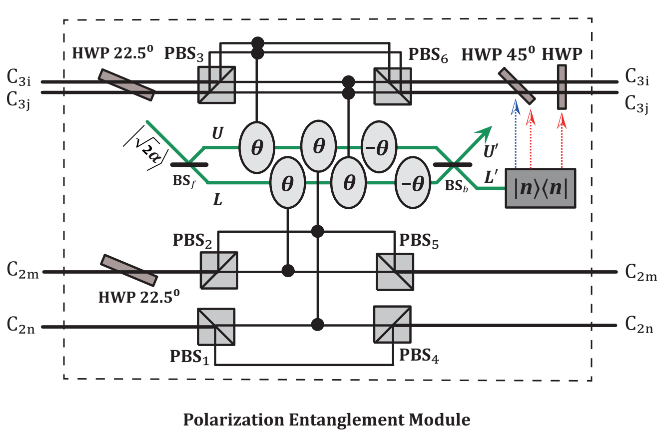

FIG. 2. Illustration plot for depicting a polarization entanglement module on the lines of the photon 2 and the photon 3. The symbols of (C_{2m},C_{2n}) and (C_{3i},C_{3j}) represent four channels which the photon 2 and the photon 3 potentially pass through, respectively, where the subscripts satisfy the conditions of $i,j,m,n=1,2$ $(i\neq j,m\neq n)$. A half wave plate (HWP) can be applied to perform a σ_z operation, while a HWP 22.5° and a HWP 45° function as a Hadamard transformation operation and a NOT (σ_x) gate operation.

Illustrated in Fig. 2, in the polarization entanglement module, photon number measurement with double cross-Kerr nonlinearities [53–55] is adopted. Half wave plates, HWP 22.5°s, are inserted into the input ports of the module, which function as Hadamard transformation operations to transform the states of the photon 2 and the photon 3 from $|H\rangle_{C_{22}}$ and $|H\rangle_{C_{31}(C_{32})}$ to $\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)_{C_{22}}$ and $\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)_{C_{31}(C_{32})}$. Affected by cross-Kerr nonlinearities, the vertical polarization mode of the photon 3 and the photon 2 via the channel C_{22} or the horizontal polarization mode of the photon 2 via the channel C_{21} will accumulate the phase shift θ on the upper coherent state $|\alpha\rangle_U$; the horizontal polarization mode of the photon 3 and the photon 2 via the channel C_{22} will accumulate the phase shift θ on the lower coherent state $|\alpha\rangle_L$.

As the consequence of the nonlinear interaction between photons and the coherent states, the state of whole system can be expressed as

$$\begin{aligned}&\frac{1}{2\sqrt{2}}|H\rangle_1|C_{12}\rangle[(|HV\rangle+|VH\rangle)_{23}|\alpha\rangle_U|\alpha\rangle_L\\&+|HH\rangle_{23}|\alpha e^{-i\theta}\rangle_U|\alpha e^{i\theta}\rangle_L\\&+|VV\rangle_{23}|\alpha e^{i\theta}\rangle_U|\alpha e^{-i\theta}\rangle_L]|C_{22}\rangle|C_{31}\rangle\\&+\frac{1}{2}|V\rangle_1|C_{11}\rangle(|HH\rangle_{23}|\alpha\rangle_U|\alpha\rangle_L\\&+|HV\rangle_{23}|\alpha e^{i\theta}\rangle_U|\alpha e^{-i\theta}\rangle_L)|C_{21}\rangle|C_{32}\rangle.\end{aligned} \quad (6)$$

Subsequently, the coherent states pass through BS_b, and the hybrid system is in the state as

$$\begin{aligned}
&\frac{1}{2\sqrt{2}}|H\rangle_1|C_{12}\rangle[(|HV\rangle+|VH\rangle)_{23}|\sqrt{2}\alpha\rangle_{U'}|0\rangle_{L'}\\
&+|HH\rangle_{23}|\sqrt{2}\alpha\cos\theta\rangle_{U'}|-\sqrt{2}i\alpha\sin\theta\rangle_{L'}\\
&+|VV\rangle_{23}|\sqrt{2}\alpha\cos\theta\rangle_{U'}|\sqrt{2}i\alpha\sin\theta\rangle_{L'}]|C_{22}\rangle|C_{31}\rangle\\
&+\frac{1}{2}|V\rangle_1|C_{11}\rangle(|HH\rangle_{23}|\sqrt{2}\alpha\rangle_{U'}|0\rangle_{L'}\\
&+|HV\rangle_{23}|\sqrt{2}\alpha\cos\theta\rangle_{U'}|\sqrt{2}i\alpha\sin\theta\rangle_{L'})|C_{21}\rangle|C_{32}\rangle. \quad (7)
\end{aligned}$$

By measuring the photon number of the coherent state exiting from the lower output port L' of BS_b, there exist three outcomes, that is, $n=0$, $n=$ even, and $n=$ odd.

If the outcome of measurement on the coherent state is $|0\rangle_{L'}$, no operation needs to be performed, and the state of the whole system is in the hyperentangled state with spatial modes and polarization modes, which can be denoted as

$$\begin{aligned}
&\xrightarrow{\text{Entanglement Module}}\frac{1}{2}|H\rangle_1|C_{12}\rangle(|H\rangle_2|C_{22}\rangle|V\rangle_3|C_{31}\rangle\\
&+|V\rangle_2|C_{22}\rangle|H\rangle_3|C_{31}\rangle)\\
&+\frac{1}{\sqrt{2}}|V\rangle_1|C_{11}\rangle|H\rangle_2|C_{21}\rangle|H\rangle_3|C_{32}\rangle. \quad (8)
\end{aligned}$$

Otherwise, if the measurement outcome witnesses an even number of photons, a σ_x operation should be performed on the photon 3, which can be fulfilled by a HWP 45°; if the measurement outcome points out an odd number of photons, a σ_y operation should be performed on the photon 3, which can be realized by the combination of a HWP 45° and a HWP.

So after the polarization entanglement operation on the photons (2,3), the state of the whole system will be collapsed into the entangled state shown in Eq. (8).

C. Channel combination process of three photons

The channel combination process includes the channel combination of the photon 1, the channel combination of the photon 2, and the channel combination of the photon 3, which combines six channels into three channels. In order to combine their channels, we let three photons pass through the optical elements of PBS_2, BS_3, and BS_4 to initiate the combination process.

1. Channel combination of the photon 1

After passing through PBS_2, the channels (C_{11}, C_{12}) of the photon 1 are combined into the channel C_1' ($P_{1\text{out}}$), that is the combination process of the photon 1.

2. Channel combination of the photon 2 and the photon 3

Due to the presence of BS_3 and BS_4, the photon 2 and the photon 3 leave the channels (C_{21},C_{22}) and the channels (C_{31},C_{32}) to the channels (C'_{21},C'_{22}) and the channels (C'_{31},C'_{32}) according to the following rules: $|C_{21}\rangle\rightarrow\frac{1}{\sqrt{2}}(|C'_{22}\rangle-|C'_{21}\rangle)$, $|C_{22}\rangle\rightarrow\frac{1}{\sqrt{2}}(|C'_{22}\rangle+|C'_{21}\rangle)$, $|C_{31}\rangle\rightarrow\frac{1}{\sqrt{2}}(|C'_{32}\rangle-|C'_{31}\rangle)$, and $|C_{32}\rangle\rightarrow\frac{1}{\sqrt{2}}(|C'_{32}\rangle+|C'_{31}\rangle)$.

Correspondingly, the state of three photons expressed as Eq. (8) is transformed to

$$\begin{aligned}
&\xrightarrow{PBS_2,\,BS_3,\,BS_4}\frac{1}{4}\{[|H\rangle_1|C_1'\rangle(|H\rangle_2|V\rangle_3+|V\rangle_2|H\rangle_3)\\
&+\sqrt{2}|V\rangle_1|C_1'\rangle|H\rangle_2|H\rangle_3]\otimes(|C'_{22}\rangle|C'_{32}\rangle-|C'_{21}\rangle|C'_{31}\rangle)\\
&+[|H\rangle_1|C_1'\rangle(|H\rangle_2|V\rangle_3+|V\rangle_2|H\rangle_3)\\
&-\sqrt{2}|V\rangle_1|C_1'\rangle|H\rangle_2|H\rangle_3]\otimes(|C'_{21}\rangle|C'_{32}\rangle-|C'_{22}\rangle|C'_{31}\rangle)\}. \quad (9)
\end{aligned}$$

With respect to four output ports (C'_{21},C'_{22}; C'_{31},C'_{32}) of two beam splitters (BS_3 and BS_4), if only one output port per beam splitter (e.g., C'_{22}; C'_{32}) is employed, the probability of the scheme will be reduced to 25% with the simple circuit [56]. If BS_3 and BS_4 are replaced with two single-way mirrors to perform coherent coupling (superposition) [47], which only allow one to transmit one photon from one side and reflect the other photon from the opposite side, the entangled state can be obtained deterministically at the terminal ports. Otherwise, there are another two methods to enhance the probability of the construction scheme. One method is that four output ports exist simultaneously on the terminal of the circuit and are detected by quantum nondemoliton measurement [57]. Assisted by proper classical feedforward, the successful probability of the scheme approaches unity [58–60]. The other method is to perform channel combination operations [61] to combine four channels into two channels of the photon 2 and the photon 3, while additional auxiliary photons are necessary. Here we adopt the channel combination operation.

Aiming to reduce the number of outport ports, we combine the channels (C'_{21},C'_{22}) and the channels (C'_{31},C'_{32}) into the channel $P_{2\text{out}}$ and the channel $P_{3\text{out}}$, which can be performed by the channel combination module, shown in Fig. 3.

In order to articulate the process of combining the channels (C'_{21},C'_{22}) into the channel $P_{2\text{out}}$, which is selected as the

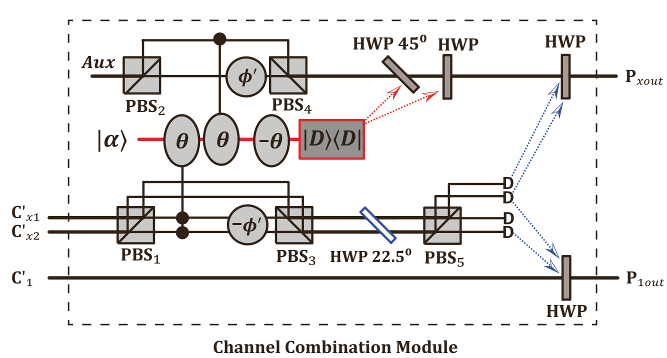

FIG. 3. Illustration plot for depicting the channel combination module on the photon x. Due to cross-Kerr nonlinearities, the auxiliary photon and the photon x passing through one of two potential channels entangle together with their spatial modes, where the subscript x denotes the photon 2 ($x=2$) or the photon 3 ($x=3$). After the photon x exits from PBS_3, the projective measurement is performed on it along the basis of $\{\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle),\frac{1}{\sqrt{2}}(|H\rangle-|V\rangle)\}$, which can be fulfilled by a HWP 22.5°, a PBS (PBS_5), and four detectors. Based on the outcomes of displacement measurement and projective measurement, classical feedforward operations need to be performed on the auxiliary photon and the photon 1.

representative to describe the channel combination in details, we change the original presentation of the system state from Eq. (9) to

$$\begin{aligned}&\tfrac{1}{4}\{|H\rangle_2|C'_{21}\rangle[|H\rangle_1|C_1'\rangle|V\rangle_3(|C'_{32}\rangle-|C'_{31}\rangle)-\sqrt{2}|V\rangle_1|C_1'\rangle|H\rangle_3(|C'_{32}\rangle+|C'_{31}\rangle)]\\&\quad+|V\rangle_2|C'_{21}\rangle|H\rangle_1|C_1'\rangle|H\rangle_3(|C'_{32}\rangle-|C'_{31}\rangle)\}\\&\quad+\tfrac{1}{4}\{|H\rangle_2|C'_{22}\rangle[|H\rangle_1|C_1'\rangle|V\rangle_3(|C'_{32}\rangle-|C'_{31}\rangle)+\sqrt{2}|V\rangle_1|C_1'\rangle|H\rangle_3(|C'_{32}\rangle+|C'_{31}\rangle)]\\&\quad+|V\rangle_2|C'_{22}\rangle|H\rangle_1|C_1'\rangle|H\rangle_3(|C'_{32}\rangle-|C'_{31}\rangle)\}.\end{aligned}\tag{10}$$

Aiming to simplify the presentation shown as Eq. (10), we let

$$|\Phi\rangle=\frac{1}{\sqrt{2}}|H\rangle_1|C_1'\rangle|V\rangle_3(|C'_{32}\rangle-|C'_{31}\rangle)-|V\rangle_1|C_1'\rangle|H\rangle_3(|C'_{32}\rangle+|C'_{31}\rangle),\quad|\Psi\rangle=\frac{1}{\sqrt{2}}|H\rangle_1|C_1'\rangle|H\rangle_3(|C'_{32}\rangle-|C'_{31}\rangle),\tag{11}$$

so Eq. (10) can be rewritten as

$$\frac{1}{2\sqrt{2}}(|\Phi\rangle|H\rangle_2|C'_{21}\rangle+|\Psi\rangle|V\rangle_2|C'_{21}\rangle)+\frac{1}{2\sqrt{2}}(\sigma_{z1}|\Phi\rangle|H\rangle_2|C'_{22}\rangle+|\Psi\rangle|V\rangle_2|C'_{22}\rangle),\tag{12}$$

where σ_{z1} denotes the single-photon transformation $\sigma_z(=|H\rangle\langle H|-|V\rangle\langle V|)$ operation performed on the photon 1.

Together with the coherent state $|\alpha\rangle$ and the auxiliary photon Aux in the state $\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)_{Aux}$, the photon 2 passes through Kerr media and undergoes the interaction contributed by cross-Kerr nonlinearities and, as a result, the whole system is in the following state:

$$\begin{aligned}&\xrightarrow{\text{Kerr media}}\tfrac{1}{4}(|\Phi\rangle|H\rangle_2|C'_{21}\rangle|H\rangle_{Aux}+|\Psi\rangle|V\rangle_2|C'_{21}\rangle|V\rangle_{Aux})|\alpha\rangle\\&\quad+\tfrac{1}{4}(\sigma_{z1}|\Phi\rangle|H\rangle_2|C'_{22}\rangle|H\rangle_{Aux}+|\Psi\rangle|V\rangle_2|C'_{22}\rangle|V\rangle_{Aux})|\alpha\rangle\\&\quad+\tfrac{1}{4}(|\Phi\rangle|H\rangle_2|C'_{21}\rangle|V\rangle_{Aux}|\alpha e^{i\theta}\rangle+|\Psi\rangle|V\rangle_2|C'_{21}\rangle|H\rangle_{Aux}|\alpha e^{-i\theta}\rangle)\\&\quad+\tfrac{1}{4}(\sigma_{z1}|\Phi\rangle|H\rangle_2|C'_{22}\rangle|V\rangle_{Aux}|\alpha e^{i\theta}\rangle+|\Psi\rangle|V\rangle_2|C'_{22}\rangle|H\rangle_{Aux}|\alpha e^{-i\theta}\rangle),\end{aligned}\tag{13}$$

where $|Aux\rangle$ is omitted due to the uniqueness of the channel of the auxiliary photon Aux, similarly hereinafter.

After the hybrid system including the photons and the coherent state leaves the realm of Kerr media, the displacement operation [41,62–64] is performed on the coherent state, so the system state can be written as

$$\begin{aligned}&\xrightarrow{\text{Displacement}}\tfrac{1}{4}(|\Phi\rangle|H\rangle_2|C'_{21}\rangle|H\rangle_{Aux}+|\Psi\rangle|V\rangle_2|C'_{21}\rangle|V\rangle_{Aux})|0\rangle\\&\quad+\tfrac{1}{4}(\sigma_{z1}|\Phi\rangle|H\rangle_2|C'_{22}\rangle|H\rangle_{Aux}+|\Psi\rangle|V\rangle_2|C'_{22}\rangle|V\rangle_{Aux})|0\rangle\\&\quad+\tfrac{1}{4}(|\Phi\rangle|H\rangle_2|C'_{21}\rangle|V\rangle_{Aux}e^{i\alpha^2\sin\theta}|\alpha(e^{i\theta}-1)\rangle+|\Psi\rangle|V\rangle_2|C'_{21}\rangle|H\rangle_{Aux}e^{-i\alpha^2\sin\theta}|\alpha(e^{-i\theta}-1)\rangle)\\&\quad+\tfrac{1}{4}(\sigma_{z1}|\Phi\rangle|H\rangle_2|C'_{22}\rangle|V\rangle_{Aux}e^{i\alpha^2\sin\theta}|\alpha(e^{i\theta}-1)\rangle+|\Psi\rangle|V\rangle_2|C'_{22}\rangle|H\rangle_{Aux}e^{-i\alpha^2\sin\theta}|\alpha(e^{-i\theta}-1)\rangle).\end{aligned}\tag{14}$$

Performing the photon number measurement on the coherent state, if zero phase shift ($n=0$) is witnessed by the measurement outcome, the system state is collapsed into

$$\xrightarrow{\text{Zero phase shift}}\frac{1}{2\sqrt{2}}(|\Phi\rangle|H\rangle_2|H\rangle_{Aux}|C'_{21}\rangle+|\Psi\rangle|V\rangle_2|V\rangle_{Aux}|C'_{21}\rangle)+\frac{1}{2\sqrt{2}}(\sigma_{z1}|\Phi\rangle|H\rangle_2|H\rangle_{Aux}|C'_{22}\rangle+|\Psi\rangle|V\rangle_2|V\rangle_{Aux}|C'_{22}\rangle).\tag{15}$$

Otherwise, if nonzero phase shift displays on the measurement setup, phase shifts $-\phi'$ and ϕ' ($\phi'=\alpha^2\sin\theta$) between the superposition terms will be removed by static phase shifters (e.g., wave plates) put into the line Aux and the line C_x'. Under the assumption of the small phase shift, if an even number of photons is indicated by the displacement measurement outcome, a HWP 45° should be inserted into the line Aux as classical feedforward to perform the $\sigma_x(=|H\rangle\langle V|+|V\rangle\langle H|)$ operation on the auxiliary photon; otherwise, if an odd number of photons is obtained, a HWP and a HWP 45° need to be inserted into the line Aux. As a result, the same state denoted as Eq. (15) can be achieved.

For completing the channel combination task, the measurement along the basis of $\{|+\rangle=\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle),|-\rangle=\frac{1}{\sqrt{2}}(|H\rangle-|V\rangle)\}$ needs to be performed on the photon x. On the terms of the basis of $\{|+\rangle,|-\rangle\}$ on the photon 2, we expand the system state denoted as Eq. (15) to

$$\begin{aligned}&\tfrac{1}{4}(|\Phi\rangle|H\rangle_{Aux}+|\Psi\rangle|V\rangle_{Aux})|+\rangle_2|C_{21}'\rangle+\tfrac{1}{4}(|\Phi\rangle|H\rangle_{Aux}-|\Psi\rangle|V\rangle_{Aux})|-\rangle_2|C_{21}'\rangle\\&\quad+\tfrac{1}{4}(\sigma_{z1}|\Phi\rangle|H\rangle_{Aux}+|\Psi\rangle|V\rangle_{Aux})|+\rangle_2|C_{22}'\rangle+\tfrac{1}{4}(\sigma_{z1}|\Phi\rangle|H\rangle_{Aux}-|\Psi\rangle|V\rangle_{Aux})|-\rangle_2|C_{22}'\rangle.\end{aligned}\tag{16}$$

The measurement along the basis of $\{|+\rangle,|-\rangle\}$ can be fulfilled by the combination of a HWP 22.5° to perform a Hadamard transformation operation on the photon 2, a polarization beam splitter PBS_5 differentiating between $|H\rangle$ polarization mode and $|V\rangle$ polarization mode, and four detectors to check the channels which the photon 2 passes through. Based on $\{|+\rangle,|-\rangle\}$ basis measurement outcomes, single photon σ_z operations (HWP) are performed on the photon 1 and/or the auxiliary photon Aux with classical feedforward, referring to Fig. 3.

At the output ports of the channel combination module, the state denoted as $\frac{1}{2}(|\Phi\rangle|H\rangle_{P_{2\text{out}}} + |\Psi\rangle|V\rangle_{P_{2\text{out}}})$ presents, which means that the channels (C'_{21},C'_{22}) are combined into the channel $P_{2\text{out}}$.

Subsequently, the channels (C'_{31},C'_{32}) need to be combined into the channel $P_{3\text{out}}$, which is the same as the above combination process, so the corresponding description is omitted for reasons of space and clarity. Thanks to symmetry, the operation order performing the channel combination operations on the photon 2 and the photon 3 may be exchanged.

After combining four channels $(C'_{21},C'_{22}; C'_{31},C'_{32})$ of the photons (2, 3) into two channels ($P_{2\text{out}}$; $P_{3\text{out}}$), at the output ports, there emerges the expected perfect W state:

$$\frac{1}{\sqrt{2}}[|H\rangle_{P_{1\text{out}}}\frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)_{P_{2\text{out}}P_{3\text{out}}} + |V\rangle_{P_{1\text{out}}}|H\rangle_{P_{2\text{out}}}|H\rangle_{P_{3\text{out}}}]. \tag{17}$$

III. FIDELITY AND FEASIBILITY OF THE GENERATION SCHEME OF THE PERFECT W STATE

Due to the unwanted coupling with environment, the photons of the coherent state may be lost and its intensity is reduced, and as a result the coherence of the entangled photons cannot be guaranteed [63,65–69]. Moreover, in the measurement process, the losses in photodetection happen frequently, which results in the detection efficiency lower than unity [70]. The detrimental factors above mentioned set the barrier for fulfilling the tasks of quantum information processing assisted by the coherent states.

The model of photon losses formally corresponds to a dissipation described by the standard master equation as given by [65]

$$\begin{aligned}\frac{\partial\rho}{\partial t} &= \hat{J}\rho + \hat{L}\rho,\\ \hat{J}\rho &= \gamma a\rho a^\dagger,\\ \hat{L}\rho &= -\frac{\gamma}{2}(a^\dagger a\rho + \rho a^\dagger a),\end{aligned} \tag{18}$$

where a and $a^\dagger$ stand for the annihilation and creation operators of the coherent state, γ represents the decay constant, and ρ denotes the density matrix of the system. And the corresponding solution is obtained as

$$\rho(t) = \tilde{D}(t)\rho(0), \tag{19}$$

where $\tilde{D}(t) = \exp[(\hat{J}+\hat{L})t] = \exp[\hat{L}t]\exp[\frac{\hat{J}}{\gamma}(1-e^{-\gamma t})]$ is the decoherence superoperator.

On the other hand, the phase shifts on the coherent state caused by the nonlinear interaction between photons and the coherent state are the consequence of a unitary evolution of the hybrid system, which can be described as a unitary evolution equation, $\tilde{U}(t)\rho(0) = U(t)\rho(0)U^\dagger(t)$, by introducing a superoperator $\tilde{U}(t)$.

In reality, the decoherence superoperators $\tilde{D}$ and the unitary superoperators $\tilde{U}$ simultaneously affect the system state during a finite temporal interval t. Adopting a reasonable assumption [68], the finite temporal interval t is divided into N parts, and during each short temporal interval $\Delta t = t/N$ ($N\sim\infty$), the decoherence superoperators $\tilde{D}$ and the unitary superoperators $\tilde{U}$ can be viewed as continuously and alternately executing on the whole system. Furthermore, the phase shifts resulting from $\tilde{U}(\Delta t)$ are $\Delta\theta = \theta/N$, $\Delta\theta = 2\theta/N$, or $\Delta\theta = 3\theta/N$, which are corresponding to different photon channel combinations. As a consequence, after the finite temporal interval t, the system evolves as

$$\rho(t) = [\tilde{D}(\Delta t)\tilde{U}(\Delta t)]^N\rho(0). \tag{20}$$

In this scheme, shown as the left-hand side of Eq. (3), the initial state of the system is $|\Psi_S\rangle = \frac{1}{2\sqrt{2}}(|H\rangle_1|C_{12}\rangle+|V\rangle_1|C_{11}\rangle)(|H\rangle_2|C_{21}\rangle+|H\rangle_2|C_{22}\rangle)(|H\rangle_3|C_{31}\rangle+|H\rangle_3|C_{32}\rangle)|\alpha\rangle$, that is, the density matrix of the initial system is $\rho(0) = |\Psi_S\rangle\langle\Psi_S|$. In the realm of Kerr media, the system is successively affected by $\tilde{U}(\Delta t)$ as

$$\begin{aligned}\tilde{U}(\Delta t)\rho(0) &= U(\Delta t)|\Psi_S\rangle\langle\Psi_S|U^\dagger(\Delta t)\\ &= \sum_{k,l=-3}^{3}|\phi_k\rangle\langle\phi_l|\otimes\left|\alpha e^{i\frac{k\theta}{N}}\right\rangle\!\left\langle\alpha e^{i\frac{l\theta}{N}}\right|,\end{aligned} \tag{21}$$

where

$$\begin{aligned}|\phi_0\rangle &= \frac{1}{2\sqrt{2}}(|HHH\rangle_{123}|C_{12}C_{22}C_{31}\rangle\\ &\quad + |VHH\rangle_{123}|C_{11}C_{21}C_{32}\rangle),\\ |\phi_{-1}\rangle &= \frac{1}{2\sqrt{2}}|HHH\rangle_{123}|C_{12}C_{21}C_{31}\rangle,\\ |\phi_1\rangle &= \frac{1}{2\sqrt{2}}|VHH\rangle_{123}|C_{11}C_{22}C_{32}\rangle,\\ |\phi_{-2}\rangle &= \frac{1}{2\sqrt{2}}|HHH\rangle_{123}|C_{12}C_{22}C_{32}\rangle,\\ |\phi_2\rangle &= \frac{1}{2\sqrt{2}}|VHH\rangle_{123}|C_{11}C_{21}C_{31}\rangle,\\ |\phi_{-3}\rangle &= \frac{1}{2\sqrt{2}}|HHH\rangle_{123}|C_{12}C_{21}C_{32}\rangle,\\ |\phi_3\rangle &= \frac{1}{2\sqrt{2}}|VHH\rangle_{123}|C_{11}C_{22}C_{31}\rangle,\end{aligned} \tag{22}$$

and $\tilde{D}(\Delta t)$ as

$$\begin{aligned}\tilde{D}(\Delta t)\tilde{U}(\Delta t)\rho(0) &= \sum_{k,l=-3}^{3}\exp\{\alpha^2(1-e^{-\gamma\frac{t}{N}})[e^{i(k-l)\frac{\theta}{N}}-1]\}\\ &\quad\times|\phi_k\rangle\langle\phi_l|\otimes\left|e^{-\frac{\gamma t}{2N}}\alpha e^{i\frac{k\theta}{N}}\right\rangle\!\left\langle e^{-\frac{\gamma t}{2N}}\alpha e^{i\frac{l\theta}{N}}\right|.\end{aligned} \tag{23}$$

Consequently, the hybrid system is changed to

$$\rho(t) = [\tilde{D}(\Delta t)\tilde{U}(\Delta t)]^N \rho(0) = \sum_{k,l=-3}^{3} e^{B_{kl}} |\phi_k\rangle\langle\phi_l| \otimes |A\alpha e^{ik\theta}\rangle\langle A\alpha e^{il\theta}|, \quad (24)$$

where

$$A = e^{-\frac{\gamma}{2}t}, \quad B_{kl} = \alpha^2\left(1 - A^{\frac{2}{N}}\right)\sum_{n=1}^{N} A^{\frac{2(n-1)}{N}}\left[e^{i(k-l)\frac{n\theta}{N}} - 1\right]. \quad (25)$$

From Eq. (25), it can be seen that the photon losses of the coherent state reduce its amplitude and enable the original pure state to evolve into a mixed state of the photons, which is similar to the lossy model of a beam splitter put behind the coherent state [66,67,69].

After performing X measurement on the dissipated coherent state, the resulting state can be obtained as follows:

$$\langle x|\rho(t)|x\rangle = \sum_{k,l=-3}^{3} C_{kl}|\phi_k\rangle\langle\phi_l| \times f(x,k\theta)f(x,l\theta)e^{i[\delta_{kl}+\varphi(x,k\theta)-\varphi(x,l\theta)]}, \quad (26)$$

where $C_{kl} = e^{\mathrm{Re}(B_{kl})}, \delta_{kl} = \mathrm{Im}(B_{kl})$; the functions $[f(x,k\theta)$, $f(x,l\theta)$, $\varphi(x,k\theta)$, and $\varphi(x,l\theta)]$ satisfy

$$f(x,n\theta) = (2\pi)^{-\frac{1}{4}} \exp\left[-\tfrac{1}{4}(x - 2A\alpha \cos n\theta)^2\right],$$
$$\varphi(x,n\theta) = (x - 2A\alpha \cos n\theta)A\alpha \sin n\theta, \quad (27)$$

where $n = 0, \pm1, \pm2, \pm3$.

There are four measurement outcomes $f(x,n\theta)$ which are corresponding to four phase shifts. The partial overlap between two adjacent Gaussian curves $f(x,n\theta)$ induces a measurement error rate depicted by the error function $\frac{1}{2}\mathrm{erfc}[A\alpha(1-\cos\theta)/\sqrt{2}]$. But as $A\alpha(1-\cos\theta)/\sqrt{2} > 2.63$, the four measurement outcomes can be distinguished with the error rate less than 10^{-4}.

The fidelity of the output state can be calculated with $F =_{\mathrm{ide}} \langle\Psi|\rho_{\mathrm{out}}|\Psi\rangle_{\mathrm{ide}}$ [69,71]. Taking the four measurement outcomes into account, the fidelity of the spatial entanglement process of this scheme is

$$F_{\mathrm{spa}} = \frac{C_{00}}{4} + \frac{1}{16}\sum_{k\neq0}(C_{kk} + C_{-kk}). \quad (28)$$

As for the polarization entanglement process, photon number measurement with double cross-Kerr nonlinearities is performed. The analysis on decoherence is similar as that of the spatial entanglement process, so we only give the calculation result as

$$F_{\mathrm{pol}} = \frac{C_{00}^2}{2} + \frac{C_{11}^2}{32} + \frac{9C_{-1-1}^2}{32} + \frac{3(C_{1-1}C_{-11} + C_{-11}C_{1-1})}{32}. \quad (29)$$

Similarly, in the process of channel combination, the fidelity can be denoted as

$$F_{\mathrm{com}} = \frac{C_{00}}{2} + \frac{1}{8}\sum_{k=1,-1}(C_{kk} + C_{-kk}). \quad (30)$$

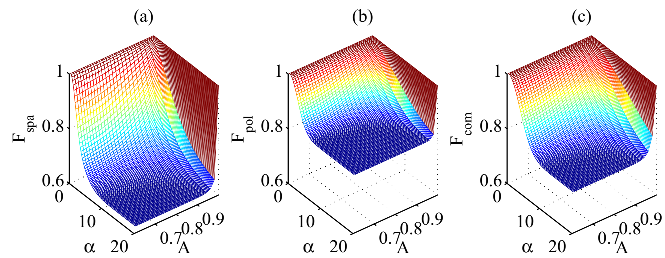

FIG. 4. Illustration plots for comparing the fidelities of three processes. The fidelity changes with the amplitude of the coherent state $|\alpha\rangle$ and the decoherence factor $A = e^{-\gamma t/2}$: (a) the fidelity of the spatial entanglement process; (b) the fidelity of the polarization entanglement process; (c) the fidelity of the combination process. Here the dimensionless parameters are set, the number splitting the temporal interval $N = 200$, and the phase shift $\theta = 20/\pi$ [72].

In Fig. 4, the fidelities which are the function of the decoherence coefficient A and the amplitude α of the coherent state in the three processes are compared, where we set the dimensionless parameters as $N = 200$, $\theta = 20/\pi$ [72]. It can be deduced that the fidelities of the three processes are unity ($F_{\mathrm{spa}} = F_{\mathrm{pol}} = F_{\mathrm{com}} = 1$) when no decoherence takes place ($A = 1$), in spite of the amplitude (α) of the coherent state. Otherwise, the fidelities decrease with the increasing amplitude of the coherent state and the reducing coefficient A induced by photon losses. By calculation, four sets of data $[F(\alpha,A)]$ are obtained as follows: $F_{\mathrm{spa}}(5,0.9) = 0.7537$, $F_{\mathrm{pol}}(5,0.9) = 0.9038$, $F_{\mathrm{com}}(5,0.9) = 0.9245$; $F_{\mathrm{spa}}(5,0.5) = 0.6728$, $F_{\mathrm{pol}}(5,0.5) = 0.8368$, $F_{\mathrm{com}}(5,0.5) = 0.8400$; $F_{\mathrm{spa}}(20,0.9) = 0.6254$, $F_{\mathrm{pol}}(20,0.9) = 0.8125$, $F_{\mathrm{com}}(20,0.9) = 0.7508$; $F_{\mathrm{spa}}(20,0.5) = 0.6250$, $F_{\mathrm{pol}}(20,0.5) = 0.8125$, $F_{\mathrm{com}}(20,0.5) = 0.7500$, from which we can see that the fidelities of the polarization entanglement process and the combination process are comparable and higher than that of the spacial entanglement process under the same conditions.

Based on the analyses above, the high fidelity perfect W state can be achieved in this scheme satisfying the requirement of weak-decoherence environment and low-intensity coherent states.

Adopting the continuous-time model, Shapiro *et al.* [73,74] analyzed the cross-Kerr nonlinear interaction and showed that single-mode cross-Kerr nonlinearities is not available for quantum information processing, for instance, the parity gate [41], due to phase noise resulting from the casual and noninstantaneous responses of Kerr media encountering a multimode light [75]. Furthermore, Shapiro *et al.* [74] gave a necessary condition of quantum information processing applying cross-Kerr nonlinearities, $\theta^2|\alpha|^2 \gg 1, \sigma^2|\alpha|^2 \ll 1, \sigma^2 \ll 1$ (σ^2 stands for phase noise variance), and pointed out that the order of 10^{-6} of σ^2 needs to be satisfied for applications in quantum information processing, which is difficult to be realized with the current techniques.

Based on the perspective of dynamical evolution of quantum states, Gea-Banacloche [76] deduced the same conclusion as Shapiro *et al.* [73] because of the presence of spontaneous emission. Different from the above viewpoints, He *et al.* [77]

showed that the goals of high fidelities, nonzero conditional phases, and high photon numbers in the application of cross-Kerr nonlinearities are not contradictory under the conditions of two photonic pulses with nonequal group velocities fully passing through each other and effectively suppressing the unwanted transverse-mode effects [78].

Additionally, quantum noise (decoherence) also precludes the perfect performance of cross-Kerr nonlinearities [79,80], but some methods have been used to attempt to solve it or alleviate its effects, such as displacement measurement [41,62–64,68] and double cross-Kerr nonlinearities [53,54,81–83].

Even if so many troubles interfere with the applications of cross-Kerr nonlinearities, numerous progresses and achievements support the feasibility of cross-Kerr nonlinearities when some conditions are satisfied [55].

Employing the proper physics systems providing larger strength of cross-Kerr nonlinearities and longer interaction time, or exploiting the appropriate measurement methods, the disadvantageous influence above can be overcome or alleviated and the error probability will be decreased. With regard to physics systems, they may be the combination of the natural systems [72,79,84–89] or artificial media [90–92]. Additionally, prolonging the transmission time in which the photons and the coherent states pass through Kerr media is also a direct method to enlarge the phase shift [93–96]. Furthermore, some other appropriate methods can also indirectly and effectively decrease the error probability [69,97,98], besides the measurement methods mentioned by this scheme.

IV. SUMMARY

In summary, we propose a scheme for generating the three-photon polarization-entangled perfect W state with the assistance of cross-Kerr nonlinearities and classical feedforward. In this scheme, three processes including the three-photon spatial entanglement process, the two-photon polarization entanglement process, and the three-photon channel combination process are applied. After the hybrid system of the photons and the coherent states passes through Kerr media, measurement is performed on the coherent states for distinguishing the phase shifts to judge different channel combinations. With the assistance of classical feedforward, single-photon operations are performed according to measurement outcomes. By virtue of the availability of optical elements and techniques involved, this generation scheme is feasible and potentially affords facilities for other practical implementations of quantum information processing based on optics.

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