PHYSICAL REVIEW A **92**, 060702(R) (2015)

Broad universal Feshbach resonances in the chaotic spectrum of dysprosium atoms

T. Maier,[1] I. Ferrier-Barbut,[1,*] H. Kadau,[1] M. Schmitt,[1] M. Wenzel,[1] C. Wink,[1] T. Pfau,[1] K. Jachymski,[2] and P. S. Julienne[3]

[1]*5. Physikalisches Institut and Center for Integrated Quantum Science and Technology, Universität Stuttgart, Pfaffenwaldring 57, 70550 Stuttgart, Germany*

[2]*Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

[3]*Joint Quantum Institute, University of Maryland and National Institute of Standards and Technology, College Park, Maryland 20742, USA*

(Received 5 June 2015; published 14 December 2015)

We report on the observation of weakly bound dimers of bosonic dysprosium with a strong universal s-wave halo character, associated with broad magnetic Feshbach resonances. These states surprisingly decouple from the chaotic background of narrow resonances, persisting across many such narrow resonances. In addition they show the highest reported magnetic moment $\mu \simeq 20\mu_B$ of any ultracold molecule. We analyze our findings using a coupled-channel theory taking into account the short range van der Waals interaction and a correction due to the strong dipole moment of dysprosium. We are able to extract the scattering length as a function of magnetic field associated with these resonances and obtain a background scattering length $a_{bg} = 91(16)\,a_0$. These results offer prospects of a tunability of the interactions in dysprosium, which we illustrate by observing the saturation of three-body losses.

DOI: 10.1103/PhysRevA.92.060702 PACS number(s): 34.50.Cx, 32.10.Dk

Magnetic Feshbach resonances have emerged as an essential tool for engineering interactions in atomic and molecular physics [1]. On the one hand they have been used to study genuine many-body systems in quantum gases, such as ultracold fermions in the Bose-Einstein condensate (BEC) to BCS crossover [2]. On the other hand they have allowed major advances in few-body physics, through the study of the Efimov effect [3], or the production of ground state molecules [4,5]. The continuous progress in cooling and control of atoms and molecules now opens the door to the study of novel condensed-matter systems using species with highly complex spectra. However, the complexity of the tunable Feshbach resonance spectrum of such species naturally raises the question as to whether precise control of interactions will be possible while avoiding harmful inelastic loss collisions. An illustration of this is found in the heavy, submerged shell elements dysprosium and erbium, both of which have been brought to quantum degeneracy [6–9]. Their complex electronic structure leads both to a large magnetic dipole moment and to a strong anisotropy of the van der Waals interaction, which give rise in turn to an extremely dense spectrum of very narrow Feshbach resonances, characterized by a chaotic distribution [10,11]. In order to use such lanthanide species to study long-range many-body interacting systems, it is essential to understand and be able to control their collisional properties.

In this Rapid Communication we demonstrate that broad Feshbach resonances actually exist for the lowest-lying state of ^{164}Dy , $J = 8$, $m_J = -8$. Furthermore, these resonances decouple from the chaotic background and thus offer a broad tunability of interactions that should be readily usable for few- and many-body studies. We present Feshbach spectroscopy measurements up to a magnetic field of 600 G. This spectroscopy reveals loss features with widths of the order of several gauss. Magnetic field modulation spectroscopy allows us to measure the binding energy of weakly bound dimers. In two cases we identify a bound state that has a persistent universal s-wave halo character over several gauss despite crossing with a large number of other bound states.

To study Feshbach resonances we prepare ultracold samples of ^{164}Dy atoms in their lowest Zeeman sublevel $m_J = -8$, with a magnetic moment $\mu_{at} = 9.93\mu_B$ where μ_B is the Bohr magneton; the details of our preparation methods are presented in [11,12]. The magnetic field was calibrated via radio-frequency spectroscopy between the two lowest Zeeman sublevels of ^{164}Dy. Feshbach spectroscopy is performed by ramping up a magnetic offset field in 15 ms to the desired target value in the range from 60 to 600 G. Typical samples contain 10^5 atoms at a temperature of 2.4 μK (see Supplemental Material [13] for details). The atoms are held for a wait time (2 s in Fig. 1) in a constant optical dipole trap where they undergo inelastic collisions inducing losses. Figure 1 shows the final atom number measured after a time of flight as a function of magnetic field with a resolution of 100 mG, mapping the Feshbach resonance spectrum. In this spectrum, an irregular pattern of several broad features appears on top of the sea of narrow resonances. Among them we identify, in particular, two distinct broad loss features located near 80 and 180 G. In the following, we focus on these two features. A magnetic field scan with fine resolution (20 mG) over these two resonances shows that inside the broad features, the dense background of narrow resonances remain (see Fig. 2, and [13]).

It is usually accepted that broad features in atom-loss spectra correspond to single broad resonances [14–17]. However, in the chaotic Feshbach spectrum of lanthanides, this correspondence is not straightforward, and the large number of interaction potentials prevents so far the association of a loss resonance to a particular bound state, except in a few cases at very low magnetic field as shown by Ref. [18]. To circumvent this shortcoming, we measure the bound states energy versus magnetic field using the standard technique of molecular association by magnetic field modulation [19,20]. We implement this spectroscopy on thermal clouds at a temperature T with 400 nK $\leqslant T \leqslant$ 700 nK, and focus on the regions $B \in$ [55 G,75 G], and $B \in$ [160 G,180 G]. The magnetic field is modulated around its bias value B at radio

*Corresponding author: i.ferrier-barbut@physik.uni-stuttgart.de

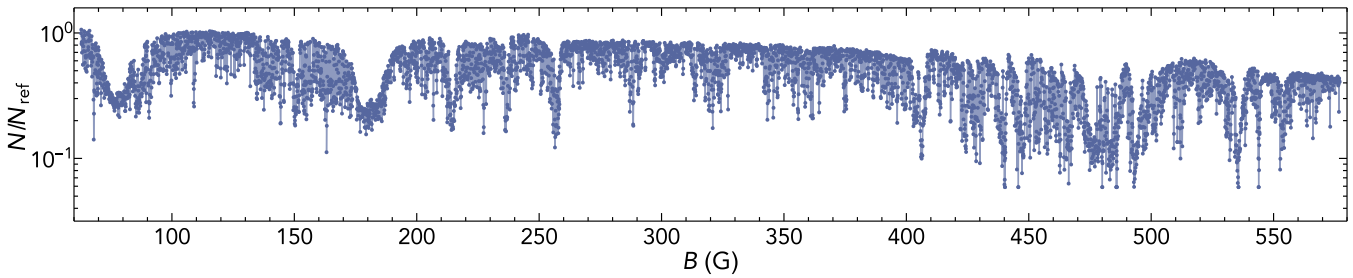

FIG. 1. (Color online) Atom-loss spectroscopy at a temperature of 2.4 μK mapping the Feshbach spectrum of ^{164}Dy in $J = 8$, $m_J = -8$ with 100 mG resolution. The atom number is normalized to a reference number $N_{\rm ref}$ taken at low field every 30 images.

frequencies during a time between 100 and 500 ms and with a modulation amplitude lying between 100 and 500 mG.

With this spectroscopy we are able to observe the states responsible for narrow resonances, but we also observe an isolated loss resonance with typical FWHM <30 kHz. This feature appears only on the low-field side of the two broad features at $B \simeq 80$ G and $B \simeq 180$ G and we associate it with weakly bound dimers with slowly varying binding energy $E_{\rm b}$. We extract $E_{\rm b}(B)$ [13] and plot it in red dots in Fig. 2(b). We put this in perspective with high-resolution atom-loss scans (a). In both cases we observe a very slow variation of $E_{\rm b}$ with magnetic field (<1 MHz over 10 G). This implies that these molecules have a magnetic moment μ very close to that of two free atoms, $\mu = 2\mu_{\rm at} - \frac{dE_{\rm b}}{dB} \simeq 20\mu_{\rm B}$. This value is the highest reported for a magnetic dipole [18]. Furthermore, we observe a a quadratic behavior for $E_{\rm b}(B)$ over several gauss, which reveals a coupling of the bound state with the two-atom continuum, conventionally termed the open channel [1]. This constitutes strong evidence that the observed bound state is in both cases the s-wave halo state that is found in the vicinity of Feshbach resonances. It is striking that this strong open channel character is found over a broad field range, despite the crossing of this state with many other bound states which are responsible for the narrow resonances dense background. In fact, near the lower resonance we observe signatures of avoided crossings with other bound states, visible in Fig. 2, which correlate with narrow resonances in the atom-loss spectrum.

We now provide a theoretical basis supporting the existence of such halo states. We base the theoretical analysis (detailed in [13]) of these observations on the dipole-modified s-wave scattering model in which a strong, open-channel-dominated resonance overlaps with many weak, narrow resonances. Our observations provide clear evidence that the effect of such a broad resonance persists across many narrow features and that

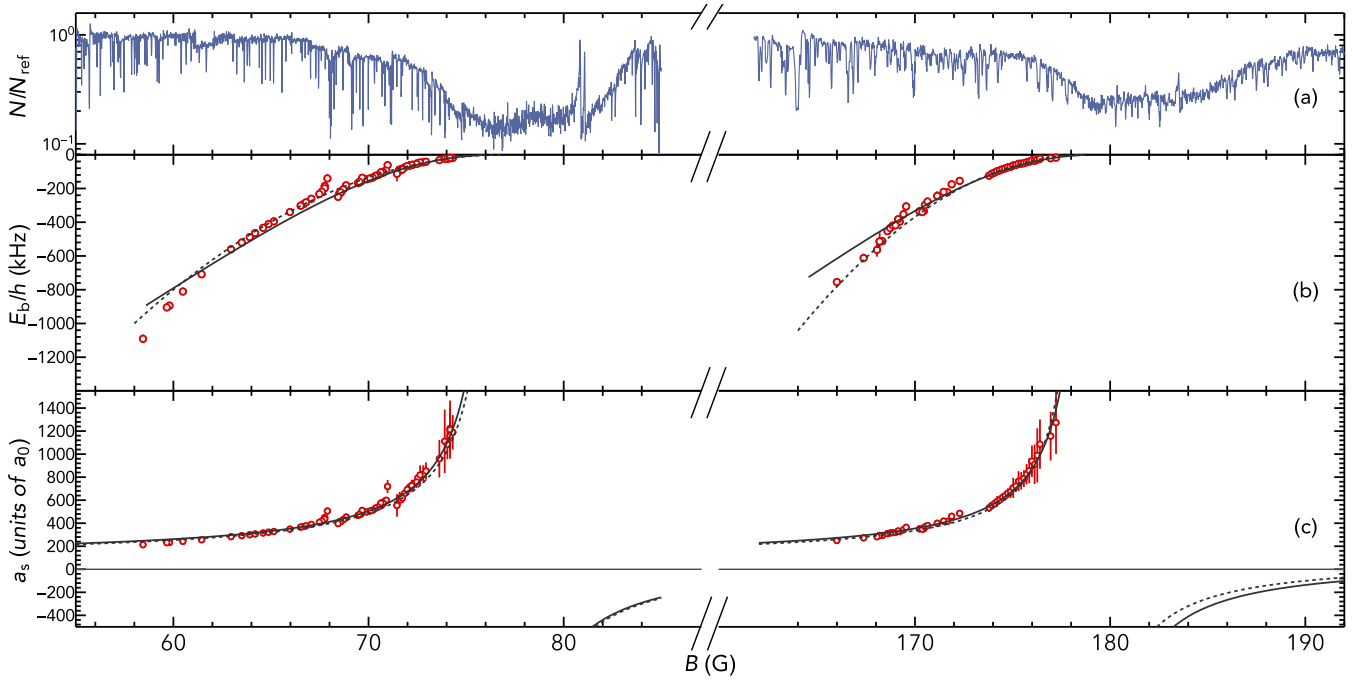

FIG. 2. (Color online) Broad Feshbach resonances of ^{164}Dy. (a) Atom-loss spectroscopy at $T = 500$ nK with a resolution of 20 mG. This spectrum shows one broad feature overlapping with many narrow resonances. (b) Binding energy of weakly bound dimers measured by magnetic field modulation spectroscopy (red circles). The solid (dashed) lines are obtained by fits of our data to the coupled-channel calculations (universal expression); from these fits we extract $a_{\rm bg}\,\Delta$ (Table I). (c) The red circles are obtained by converting the $E_{\rm b}(B)$ data using the coupled-channel calculations for $a(E_{\rm b})$. The sold lines are a fit to this data using Eq. (1). The dashed lines are the scattering length resulting from the fit of $E_{\rm b}(B)$ by the universal expression assuming $a_{\rm bg} = 91a_0$ (see main text).

TABLE I. Resonance parameters obtained from fitting the binding energy data to the universal formula (2) and to numerical coupled channels calculation.

Universal B_0	Universal $a_{bg}\Delta$	Numerical CC B_0	Numerical CC $a_{bg}\Delta$
76.9(5) G	2810(100) G a_0	76.8(5) G	2700(100) G a_0
178.8(6) G	2150(150) G a_0	179.1(6) G	2540(110) G a_0

away from narrow poles the scattering length a takes the usual expression

$$a(B) = a_{bg}\left(1 - \frac{\Delta}{B - B_0}\right), \tag{1}$$

where B_0 is the resonance pole, Δ its width, and a_{bg} the background scattering length. Furthermore, in the pole vicinity, binding energies $E_b \lesssim 1$ MHz are well approximated by the universal expression

$$E_b = -\frac{\hbar^2}{ma^2} \tag{2}$$

with m the mass of a dysprosium atom and $\hbar$ Planck's constant.

In our theoretical model we account for slight deviations from the universal relation (2) that result from the form of the long-range interaction potential which contains a typical van der Waals $-C_6/r^6$ term and a contribution from dipole-dipole interactions which for the s wave has $\propto 1/r^4$ character [21]. We construct a two-channel model with short-range coupling to a ramping bound state in the closed channel. The model parameters are a_{bg}, B_0, the magnetic moment difference between channel states $\delta\mu$, and the dimensionless pole strength $s_{res} \propto a_{bg}\Delta\delta\mu$ [1,13]. In general, large s_{res} values indicate open-channel-dominated resonances with highly universal behavior.

We use this coupled-channel (CC) model to describe the observed data. We found out that, as expected, the observed resonances have a high s_{res}. The data allows us to extract only a lower bound $s_{res} \gtrsim 10$ and we cannot obtain $\delta\mu$ either. Rather, we can only extract the product $a_{bg}\,\Delta$. We obtain $a_{bg}\,\Delta$ by a direct fit to our $E_b(B)$ data [13], the result of which is represented in Fig. 2(b) as a solid line. Second, we calculate the binding energy dependence on the scattering length $E_b(a)$ assuming that a_{bg} is close to $100\,a_0$, which will be justified below. We use it to translate the experimental binding energies to scattering lengths ([red circles in Fig. 2(c)]. Comparing the result to (1) also allows then to extract $a_{bg}\,\Delta$. As expected these two methods agree, with a fit result given in Table I. In addition, we also simply fit our $E_b(B)$ data by the universal quadratic dependence $[E_b = -(B - B_0)^2/(a_{bg}\Delta)^2]$ in the vicinity of the pole ($B > 66$ G and $B > 173$ G for the two resonances, respectively), with a fit result shown by the dashed line in Fig. 2(b) and in Table I. The coupled-channel theory and the universal fit are in close agreement with each other for the lower-field resonance and in reasonable agreement for the higher-field one. We assume $a_{bg} = 91 a_0$ (as justified below) to calculate $a(B)$ from the value of $a_{bg}\Delta$ given by the universal fit, in close agreement with the CC results. Our measure of a is valid when the magnetic field is tuned away from the narrow resonances as is visible in Fig. 2(c) (see also [13] for a close zoom).

The $E_b(B)$ data displays a quadratic field dependence well beyond the pole vicinity. This indicates that the observed broad resonances have decisive impact on the scattering properties over a large magnetic field region, although multiple couplings to other states can be found in the data. The investigation of the coupling between the halo state and other bound states will be the topic of future research.

Furthermore, we observe several indications that the broad resonance provides a background scattering length for the narrow resonances (see [13]) that indeed crosses zero on the high-field side of the 76.9 G resonance. For nonmagnetic atoms, a vanishing a results in an absence of elastic collisions at low temperature, rendering evaporation ineffective [22]. As a consequence, the final atom number and temperature after a given wait time are maximal. In the present case of Dy, though a vanishes, the dipolar interaction still induces collisions characterized by the dipolar length $D = m\mu_0\mu_{at}^2/8\pi\hbar^2 = 196\,a_0$ and the collisional cross section $\sigma_{dip} = 32\pi D^2/45$ [21,23]. Nonetheless, when $a = 0$, the cross section and then thermalization is minimal. This leads to a slower evaporation, characterized by a maximum in the background (in-between narrow resonances) atom number and temperature after a holding time in a constant trap. We do observe a maximum in these two observables, located for the atom number at $B_{Max,N} = 109(5)$ G and for temperature at $B_{Max,T} = 107(5)$ G (see [13] for a detailed figure). We join this observation with a second analysis, where we make use of the asymmetric line shape of the narrow resonances. Indeed for a narrow resonance, the sign of its "local" background scattering length $\tilde{a}_{bg}$ [13,24] influences the symmetry of its line shape in the atom-loss spectrum. If $\tilde{a}_{bg} > 0$ ($\tilde{a}_{bg} < 0$) the local zero crossing attached to this resonance is on the high- (low-) field side of its pole, and the three-body loss spectrum is expected to show a minimum near such zero crossing. Most of the resonances we observe are too narrow to asses the symmetry of their line shape given our resolution; however, we can still observe several resonances at different fields, and they show that below the 76.9 G resonance, $\tilde{a}_{bg} > 0$. For higher fields we observe $\tilde{a}_{bg} < 0$, up to a field of about 109 G where we actually observe a broad enough resonance, with a very symmetric line shape, corresponding to $\tilde{a}_{bg}$ close to zero (see [13] for spectra). This joint analysis shows that there is an overall "background" scattering length that reaches zero at a field $B = 108(5)$ G. Assuming that the broad s-wave halo resonance continues to provide the local B-dependent background to the narrow resonances [24], the universal model (1) then implies a width $\Delta = 3(6)$ G and $a_{bg} = 91(15)\,a_0$ for this resonance, justifying the assumptions made above. This value is also in agreement with the rethermalization measurements of Ref. [25] performed at low fields. While the data suggests the persistence of such strong s-wave halo influence across a set of narrow resonances consistently with [24], continuing experimental and theoretical work is needed to explore this effect.

We have characterized two Feshbach resonances with $s_{res} \gg 1$; we thus expect to observe effects characteristic for broad resonances. We now show evidence that our observations of atom losses in the vicinity of the poles are indeed

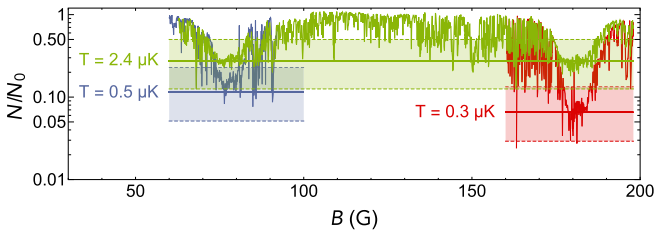

FIG. 3. (Color online) Atom loss spectroscopy for different experimental starting conditions and wait times. Final atom number N normalized to the initial one N_0 as a function of magnetic field, with a resolution of 100 mG. Green: initial temperature, $T_0 = 2.4\,\mu$K; initial density: $n_0 = 6.1(2.0) \times 10^{12}$ cm^{-3}, wait time $t = 2s$; blue: $T_0 = 0.5\,\mu$K, $n_0 = 3.7(1.3) \times 10^{12}$ cm^{-3}, $t =$ 500 ms; red: $T_0 = 0.3\,\mu$K, $n_0 = 4.6(1.6) \times 10^{12}$ cm^{-3} $t = 300$ ms. The temperature dependence of the saturation is well reproduced by the model of universal loss dynamics of unitary Bose gases of Ref. [28] (solid horizontal lines). The shaded regions represent the uncertainty on the results of the model given a one-standard deviation on all experimental parameters entering the model (temperature, atom number, trap frequency, and depth).

compatible with universal loss dynamics found at the center of broad resonances. For this we study atom losses at different temperatures. We observe that approaching the poles, the final atom number reaches a minimum that is the same on the two broad resonances. Furthermore, at lower temperature, the saturation is reached in a narrower field region and at a lower level despite a smaller central density in the initial conditions, Fig. 3.

The temperature dependence of three-body losses of Bose gases is known in the unitary regime where $a/\lambda_{\mathrm{dB}} \gg 1$, with λ_{dB} the thermal de Broglie wavelength. For a unitary Bose gas, the three-body loss parameter L_3 takes a $1/T^2$ behavior: $L_3 = \frac{\hbar^5}{m^3}\, 36\sqrt{3}\,\pi^2 \frac{1-e^{-4\eta_*}}{(k_B T)^2}$ [26,27]. The elasticity parameter η_* characterizes the efficiency with which three atoms in contact recombine to a dimer and a free atom.

We compare our findings with the loss dynamics model developed in [28], which predicts the final atom number taking into account two-body evaporation and three-body recombination. This model requires the knowledge of the trap depth, frequencies and initial atom number and temperature. These observables are calibrated (see [13]), and the only unknown parameter of this model is η_*. We then compare this model with a fixed η_* to the three data sets represented in Fig. 3. Within experimental uncertainties, we find that this model well reproduces our final atom number at resonance for a fixed η_* that we find to be

$$\eta_* = 0.07^{+0.17}_{-0.05}. \tag{3}$$

This value is close to the lowest reported values for alkali-metal atoms [29–34] and implies lifetimes comparable to these species in the unitary regime. Having this experimental information is valuable since no theoretical prediction is available. We observe that η_* is identical for our two broad resonances. A systematic study of $L_3(T)$ at resonance would yield a more precise measure.

In conclusion, it is remarkable that despite an apparent chaotic distribution of Feshbach resonances, isolated states can decouple from this background and lead to a simple description. This emergence of structure from a chaotic background is a generic feature of quantum-chaotic systems [35], with the emergent states associated with classical quasiperiodic trajectories in phase space [36]. Further work is needed to understand the origin of our broad resonances. However, the simple universal description provided here is sufficient to predict how to control dipolar gases in an understood way [37]. Between narrow resonances we obtain pure BECs with up to 25×10^3 atoms with long lifetimes ($\gtrsim$1 s). The scattering length obtained in this work can thus be confirmed with precision by the measurement of the density distribution of Bose-Einstein condensates. Such measurements are ongoing with our setup. This opens prospects of studying the few-body physics of bosonic dipoles; for instance, the energy of Efimov states is thought to be impacted by the dipole-dipole interaction [38].

We thank Axel Griesmaier for support at the early stage of the experiment. This work is supported by the German Research Foundation (DFG) within SFB/TRR21. H.K. acknowledges support by the "Studienstiftung des deutschen Volkes." K.J. acknowledges financial support by the Foundation for Polish Science International PhD Projects Programme cofinanced by the EU European Regional Development Fund and START program. P.S.J. acknowledges support by an AFOSF MURI grant.

T.M. and I.F.-B. contributed equally to this work.

[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).

[2] *The BCS-BEC Crossover and the Unitary Fermi Gas*, edited by W. Zwerger, Lecture Notes in Physics (Springer, Berlin, 2012).

[3] F. Ferlaino, A. Zenesini, M. Berninger, B. Huang, H.-C. Nägerl, and R. Grimm, Few-Body Syst. **51**, 113 (2011).

[4] J. G. Danzl, E. Haller, M. Gustavsson, M. J. Mark, R. Hart, N. Bouloufa, O. Dulieu, H. Ritsch, and H. C. Nägerl, Science **321**, 1062 (2008).

[5] K.-K. Ni, S. Ospelkaus, M. H. G. de Miranda, A. Pe'er, B. Neyenhuis, J. J. Zirbel, S. Kotochigova, P. S. Julienne, D. S. Jin, and J. Ye, Science **322**, 231 (2008).

[6] K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, and F. Ferlaino, Phys. Rev. Lett. **108**, 210401 (2012).

[7] K. Aikawa, A. Frisch, M. Mark, S. Baier, R. Grimm, and F. Ferlaino, Phys. Rev. Lett. **112**, 010404 (2014).

[8] M. Lu, N. Q. Burdick, S. H. Youn, and B. L. Lev, Phys. Rev. Lett. **107**, 190401 (2011).

[9] M. Lu, N. Q. Burdick, and B. L. Lev, Phys. Rev. Lett. **108**, 215301 (2012).

[10] A. Frisch, M. Mark, K. Aikawa, F. Ferlaino, J. L. Bohn, C. Makrides, A. Petrov, and S. Kotochigova, Nature (London) **507**, 475 (2014).

[11] T. Maier, H. Kadau, M. Schmitt, M. Wenzel, I. Ferrier-Barbut, T. Pfau, A. Frisch, S. Baier, K. Aikawa, L. Chomaz, M. J. Mark, F. Ferlaino, C. Makrides, E. Tiesinga, A. Petrov, and S. Kotochigova, Phys. Rev. X **5**, 041029 (2015).

[12] T. Maier, H. Kadau, M. Schmitt, A. Griesmaier, and T. Pfau, Opt. Lett. **39**, 3138 (2014).

[13] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.92.060702 for a description of the theoretical calculations and details on the atom loss and binding energy measurements.

[14] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, Nature (London) **392**, 151 (1998).

[15] V. Vuletic, A. J. Kerman, C. Chin, and S. Chu, Phys. Rev. Lett. **82**, 1406 (1999).

[16] M. Repp, R. Pires, J. Ulmanis, R. Heck, E. D. Kuhnle, M. Weidemüller, and E. Tiemann, Phys. Rev. A **87**, 010701 (2013).

[17] S.-K. Tung, C. Parker, J. Johansen, C. Chin, Y. Wang, and P. S. Julienne, Phys. Rev. A **87**, 010702 (2013).

[18] A. Frisch, M. Mark, K. Aikawa, S. Baier, R. Grimm, A. Petrov, S. Kotochigova, G. Quéméner, M. Lepers, O. Dulieu, and F. Ferlaino, Phys. Rev. Lett. **115**, 203201 (2015).

[19] S. T. Thompson, E. Hodby, and C. E. Wieman, Phys. Rev. Lett. **95**, 190404 (2005).

[20] C. Langmack, D. H. Smith, and E. Braaten, Phys. Rev. Lett. **114**, 103002 (2015).

[21] J. L. Bohn, M. Cavagnero, and C. Ticknor, New J. Phys. **11**, 055039 (2009).

[22] S. Jochim, M. Bartenstein, G. Hendl, J. H. Denschlag, R. Grimm, A. Mosk, and M. Weidemüller, Phys. Rev. Lett. **89**, 273202 (2002).

[23] We have $D = 3/2a_{\mathrm{dd}}$ where a_{dd} is defined in [39].

[24] K. Jachymski and P. S. Julienne, Phys. Rev. A **88**, 052701 (2013).

[25] Y. Tang, A. Sykes, N. Q. Burdick, J. L. Bohn, and B. L. Lev, Phys. Rev. A **92**, 022703 (2015).

[26] B. S. Rem, A. T. Grier, I. Ferrier-Barbut, U. Eismann, T. Langen, N. Navon, L. Khaykovich, F. Werner, D. S. Petrov, F. Chevy, and C. Salomon, Phys. Rev. Lett. **110**, 163202 (2013).

[27] A 7% modulation of L_3 for identical bosons is omitted in this expression (see [26]).

[28] U. Eismann, L. Khaykovich, S. Laurent, I. Ferrier-Barbut, B. S. Rem, A. T. Grier, M. Delehaye, F. Chevy, C. Salomon, L.-C. Ha, and C. Chin, arXiv:1505.04523.

[29] T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H. C. Nägerl, and R. Grimm, Nature (London) **440**, 315 (2006).

[30] M. Zaccanti, B. Deissler, C. D'Errico, M. Fattori, M. Jona-Lasinio, S. Müller, G. Roati, M. Inguscio, and G. Modugno, Nat. Phys. **5**, 586 (2009).

[31] R. J. Fletcher, A. L. Gaunt, N. Navon, R. P. Smith, and Z. Hadzibabic, Phys. Rev. Lett. **111**, 125303 (2013).

[32] N. Gross, Z. Shotan, S. Kokkelmans, and L. Khaykovich, Phys. Rev. Lett. **105**, 103203 (2010).

[33] P. Dyke, S. E. Pollack, and R. G. Hulet, Phys. Rev. A **88**, 023625 (2013).

[34] R. J. Wild, P. Makotyn, J. M. Pino, E. A. Cornell, and D. S. Jin, Phys. Rev. Lett. **108**, 145305 (2012).

[35] E. J. Heller, Phys. Rev. Lett. **53**, 1515 (1984).

[36] M. P. Jacobson, C. Jung, H. S. Taylor, and R. W. Field, J. Chem. Phys. **111**, 600 (1999).

[37] T. Lahaye, T. Koch, B. Fröhlich, M. Fattori, J. Metz, A. Griesmaier, S. Giovanazzi, and T. Pfau, Nature (London) **448**, 672 (2007).

[38] Y. Wang, J. P. D'Incao, and C. H. Greene, Phys. Rev. Lett. **106**, 233201 (2011).

[39] T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, and T. Pfau, Rep. Prog. Phys. **72**, 126401 (2009).