Total transmission of inhomogeneous electromagnetic waves at planar interfaces

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We present the total transmission effect of inhomogeneous electromagnetic waves between dissipative media. The total transmission of a p-polarized plane wave at the interface between two lossless dielectrics happens at the Brewster angle. However, when a dielectric-conductor interface is considered, the effect cannot be achieved and there is only a minimum, different from zero, of the reflection coefficient. We prove that, by considering an inhomogeneous plane wave, the total transmission can be obtained both at the interface between two dissipative media and at the dielectric-conductor interface.

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I. INTRODUCTION

The Brewster [1] angle is the angle of total transmission of a p-polarized plane wave at the interface between two lossless dielectrics [2]. In fact, if the second medium presents losses, i.e., the plane boundary is a dielectric-conductor interface, then the Fresnel reflection coefficient cannot be zero for any incident angle. In this case, the so-called pseudo-Brewster angle has been introduced. In the literature, two different definitions of the pseudo-Brewster angle have been given [3–5]: In one case it is the angle for which the reflection coefficient in p polarization is minimum; in the other case, called the second Brewster angle, it is the angle for which the ratio between the reflection coefficients in p and s polarization is minimum. Moreover, another angle, called the principal angle or third Brewster angle, can be defined as the angle at which the change of the phase difference between the parallel and perpendicular components of the wave after the reflection is 90° [6,7]. The electromagnetic interaction with an interface between dissipative materials has been widely studied in the literature [8,9], and the analysis of the complex Fresnel coefficients at the dielectric-conductor interface is an important topic in terms of both theory and application and it has already been the subject of recent works [10].

In this paper we analyze the interaction between an inhomogeneous wave and a planar boundary between dissipative media. This topic has been widely studied in the literature because of its applications, concerning surface-wave propagation, surface polaritons, and lateral and leaky-wave excitation [11]. We analyze the possibility of obtaining the total transmission of an inhomogeneous wave both at the boundary between two dissipative media and at a dielectric-conductor interface. Such total transmission can be obtained only if the incident wave is suitably inhomogeneous, i.e., the angle between the constant-phase and the constant-amplitude planes assumes a well-defined value, and at a critical angle, which can be defined as a Brewster angle, similarly to the case of lossless materials. The connection between the Brewster angle and the propagation of inhomogeneous waves bounded to the interface has been pointed out in the literature [12]. Here we show how the effect can be obtained with waves propagating far away from the interface. The effect is of extreme interest especially in the case of the dielectric-conductor interface, when an inhomogeneous incident wave is considered. Such an incident wave can be obtained, for example, through a leaky-wave antenna, i.e., an antenna able to generate an inhomogeneous wave in free space. Such antennas have been widely studied in the literature, in both microwave and optics regimes [13,14].

II. THEORETICAL FORMULATION

Let us consider a Cartesian reference frame (x,y), with the y axis on the interface between two media. The half space $x<0$ is filled with medium 1, with relative permittivity ε_1, while the half space $x>0$ is filled with medium 2, with relative permittivity ε_2 (see Fig. 1). In the following, where we talk about permittivity, we always intend the relative one. We suppose nonmagnetic and dissipative media, i.e., with relative permeabilities $\mu_1=\mu_2=1$ and with complex relative electric permittivities $\varepsilon_s=\varepsilon_s'+\iota\varepsilon_s''$ with $\varepsilon_s',\varepsilon_s''\in\mathbb{R}$ and $s=1,2$. We consider a linearly polarized plane wave, in s or p polarization, coming from medium 1, incident on the interface between the two media. Because the incident plane wave propagates in a dissipative medium, it is characterized by a phase and an attenuation vector $\boldsymbol{\beta}_1$ and $\boldsymbol{\alpha}_1$, respectively. Here we suppose that the phase and attenuation vectors and the unit vector perpendicular to the interface are coplanar. This hypothesis is not needed for our purpose, but allows us to give simple analytical expressions of the transmitted angles. The phase vector forms an angle ξ_1 with the x axis, while the attenuation vector forms an angle ζ_1 with the same axis (see Fig. 1). We define $\eta_1=\zeta_1-\xi_1$ as the angle between the phase and the attenuation vectors, sometimes called the inhomogeneity angle, because when $\eta=0$ the wave is homogeneous. If the medium 1 is lossy, then the magnitudes of the phase and attenuation vectors β_1 and α_1 are related to the medium parameters and to the inhomogeneity angle by well-known relations [15]. In these conditions, also the transmitted wave has a propagation vector composed of a phase and an attenuation vector $\boldsymbol{\beta}_2$ and $\boldsymbol{\alpha}_2$, respectively, characterized by the angles ξ_2, ζ_2, and η_2. The expressions of the magnitudes and of the angles of the transmitted wave have been presented in [16,17].

To find the total transmission effect, we have to impose the cancellation of the reflection coefficient. The expressions of the Fresnel coefficients in the case of lossy media are exactly the same as in the lossless case [7]. If we write the reflection coefficient in the s polarization, it can be seen that

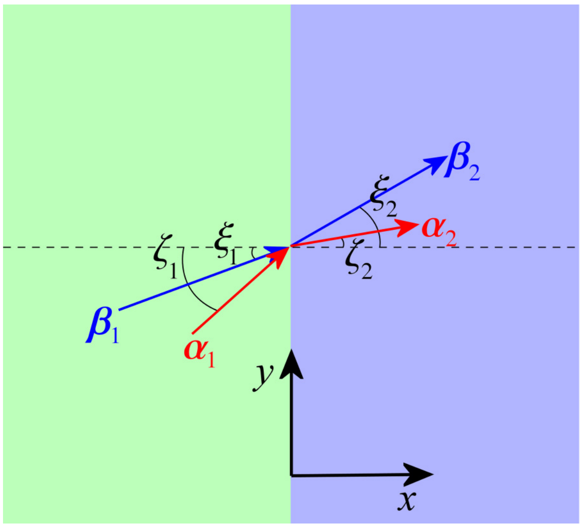

FIG. 1. (Color online) Geometry of the problem.

it cannot be equal to zero for any incident angle, as in the lossless case. On the other hand, imposing the cancellation of the Fresnel reflection coefficient in the p polarization, an expression of the tangential component of the wave vector can be found. Such a condition is well known in the literature, because it is the same expression of the wave vector of a surface wave [18]

$$k_{1y} = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_2+\varepsilon_1}} = k_0\gamma. \tag{1}$$

The analysis of this expression is important and requires us to consider different aspects. First of all, we have to establish whether the first medium is lossless or lossy. In the former case, if the incident wave is homogeneous, then the left-hand side of the equation is a purely real number: As a consequence, if the second medium is dissipative, i.e., its permittivity is a complex number, the equation cannot be satisfied. On the other hand, if the incident wave is inhomogeneous, the left-hand side can be a complex number and the equation can be satisfied. In the latter case, both sides of the equation can be complex numbers and the equation can be satisfied. Let us consider the real and imaginary parts of Eq. (1) separately:

$$\beta_1 \sin\xi_B = k_0\gamma', \tag{2}$$

$$\alpha_1 \sin\zeta_B = k_0\gamma'', \tag{3}$$

where γ' and γ'' are the real and imaginary parts of the complex number γ, respectively. We start from the case of two dissipative materials, i.e., we suppose that both ε_1 and ε_2 are complex numbers. Inserting Eqs. (2) and (3) in the real and imaginary parts of the dispersion equation of a dissipative free-space [15], we obtain

$$\frac{\gamma'^2}{\sin^2\xi_B} - \frac{\gamma''^2}{\sin^2\zeta_B} = \varepsilon_1', \tag{4}$$

$$\frac{\varepsilon_1''\sin\xi_B\sin\zeta_B}{2\cos(\zeta_B-\xi_B)} = \gamma'\gamma''. \tag{5}$$

With some algebra, we can find the following expressions:

$$\cot^2\zeta_B = \frac{\gamma'^2 - (\varepsilon_1' - \gamma'^2 + \gamma''^2)\tan^2\xi_B}{\gamma''^2\tan^2\xi_B}, \tag{6}$$

$$(\varepsilon_1'' - 2\gamma'\gamma'')\tan\xi_B = 2\gamma'\gamma''\cot\zeta_B. \tag{7}$$

Squaring Eq. (7) and substituting the expression (6), a biquadratic equation for $\tan\xi_B$ can be obtained:

$$[\mathrm{Im}(\varepsilon_1-\gamma^2)]^2\tan^4\xi_B + 4\gamma'^2\mathrm{Re}(\varepsilon_1-\gamma^2)\tan^2\xi_B - 4\gamma'^4 = 0. \tag{8}$$

Solving for $\tan^2\xi_B$ and with some algebra, the following expression can be found:

$$\tan^2\xi_B = \frac{-2\gamma'^2\mathrm{Re}(\varepsilon_1-\gamma^2)}{[\mathrm{Im}(\varepsilon_1-\gamma^2)]^2}\left\{1-\sqrt{1+\left[\frac{\mathrm{Im}(\varepsilon_1-\gamma^2)}{\mathrm{Re}(\varepsilon_1-\gamma^2)}\right]^2}\right\}. \tag{9}$$

In the square root the tangent of the phase ϕ of the complex number $\varepsilon_1 - \gamma^2$ appears. With some algebra, the expression (9) assumes the form

$$\tan^2\xi_B = \frac{2\gamma'^2}{|\varepsilon_1-\gamma^2|}\frac{1-\cos\phi}{\sin^2\phi}. \tag{10}$$

Noting that $\varepsilon_1 - \gamma^2 = \varepsilon_1^2/(\varepsilon_1+\varepsilon_2)$ and that $\gamma' = |\gamma|\cos\phi_\gamma$, where ϕ_γ is the phase of the complex number γ, we obtain the following expression:

$$\tan\xi_B = \left|\frac{n_2}{n_1}\right|\sqrt{\frac{2\cos\phi_\gamma}{1+\cos\phi}}, \tag{11}$$

where n_1 and n_2 are the complex refractive indices of the two media. Similarly, an expression for the angle of total transmission of the attenuation vector can be found by inserting Eq. (11) into Eq. (6), obtaining

$$\tan\zeta_B = \left|\frac{n_2}{n_1}\right|\sqrt{\frac{2\sin\phi_\gamma}{1-\cos\phi}}. \tag{12}$$

Expressions (11) and (12) are of extreme interest: In fact, when the media are lossless, the permittivities become real and the arguments ϕ and ϕ_γ become zero. As a consequence, the formula (11) reduces to the well-known expression of the Brewster angle, while formula (12) becomes indeterminate, because the angle of the attenuation vector cannot be defined for a homogeneous wave in a lossless material.

Equation (11) can be seen as a generalized expression of the Brewster angle in the case of two dissipative media. It is important to say that, once the angles ξ_B and ζ_B are fixed, the angle $\eta_B = \zeta_B - \xi_B$ is fixed too. Now, while the angles ξ_B and ζ_B depend on the direction perpendicular to the interface, the inhomogeneity angle is an intrinsic property of the incident wave [19]. Therefore, given two dissipative media, only a particular inhomogeneous wave, with a fixed value of the angle η_B, depending on the properties of the materials, can be totally transmitted from medium 1 to medium 2. An expression of $\tan\eta_B$ can be found starting from Eq. (5): With the trigonometric identity $\cot\xi_B - \cot\zeta_B = \sin(\zeta_B -$

$\xi_B)/\sin\xi_B\sin\zeta_B$, the following expression can be obtained:

$$\tan\eta_B = \frac{2\gamma'\gamma''}{\varepsilon_1''}(\cot\xi_B - \cot\zeta_B). \tag{13}$$

An inhomogeneous wave, with inhomogeneity angle η_B, is totally transmitted if its incident phase vector impinges at an angle given by Eq. (11). For these reasons, we can call the angle ξ_B the Brewster angle for a plane wave incident at the interface between two dissipative dielectrics.

The interaction at the interface between two lossy media, studied above, can be interesting in several applications, e.g., in the surface-polariton propagation in stratified structures. However, the case of a dielectric-conductor interface, i.e., when medium 1 is lossless, is of extreme interest too. If we consider the expression of the tangential component of the incident wave vector (1) and we consider a homogeneous incident wave, then the left-hand side of the equation is real and the right-hand side of the equation is complex. This is the reason why the total transmission is not possible at the dielectric-conductor interface when a homogeneous incident wave is considered. On the other hand, if an incident inhomogeneous wave is considered, the left-hand side of Eq. (1) is complex and the condition can be matched. Before analyzing this scenario, let us point out that an inhomogeneous wave in a lossless medium is not a novelty in the literature. In fact, the possibility of having an inhomogeneous wave in a dielectric, with $\alpha_1 \neq 0$ and $\eta_1 = \pi/2$, is well known [20]. These waves are often called leaky waves, because they can be obtained by some leakage effects on a guiding structure. The generation of leaky waves is a widely studied topic in the literature and there are examples of leaky-wave antennas in both microwave and optical regimes [13,14,21]. It is important to recall that the same expressions for the magnitude β_1 and α_1 that we apply in the lossy case do not hold for the leaky waves. When an inhomogeneous wave propagates in a lossless medium, these magnitudes depend on the antenna. This means that by designing the antenna we fix, for example, β_1, and the relevant α_1 is obtained by the dispersion equation.

Now we show how the total transmission in a dissipative medium can be obtained considering an inhomogeneous plane wave incident from a lossless material. Equations (2) and (3), when the first medium is lossless and for an inhomogeneous incident wave, i.e., with the conditions $\varepsilon_1 \in \mathbb{R}$, $\alpha_1 \neq 0$, and $\eta_1 = \pi/2$, can be written as follows:

$$\beta_1 \sin\xi_B = k_0\gamma', \tag{14}$$

$$\alpha_1 \cos\xi_B = k_0\gamma''. \tag{15}$$

Squaring Eqs. (14) and (15) and summing side to side, we obtain

$$\left(\frac{k_0\gamma'}{\beta_1}\right)^2 + \left(\frac{k_0\gamma''}{\alpha_1}\right)^2 = 1. \tag{16}$$

If one considers the dispersion equation in a lossless free space, Eq. (16) becomes a biquadratic equation in β_1. The solution of such an equation is the following expression:

$$\beta_B = k_0\sqrt{\frac{\varepsilon_1 + |\gamma|^2}{2}}\sqrt{1 + \sqrt{1 - \varepsilon_1\left(\frac{2\gamma'}{\varepsilon_1 + |\gamma|^2}\right)^2}}. \tag{17}$$

It can easily be proved that this magnitude always exists if $\gamma'' \neq 0$, i.e., if medium 2 is dissipative. Equation (17) represents the magnitude of the phase vector that an incident inhomogeneous wave from a lossless dielectric must have in order to be totally transmitted in a dissipative material; for this reason we can call it the Brewster magnitude. At this point we can obtain the total transmission angle from the ratio between Eqs. (14) and (15), giving

$$\tan\xi_B = \frac{\alpha_B}{\beta_B}\cot\phi_\gamma, \tag{18}$$

where α_B is the magnitude of the attenuation vector relevant to the Brewster magnitude. At a dielectric-conductor interface there is only one inhomogeneous wave that can be totally transmitted from medium 1 to medium 2 and it is the one with $\beta_1 = \beta_B$. This condition is analogous to the condition $\eta_1 = \eta_B$ in the case of two dissipative media.

Finally, from the above considerations, the following question can be asked: Is it possible to obtain the total transmission of an inhomogeneous wave at the interface between two lossless dielectrics? As is well known, it is possible for homogeneous waves, but, as we saw, the inhomogeneous waves show behaviors extremely different from them. To answer the question, we just have to consider again Eq. (1). If medium 1 is lossless, then $\eta_1 = \pi/2$. Moreover, if medium 2 is lossless too, then $\gamma \in \mathbb{R}$, i.e., $\gamma'' = 0$. With these conditions, the following result can be found:

$$\beta_B = k_0\gamma, \tag{19}$$

$$\xi_B = \pi/2. \tag{20}$$

These are the characteristics of a surface wave, with a fixed magnitude of the phase vector. It is extremely interesting to note that this magnitude is the magnitude of the tangential component of the wave vector of a surface polariton [22]. The connection between surface waves at the planar boundary between two lossless dielectrics and the Brewster angle has been already studied in the literature [12].

III. RESULTS

In this section we propose some examples of total transmission of inhomogeneous plane waves at the interface between two dissipative materials and at dielectric-conductor interfaces. As a first example, let us consider an interface between aluminum and gold in the infrared range and in particular at the wavelength $\lambda = 7.7$ μm, where the two materials have, respectively, the following permittivities: $\varepsilon_1 = 0.12 + i\ 0.04$ and $\varepsilon_2 = 0.84 + i\ 1.91$ [23]. Computing the inhomogeneity Brewster angle, we find $\eta_B = 31.22°$. In Fig. 2 the reflection coefficient as a function of the angle of the incident phase vector is shown in two cases: when the incident wave is homogeneous, with $\eta_1 = 0$, and when it is inhomogeneous, with $\eta_1 = \eta_B$. We see that when the incident wave is homogeneous, the reflection coefficient in p polarization has

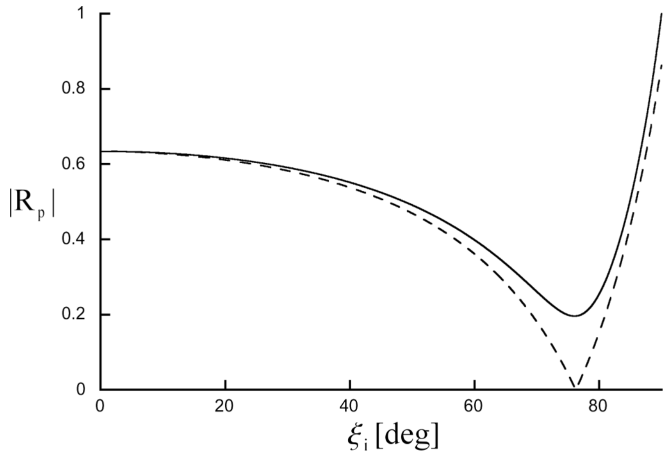

FIG. 2. Magnitude of the Fresnel reflection coefficient for an interface between aluminum and gold at $\lambda = 7.7$ μm, with permittivities $\varepsilon_1 = 0.12 + i\ 0.04$ and $\varepsilon_2 = 0.84 + i\ 1.91$, respectively. Two cases are considered: when the incident wave is homogeneous (solid line) and when the incident wave is inhomogeneous with $\eta_1 = \eta_B$ (dashed line).

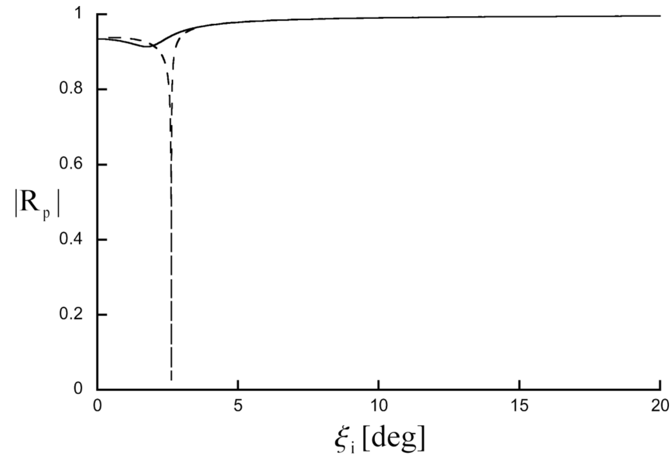

FIG. 3. Magnitude of the Fresnel reflection coefficient for an interface between seawater and a loamy wet soil, with permittivities and conductivities $\varepsilon_1 = 81$, $\sigma_1 = 100$ S m^{-1} and $\varepsilon_2 = 15$, $\sigma_2 = 0.1$ S m^{-1}, respectively. Two cases are considered: when the incident wave is homogeneous (solid line) and when the incident wave is inhomogeneous with $\eta_1 = \eta_B$ (dashed line).

a minimum, different from zero, for the pseudo-Brewster angle $\xi_{\mathrm{PB}} = 76.04^\circ$. However, when the incident wave is inhomogeneous, with $\eta_1 = \eta_B$, the reflection coefficient has a minimum in zero, for the Brewster angle $\xi_B = 76.32^\circ$. On the one hand, we see that the pseudo-Brewster angle and the Brewster angle are close to each other. On the other hand, we can see that simply changing the inhomogeneity angle, the reflection coefficient decreases and the minimum reaches zero for $\eta_1 = \eta_B$.

In order to give another example, we consider the interface between seawater and a loamy wet soil, at a frequency of 100 MHz. At such a frequency the permittivities and conductivities of the materials are the following [24]: $\varepsilon_1 = 81$, $\sigma_1 = 100$ S m^{-1} and $\varepsilon_2 = 15$, $\sigma_2 = 0.1$ S m^{-1}. With these media, the Brewster inhomogeneity angle is $\eta_B = -1.40^\circ$. In Fig. 3 the reflection coefficient for two different incident waves, with $\eta_1 = 0$ and $\eta_1 = \eta_B$, respectively, is shown. We see again that the homogeneous wave has a slightly pronounced minimum at the pseudo-Brewster angle $\xi_{\mathrm{PB}} = 1.76^\circ$, while the inhomogeneous wave presents an extremely pronounced minimum at the Brewster angle $\xi_B = 2.64^\circ$. Again, the two angles are very close. In the last case the reflection coefficient is close to unity for almost all the incident angles because of the large difference between the electric densities of the two media. This fact makes the total transmission effect extremely peaked. From Fig. 3 we see how different the behaviors of a homogeneous and an inhomogeneous wave can be at the planar interface between two media.

Now we show some examples of total transmission of inhomogeneous plane waves when the first medium is lossless. Let us consider the interface between air, i.e., $\varepsilon_1 = 1$, and gold at $\lambda = 7.7$ μm, with permittivity $\varepsilon_2 = 0.84 + i1.91$. The normalized Brewster magnitude with respect to the vacuum wave number is $\beta_B/k_0 = 1.04$. This magnitude is extremely close to unity, as it is in the conventional leaky-wave antennas [13]. In Fig. 4 the reflection coefficient in p polarization is shown for a homogeneous incident wave and for an inhomogeneous wave with the Brewster amplitude $\beta_1 = \beta_B$. We computed the reflection coefficient by the analytical expression and by an electromagnetic simulation implemented on the software COMSOL MULTIPHYSICS. We can note, in the case of an air-conductor interface, a behavior similar to the previously mentioned case of two lossy media. When the incident wave is homogeneous, the reflection coefficient presents a minimum different from zero. This minimum is exactly at the pseudo-Brewster angle $\xi_{\mathrm{PB}} = 52.57^\circ$, well known in the literature. However, when the incident wave is inhomogeneous, i.e., it is a leaky wave, then the minimum

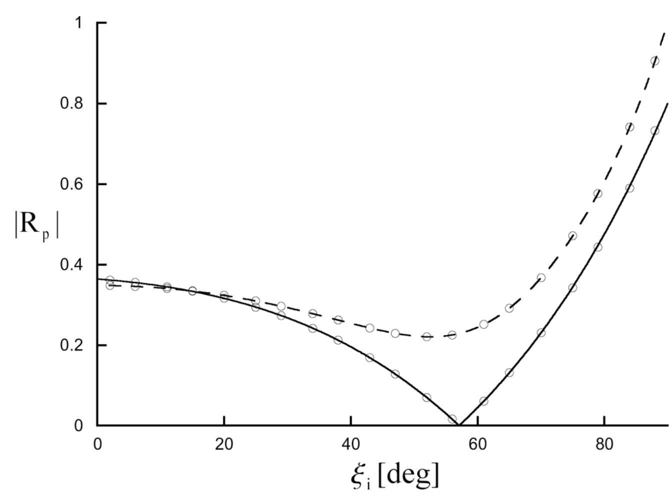

FIG. 4. Magnitude of the Fresnel reflection coefficient for an interface between air and gold at $\lambda = 7.7$ μm, with permittivity $\epsilon_2 = 0.84 + i\ 1.91$. Two cases are considered: when the incident wave is homogeneous (dashed line) and when the incident wave is inhomogeneous with $\beta_1 = \beta_B$ (solid line). In both cases the reflection coefficient has been computed by an electromagnetic simulation (circles).

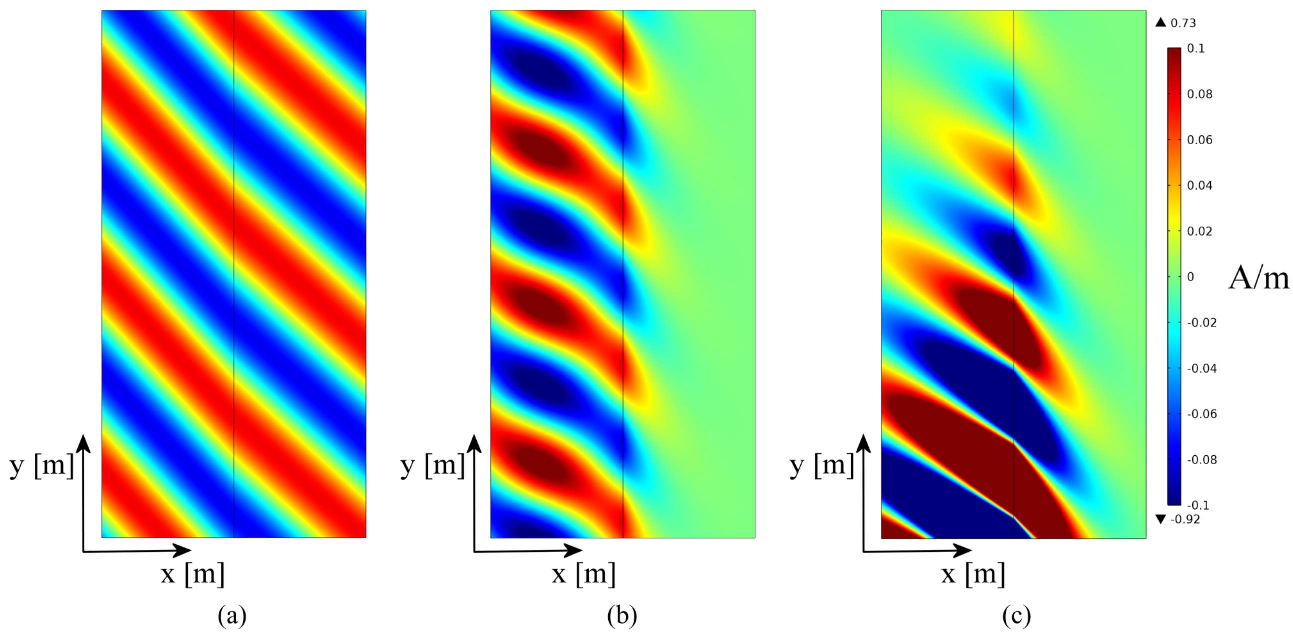

FIG. 5. (Color online) Two-dimensional maps (plotted on the same scale) of the real part of the magnetic field of a plane wave in p polarization incident at the interface between a vacuum and medium 2 with relative permittivity ε_2. (a) Medium 2 is lossless, with $\varepsilon_2 = 0.84$, the incident wave is homogeneous, and the incident angle is the Brewster angle. (b) Medium 2 is dissipative, with $\varepsilon_2 = 0.84 + i1.91$ (gold at wavelength $\lambda = 7.7\ \mu$m), the incident wave is homogeneous, and the incident angle is the pseudo-Brewster angle. (c) Medium 2 is dissipative [as in (b)], the incident wave is inhomogeneous with $\beta_1 = \beta_B$, and the incident angle is the Brewster angle.

goes to zero and we find that an incident angle $\xi_B = 57.11^\circ$ exists for which the wave is totally transmitted.

In Fig. 5 two-dimensional maps relevant to plane waves in p polarization, obtained by electromagnetic simulations, are shown. The real part of the magnetic-field component orthogonal to the plane of incidence is represented. Three cases are considered: Medium 2 is lossless, with $\varepsilon_2 = 0.84$, the incident wave is homogeneous, and the incident angle is the Brewster angle [Fig. 5(a)]; medium 2 is dissipative, with $\varepsilon_2 = 0.84 + i1.91$ (gold at wavelength $\lambda = 7.7\ \mu$m), the incident wave is homogeneous, and the incident angle is the pseudo-Brewster angle [Fig. 5(b)]; and medium 2 is dissipative [the same as in Fig. 5(b)], the incident wave is inhomogeneous with $\beta_1 = \beta_B$, and the incident angle is the Brewster angle [Fig. 5(c)]. We note that when the incident wave is homogeneous and medium 2 is lossless, the wave in the first medium, on the left-hand side of Fig. 5(a), presents undistorted constant-phase planes, since there is no reflected wave. In contrast, when the incident wave is homogeneous and medium 2 is dissipative, the wave in the first medium, on the left-hand side of Fig. 5(b), is the superposition of the incident and the reflected waves; as a consequence, the distortion of the constant-phase planes is apparent. On the other hand, when the incident wave is inhomogeneous and medium 2 is dissipative, the wave in the first medium, on the left-hand side of Fig. 5(c), presents again undistorted constant-phase planes, the reflected wave being absent.

As another important example, let us consider the interface between air ($\epsilon_1 = 1$) and seawater at the frequency of 100 MHz, with permittivity and conductivity $\varepsilon_2 = 81$ and $\sigma_2 = 100\ \mathrm{S\,m^{-1}}$, respectively. In this case the Brewster magnitude is $\beta_B/k_0 = 1.000\,014$, the Brewster angle $\xi_B = 89.70^\circ$, and the pseudo-Brewster angle $\xi_{\mathrm{PB}} = 89.58^\circ$. In Fig. 6 the reflection coefficient is shown in both cases of a homogeneous and an inhomogeneous incident wave. We find that both the Brewster and the pseudo-Brewster angles are near the grazing incidence. Moreover, at the Brewster angle the reflection coefficient presents an extremely narrow peak. Both these facts are due to the extremely high ratio between the refractive indices of the seawater and the air at the working frequency.

For the sake of completeness, we want to analyze two more scenarios. The first is the case in which medium 1 is dissipative and medium 2 is lossless. The same procedure

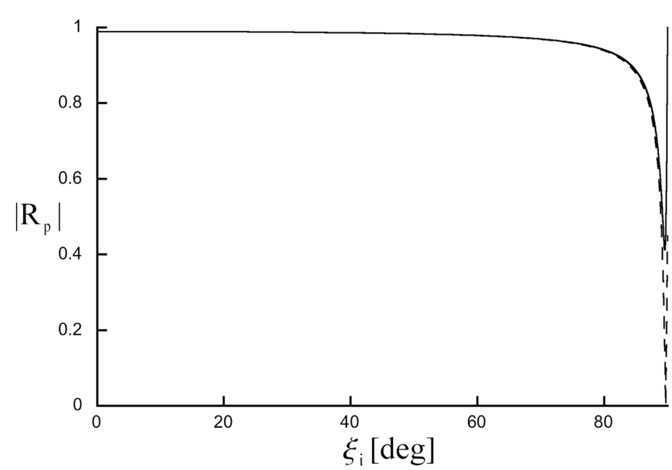

FIG. 6. Magnitude of the Fresnel reflection coefficient for an interface between air and seawater at a frequency of 100 MHz ($\varepsilon_1 = 1$, $\varepsilon_2 = 81$, and $\sigma_2 = 100\ \mathrm{S\,m^{-1}}$). Two cases are considered: when the incident wave is homogeneous (solid line) and when the incident wave is inhomogeneous with $\beta_1 = \beta_B$ (dashed line).

followed for two lossy media can be adopted. Actually, we obtain that the expressions of ξ_B and ζ_B, given in Eqs. (11) and (12), respectively, are still valid. However, in this case, we find two different behaviors of the transmitted wave. If, on the one hand, the incident wave, from the lossy medium, has the attenuation vector perpendicular to the interface, i.e., $\zeta_1 = 0$, then the transmitted wave will be a homogeneous wave in a lossless medium, i.e., $\alpha_2 = 0$. On the other hand, if $\zeta_1 \neq 0$, then the transmitted wave will be an inhomogeneous wave in a lossless medium, i.e., with $\eta_2 = \pi/2$.

IV. CONCLUSION

In this paper we generalized the concept of the Brewster angle for the case of plane waves incident on the interface between dissipative materials. In the case of two dissipative materials the total transmission is possible when the incident plane wave is p polarized and for a suitable inhomogeneous angle, i.e., the angle between the phase and the attenuation vectors. When such a condition is satisfied, the total transmission occurs for a particular angle of incidence of the phase vector, which we called Brewster angle. On the other hand, when the first medium is lossless and the second medium is lossy, then the total transmission can be obtained when the incident wave is inhomogeneous. Examples of inhomogeneous plane waves in a lossless dielectric are those generated by leaky-wave antennas. In this case the total transmission occurs when the magnitude of the incident phase vector assumes a particular value that we called the Brewster magnitude. When such a condition is satisfied, then the wave is totally transmitted for a particular incident angle of the phase vector. We showed examples of total transmission in both cases of two dissipative media and of the dielectric-conductor interface on both optical and radio-frequency regimes.

[1] D. Brewster, On the laws which regulate the polarization of light by reflection from transparent bodies, Philos. Trans. R. Soc. London **105**, 125 (1815).

[2] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).

[3] R. F. Potter, Analytical determination of optical constants based on the polarized reflectance at a dielectric-conductor interface, J. Opt. Soc. Am. **54**, 904 (1964).

[4] G. P. Ohman, The pseudo-Brewster angle, IEEE Trans. Antennas Propag. **AP-25**, 903 (1977).

[5] S. Y. Kim, and K. Vedam, Analytical solution of the pseudo-Brewster angle, J. Opt. Soc. Am. A **3**, 1772 (1986).

[6] H. B. Holl, Specular reflection and characteristics of reflected light, J. Opt. Soc. Am. **57**, 683 (1967).

[7] R. M. A. Azzam, and T. F. Thonn, Maximum minimum reflectance of parallel-polarized light at interfaces between transparent and absorbing media, Appl. Opt. **22**, 4155 (1983).

[8] C. von Fragstein, Energieübergang an der grenze zweier absorbierende medien mit einer anwendung auf die wärmestrahlung in absorbierenden körpern, Ann. Phys. (Leipzig) **442**, 63 (1950).

[9] C. von Fragstein, Ist eine lichtbewegung stets umkehrbar?, Opt. Acta **2**, 16 (1955).

[10] R. M. A. Azzam, Complex reflection coefficients of p- and s-polarized light at the pseudo-Brewster angle of a dielectric-conductor interface, J. Opt. Soc. Am. A **30**, 1975 (2013).

[11] F. Frezza and N. Tedeschi, Electromagnetic inhomogeneous waves at planar boundaries: Tutorial, J. Opt. Soc. Am. A **32**, 1485 (2015).

[12] H. M. Barlow and J. Brown, *Radio Surface Waves* (Clarendon, Oxford, 1962).

[13] D. R. Jackson and A. A. Oliner, in *Modern Antenna Handbook*, edited by C. Balanis (Wiley, New York, 2008), Chap. 7, pp. 325–368.

[14] S. Campione, C. Guclu, Q. Song, O. Boyraz, and F. Capolino, An optical leaky wave antenna with Si perturbations inside a resonator for enhanced optical control of the radiation, Opt. Express **20**, 21305 (2012).

[15] L. J. Adler, R. B. Chu, and R. M. Fano, *Electromagnetic Energy Transmission and Radiation* (MIT Press, Cambridge, 1969).

[16] J. E. Roy, New results for the effective propagation constants of nonuniform plane waves at the planar interface of two lossy media, IEEE Trans. Antennas Propag. **51**, 1206 (2003).

[17] F. Frezza and N. Tedeschi, Deeply penetrating waves in lossy media, Opt. Lett. **37**, 2616 (2012).

[18] P. Halevi, in *Electromagnetic Surface Modes*, edited by A. D. Boardman (Wiley, New York, 1982), Chap. 7, pp. 249–304.

[19] F. Frezza, and N. Tedeschi, On the electromagnetic power transmission between two lossy media: Discussion, J. Opt. Soc. Am. A **29**, 2281 (2012).

[20] T. Tamir, Inhomogeneous wave types at planar interfaces: III—Leaky waves, Optik **38**, 269 (1973).

[21] Y. Wang, A. S. Helmy, and G. V. Eleftheriades, Ultra-wideband optical leaky-wave slot antennas, Opt. Express **19**, 12392 (2011).

[22] M. A. Noginov, G. Zhu, M. Mayy, B. A. Ritzo, N. Noginova, and V. A. Podolskiy, Stimulated emission of surface plasmon polaritons, Phys. Rev. Lett. **101**, 226806 (2008).

[23] M. J. Weber, *Handbook of Optical Materials* (CRC, Boca Raton, 2003).

[24] D. J. Daniels, *Ground Penetrating Radar*, 2nd ed. (IEEE, Piscataway, 2004).