Strong squeezing in periodically modulated optical parametric oscillators

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We consider specific signatures of squeezing for time-modulated light fields and propose the scheme of an optical parametric oscillator driven by a continuously modulated pump field. The application of a periodically modulated driving field instead of a continuous wave field drastically improves the degree of quadrature integral squeezing. This quantity goes below the standard limit of 50% relative to the level of vacuum fluctuations. We develop semiclassical and quantum theories of an optical parametric oscillator under the influence of a pump field with harmonically modulated amplitude for all operational regimes, including numerical simulations at the threshold point. The results can be directly applied in time-resolved quantum communication protocols.

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I. INTRODUCTION

Squeezed states of light play an important role in the development of quantum physics. These states have been widely employed to achieve a measurement sensitivity beyond the standard quantum limits in applications such as precision interferometry and atomic spectroscopy. Squeezed light was first produced using atomic sodium as a nonlinear medium [1], optical fibers [2], and nonlinear crystals [3]. Squeezed-state generation using a Josephson parametric amplifier has been also considered [4], see also Ref. [5]. Novel discoveries made with squeezed states include quantum information processing with continuous variables [6]. Single-mode squeezed states have also served as a continuous-variable entanglement source, since combining two single-mode squeezed states at a beam splitter creates an entangled two-mode state [7–9]. Substantial squeezing has been observed in modern experiments in applications to gravitational wave detectors and biological measurements. Different schemes for the generation of squeezed light states have been proposed and realized over the years. Among these, we can mention the generation of squeezed microwave fields with up to 10 dB of noise suppression [10] in the field of quantum information processing with superconducting circuits. Optomechanical squeezing of 1.7 dB below the shot-noise level has been also observed in an optical cavity with an embedded, mechanically compliant dielectric membrane [11] and squeezing of a strongly interacting optoelectromechanical system using a parametric drive has been also demonstrated [12].

In this field, a wide variety of quantum communication applications has been investigated operating with continuous wave squeezed light beams. In addition, squeezing as well as quantum correlations have been mainly demonstrated in the spectral domain and not in the time domain. It should be noted that such spectral measurements (rather than time-dependent ones) have been performed even for the case of pulsed squeezing experiments [13–17]. The method of time-resolved homodyne measurement of individual pulsed squeezed states as well as that of individual quadrature-entangled pulses have been developed [18,19]. Such single-shot homodyne detection has already been performed in the pioneering experiments on quantum tomography and quantum correlations [20,21] and also in papers [22,23]. These approaches open possibilities for homodyne measurement of quadrature noise variances in the time domain (see, for example, Refs. [24–26]) and hence for elaboration of time-resolved quantum information protocols operating in a pulsed regime in addition to the ordinary ones elaborated in spectral domains for continuous waves.

In this paper, we continue investigations of the squeezing in time domain. In this way, we propose and investigate in detail periodically modulated squeezed states of light generated in optical parametric oscillators (OPO) driven by a continuously time-modulated pump field. OPO, based on the processes of down conversion in a cavity, have proven to be efficient sources of squeezed light [3,27,28]. Excellent agreement between theory [29–36] and experiment [3,27] of squeezing is obtained in the region below threshold of OPO and in the spectral domain. A systematic quantum theory of OPO including the near-threshold region has been developed in Refs. [37,38].

In our scheme, quantum systems can display qualitatively different forms of behavior when driven by fast time-periodic modulations. Particularly, the application of a sequence of tailored pulses as well as time-modulated cw field leads to improving the degree of quantum effects in open-cavity nonlinear systems and to the onset of qualitatively new quantum effects. This approach was recently exploited for the generation of Fock states in a periodically driven Kerr nonlinear resonator (KNR) [39,40] and for the demonstration of multiphoton blockades in pulsed regimes of a dissipative KNR beyond stationary limits [41,42]. The improvement of optomechanical and electromechanical entanglement by time modulation has also been shown [43]. It has been demonstrated that amplitude modulation can improve the performance of single photon sources based on quantum dots [44]. The idea

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to enrich quantum physical systems by designing a time modulation has been explored in several other fields of research including periodically driven nonlinear oscillators [45–47] and periodically driven quantum matter [48]. Below we exploit such an approach to improve the degree of one-mode squeezing.

In this paper we consider the total or integral one-mode squeezing in OPO, which is analyzed through variances $V_\theta(t)$ of quadrature amplitudes of electromagnetic field $X_\theta(t) = \frac{1}{\sqrt{2}}[a^+(t)e^{-i\theta} + a(t)e^{i\theta}]$, where $a(t)$, $a^+(t)$ are the boson operators, $V(x) = \langle x^2\rangle - \langle x\rangle^2$ denotes the variance. In this case the integral one-mode squeezing for the intracavity mode of subharmonics generated by OPO reaches only 50% relative to the level of vacuum fluctuations, $V \geqslant 0.5V_0$, and the equality holds if the pump field intensity is close to the generation threshold. This has been demonstrated in the framework of a linear treatment of quantum fluctuations [49,50] as well as in the exact quantum theory developed in Refs. [37,38]. An analogous limitation applies to the two-mode squeezed light beams generated in nondegenerate optical parametric oscillators (NOPO) as was shown using a Fokker-Planck equation approach [51]. It is obvious (see, for example, a short discussion in Ref. [50]) that such a limitation on the degree of squeezing is due to dissipation and decoherence as well as cavity induced feedback. It is important to note again that the limit $V \geqslant 0.5V_0$ applies for integral squeezing but not for the spectral squeezing.

In this paper we obtain a remarkable result, that the application of a pump field with periodically varying amplitude instead of a continuous-wave (CW) pump field significantly improves the degree of squeezing in OPO. We demonstrate the phenomenon for OPO in a doubly resonant optical ring cavity (see Fig. 1) and show that the improvement of total squeezing takes place for both below- and above-threshold operational regimes.

We would like to underline again the difference between the focus of our paper and most of the previous works devoted to the study of squeezing. It is an established standard to describe squeezing with the spectra of quantum fluctuations, as has been done in connection with pulsed squeezing experiments [13–17]. Unlike that, we follow the philosophy of Refs. [18,20] and analyze the integral squeezing characteristics of time-modulated light, including periodically pulsed light beams.

We perform our calculations within the framework of stochastic equations of motion and a linearized treatment of quantum fluctuations. One should keep in mind, however, that the linearized approach does not apply in the threshold regime with large quantum fluctuations. In order to verify the accuracy of our analytical calculations as well as to investigate the threshold regime we also perform numerical simulations based on the quantum state diffusion method [52].

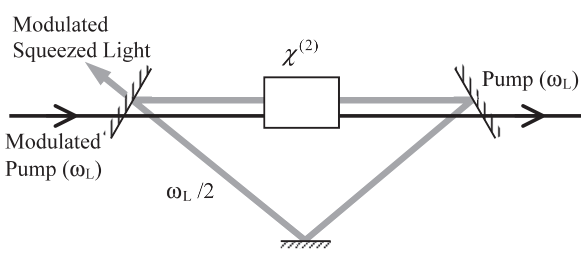

FIG. 1. The principal scheme of OPO in a cavity supporting the pump mode at frequency ω_L and a subharmonic mode.

The paper is organized as follows. In Sec. II we derive the stochastic equations for the complex c-number variables corresponding to the operators a, a_L, and present a semiclassical analysis of the periodically modulated OPO. Section III is devoted to the perturbative analysis of quantum fluctuations employing the ratio of nonlinearity to damping rate, $k/\gamma \ll 1$, as the small parameter. In Sec. IV we investigate the time-modulated dynamics and squeezed variance for the case of continuously modulated OPO. We also discuss in Sec. V the critical effects in the near-threshold regime. We summarize our results and give our conclusion in Sec. VI.

II. EQUATIONS AND TIME-MODULATED SEMICLASSICAL DYNAMICS

The Hamiltonian describing the system within the framework of the rotating wave approximation and in the interaction picture is

$$H = H_{\text{sys}} + H_{\text{int}} + H_R, \tag{1}$$

where the intracavity or system Hamiltonian is given by

$$H_{\text{sys}} = i\hbar f(t)(e^{i(\Phi_L - \omega_L t)}a_L^+ - e^{-i(\Phi_L - \omega_L t)}a_L) + i\hbar k(e^{i\Phi_k}a_L a^{+2} - e^{-i\Phi_k}a_L^+ a^2). \tag{2}$$

Here, a_L and a are the boson operators for cavity modes at the frequencies ω_L and $\omega_L/2$. The pump mode a_L is driven by an amplitude-modulated external field at the frequency ω_L with time periodic, real valued amplitude $f(t+T) = f(t)$. Down conversion of pump photons to resonant subharmonic-mode photons at frequency $\omega_L/2$ occurs due to a $\chi^{(2)}$ nonlinearity placed in the cavity. The constant $ke^{i\Phi_k}$ determines the efficiency of the down-conversion process $\omega_L \to \frac{\omega_L}{2} + \frac{\omega_L}{2}$ in the $\chi^{(2)}$ medium. The term H_R is the reservoir Hamiltonian, which describes the free evolution of the extracavity modes. The term

$$H_{\text{int}} = \hbar(a_L\Gamma_L^+ + a\Gamma^+ + \text{H.c.}) \tag{3}$$

is the Hamiltonian describing the interaction between the system and the reservoir, and $\Gamma_L, \Gamma_L^+, \Gamma, \Gamma^+$ are reservoir operators that create and destroy photons in the loss reservoir coupled to internal pump and subharmonic modes.

Due to presence of dissipation in this problem, the adequate description is given by a master equation for the reduced density operator ρ of the system. Within the framework of the rotating wave approximation and in the interaction picture it takes the form

$$\frac{\partial\rho}{\partial t} = \frac{1}{i\hbar}[H',\rho] + \gamma_L(2a_L\rho a_L^+ - a_L^+a_L\rho - \rho a_L^+ a_L) + \gamma(2a\rho a^+ - a^+a\rho - \rho a^+ a), \tag{4}$$

where for the case of zero detunings

$$H' = i\hbar f(t)(a_L^+ - a_L) + i\hbar k(a_L a^{+2} - a_L^+ a^2), \tag{5}$$

while γ_L, γ are the damping rates for the pump and subharmonic modes, respectively. Let us also note that this equation is rewritten through the transformed boson operators $a_L \to a_L \exp(-i\Phi_L)$, $a \to a\exp[-\frac{i}{2}(\Phi_L + \Phi_k)]$. That leads to a cancellation of the phases on the intermediate stages of the calculations. As a result, the Hamiltonian H' depends only on real-valued coupling constants.

We perform the calculations within the framework of the positive-P representation [53,54], and obtain stochastic equations for the complex c-number variables α, α_L and β, β_L, corresponding to the operators a, a_L and a^+, a_L^+. We consider the regime of adiabatic elimination of the pump mode for the case of high cavity losses for the pump mode, $\gamma_L \gg \gamma$. In this approach the stochastic amplitudes are given by the following equations:

$$d\alpha = -(\gamma + \lambda\alpha\beta)\alpha dt + \varepsilon(t)\beta dt + dW_\alpha, \tag{6}$$

$$d\beta = -(\gamma + \lambda\alpha\beta)\beta dt + \varepsilon(t)\alpha dt + dW_\beta, \tag{7}$$

$$\alpha_L(t) = [f(t) - k\alpha^2]/\gamma_L, \tag{8}$$

$$\beta_L(t) = [f(t) - k\beta^2]/\gamma_L, \tag{9}$$

and dW_α, dW_β are Gaussian noise terms (the Wiener increments) with the following nonzero correlators

$$\langle dW_\alpha(t)dW_\alpha(t)\rangle = [\varepsilon(t) - \lambda\alpha^2]dt, \tag{10}$$

$$\langle dW_\beta(t)dW_\beta(t)\rangle = [\varepsilon(t) - \lambda\beta^2]dt, \tag{11}$$

where $\varepsilon(t) = f(t)k/\gamma_L$, $\lambda = k^2/\gamma_L$.

The equations of motion have time-dependent coefficients. Nevertheless, surprisingly, it is possible to find their analytical solution in the semiclassical approach for an arbitrary, but real modulation amplitude $f(t)$. The presentation of both semiclassical and quantum theories of such a time-modulated OPO is another important goal of this paper.

First, we shall study the solution of stochastic equations in semiclassical treatment, neglecting the noise terms, for mean photon numbers n and phases φ of the modes [$n = \alpha\beta$, $\varphi = \frac{1}{2i}\ln(\alpha/\beta)$] for time intervals exceeding the transient time, $t \gg \gamma^{-1}$. An analysis shows that similar to the standard OPO, the considered system also exhibits a threshold behavior, which is easily described through the period-averaged pump field amplitude $\overline{f(t)} = \frac{1}{T}\int_0^T f(t)dt$, where $T = 2\pi/\delta$ is the period and δ is the frequency of modulation, $\delta \ll \omega_L$, which will be specified below. The below-threshold regime with a stable trivial zero-amplitude solution $\alpha = \beta = 0$ is realized for $\overline{\varepsilon} < \varepsilon_{\text{th}} = \gamma$, or $\overline{f} = \frac{1}{T}\int_0^T f(t)dt < f_{\text{th}}$, where $f_{\text{th}} = \gamma\gamma_L/k$ is the threshold value of the pump field amplitude. When $\overline{f} > f_{\text{th}}$ a stable nontrivial solution exists with the following properties. The equations for the mean photon number of the subharmonic $n_c = \langle a^+a\rangle = |\alpha|^2$ and the phase φ read as

$$\frac{d}{dt}n_c = 2\varepsilon(t)n_c\cos(2\varphi) - 2\lambda n_c^2 - 2\gamma n_c, \tag{12}$$

$$\frac{d}{dt}\varphi = -\varepsilon(t)\sin(2\varphi), \tag{13}$$

with the following solution for the phase:

$$\cos[2\varphi(t)] = \tanh\left[2\int_{t_0}^t \varepsilon(t')dt' + c_0\right]. \tag{14}$$

Let us consider this result for asymptotic time intervals that are long compared to the transient time, $t \gg \gamma^{-1}$. Note, that the formula

$$\int_{t_1}^{t_2} \varepsilon(t)dt = (t_2 - t_1)\overline{\varepsilon} + \epsilon(t_2) - \epsilon(t_1) \tag{15}$$

applies for a periodic function $\varepsilon(t + T) = \varepsilon(t)$, where $\overline{\varepsilon} = \frac{1}{T}\int_0^T \varepsilon(t)dt$ is the period-averaged amplitude and $\epsilon(t)$ is a periodic function $\epsilon(t + T) = \epsilon(t)$. In particular, for the case of harmonic modulation $f(t) = f_0 + f_1\cos\delta t$, the mean value is $\overline{f} = f_0$ and hence

$$\varepsilon(t) = \overline{\varepsilon} + \varepsilon_1\cos\delta t, \tag{16}$$

where $\overline{\varepsilon} = k\overline{f}/\gamma_L$, $\varepsilon_1 = kf_1/\gamma_L$. In this case we have

$$\int_{t_1}^{t_2} \varepsilon(t)dt = (t_2 - t_1)\overline{\varepsilon} + \frac{\varepsilon_1}{\delta}[\sin(\delta t_2) - \sin(\delta t_1)] \tag{17}$$

and

$$\epsilon(t) = \frac{\varepsilon_1}{\delta}\sin(\delta t). \tag{18}$$

Using these equations we conclude from (14) that for time intervals exceeding the transient time $t - t_0 \gg \gamma^{-1}$, $\cos[2\varphi(t)] \to 1$ and hence the phase becomes $\varphi = \pi m$, ($m = 0, \pm 1, \pm 2, \ldots$). Restoring the previous phase structure of intercavity interaction we conclude that the phase of the subharmonic mode has a twofold symmetry

$$\varphi = \tfrac{1}{2}(\Phi_L + \Phi_K) \pm \pi m. \tag{19}$$

Then, for $n_c = 1/Z$, Eq. (12) is reduced to

$$\frac{d}{dt}Z = -2[\varepsilon(t) - \gamma]Z + 2\lambda, \tag{20}$$

and the mean photon number reads as

$$n_c^{-1}(t) = Z_h + 2\lambda\int_{t_0}^t e^{-2\int_\tau^t[\varepsilon(t') - \gamma]dt'}d\tau. \tag{21}$$

Here, $Z_h = Z(t_0)\exp\{-2\int_{t_0}^t[\varepsilon(t') - \gamma]dt'\}$ is the solution of the homogeneous equation corresponding to (20).

As the analysis shows, beyond the transient time intervals, $t - t_0 \gg \gamma^{-1}$, the term Z_h can be neglected and we arrive at the result

$$n_c^{-1}(t) = 2\lambda\int_{t_0}^t \exp\left\{-2\int_\tau^t[\varepsilon(t') - \gamma]dt'\right\}d\tau. \tag{22}$$

Note, that in this case $n_c(t)$ is a periodic function of time. For the case of a monochromatic pump field, $f(t) = \text{const}$, we obtain the usual result for the photon number of an ordinary OPO in the steady state,

$$n_{\text{st}} = (\overline{\varepsilon} - \gamma)/\lambda = (\overline{f} - f_{\text{th}})/k. \tag{23}$$

It is interesting to consider the period-averaged mean photon number $\overline{n}_c = \frac{1}{T}\int_0^T n_c(t)dt$. Straightforward calculations show that $\overline{n}_c$ depends on the period-averaged amplitude $\overline{f}$ and coincides with the analogous result for non-modulated OPO, $\overline{n}_c = n_{\rm st} = (\overline{f} - f_{\rm th})/k$.

III. PERTURBATION THEORY AND SQUEEZED VARIANCE

The aim of the present section is to study the quantum-statistical properties of an OPO in a linearized treatment of quantum fluctuations. We assume that the quantum fluctuations are sufficiently small so that Eqs. (6) and (7) can be linearized around the stable semiclassical steady state $\alpha(t) = \alpha^0 + \delta\alpha(t)$, $\beta(t) = \beta^0 + \delta\beta(t)$. This method is appropriate for analyzing the quantum-statistical effects, i.e., squeezed variances, for all operational regimes with the exception of the vicinity of the threshold, where the level of quantum noise increases substantially.

Nevertheless, in our analysis we will use the modified perturbative approach expanding measurable quantities in power series in the small parameters of the theory. Indeed, in current experiments the ratio of nonlinearity to damping is small, $k/\gamma \ll 1$ (typically 10^{-4} or less), and hence $\lambda/\gamma = k^2/(\gamma\gamma_{\rm L}) \ll 1$ is a small parameter of the theory.

It should be also noted that the assumption of vanishing boundary terms in the standard procedure of deriving a Fokker-Planck equation in the positive-P representation is generally valid only when $k/\gamma \ll 1$. In this case the boundary terms, which allow the master equation to be rewritten as a Fokker-Planck equation, are exponentially suppressed.

In this approach, the limit $\lambda \to 0$ corresponds to neglecting noise terms in the equations (6) and (7). This means that in the limit $\lambda/\gamma \ll 1$ the solutions of Eqs. (6), (7) are transformed to the semiclassical solutions. Indeed, the solutions of the deterministic parts of (6), (7) are proportional to $\lambda^{-1/2}$ ($n_c \sim \lambda^{-1}$, accordingly $\alpha \sim \lambda^{-1/2}$, $\beta \sim \lambda^{-1/2}$) while the noise terms containing the terms $\lambda\alpha^2$ and $\lambda\beta^2$ remain finite for $k \to 0$.

In order to develop this perturbative analysis systematically and in order to avoid the $\lambda^{-1/2}$ terms for the stochastic amplitudes α, β in the zero order of perturbation theory, we introduce the scaled amplitudes as $\alpha' = s\alpha$, $\beta' = s\beta$ with the scaling parameter $s = \sqrt{\lambda/\gamma}$. This leads to the equations

$$d\alpha' = -\gamma(1+\alpha'\beta')\alpha' dt + \varepsilon(t)\beta' dt + dW_{\alpha'}, \tag{24}$$

$$d\beta' = -\gamma(1+\alpha'\beta')\beta' dt + \varepsilon(t)\alpha' dt + dW_{\beta'}, \tag{25}$$

where $dW_{\alpha'} = s\,dW_\alpha$, $dW_{\beta'} = s\,dW_\beta$, and the correlators are

$$\langle dW_{\alpha'}(t) dW_{\alpha'}(t)\rangle = s^2[\varepsilon(t) - \gamma\alpha'^2]dt, \tag{26}$$

$$\langle dW_{\beta'}(t) dW_{\beta'}(t)\rangle = s^2[\varepsilon(t) - \gamma\beta'^2]dt. \tag{27}$$

Note, that in these variables the noise terms vanish while the scaled photon number $n' = s^2 n$ remains finite for $\lambda \to 0$. Further, we omit the primes for simplicity.

Another important difference of our approach, in comparison with the standard ones, is connected with the choice of appropriate stochastic variables [45,46]. Since in the present paper our primary interest is to calculate the mean photon number and the squeezed variance, it is convenient to perform the perturbative analysis of the system for the following combination of stochastic amplitudes: $n = \alpha\beta$, $R = n - \frac{1}{2}(\alpha^2 + \beta^2)$. It is easy to check that the variance $V_\theta(t)$ is expressed through R. Indeed, using the periodic nature of intracavity interaction and the relationships between normally ordered operator averages and stochastic moments with respect to the P function, we obtain

$$\begin{aligned} V_\theta(t) &= \tfrac{1}{2}(1 + 2\langle a^+ a\rangle - \langle a^2\rangle e^{i\Phi} - \langle a^{+^2}\rangle e^{-i\Phi}) \\ &= \tfrac{1}{2}(1 + 2\langle\alpha\beta\rangle - \langle\alpha^2\rangle e^{i\Phi} - \langle\beta^2\rangle e^{-i\Phi}), \end{aligned} \tag{28}$$

where $\Phi = \theta + \Phi_k$. Thus, for $\Phi = 2\pi n$, $(n = 0, \pm 1, \pm 2, \ldots)$, we have $V(t) = \frac{1}{2} + \langle R\rangle$. The remarkable advantage of this approach is that it allows us to obtain results for both below and above threshold operational regimes in the same way. Using Ito rules for changing the stochastic variables, we obtain from (24) and (25),

$$dn = -2\gamma n dt - 2\gamma n^2 dt + 2\varepsilon(t) n dt - 2\varepsilon(t) R + dW_n, \tag{29}$$

$$\begin{aligned} dR = &-2[\gamma + \gamma n + \varepsilon(t)]R dt \\ &- s^2[\varepsilon(t) - \gamma n + \gamma R] + dW_R, \end{aligned} \tag{30}$$

where the stochastic correlations are given by

$$\langle dW_n(t) dW_n(t)\rangle = s^2[\varepsilon(t) n - \gamma n^2 - \varepsilon(t) R]dt, \tag{31}$$

$$\langle dW_R(t) dW_R(t)\rangle = -2s^2 R[\varepsilon(t) R - \gamma n + \gamma R]dt, \tag{32}$$

$$\langle dW_R(t) dW_n(t)\rangle = -s^2 R[\varepsilon(t) + \gamma n]dt. \tag{33}$$

We expand the scaled variables in a truncated power series of the parameter s, $n = n^{(0)} + s n^{(1)} + s^2 n^{(2)}$ and $R = R^{(0)} + s R^{(1)} + s^2 R^{(2)}$, to perform the perturbative analysis of the moments. Similar expansion is used for the noise terms W_n, W_R. It is not difficult to check that amongst the correlators with first-order noise terms the only nonzero one is

$$\left\langle dW_n^{(1)}(t) dW_n^{(1)}(t)\right\rangle = n_0'[\varepsilon(t) - \gamma n_0']dt. \tag{34}$$

The zero-order form of the equations (29), (30) is

$$\begin{aligned} dn^{(0)} = &-2\gamma n^{(0)} dt - 2\gamma (n^{(0)})^2 dt \\ &+ 2\varepsilon(t) n^{(0)} dt - 2\varepsilon(t) R^{(0)}, \end{aligned} \tag{35}$$

$$dR^{(0)} = -2[\gamma + \gamma n^{(0)} + \varepsilon(t)]R^{(0)} dt. \tag{36}$$

Thus, in the asymptotic regime we obtain $R^{(0)} = 0$ and hence the semiclassical results for the mean photon numbers read as

$$n^{(0)} = \begin{cases} 0; & \overline{\varepsilon} < \varepsilon_{\rm th} \\ s^2 n_c; & \overline{\varepsilon} > \varepsilon_{\rm th}. \end{cases} \tag{37}$$

The first-order equations (29), (30) are given by

$$\begin{aligned} dn^{(1)} = &-2[\gamma + 2\gamma n^{(0)} - \varepsilon(t)]n^{(1)} dt \\ &- 2\varepsilon(t) R^{(1)} dt + dW_n^{(1)}, \end{aligned} \tag{38}$$

$$dR^{(1)} = -2[\gamma + \gamma n^{(0)} + \varepsilon(t)]R^{(1)} dt. \tag{39}$$

We conclude that $R^{(1)} = 0$ for $t \to \infty$. As $\langle W^{(1)}\rangle = 0$, we have for $\langle n^{(1)}\rangle$

$$d\langle n^{(1)}\rangle = -2[\gamma + 2\gamma n^{(0)} - \varepsilon(t)]\langle n^{(1)}\rangle dt. \tag{40}$$

Therefore we conclude that for both regimes of generation $\langle n^{(1)}\rangle = 0$. However, the noise terms $W^{(1)}$ lead to the nonvanishing variance of the photon number. Using Ito rules we arrive for the variance $\langle (n^{(1)})^2\rangle$ at

$$d\langle (n^{(1)})^2\rangle = -4[\gamma + 2\gamma n^{(0)} - \varepsilon(t)]\langle (n^{(1)})^2\rangle dt + n^{(0)}[\varepsilon(t) - \gamma n^{(0)}]dt, \tag{41}$$

with the following formal solution

$$\langle (n^{(1)}(t))^2\rangle = \int_{-\infty}^{t} \exp\left\{-4\int_{\tau}^{t}[\gamma + 2\gamma n^{(0)}(t') - \varepsilon(t')]dt'\right\} \times n^{(0)}(\tau)[\varepsilon(\tau) - \gamma n^{(0)}(\tau)]d\tau. \tag{42}$$

In the below-threshold regime, $n^{(0)} = 0$ and the variance $\langle [n^{(1)}(t)]^2\rangle$ is equal to zero, too.

Let us turn to the second-order equations. Because $R^{(0)} = R^{(1)} = 0$, the variance in the scaled variables $V(t) = \frac{1}{2} + \langle R\rangle$ is determined by the second-order term $R^{(2)}$. The corresponding equation for R reads as

$$d\langle R^{(2)}\rangle = -2[\gamma + \gamma n^{(0)} + \varepsilon(t)]\langle R^{(2)}\rangle dt - [\varepsilon(t) - \gamma n^{(0)}]dt. \tag{43}$$

This equation leads to the following result for both regimes of generation:

$$\langle R^{(2)}\rangle = -\int_{-\infty}^{t} \exp\left\{-2\int_{\tau}^{t} [\gamma + \varepsilon(t') + \gamma n^{(0)}(t')]dt'\right\} \times [\varepsilon(\tau) - \gamma n^{(0)}(\tau)]d\tau. \tag{44}$$

Finally, for the variance of the quadrature amplitude $V(t) = \frac{1}{2} + \langle R^{(2)}\rangle$ we obtain the linear equation

$$\frac{dV(t)}{dt} = -2[\gamma + \gamma n_c(t) + \varepsilon(t)]V(t) + \gamma + 2\gamma n_c(t), \tag{45}$$

which is rewritten through the original unscaled photon number $n_c(t)$. The solution in the asymptotic regime is given by

$$V(t) = \gamma\int_{-\infty}^{t} \exp\left\{-2\int_{\tau}^{t} [\gamma + \varepsilon(t') + \gamma n_c(t')]dt'\right\} \times [1 + 2n_c(\tau)]d\tau. \tag{46}$$

It is not difficult to transform this expression to the form where its periodic dependence is obvious,

$$V(t) = \gamma\int_{-\infty}^{0} \exp\left\{-2\int_{\tau}^{0} [\gamma + \varepsilon(t'+t) + \gamma n_c(t'+t)]dt'\right\} \times [1 + 2n_c(\tau + t)]d\tau. \tag{47}$$

The analysis of the below-threshold regime is simple and leads to Eq. (47) with $n_c = 0$,

$$V(t) = \gamma\int_{-\infty}^{0} \exp\left\{-2\int_{\tau}^{0} [\gamma + \varepsilon(t'+t)]dt'\right\}d\tau. \tag{48}$$

Equations (47) and (48) for the time-dependent variances constitute the main results of the paper. In particular, when $\varepsilon(t) = \overline{\varepsilon} = \text{const}$, Eq. (48) takes the form $V = \gamma/[2(\gamma + \overline{\varepsilon})]$, which coincides with the result for the ordinary OPO below threshold. For the general case the integral in (48) can not be handled but a lower bound for $V(t)$ can be obtained in the form

$$V(t) \geqslant \frac{\gamma}{2(\gamma + \varepsilon_{\max})}. \tag{49}$$

Next, we present the applications of these general results to time-modulated OPO.

IV. PERIODICALLY MODULATED SQUEEZING

In the previous section we have derived results for the mean photon number, Eq. (22), and the quadrature variance, Eq. (47), of a single-mode light beam generated in time-modulated OPO. Equations (46) and (47) apply for arbitrary n_c, particularly for $n_c = 0$, which allows us to use them in both the below- and above-threshold regimes. As applications of these results we consider in this section the experimentally available scheme of OPO under the action of a pump field with continuous, harmonically modulated amplitude, $\varepsilon(t) = \overline{\varepsilon} + \varepsilon_1\cos(\delta t)$ with $\delta \ll \omega_L$.

Such amplitude modulation can be realized electronically using standard techniques, in particular, with the help of an electro-optic amplitude modulator. Alternatively, amplitude modulation can be achieved in OPO driven by polychromatic pump field. We consider the setup of Fig. 1, assuming the pump mode is driven by the field $E_{\text{ext}}(t) = f_0\cos(\omega_L t + \Phi_L) + (f_1/2) \times \{\cos[(\omega_L + \delta)t + \Phi_L + \Phi] + \cos[(\omega_L - \delta)t + \Phi_L - \Phi]\}$ at the central frequency ω_L and two satellites $\omega_L + \delta$, $\omega_L - \delta$, for $\delta \ll \omega_L$. It is easy to check that the Hamiltonian of this system is indeed given by Eq. (1) and the modulation amplitude is equal to $f(t) = f_0 + f_1\cos(\delta t + \Phi)$. Above the threshold, $\overline{f} = f_0 > f_{\text{th}}$, the mean photon number, Eq. (22), in the case of harmonic modulation reads as

$$n_c^{-1}(t) = 2\lambda\int_{-\infty}^{0} \exp\left[2\gamma\tau\left(\frac{\overline{f}}{f_{\text{th}}} - 1\right)\right] \times \exp\left\{\frac{2\gamma f_1}{\delta f_{\text{th}}}\{\sin[\delta(t+\tau)] - \sin(\delta t)\}\right\}d\tau. \tag{50}$$

This result is illustrated in Fig. 2(a) for different levels of modulation. For $f_1 = 0$ it reaches the standard result $n_c = n_{\text{st}} = (f_0 - f_{\text{th}})/k$.

It is not difficult to obtain from Eq. (50) the lower and upper bounds for the mean photon number,

$$n_{\text{st}}\exp(-4\varepsilon_1/\delta) \leqslant n_c(t) \leqslant n_{\text{st}}\exp(+4\varepsilon_1/\delta). \tag{51}$$

Using these inequalities we conclude that the amplitude of oscillations of the photon number decreases with increasing modulation frequency δ, and increases with the parameter ε_1.

Next, we turn to the study of the squeezed quadrature variance in the presence of harmonic modulation. Below the threshold from (48) we obtain

$$V(t) = \gamma\int_{-\infty}^{t} \exp\left\{-2(\gamma + \overline{\varepsilon})(t-\tau) + \frac{2\varepsilon_1}{\delta}[\sin(\delta t) - \sin(\delta\tau)]\right\}d\tau. \tag{52}$$

For the case of a weak modulation level, $\varepsilon_1/\delta \ll 1$, the exponent can be expanded in a power series of the ratio ε_1/δ.

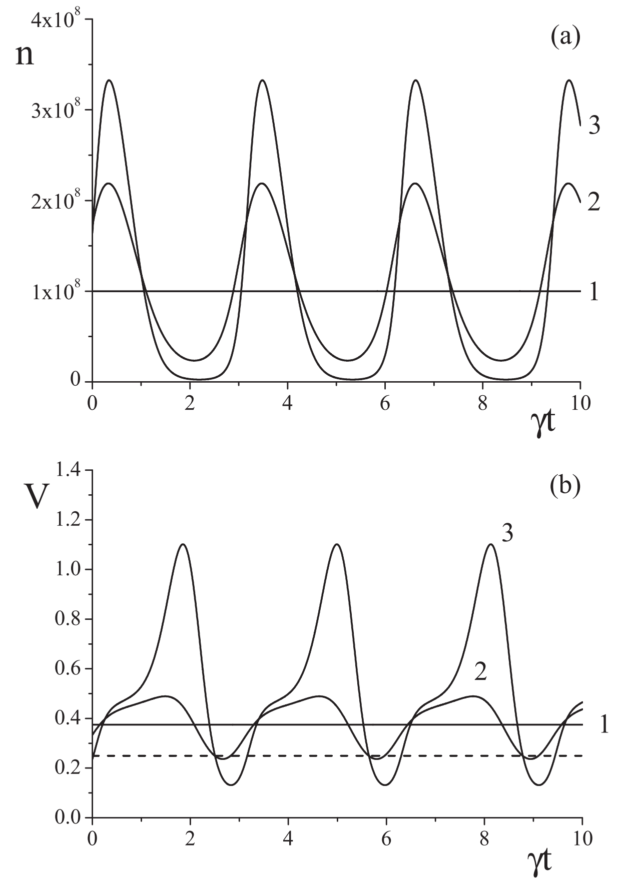

FIG. 2. (a) Mean photon number and (b) variance versus dimensionless time for the parameters: $k^2/\gamma_L\gamma = 10^{-8}$, $\delta = 2\gamma$, $f_0 = 2\gamma\gamma_L/k = 2f_{\rm th}$; $f_1 = 0$ (curve 1) corresponds to the stationary limit, $f_1 = 0.75f_0$ (curve 2), and $f_1 = 1.5f_0$ (curve 3). The dashed line in Fig. 2(b) corresponds to the stationary limit $V = 1/4$.

Then, the variance up to the first order is

$$V(t) \approx \frac{\gamma}{2(\gamma+\overline{\varepsilon})}\left\{1 - \frac{2\varepsilon_1}{\delta}\frac{\sin(\delta t) + \left[\frac{2(\overline{\varepsilon}+\gamma)}{\delta}\right]\cos(\delta t)}{1+\left[\frac{2(\overline{\varepsilon}+\gamma)}{\delta}\right]^2}\right\}. \tag{53}$$

The maximal degree of squeezing in this case is achieved for the time intervals $\delta t = \arctan\frac{\delta}{2(\gamma+\overline{\varepsilon})} + 2\pi k$, and reads as

$$V_{\rm min} = \frac{\gamma}{2(\gamma+\overline{\varepsilon})}\left\{1 - \frac{2\varepsilon_1}{\delta}\frac{1}{\sqrt{1+\left[\frac{2(\overline{\varepsilon}+\gamma)}{\delta}\right]^2}}\right\}. \tag{54}$$

Note that, near the threshold the level of integral squeezing is less than 25%.

Typical numerical results for the above-threshold regime are presented in Fig. 2(b), based on Eq. (47). The variance shows a time-dependent modulation with a period $2\pi/\delta$. The drastic difference between the degree of single-mode squeezing for modulated and stationary dynamics is also clearly seen in Fig. 2(b), where curve 1 represents an example for the stationary case ($f_1 = 0$). This variance, however, is still above the stationary limit of 1/4 for the chosen parameters. The stationary variance near the threshold is bounded by the condition $V \geq \frac{1}{2}V_0$, where the variance V_0 corresponds to the level of vacuum fluctuations and is normalized as $V_0 = 1/2$.

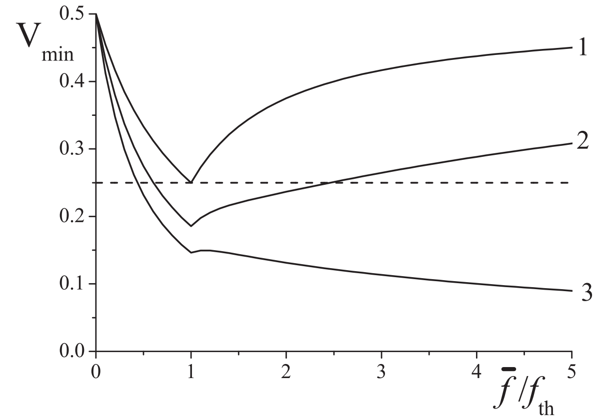

FIG. 3. The minimum levels of the variance versus $\overline{f}/f_{\rm th}$ for three levels of modulation: $f_1 = 0$ (curve 1), $f_1 = 0.75\overline{f}$ (curve 2), and $f_1 = 1.5\overline{f}$ (curve 3). The parameters are as in Fig. 2. The dashed line corresponds to the stationary limit.

However, the variance for the case of modulated dynamics can be less than $\frac{1}{4}$ for definite time intervals. To indicate these effects we also show in Fig. 2(b) the stationary limit $V = 1/4$ as a dashed line.

The minimum values of the variance $V_{\rm min} = V(t_m)$ at fixed time intervals $t_m = t_0 + 2\pi m/\delta$, $(m = 0,1,2\ldots)$ are shown in Fig. 3. (curves 2 and 3) for various operational regimes. As it is expected, the degree of squeezing increases with the ratio $f_1/\overline{f}$. Curve 1 represents the stationary case where the variance is constant and hence $V_{\rm min} = V$. Therefore, this result demonstrates the transition of the variance through the threshold. Another peculiarity here is that the stationary variance (curve 1) has a characteristic threshold behavior, which disappears in the case of strong modulation (curve 3). We conclude that there is nothing of principle that prohibits reaching approximately perfect squeezing for the case of strong modulation. An analysis shows that the production of strong squeezing occurs for the period of modulation comparable with the characteristic time of dissipation, $\delta \approx \gamma$, and disappears for asymptotic cases of slow ($\delta \ll \gamma$) and fast ($\delta \gg \gamma$) modulations.

At the end of this section we briefly discuss the squeezing in the framework of external-operator moments, since measurements are usually performed on output fields that are external to the cavity. We consider the output behavior of OPO assuming that all losses occur through the output coupler (see Fig. 1). For this goal the method of external-field measurements based on input-output relations [31–33] is used. In this case the output photon field of the subharmonic is $\Phi^{\rm out}(t) = \sqrt{2\gamma}a(t)$, while the output photon field of the pump mode is equal to $\Phi_L^{\rm out}(t) = \sqrt{2\gamma}a_L(t) - \Phi_L^{\rm in}(t)$, where $\Phi_L^{\rm in}(t)$ is the input photon field. The output-quadrature field variable is defined as $X^{\rm out}(\theta) = \frac{1}{\sqrt{2}}[\Phi^{\rm out}(t)e^{i\theta} + \Phi^{\rm out+}(t)e^{-i\theta}]$, while the output measured time-dependent variance can be written through the normal ordered moments as $V_\theta^{\rm out}(t) = \langle : X^{\rm out}(\theta)X^{\rm out}(\theta) : \rangle - \langle X^{\rm out}(\theta)\rangle^2$ (see, for example Ref. [55]). Using these formulae we get the variance $V^{\rm out}(t) = 2\gamma(V - \frac{1}{2})$, as well as the mean photon number per unit time $n^{\rm out}(t) = 2\gamma n_c(t)$. Thus, the time-dependent output variance corresponding to an instantaneous measurement of the output field moments is easily calculated in terms of the intracavity quadrature variance. Integral output squeezing is realized if $V^{\text{out}} < 0$ [55], while the lower bound for V^{out} is determined by the stationary limit and reads $V^{\text{out}}/2\gamma > -1/4$. Thus, the normalized output time-dependent variance for the case of modulated OPO can be less than $-1/4$, ($V^{\text{out}}/2\gamma < -\frac{1}{4}$), for definite time intervals.

V. CRITICAL FLUCTUATIONS AND WIGNER FUNCTION

We now consider what happens at or near the classical threshold, $\overline{f} \approx f_{\text{th}}$, where the perturbative analysis in general gives divergent results. It is well known that the linearized theory is applicable only outside the critical region, although the variance (46) is surprisingly well defined also at the threshold.

In order to verify the accuracy of our analytical calculations, we use the numerical quantum trajectory simulations, which are valid in all regions. We investigate also the variance of the critical fluctuations at the threshold point as well as the Wigner functions.

Our approach is based on the quantum state diffusion simulation method [52], (details of analogous calculations for an anharmonic oscillator in a time-modulated field can be found in Refs. [39,40,56,57]). The numerical simulations are performed in the truncated Fock basis of the subharmonic mode $\omega_L/2$ in the regime of strong nonlinear coupling $k \lesssim \gamma$ or $\lambda/\gamma \lesssim 1$. We note that OPOs with such extremely large nonlinearities are not realized in practice and in this part of the article we do not intend to give results close to an experimental situation but discuss the fundamental problems of critical fluctuations. As our analysis shows, this simulation tends to disagree with analytical results obtained for the ranges of comparatively high values of the parameter λ/γ and especially at the threshold. Figure 4 shows the detailed results of the simulation for the numerical values of the variance, for $\lambda/\gamma = 0.06$. As we see, in the deep quantum operational regime, $\lambda/\gamma = 0.06$, the analytical solution (curve 1) disagrees with the numerical results (curve 2) in the vicinity of the threshold. However, for the typical values $\lambda/\gamma \ll 1$ the

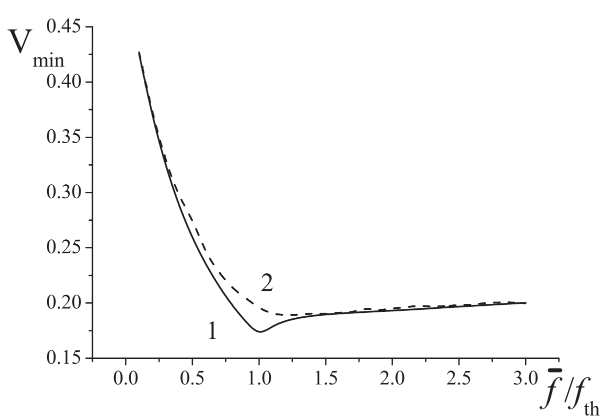

FIG. 4. The minimum values of the variances around the transition through the threshold of generation as the result of analytical (curve 1) and numerical calculations (curve 2). The parameters are: $\delta = 2\gamma$, $\lambda = 0.06\gamma$, $\overline{f} = f_1$.

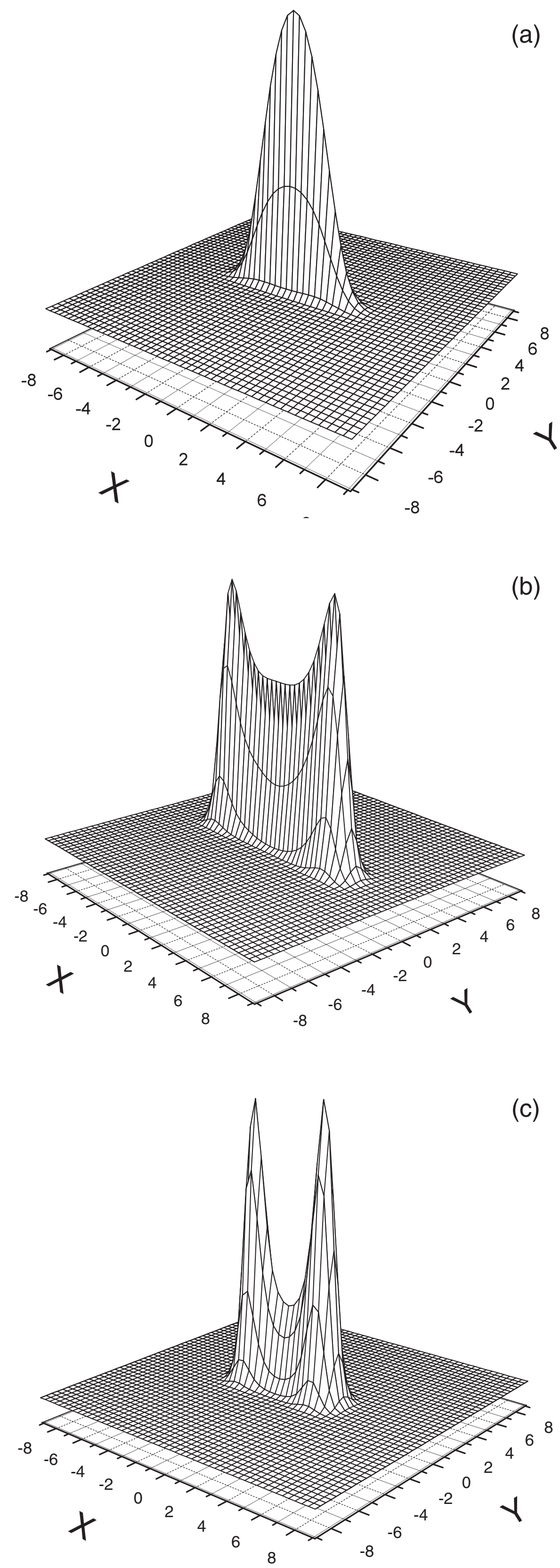

FIG. 5. Wigner functions in the critical operational ranges of OPO. The parameters are: $\delta = 2\gamma$, $\lambda = 0.06\gamma$, $\overline{f} = 1.2 f_{\text{th}}$, $f_1 = 1.5\overline{f}$; (a) $V = 0.16$; (b) $V = 0.27$; (c) $V = 0.4$.

analytical and numerical results are almost indistinguishable for all operational regimes even for a narrow critical range.

As it is well known, a subharmonic mode in ordinary OPO above threshold has a twofold symmetry in phase space. In the steady state and in the semiclassical approach there are two stable states of $\omega_L/2$ mode with equal photon numbers, but with two different phases. Accordingly, the Wigner function in a quantum dissipative regime has a twofold symmetry in phase space under rotation of the phase space by an angle π around its origin, $W(r,\theta) = W(r,\theta+\pi)$, where r, θ are the polar coordinates of the complex phase space. Moreover, the Wigner function above threshold has two humps corresponding to the two states. The results for the time evolution of the Wigner function for the case of time-modulated OPO above threshold, $\overline{\varepsilon} > \gamma$, are depicted in Fig. 5 for fixed moments of time during the period of modulation. These calculations show that the time evolution of the Wigner function has a periodic structure with two humps. The distance between the humps changes during the period of modulation: it is maximal for the maximal mean photon number Fig. 5(b), while the situation for two other photon numbers is shown in Fig. 5(a) and Fig. 5(c).

VI. CONCLUSION

We have considered a model of OPO driven by a time-modulated pump field. This device has been proposed for the generation of periodically modulated squeezed light. It has been shown that a time-modulated OPO provides a very effective mechanism for the improvement of the degree of quadrature squeezing even in the presence of decoherence and cavity induced feedback. The total or integral squeezing in terms of time-dependent intracavity quadrature variances goes below the level of 50% relative to the level of vacuum fluctuations, $V < 1/4$. It has been shown that a similar conclusion also holds for the output measured integral variance. This is in contrast to the case of the usual OPO operated with CW pump where these spectrally integrated quantities are bounded by the stationary limit. This progress is due to the fact that the scheme considered operates under appropriately tailored nonstationary conditions unlike many other squeezed light sources. It is essential to note that this improvement relates to the total or integral squeezing of the output photon field of subharmonic rather than the spectral squeezing. Indeed, most of the squeezing experiments have been performed in the spectral domain in terms of the output measured spectral variances. Moreover, a squeezing level that is significantly lower than the integral squeezing has been achieved at low-frequency spectral ranges. Nevertheless, as noted in Ref. [55], states with even perfect squeezing of a certain spectral component may be nonsqueezed in the full sense. Unlike that, we have considered total or integral squeezing in terms of both time-dependent intracavity modes and time-dependent output photon fields instead of spectral component squeezing.

In our analysis we have considered, as a first stage, the simplest model of a two-mode OPO, but have not developed a general theory of time-modulated OPOs, which should include also a quantum theory of time modulation. Such theory, being created, should operate with so-called wave-packet field operators (see, for example Ref. [58]) and with an input wave packet envelope, instead of operating with ordinary monomode bosonic operators. In our analysis, we have not investigated the dependence between time-dependent and spectral quantum characteristics of OPO. So far such analysis have been performed mainly for steady-state regimes. The analysis of quantum fluctuations of temporal modes also deserves special attention for more accurate identification of squeezing. For instance, we need to specify a certain temporal mode using a temporal filter. A simple example is the square filter of duration τ leading to the measurement of the quadrature operator in the following form:

$$X_\theta(f) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt X_\theta(t). \tag{55}$$

It seems that using the temporal filter might lead to small degradation of the degree of time-modulated squeezing. However, if the filter duration is shorter than the period of modulation, $\tau \ll T$, these effects might be inessential. Most recently, such time-resolved measurement [59] has been experimentally demonstrated for investigations of entangling photons (see, for example, Refs. [60,61]). Recently, a complete analysis of photon temporal modes for quantum information science has been given [62]. Nevertheless, we note that a complete theoretical study of the above-mentioned problems in application to time-modulated OPO is beyond the scope of this paper and is a topic for future work.

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