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Thermalization of a two-level atom in a planar dielectric system out of thermal equilibrium

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We study the thermalization of an elementary quantum system modeled by a two-level atom interacting with stationary electromagnetic fields out of thermal equilibrium near a dielectric slab. The slab is held at a temperature different from that of the region where the atom is located. We find that when the slab is nonabsorbing and nondispersive, out-of-thermal-equilibrium effects exist only when its thickness is infinite. In other words, no out-of-thermal-equilibrium effects appear for a real dielectric slab of a finite thickness d. Furthermore, a finite thick dielectric slab with a tiny imaginary part in the relative permittivity $\mathrm{Im}\,\epsilon$ behaves like a half-space dielectric substrate if $\frac{\mathrm{Im}\,\epsilon}{\sqrt{\mathrm{Re}\,\epsilon-1}}\frac{d}{\lambda_0} > 1$ is satisfied, where λ_0 is the transition wavelength of the atom. This condition can serve as a guide for an experimental verification, using a dielectric substrate of a finite thickness, of the effects that arise from out-of-thermal-equilibrium fluctuations with a half-space (infinite thickness) dielectric.

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I. INTRODUCTION

Physical systems out of thermal equilibrium but in a stationary configuration, such as that of a substrate and an environment held respectively at different temperatures, may exhibit remarkable and measurable quantum phenomena and thus have recently attracted increasing interest both theoretically and experimentally. In this respect, Antezza *et al.* [1] investigated, in the large distance limit, the Casimir-Polder (CP) force [2] in such an out-of-thermal-equilibrium situation. They found that the CP force shows new qualitative and quantitative behaviors. Specifically, the force decays as $1/z^3$ and is proportional to ΔT^2, where z is the distance between an atom and the surface of the substrate and $\Delta T^2 \equiv T_s^2 - T_e^2$, with T_e and T_s the temperatures of the thermal bath in the right half space and the substrate in the left half space, respectively. This behavior of the force differs clearly from that of an atom both in vacuum, which has a $1/z^5$ dependence [2], and in a thermal equilibrium environment, which behaves like T/z^4 and is attractive [3]. Actually, the out-of-thermal-equilibrium CP force can be either attractive or repulsive depending on the difference of two temperatures. Later, Zhou and Yu analyzed in detail the behaviors of the out-of-thermal-equilibrium CP force of an atom near the surface of a half-space real dielectric substrate in different distance regimes [4], where a real dielectric refers to a nonabsorbing and nondispersive dielectric whose permittivity is real and frequency independent. In addition, the CP force of a diamagnetic atom out of thermal equilibrium has also been investigated in [5]. Remarkably, the new behavior of the CP force out of thermal equilibrium has been measured in experiment by positioning a nearly pure ^{87}Rb Bose-Einstein condensate a few microns from a dielectric substrate, which consists of uv-grade fused silica with a 2-mm thickness [6].

On the other hand, the dynamics of an elementary quantum system in a stationary environment out of thermal equilibrium has been studied by Bellomo *et al.* [7] and it has been found that the quantum system modeled by a two-level atom can be thermalized to a steady state with an effective temperature between the temperature of the wall and that of the environment. A similar result has also been obtained for an atom placed outside a radiating Schwarzschild black hole [8]. For two quantum emitters interacting with a common stationary electromagnetic field out of thermal equilibrium, Bellomo and Antezza found that the absence of equilibrium allows the generation of steady entangled states between the emitters, which is inaccessible at thermal equilibrium [9,10]. In addition, the photon heat tunneling was discussed in [11,12]. Other aspects about the out-of-thermal-equilibrium effects have been discussed in [13–27].

To simplify the theoretical calculations, a half-space, even real, dielectric substrate is usually assumed when analyzing the nonequilibrium thermal system. However, in reality, such a dielectric substrate never exists. In fact, in experiment, a dielectric slab with a finite thickness and absorption and dispersion is generally used. As a result, questions naturally arise as to when a generic finite slab can be regarded as an infinite substrate on which the theoretical calculations are based and how the novel out-of-thermal-equilibrium effects depend on the dielectric property. In this paper we try to answer these questions in terms of the thermalization of a polarizable two-level atom in a thermal bath near a planar dielectric slab out of thermal equilibrium. We will show that for a nonabsorbing and nondispersive dielectric with a finite thickness no out-of-thermal-equilibrium effects appear as far as the thermalization of the atom is concerned. So, to have nonvanishing out-of-thermal-equilibrium effects, one has to have a real dielectric substrate with an infinite thickness or a complex dielectric substrate. Since the infinitely thick substrate does not really exist, we give the condition when a dielectric with a tiny nonzero imaginary part in the relative permittivity with a finite thickness can be regarded as a half-space dielectric. This puts on a solid foundation the experimental test using a finite dielectric substrate of theoretical predictions for novel effects from out of thermal equilibrium based upon a half-space dielectric.

II. OPEN QUANTUM SYSTEM

We examine in the framework of open quantum systems the thermalization of a two-level atom near a dielectric substrate

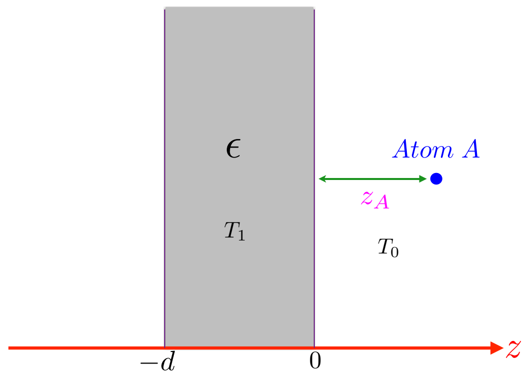

FIG. 1. (Color online) Scheme of the system considered.

in a stationary configuration out of thermal equilibrium. We assume that two stationary states of the atom are represented by $|1\rangle$ and $|2\rangle$, respectively, and the energy spacing is $\hbar\omega_0$. A planar dielectric slab with thickness d is placed in a thermal bath at temperature T_0 and its right surface coincides with the $z = 0$ plane. The slab is assumed to be in local thermal equilibrium at a different temperature T_1 (see Fig. 1). The atom is at the position $z_A > 0$ in the empty space. So the whole system is out of thermal equilibrium but in a stationary regime and the total Hamiltonian that governs the evolution of the system takes the form

$$H = H_A + H_B + H_I, \tag{1}$$

where $H_A = \sum_{m=1}^{2} \hbar\omega_m |m\rangle\langle m|$ is the Hamiltonian of the atom, H_B is the Hamiltonian describing the environment the atom is coupled to, and H_I denotes the interaction between the atom and the environment, which takes the form $H_I = -\boldsymbol{D}(t)\cdot\boldsymbol{E}(\boldsymbol{r},t)$ in the multipolar coupling scheme. Here $\boldsymbol{D}(t)$ is the electric dipole moment of the atom and $\boldsymbol{E}(\boldsymbol{r},t)$ is the electric-field strength. In fact, H_I can also be rewritten as

$$H_I = -\sum_{i,\omega} e^{-i\omega t} A_i(\omega) E_i(\mathbf{r},t), \tag{2}$$

where $i \in \{x,y,z\}$ and $A_i(\omega) = \sum_{\varepsilon'-\varepsilon=\omega} \Pi(\varepsilon) D_i \Pi(\varepsilon')$, with $\Pi(\varepsilon)$ denoting the projection onto the eigenspace belonging to the eigenvalue ε of H_A, which means that $A_i(\omega)$ are the eigenoperators of H_A. For a two-level atom, $\boldsymbol{D}(t)$ can be expressed as

$$\boldsymbol{D}(t) = \boldsymbol{d}_{21}|2\rangle\langle 1| e^{-i\omega_0 t} + \boldsymbol{d}_{21}^{*}|1\rangle\langle 2| e^{i\omega_0 t}, \tag{3}$$

which implies that $\boldsymbol{A}(\omega) = \sum_{\omega} \boldsymbol{d}_{21}|2\rangle\langle 1| = \boldsymbol{A}^{\dagger}(-\omega)$.

In the interaction picture, the total density matrix $\rho_{\rm tot}(t)$ of the system satisfies the von Neumann equation

$$\frac{d}{dt}\rho_{\rm tot}(t) = -\frac{i}{\hbar}[H_I, \rho_{\rm tot}(t)], \tag{4}$$

with the initial state being described by $\rho_{\rm tot}(0) = \rho(0)\otimes\rho_B$, where $\rho(0)$ is the initial density matrix of the atom and ρ_B is that of the environment. Tracing $\rho_{\rm tot}(t)$ over the degrees of freedom associated with the environment, one can obtain the reduced density matrix $\rho(t)$ for the two-level atom, namely, $\rho(t) = {\rm Tr}_B[\rho_{\rm tot}(t)]$, which, in the limit of weak coupling, obeys the master equation [7,8,28,29]

$$\begin{aligned}\frac{d}{dt}\rho(t) = &-\frac{i}{\hbar}[H_A + H_{LS}, \rho(t)] + \Gamma(\omega_0)[\rho_{22}|1\rangle\langle 1| \\ &- \frac{1}{2}\{|2\rangle\langle 2|, \rho(t)\}] \\ &+ \Gamma(-\omega_0)[\rho_{11}|2\rangle\langle 2| - \frac{1}{2}\{|1\rangle\langle 1|, \rho(t)\}],\end{aligned} \tag{5}$$

where H_{LS} is the so-called Lamb-shift Hamiltonian since it produces shifts of the atomic energy levels and $\Gamma(-\omega_0)$ and $\Gamma(\omega_0)$ are, respectively, the downward and upward transition rates, which are defined as

$$\begin{aligned}\Gamma(\omega_0) &\equiv \sum_{i,j} \gamma_{ij}(\omega_0)[\boldsymbol{d}_{21}]_i^{*}[\boldsymbol{d}_{21}]_j, \\ \Gamma(-\omega_0) &\equiv \sum_{i,j} \gamma_{ij}(-\omega_0)[\boldsymbol{d}_{21}]_i[\boldsymbol{d}_{21}]_j^{*}.\end{aligned} \tag{6}$$

Here $\gamma_{ij}(\omega)$ is given by [7]

$$\gamma_{ij}(\omega) = \frac{2\pi}{\hbar^2}\int_0^{\infty} d\omega' \times \begin{cases} \langle E_i(\boldsymbol{r},\omega)E_j^{\dagger}(\boldsymbol{r},\omega')\rangle, & \omega > 0 \\ \langle E_i^{\dagger}(\boldsymbol{r},-\omega)E_j(\boldsymbol{r},\omega')\rangle, & \omega < 0. \end{cases} \tag{7}$$

For a nonmagnetic medium, the electric-field operator can be expressed as

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r},\omega) = i\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0}}\int d^3r'\sqrt{{\rm Im}\,\epsilon(\boldsymbol{r}',\omega)} \\ \times \mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\boldsymbol{f}(\boldsymbol{r}',\omega).\end{aligned} \tag{8}$$

Here ϵ_0 and ϵ are the vacuum and relative permittivity, respectively, $\mathbf{G}$ is the classical Green's tensor, which satisfies a useful integral relation

$$\begin{aligned}\int d^3s\,{\rm Im}\,\epsilon(\boldsymbol{s},\omega)\mathbf{G}(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathbf{G}^{*\rm T}(\boldsymbol{r}',\boldsymbol{s},\omega) \\ = \frac{c^2}{\omega^2}{\rm Im}\,\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega),\end{aligned} \tag{9}$$

and $\boldsymbol{f}(\boldsymbol{r},\omega)$ and $\boldsymbol{f}^{\dagger}(\boldsymbol{r},\omega)$ are the annihilation and creation operators of the elementary electric excitations, respectively. They obey the bosonic commutation relations $[\boldsymbol{f}(\boldsymbol{r},\omega),\boldsymbol{f}^{\dagger}(\boldsymbol{r}',\omega')] = \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')$ and $[\boldsymbol{f}(\boldsymbol{r},\omega),\boldsymbol{f}(\boldsymbol{r}',\omega')] = [\boldsymbol{f}^{\dagger}(\boldsymbol{r},\omega),\boldsymbol{f}^{\dagger}(\boldsymbol{r}',\omega')] = \mathbf{0}$, where $\mathbf{0}$ represents a zero matrix. For the thermal state describing the system in a stationary configuration out of thermal equilibrium we are considering, one has

$$\begin{aligned}\langle\{\beta_i\}|\boldsymbol{f}(\boldsymbol{r},\omega)\boldsymbol{f}^{\dagger}(\boldsymbol{r}',\omega')|\{\beta_i\}\rangle \\ = [1 + N(\omega,\beta_i)]\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega'),\end{aligned} \tag{10}$$

$$\langle\{\beta_i\}|\boldsymbol{f}^{\dagger}(\boldsymbol{r},\omega)\boldsymbol{f}(\boldsymbol{r}',\omega')|\{\beta_i\}\rangle = N(\omega,\beta_i)\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega'), \tag{11}$$

where $\beta_i = \hbar c/kT_i$, with $i = 0$ or 1, and $N(\omega,\beta_i) = \frac{1}{e^{\beta_i \omega/c}-1}$. Substituting Eq. (8) into Eq. (7) and considering the relations given in Eqs. (10) and (11), we have

$$\gamma_{ij}(\omega) = \frac{2\mu_0\omega^4}{\hbar c^2}\int d^3r'[1+N(\omega,\beta)]\,\mathrm{Im}\,\epsilon(\boldsymbol{r}',\omega) \times G_{ik}(\boldsymbol{r},\boldsymbol{r}',\omega)G^*_{jk}(\boldsymbol{r},\boldsymbol{r}',\omega) \tag{12}$$

and

$$\gamma_{ij}(-\omega) = \frac{2\mu_0\omega^4}{\hbar c^2}\int d^3r' N(\omega,\beta)\,\mathrm{Im}\,\epsilon(\boldsymbol{r}',\omega) \times G_{ik}(\boldsymbol{r},\boldsymbol{r}',\omega)G^*_{jk}(\boldsymbol{r},\boldsymbol{r}',\omega). \tag{13}$$

Here μ_0 is the vacuum permeability and $\mu_0\epsilon_0 = 1/c^2$ is used.

For an atom near a dielectric slab described in Fig. 1, Eqs. (12) and (13) become

$$\gamma_{ij}(\omega) = \frac{2\mu_0\omega^2}{\hbar}[1+N(\omega,\beta_0)]\,\mathrm{Im}\,G_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega) + \frac{2\pi}{\hbar}[N(\omega,\beta_1)-N(\omega,\beta_0)]g_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega) \tag{14}$$

and

$$\gamma_{ij}(-\omega) = \frac{2\mu_0\omega^2}{\hbar}N(\omega,\beta_0)\,\mathrm{Im}\,G_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega) + \frac{2\pi}{\hbar}[N(\omega,\beta_1)-N(\omega,\beta_0)]g_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega), \tag{15}$$

where Eq. (9) has been used and

$$g_{ij}(\boldsymbol{r},\boldsymbol{r},\omega) \equiv \frac{\mu_0\omega^4}{\pi c^2}\int d^2\boldsymbol{r}'_\parallel \int_{-d}^{0} dz'\,\mathrm{Im}\,\epsilon\, G_{ik}(\boldsymbol{r},\boldsymbol{r}',\omega) \times G^*_{jk}(\boldsymbol{r},\boldsymbol{r}',\omega), \tag{16}$$

with $\boldsymbol{r}'_\parallel = \{x',y'\}$. The first term on the right-hand side of Eqs. (14) and (15) gives the contributions of the zero-point fluctuations and the thermal fluctuations in thermal equilibrium at a temperature T_0, while the second term arises from the out-of-thermal-equilibrium nature of the system. For the system we are considering, only the diagonal elements of $\mathrm{Im}\,G_{ij}(\boldsymbol{r},\boldsymbol{r},\omega)$ and $g_{ij}(\boldsymbol{r},\boldsymbol{r},\omega)$ are nonvanishing.

III. THERMALIZATION

Using Eqs. (14) and (15), we can show that the transition rates $\Gamma(\omega_0)$ and $\Gamma(-\omega_0)$ can be reexpressed as [7]

$$\begin{pmatrix}\Gamma(\omega_0)\\ \Gamma(-\omega_0)\end{pmatrix} = \alpha(\omega_0)\Gamma_0(\omega_0)\begin{pmatrix}1+N_{\mathrm{eff}}(\omega_0)\\ N_{\mathrm{eff}}(\omega_0)\end{pmatrix}, \tag{17}$$

where $\Gamma_0(\omega_0) = \frac{\omega_0^3|\mathbf{d}_{12}|^2}{3\pi\epsilon_0\hbar c^3}$ is the vacuum spontaneous-emission rate related to the transition between the ground and excited states,

$$\alpha(\omega_0) = \frac{6\pi c}{\omega_0}\sum_{i,j}\frac{[\boldsymbol{d}_{21}]_i[\boldsymbol{d}_{21}]^*_j}{|\boldsymbol{d}_{21}|^2}\mathrm{Im}G_{ij}(\boldsymbol{r},\boldsymbol{r},\omega_0), \tag{18}$$

and

$$\begin{aligned} N_{\mathrm{eff}}(\omega_0) &= N(\omega_0,\beta_0) + \frac{6\pi^2 c}{\mu_0\omega_0^3\alpha(\omega_0)}[N(\omega_0,\beta_1)-N(\omega_0,\beta_0)] \times \sum_{i,j}\frac{[\boldsymbol{d}_{21}]_i[\boldsymbol{d}_{21}]^*_j}{|\boldsymbol{d}_{21}|^2}g_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)\\ &= N(\omega_0,\beta_0) + \frac{2\pi^2 c}{\mu_0\omega_0^3\alpha(\omega_0)} \times [N(\omega_0,\beta_1)-N(\omega_0,\beta_0)]\,\mathrm{g}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0). \end{aligned} \tag{19}$$

Here the last line holds for an isotropically polarizable atom and $g = g_{xx}+g_{yy}+g_{zz}$. So $N_{\mathrm{eff}}(\omega_0)$ depends on the temperature T_i $(i = 0,1)$ and the dielectric property of a slab encoded in the function $g(\boldsymbol{r},\boldsymbol{r},\omega)$. As discussed in [7,8], after evolving for a sufficiently long period of time, the atom will be thermalized to a steady state with an effective temperature

$$T_{\mathrm{eff}} = \frac{\hbar\omega_0}{k}\{\ln[1+N^{-1}_{\mathrm{eff}}(\omega_0)]\}^{-1}. \tag{20}$$

It is easy to see that if the substrate is in thermal equilibrium with the thermal radiation in the empty space where the atom is located, then T_{eff} reduces to T_0 as expected.

In order to analyze in detail the thermalization temperature of the atom, we first need to examine the behavior of $\mathrm{g}(\boldsymbol{r},\boldsymbol{r},\omega)$, which depends on the Green's function $\mathbf{G}(\omega,\boldsymbol{r},\boldsymbol{r}')$, where $\boldsymbol{r}$ indicates the position of the atom and thus it is restricted to the empty right half space, while $\boldsymbol{r}'$ is in the slab. For the system considered, the Green's function can be expanded as

$$\mathbf{G}(\omega,\boldsymbol{r},\boldsymbol{r}') = \int d^2\boldsymbol{k}\, e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_\parallel-\boldsymbol{r}'_\parallel)}\mathbf{G}(\boldsymbol{k},\omega,z,z'), \tag{21}$$

where $\boldsymbol{k} = (k_x,k_y)$. Since z and z' are in different regions, from Refs. [30,31] we have that

$$\mathbf{G}(\boldsymbol{k},\omega,z,z') = \frac{i}{8\pi^2 b_0(k)}\sum_{\sigma=s,p}\xi^\sigma\frac{t^\sigma(k)e^{ib_0(k)z}}{D^\sigma(k)} \times \hat{\boldsymbol{e}}^+_{\sigma_0}(\boldsymbol{k})[\hat{\boldsymbol{e}}^-_{\sigma_1}(-\boldsymbol{k})e^{-ib_1(k)z'} + r^\sigma_-(k)\hat{\boldsymbol{e}}^+_{\sigma_1}(-\boldsymbol{k})e^{ib_1(k)(z'+2d)}], \tag{22}$$

where $\xi^p = 1$, $\xi^s = -1$, $b_0(k) = \sqrt{k_0^2-k^2}$, $b_1(k) = \sqrt{k_1^2-k^2}$, $k = |\boldsymbol{k}|$, $k_0 = \frac{\omega}{c}$, $k_1 = \sqrt{\epsilon}\frac{\omega}{c}$, and

$$D^\sigma(k) = 1 - r^\sigma_- r^\sigma_+(k)e^{2ib_1(k)d},$$

with $r^\sigma_+(k)$ and $r^\sigma_-(k)$ being the reflection coefficients at the right and left boundaries of the slab, which have the forms

$$r^s_\pm(k) = \frac{b_1(k)-b_0(k)}{b_1(k)+b_0(k)}, \quad r^p_\pm(k) = \frac{b_1(k)-\epsilon b_0(k)}{b_1(k)+\epsilon b_0(k)}. \tag{23}$$

Here $t^\sigma(k) = \sqrt{\frac{1}{\epsilon}}[1-r^\sigma_+(k)]$ is the transmission coefficient between the empty space and the slab. In addition, we define

$$\hat{\boldsymbol{e}}^\pm_{p_i}(\boldsymbol{k}) = \frac{1}{k_i}(\mp b_i\hat{\boldsymbol{k}} + k\hat{\boldsymbol{z}}), \quad \hat{\boldsymbol{e}}^\pm_{s_i}(\boldsymbol{k}) = \hat{\boldsymbol{k}}\times\hat{\boldsymbol{z}}. \tag{24}$$

Substituting Eqs. (21) and (22) into Eq. (16), for a system described in Fig. 1 and an isotropically polarizable atom, one has $g(\boldsymbol{r},\boldsymbol{r},\omega) = g(z,z,\omega)$ with

$$g(z,z,\omega) = \frac{\mu_0\omega^2}{8\pi^2}\int_0^\infty \frac{kdk}{|b_0(k)|^2} e^{-2\,\mathrm{Im}\,b_0(k)z}[\,\mathrm{Re}\,b_1(k)[A_+(k)+A(k)](1-e^{-2\,\mathrm{Im}\,b_1(k)d}) + e^{-2\,\mathrm{Im}\,b_1(k)d}(\,\mathrm{Re}\,b_1(k)[A_+(k)|r_-^p(k)|^2 + A(k)|r_-^s(k)|^2](1-e^{-2\,\mathrm{Im}\,b_1(k)d}) + 2\,\mathrm{Im}\,b_1(k)[A_-(k)\,\mathrm{Re}\,r_-^p(k) + A(k)\,\mathrm{Re}\,r_-^s(k)]\sin[2\,\mathrm{Re}\,b_1(k)d] + 2\,\mathrm{Im}\,b_1(k)[A_-(k)\,\mathrm{Im}\,r_-^p(k) + A(k)\,\mathrm{Im}\,r_-^s(k)]\{\cos[2\,\mathrm{Re}\,b_1(k)d]-1\})], \quad (25)$$

where

$$A_\pm(k) = \left|\frac{t^p(k)}{D^p(k)}\right|^2 \frac{[k^2 \pm |b_1(k)|^2][k^2+|b_0(k)|^2]}{|k_0k_1|^2}, \quad A(k) = \left|\frac{t^s(k)}{D^s(k)}\right|^2. \quad (26)$$

This expression shows that $\mathrm{Im}\,b_0(k)$ must be nonzero, otherwise $g(z,z,\omega)$ will become a constant independent of z. From the definition of $b_0(k)$ and $b_1(k)$ we obtain that

$$2\,\mathrm{Im}^2\,b_0(k) = -\left(\frac{\omega^2}{c^2}-k^2\right) + \left|\frac{\omega^2}{c^2}-k^2\right| \quad (27)$$

and

$$\mathrm{Im}^2\,b_1(k) = \frac{1}{2}\left[-\left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon - k^2\right) + \sqrt{\frac{\omega^4}{c^4}\mathrm{Im}^2\,\epsilon + \left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon - k^2\right)^2}\right], \quad (28)$$

$$\mathrm{Re}^2\,b_1(k) = \frac{1}{2}\left[\left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon - k^2\right) + \sqrt{\frac{\omega^4}{c^4}\mathrm{Im}^2\,\epsilon + \left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon - k^2\right)^2}\right]. \quad (29)$$

A nonzero $\mathrm{Im}\,b_0(k)$ means that $k^2 > \frac{\omega^2}{c^2}$ and thus only the $k > \frac{\omega}{c}$ interval in k integration from 0 to ∞ contributes. Let us note that $\mathrm{Im}^2\,b_1(k)$ is an increasing function of k^2, while $\mathrm{Re}^2\,b_1(k)$ is a decreasing one, as shown graphically in Fig. 2.

If the slab consists of real dielectrics, i.e., $\mathrm{Im}\,\epsilon = 0$, Eqs. (28) and (29) tell us that $\mathrm{Im}\,b_1(k) = 0$ if $\mathrm{Re}\,b_1(k) \neq 0$ and vice versa. As a result, it is easy to see that $g(z,z,\omega) = 0$. Only when the slab thickness is infinite, that is, $d \to \infty$ and $r_-^\sigma = 0$, is $g(z,z,\omega)$ nonzero and it then becomes

$$g(z,z,\omega) = \frac{\mu_0\omega^2}{8\pi^2}\int_0^\infty \frac{kdk}{|b_0(k)|^2} e^{-2\,\mathrm{Im}\,b_0(k)z} \times \mathrm{Re}\,b_1(k)[\bar{A}_+(k)+\bar{A}(k)], \quad (30)$$

where

$$\bar{A}_+(k) = |t^p(k)|^2 \frac{[k^2+|b_1(k)|^2][k^2+|b_0(k)|^2]}{|k_0k_1|^2}, \quad \bar{A}(k) = |t^s(k)|^2. \quad (31)$$

This demonstrates that there is no out-of-thermal-equilibrium effect for any real dielectric substrate of finite thickness even when the substrate is held at a different local temperature. In other words, an infinite thickness is the only way to have an out-of-thermal-equilibrium effect for a real dielectric substrate.

Another way to have a nonzero out-of-thermal-equilibrium effect is that the slab consists of the dispersive and absorbing dielectric ($\mathrm{Im}\,\epsilon \neq 0$). This is similar to what happens to the decay rate of the excited state of an atom in front of a dielectric plate, which is proportional to the imaginary part of the permittivity and also equals zero when $\mathrm{Im}\,\epsilon = 0$ [32].

From Eq. (25) one can see that if $2\,\mathrm{Im}\,b_1(k)d > 1$, the terms depending on d can be neglected since they are exponentially suppressed as compared to the other term, which then gives the dominant contribution. In this case, the result of the integral becomes effectively independent of d and approximates to that in the case of a half-space dielectric substrate, which has the same form as that given in Eq. (30). Since $\mathrm{Im}^2\,b_1(k)$ is an increasing function of k^2 and $k^2 > \frac{\omega^2}{c^2}$ is required, the minimum value of $\mathrm{Im}\,b_1(k)$ is achieved at $k^2 = \frac{\omega^2}{c^2}$,

$$\min\{\mathrm{Im}\,b_1(k)\} = \frac{1}{\sqrt{2}}\frac{\omega}{c}[-(\mathrm{Re}\,\epsilon - 1) + \sqrt{\mathrm{Im}^2\,\epsilon + (\mathrm{Re}\,\epsilon-1)^2}]^{1/2}. \quad (32)$$

So the condition for the thermalization of a two-level atom with a typical transition ω_0 near a dielectric slab of finite thickness d out of thermal equilibrium to behave like that near an infinitely thick half-space dielectric substrate is

$$\frac{\sqrt{2}d}{\lambda_0}[-(\mathrm{Re}\,\epsilon - 1) + \sqrt{\mathrm{Im}^2\,\epsilon + (\mathrm{Re}\,\epsilon-1)^2}]^{1/2} > 1, \quad (33)$$

where $\lambda_0 = \frac{c}{\omega_0}$ is the transition wavelength of the atom. Since the dielectrics with a very small but nonzero $\mathrm{Im}\,\epsilon$, such as fused silica and sapphire, are used in the experiment to observe the novel feature for the CP force out of thermal equilibrium [6], we expand the condition in the limit of $\mathrm{Im}\,\epsilon \sim 0$ and obtain

$$\frac{\mathrm{Im}\,\epsilon}{\sqrt{\mathrm{Re}\,\epsilon - 1}}\frac{d}{\lambda_0} > 1. \quad (34)$$

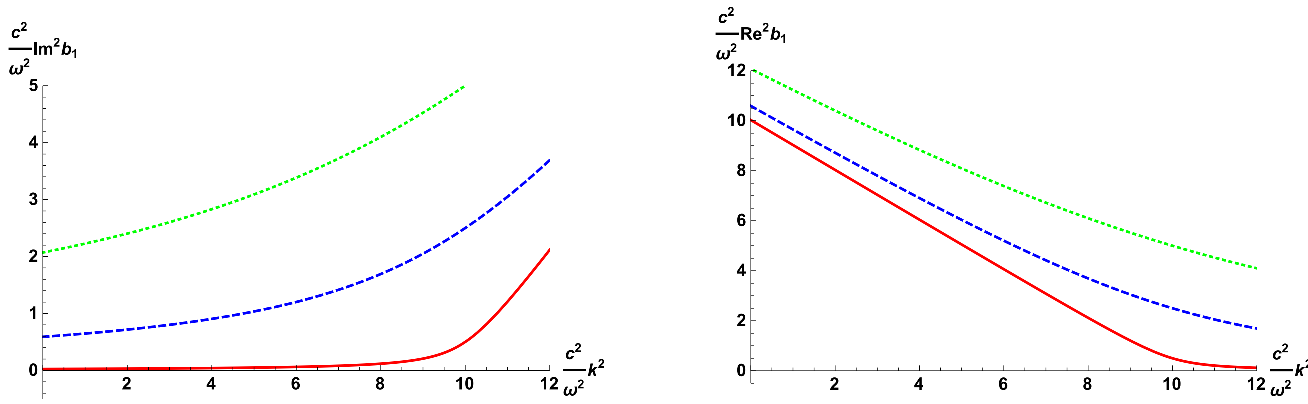

FIG. 2. (Color online) Evolutionary curves of $\frac{c^2}{w^2}\,\mathrm{Im}^2\, b_1$ (left) and $\frac{c^2}{w^2}\,\mathrm{Re}^2\, b_1$ (right) with respect to $\frac{c^2}{w^2}k^2$ with $\mathrm{Re}\,\epsilon = 10$. The red solid, blue dashed, and green dotted lines correspond to $\mathrm{Im}\,\epsilon = 1$, 5, and 10, respectively.

Obviously, for a given atom we can always find a finite d so that the condition is satisfied as long as $\mathrm{Im}\,\epsilon$ is not vanishing no matter how small it is. Since mathematically an infinitely thick slab does not exist, the above relation can serve as a guide for an experimental verification of the effects that arise from out-of-thermal-equilibrium fluctuations and justifies testing experimentally the novel property theoretically found from a half-space dielectric out of thermal equilibrium using a dielectric substrate of finite thickness with a tiny imaginary part in the relative permittivity.

A few comments are in order. First, our results can be generalized to a system of a multilayer dielectric body with each layer of a different permittivity in local thermal equilibrium at a different temperature. For the multilayered substrate consisting of only real dielectric, if the outermost left layer is a perfect mirror or empty space, the system has no out-of-thermal-equilibrium effect, at least as far as the thermalization of the atom is concerned. If the outermost left layer is a half-space substrate at a certain temperature, only this temperature and that of the thermal bath in the right half empty space affect the thermalization of the atom. Second, although our calculations are performed under the assumption of an isotropically polarizable atom, our conclusions also hold for an anisotropically polarizable atom since the only difference for such a case is that the definitions of $A_{\pm}(k)$ and $A(k)$ in Eq. (25) are different. Finally, here we only investigate the thermalization of an atom in front of a slab. The out-of-thermal-equilibrium CP force, especially its explicit dependence on the thickness d, the distance z_A, and the temperatures T_i in different limits like what was discussed in [32] for the thermal CP force, is an interesting topic that is currently under investigation.

IV. CONCLUSION

We have studied the thermalization of a two-level atom near a planar dielectric substrate in a stationary environment out of thermal equilibrium in which the atom is located in an empty space filled with a thermal bath at a temperature different from the local thermal equilibrium temperature of the substrate. We demonstrate that when the planar dielectric substrate is a real dielectric of finite thickness, no out-of-thermal-equilibrium effects appear as far as the thermalization of the atom is concerned. That is to say, the atom thermalizes as if the substrate is in thermal equilibrium with the thermal bath in the empty space where the atom is located. We also show that a planar dispersive and absorbing dielectric substrate with a finite thickness and a tiny imaginary part in the relative permittivity, in its influence on the thermalization of the atom, behaves like a half-space dielectric under certain conditions and we concretely derived this condition in our paper, which can serve as a guide for an experimental verification, using a dielectric substrate of a finite thickness, of the effects that arise from out-of-thermal-equilibrium fluctuations with a half-space (infinite thickness) dielectric.

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