Synchronization of qubit ensembles under optimized π-pulse driving

Sergey V. Remizov,[1,2] Dmitriy S. Shapiro,[1,2,*] and Alexey N. Rubtsov[1,3,4]
[1]*Dukhov Research Institute of Automatics (VNIIA), 127055 Moscow, Russia*
[2]*V. A. Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, 125009 Moscow, Russia*
[3]*Russian Quantum Center, Skolkovo, 143025 Moscow Region, Russia*
[4]*Department of Physics, Moscow State University, 119991 Moscow, Russia*
(Received 18 August 2015; published 5 November 2015)

We propose a technique of a simultaneous excitation of disordered qubits providing an effective suppression of inhomogeneous broadening in their spectral density. The technique is based on applying of an optimally chosen π pulse with a smooth nonrectangular shape. We study excitation dynamics of an off-resonant qubit subjected to a strong classical electromagnetic driving field with a large reference frequency and slow envelope. Within this solution we optimize the envelope to achieve a preassigned accuracy in qubit synchronization.

DOI: 10.1103/PhysRevA.92.053814 PACS number(s): 42.50.Ct, 03.67.Ac, 42.50.Hz

I. INTRODUCTION

The investigation of qubit ensembles reveals analogies with quantum optics effects [1–3] and possibilities for construction of quantum computers and simulators [4–7]. Solid-state realizations of qubit ensembles are superconducting Josephson circuits [5,8,9], nitrogen-vacancy (NV) centers in diamond samples [10–12], or nuclear and electron spins realized as ^{31}P donors in ^{28}Si crystals [13] and Cr^{3+} spins in Al_2O_3 [14]. The coupling of qubit ensembles with a superconducting microwave resonators results in the formation of subwavelength quantum metamaterials [15–19]. The long-range interaction through a photon mode results in the formation of collective qubit states in these metamaterials [20,21], as the Dicke model describes. One of the crucial distinctions of artificial qubits from natural atoms is that their excitation energies are in many cases tunable *in situ* by external magnetic fields. Beside of the tunability, another property is a disorder in excitation frequencies and, as a consequence, inhomogeneous broadening of the density of states in qubit ensembles. This is related to fundamental mechanisms such as an exponential dependence of excitation energy on Josephson and charging energies in superconducting qubits or spatial fluctuations of background magnetic moments [22] in systems with NV centers.

A disordered spectrum of collective modes offers multimode quantum memory, where information about a photon state is encoded as a tunable collective qubit mode [23,24]. The storage and retrieval protocols were proposed in Refs. [25–27] and are based on spin-refocusing techniques [28,29] or successive magnetic-field gradients. In the context of quantum memory the unavoidable spectral broadening in qubit excitation frequencies provides multimode performance from one side, but from the other side it is one of the limiting factors affecting coherence times. Therefore, the development of techniques of effective suppression of the disorder in qubit frequencies and synchronization of their dynamics is an important problem. For instance, one of the options to suppress the disorder is an atomic frequency comb (AFC) technique applied to rare-earth-metal-ion qubit ensembles [30]. This method is based on frequency-selective optical pumping and subsequent transitions to metastable auxiliary hyperfine states. Also, adiabatic passage (AP) methods such as Stark-shift-chirped rapid AP (SCRAP) [31,32] or stimulated Raman AP (STIRAP) [33] can be applied. Another way of solving this problem was demonstrated in Ref. [11] as a "cavity protection" effect in NV centers. The effect is related to decreasing the relaxation rate of collective qubit modes proportional to the spectral broadening.

Our research is inspired by one of the key ideas of Ref. [11]: the succession of microwave rectangular pulses can serve as an efficient method for exciting disordered NV centers from the ground to the excited state. In our paper we study the possibility of a simultaneous qubit excitation by a *single* nonrectangular π pulse, rather than the sequence mentioned above. We observe that the optimized nonrectangular shape of a π pulse provides an efficient tool for suppression of the disorder effects as well. It allows us to excite qubits within a wide detuning range with almost 100% probability. In contrast to AFC methods, this technique does not require using auxiliary level transitions. It should be noted that the π-pulse duration time in our approach is proportional to the inverse Rabi frequency, which means that the operation time of our technique is significantly shorter than the timescales of the SCRAP and STIRAP methods.

We assume that the π pulse is realized as electromagnetic signal $f(t)e^{-i\omega t}$ with a carrying frequency ω being almost in resonance with the qubit excitation frequency. In our study we optimize the envelope shape $f(t)$ in a class of smooth functions, which guarantees that higher energy levels of a qubit are not affected. In particular, we use a superposition of sine functions, which guarantees the continuity of $f(t)$ over the entire π-pulse duration period, including its starting and final moments. Moreover, this feature is relevant for experimental realization of π pulses based on GHz signal generators. Meanwhile, the use of sine functions is not crucial in this technique and another envelope shape can be used, such as a set of rectangular pulses.

This technique can be applied to disordered systems with strong qubit-cavity couplings like NV centers or superconducting metamaterials, as well as to the atomic-clock devices [34] as a tool for the preparation of a specific atomic state.

*shapiro@cplire.ru

II. DEFINITIONS

We address the possibility of a simultaneous excitation of disordered qubit ensemble coupled to a photon transmission line being the source of the driving. Qubits are assumed to be noninteracting with each other and long lived in comparison to the π-pulse duration time: $\tau \ll \tau_\varphi$. The absence of qubit-qubit interactions means that we can study the dynamics of a single off-resonant driven qubit. We fix the carrying frequency ω and assume that the qubit energy ϵ can be varied to reflect the spectral broadening. Neglecting the qubit decoherence, we solve the Schrödinger equation $i\partial_t|\psi\rangle = (H_q + H_{\rm ext})|\psi\rangle$, where the unperturbed qubit Hamiltonian is $H_q = \epsilon(\sigma_0 + \sigma_z)/2$ and the external driving is $H_{\rm ext} = [f(t)e^{-i\omega t}\sigma_+ + f^*(t)e^{i\omega t}\sigma_-]/2$. We define the wave function of the qubit state in the ω rotating frame as

$$|\psi(t)\rangle = \begin{pmatrix} \alpha(t)e^{-i(\omega+\delta/2)t} \\ \beta(t)e^{-i\delta t/2} \end{pmatrix},$$

where the detuning frequency is $\delta = \epsilon - \omega$. The Hamiltonian H of the driven qubit in this rotating frame reads

$$H = \frac{1}{2}\begin{pmatrix} \delta & f(t) \\ f^*(t) & -\delta \end{pmatrix}. \tag{1}$$

We assume that the qubit is close to the cavity resonance $\epsilon \approx \omega$ and consider the evolution of the qubit wave function within the time interval $0 < t < \tau$ starting from the ground state $|\psi(0)\rangle = |g\rangle$ at the initial moment of time $t = 0$.

The π-pulse time τ is considered as a fixed value. At this point we define the frequency

$$\Omega = \frac{\pi}{\tau},$$

which is the main scale in our consideration along with the detuning δ. The frequency Ω has a transparent physical meaning: this is frequency of Rabi oscillations of the resonant qubit with $\delta = 0$ under the constant driving amplitude given by $F_0(t) = \Omega e^{-i\omega t}$. Hence, time τ is the half of the Rabi period associated with a rectangular π pulse $F_0(0 < t < \tau)$ exciting the system from $|g\rangle$ to $|e\rangle$. Nonzero detuning, related to the inhomogeneous broadening or spread in qubit frequencies, does not allow us to achieve full qubit excitation if the envelope shape $f(t)$ is constant. In the following consideration we modify $f(t)$ in the time interval $0 < t < \tau$ into a more complicated nonrectangular shape $f(t) \neq$ const. to achieve higher efficiency in near-to-resonance qubit excitation.

The Schrödinger equation with the Hamiltonian (1) allows analytical solutions only in several particular cases. The basic one is the constant driving amplitude $f(t) = f =$ const. and arbitrary detuning δ which corresponds to damped Rabi oscillations of the frequency $\Omega_R = (f^2 + \delta^2)^{1/2}$. In this case the evolution of the wave function being in the ground state at $t = 0$ reads

$$|\psi(t)\rangle = \begin{pmatrix} -\frac{if}{\Omega_R}\sin\Omega_R t/2 \\ \cos\Omega_R t/2 + \frac{i\delta}{\Omega_R}\sin\Omega_R t/2 \end{pmatrix}. \tag{2}$$

One can see that detuning reduces the maximum of the excitation probability. This effect results in an impossibility of a synchronization of the qubit ensemble by the rectangular shape of the driving envelope.

Our further consideration is based on another exact solution, which holds for the on-resonance driving regime $\delta = 0$ and arbitrary real-valued $f(t)$. The time evolution of the ground state within this solution reads

$$|\psi(t)\rangle = \begin{pmatrix} -i\sin\frac{\varphi(t)}{2} \\ \cos\frac{\varphi(t)}{2} \end{pmatrix}, \tag{3}$$

where the phase $\varphi(t)$ is given by the time integral

$$\varphi(t) = \int_0^t f(t_1)dt_1.$$

The π-pulse condition, which is the inversion of a qubit occupation number, for this resonant case holds for

$$\varphi(\tau) = \pi. \tag{4}$$

The last constraint (4) provides the class of real-valued functions $f(t)$ that we address in the optimization procedures below.

III. PERTURBATIVE SOLUTION

As mentioned above, the exact solution is not known for arbitrary $f(t)$ and nonzero detuning $\delta \neq 0$. Hence, we develop a perturbation theory based on treating the $\delta\sigma_z/2$ terms in the Hamiltonian (1) as a small perturbation and considering the exact solution (3) at $\delta = 0$ as the zeroth-order approximation. We end up with the following recursive equations forming the perturbation theory in δ:

$$i\dot{\alpha}^{(n)}(t) - \frac{f(t)}{2}\beta^{(n)}(t) = \frac{\delta}{2}\alpha^{(n-1)}(t), \tag{5}$$

$$i\dot{\beta}^{(n)}(t) - \frac{f(t)}{2}\alpha^{(n)}(t) = -\frac{\delta}{2}\beta^{(n-1)}(t). \tag{6}$$

The full solution reads

$$\begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = \begin{pmatrix} x(t)\cos\frac{\varphi(t)}{2} - iy(t)\sin\frac{\varphi(t)}{2} \\ y(t)\cos\frac{\varphi(t)}{2} - ix(t)\sin\frac{\varphi(t)}{2} \end{pmatrix}. \tag{7}$$

Assuming that the qubit was in the ground state at the initial moment of time $|\psi(0)\rangle = |g\rangle$, the solution for $x(t)$ and $y(t)$ is given by a δ^n series with nested integrals as the coefficients:

$$x(t) = -\frac{\delta}{2}\int_0^t dt_1 \sin\varphi(t_1) + \frac{\delta^2 i}{4}\int_0^t dt_1 \int_0^{t_1} dt_2 \sin[\varphi(t_2) - \varphi(t_1)] + \cdots, \tag{8}$$

$$y(t) = 1 + \frac{\delta i}{2}\int_0^t dt_1 \cos\varphi(t_1) - \frac{\delta^2}{4}\int_0^t dt_1 \int_0^{t_1} dt_2 \cos[\varphi(t_2) - \varphi(t_1)] + \cdots. \tag{9}$$

These are general equations of the perturbation theory we use to find the optimal shape of a π pulse $f(t)$ providing simultaneous qubit excitations. Assuming the envelope of the driving to be a smooth function at $t = 0$ and at $t = \tau$, we model $f(t)$ as a superposition of a finite number N of sine functions, where Ω comes as a factor both in the sine arguments and in the driving amplitude,

$$f(t) = \Omega \sum_{n=1}^{N} k_{2n-1} \sin(n - 1/2)\Omega t. \tag{10}$$

According to Eq. (7) the wave function after the π pulse takes the following form:

$$\begin{pmatrix} \alpha(\tau) \\ \beta(\tau) \end{pmatrix} = \begin{pmatrix} -iy(\tau) \\ -ix(\tau) \end{pmatrix}. \tag{11}$$

We optimize numerically a finite set of coefficients k_{2n-1} and require that the ground-state amplitude at $t = \tau$ is zero up to δ^N in the δ expansion, i.e., $\beta(\tau) = O(\delta^N)$. It is possible if two conditions are fulfilled: (i) the resonant qubit is excited, i.e., $\varphi(\tau) = \pi$, which is expressed in terms of k_{2n-1} as

$$4 \sum_{n=1}^{N} \frac{k_{2n-1}}{2n-1} = \pi, \tag{12}$$

and (ii) the off-resonant qubit excitation almost does not depend on detuning, i.e., $x(\tau) = O(\delta^N)$ in Eq. (11). The requirement $x(\tau) = O(\delta^N)$ can be reduced to a set of $N - 1$ equations corresponding to the vanishing of the δ^n terms at $n \leqslant N - 1$ in the expansion (8), if we represent it as $x(\tau,\delta) = \sum_{n=1} c_n \delta^n$, where

$$c_1 = \int_0^\tau dt_1 \sin \varphi(t_1) = 0, \tag{13}$$

$$c_2 = \int_0^\tau dt_1 \int_0^{t_1} dt_2 \sin[\varphi(t_2) - \varphi(t_1)] = 0.$$

$$\vdots \tag{14}$$

To summarize, our perturbative solution consists of a finite system of N equations [(12), (13), (14), ...] providing a smooth solution for $f(t)$. The precision order of the technique is given by N and ensures that the excitation probability $n_\uparrow(\tau,\delta)$ of an off-resonant qubit is close to unity up to the small correction $n_\uparrow(\tau,\delta) = 1 - |x(\tau,\delta)|^2$. This correction is nothing but the precision of the technique and is given by the probability of the ground-state qubit occupation $n_0(\tau,\delta) \equiv |x(\tau,\delta)|^2$.

IV. RESULTS

A. π-pulse optimization scheme at $N = 3$

In this section we provide numerical results for optimizing a π pulse constructed from $N = 3$ terms:

$$f(t) = \Omega(k_1 \sin t\Omega/2 + k_3 \sin 3t\Omega/2 + k_5 \sin 5t\Omega/2).$$

We start from the numerical solution for k_3 and k_5 by using Eqs. (13) and (14); after that we choose k_1 according to the π-pulse condition constraint (12). In Fig. 1 we plot two sets of curves in coordinates (k_3,k_5): (i) Solid (red) curves correspond to vanishing of the term linear in δ in $x(\tau)$, i.e., $c_1 = 0$, according to Eq. (13). Hence, in this case the residual part for the ground-state amplitude is $x(\tau) = O(\delta^2)$. (ii) Dashed (blue) curves correspond to vanishing of the quadratic term $c_2 = 0$ in $x(\tau)$, given by Eq. (14). Note that, under the last condition $c_2 = 0$, the term linear in δ may survive ($c_1 \neq 0$).

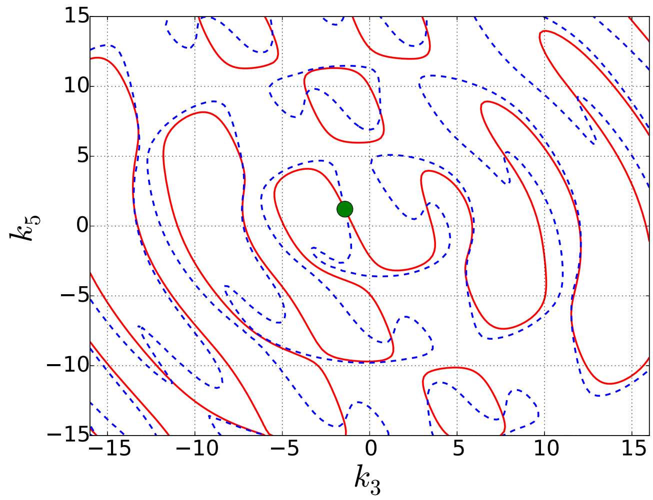

FIG. 1. (Color online) Curves in (k_3,k_5) plane formed by optimal values of sine amplitudes. Solid (red) curve corresponds to quadratic dependence of the ground-state amplitude upon detuning. Crossing points of solid (red) and dashed (blue) curves correspond to the cubic dependence of the ground-state amplitude upon detuning. Such a point nearest zero is marked by a dot (green) and is studied in detail. The value of k_1 is selected so that the π pulse is formed.

The crossing points of these dashed (blue) and solid (red) set of curves satisfy both conditions (i) and (ii). These points correspond to the synchronization of qubit excitations with the precision $n_0(\tau,\delta) \propto (\delta/\Omega)^6$. The third parameter k_1 is found from Eq. (12) where we set $N = 3$ and take k_3,k_5 in accordance with dashed (blue) and solid (red) curve crossing points, e.g., that one is marked as a dot (green). This green point corresponds to the envelope $f(t)$ having the smallest maximum value which provides the effective suppression of the disorder.

B. Synchronization of qubit excitation vs detuning

The higher-order schemes are build straightforwardly around the above solution at $N = 3$. In this section we collect all the results for $N = 1,2,3,4$ order schemes in the driving envelope function (10). In the left column of Fig. 2 we plot the optimized shapes of π pulses found within the above perturbative approach for a given truncation number N. In the right column we show plots illustrating time evolution of the qubit excitation dynamics $n_\uparrow(t)$ within the π-pulse duration time $0 < t < \tau$. Solid curves in the right column in Fig. 2 correspond to resonant driving $\delta = 0$, while dashed curves are related to nonzero detuning. The dynamics starts from the ground state at $t = 0$ and grows significantly at the half of the π-pulse duration time $\tau/2$. The increase of $n_\uparrow(t)$ resembles the response to singular driving at $t = \tau/2$, because the limit of $f(t)$ at $N \to \infty$ corresponds to the ideal π pulse with $f_{N\to\infty}(t) = \pi\delta(t - \tau/2)$, which is obviously not achievable experimentally. We stress that we work in the regime of finite N and finite amplitudes and treat the efficiency of this approach by means of deviation of the resulting $n_\uparrow(\tau)$ from unity with respect to nonzero detuning δ. The last two plots in Fig. 2 illustrate good efficiency of the corresponding π-pulse shapes: at $N = 3$ and 4 we observe that the dashed curves are very close

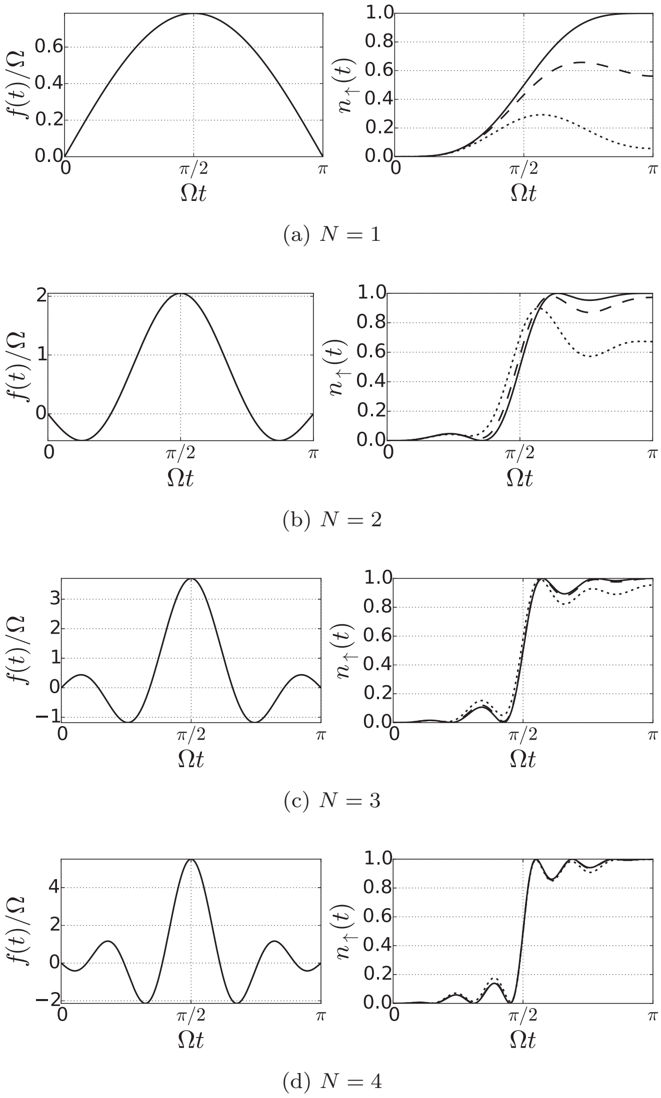

FIG. 2. The shapes of driving amplitudes (on left) and the time dependence of excited level occupation number $n_\uparrow(t)$ (on right) plotted for schemes of a different orders. The driving envelope $f(t)$ shown in panel (a) corresponds to a nonoptimized π pulse and is plotted as a reference. Solid curves in the right column correspond to resonant driving $\delta = 0$, while dashed and dotted curves are related to the detuning $\delta = \Omega$ and 2Ω, respectively.

to the solid ones at $t = \tau$. This means that the inhomogeneous broadening is effectively suppressed in a Rabi frequency range $\propto\Omega$ and this technique offers the synchronization of qubit ensemble starting even from $N = 3$.

In Fig. 3 we plot the numerical results for the dependencies of ground-state-amplitude absolute values $\sqrt{n_0(\tau)}$ after a π pulse as a function of detuning δ associated with a spectral broadening. It can be seen that the increase of the scheme order N results in flattening of the curves for $\sqrt{n_0(\tau)}$ around the point $\delta = 0$. This flattening is a quantitative demonstration of the inhomogeneous-broadening suppression.

Figure 4 is plotted in double logarithmic scale and illustrates the precision $n_0(\tau)$ of the π-pulse technique proposed. This figure allows one to estimate the residual value of the ground-state amplitude for the given scheme order. For instance,

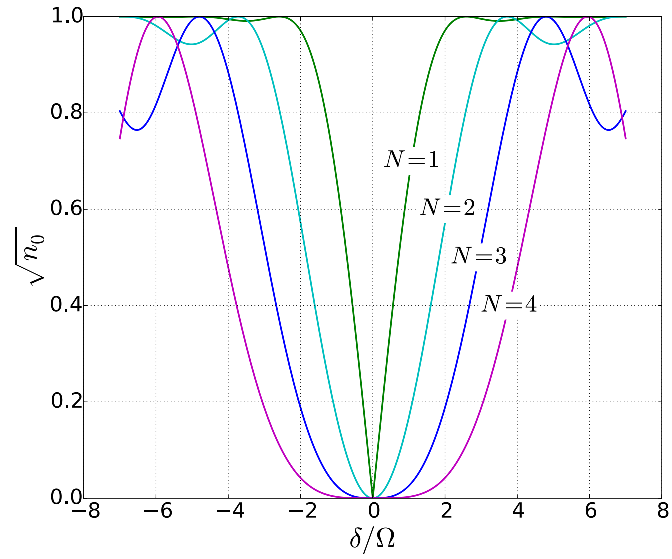

FIG. 3. (Color online) Dependence of ground-state amplitude after optimized π pulse as a function of detuning in schemes of different orders N.

for $N = 3$ the optimized π pulse allows us to achieve the probability of qubit ensemble excitation up to $n_\uparrow \approx 1$ to 10^{-3} at detuning values up to Ω. Besides that, Fig. 4 illustrates a robustness of the scheme against intensity fluctuations of the driving field. The discrepancy of the k_n amplitudes from their optimal values results in decreasing of the effective scheme order. This is illustrated by the curve marked $N = 3^*$ where

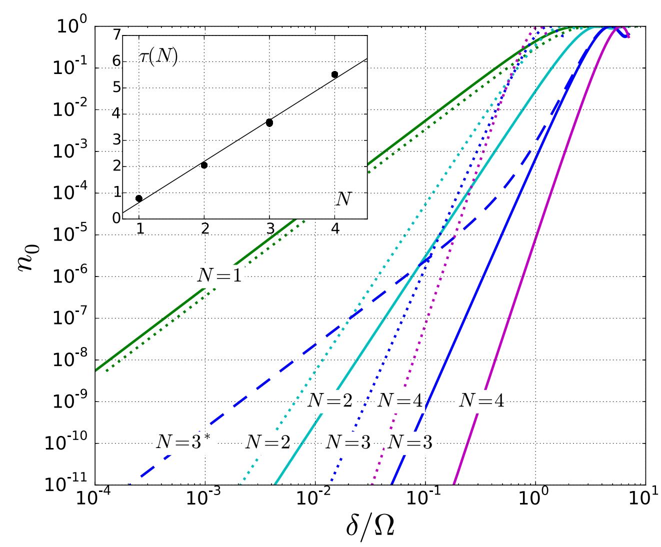

FIG. 4. (Color online) Dependence of residual ground-state occupation number after optimized π pulse on detuning value plotted in double logarithmic scale. Solid and dotted curves correspond to locked π-pulse duration time $\tau = \pi/\Omega$ and locked maximum value Ω of the intensity, respectively. For the latter case the dependence $\tau(N)$ is plotted in the inset. Blue dashed curve marked as $N = 3^*$ corresponds to slightly changed values of k_3 and k_5 by 2% in the third-order scheme and illustrates the robustness of the technique.

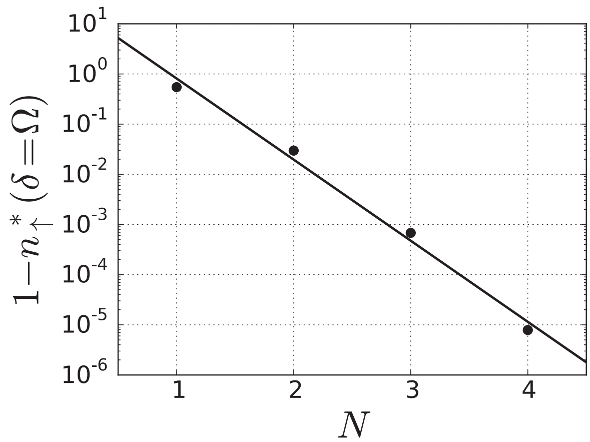

FIG. 5. Tolerance in qubit occupation number at detuning $|\delta| < \Omega$ as a function of optimization scheme order N. These data are extracted from the linear parts of the curves in Fig. 4.

values of k_3 and k_5 in the third-order scheme are increased by 2% from the optimal values. One can see from the dashed curve $N = 3^*$ that the scheme is stable up to the precision $n_0 \approx 10^{-4}$. For higher precision the slope of the curve $N = 3^*$ changes: it goes parallel to the $N = 1$ curve related to the first-order scheme and reflects an effective decrease of the scheme order from the third to the first.

The driving amplitude is limited in a real experiment. That means that we must rescale the π-pulse duration time in order to satisfy the π-pulse condition if we keep maximum of the intensity being locked to a certain value. To illustrate this, we use the envelope shapes evaluated above and rescale their maximum value down to Ω, i.e., $\max|f(t)| = \Omega$. After that we find optimized value of π-pulse duration times $\tau(N)$ at different scheme orders. The dependencies calculated for the rescaled envelope shapes and corresponding $\tau(N)$ are plotted as dotted curves in Fig. 4. The dependence $\tau(N)$ is close to linear and is shown in the inset of Fig. 4.

From the curves for $\sqrt{n_0(\tau)}$ shown in Fig. 4 we extract the coefficients of power-law dependencies of residual ground-state amplitude for a given N. As one can see, the dimensionless combination $n_0(\delta)(\delta/\Omega)^{-2N}$ does not depend on δ. Dependence on N of a logarithmically scaled value of this combination can be fit by a straight line, as shown in Fig. 5,

$$n_0(\delta) = 1 - n_\uparrow \approx 34\left(0.16\frac{\delta}{\Omega}\right)^{2N}. \tag{15}$$

Equation (15) is one of the central results which show the quantitative dependence of the precision on the order N and on detuning. The small scaling factor 0.16 for δ/Ω shows that this technique based on the sine representation (10) could provide the synchronization of the ensemble even if the driving-signal amplitude is less than the broadening by one order of magnitude.

V. DISCUSSION

The efficient excitation of NV centers in diamond reported in Ref. [11] were achieved through the sequences of rectangular pulses with periodical switching of the amplitude sign. This technique demonstrates the possibility of excitation of strongly off-resonant qubits by a weak driving signal. We studied an opposite regime when the single nonrectangular π pulse effectively suppresses the disorder. The mechanism we addressed is related to synchronous excitation dynamics of a two-level system under a particular nonrectangular envelope shape $f(t)$ of the π pulse. We considered a smooth nonrectangular shape of the π pulse given by external electromagnetic driving $F(0 < t < \tau) = f(t)e^{-i\omega t}$ with the envelope representable as the sum of N sine functions $f(t) = \Omega\sum^N k_{2n+1}\sin(2n+1)\pi t/(2\tau)$ where the amplitude and the pulse duration are locked with each other $\tau = \pi/\Omega$. The off-resonant response of a qubit to a nonrectangular signal cannot be calculated exactly and we found the perturbative solution. Within this solution we proposed the method based on optimization of the set of N parameters k_{2n+1}, which provide synchronous excitation of off-resonant qubits. Note that this optimization is not a direct expansion in a sine basis of ideal π pulse in a delta-functional form. The precision of this method, expressed in terms of qubit excitation number $n_\uparrow$, is controlled by the order N of the scheme and is proportional to $(\delta/\Omega)^{2N}$. This scheme is efficient for the qubit energies falling into tunable spectral range estimated as the driving amplitude strength Ω. Within our solution we demonstrated that the π pulse formed by $N = 4$ sine functions shows simultaneous excitation of qubits with the probability up to $n_\uparrow \approx 1$ to 10^{-5} for qubit frequencies ranging in $\approx\omega \pm \Omega$.

The sine expansion we used in this approach was based on the experimental requirement of continuity of $f(t)$ at initial and finite moments of time. Optimal envelope shape $f(t)$ can also be found in another basis, i.e., cosine series or rectangular-based blocks. Our calculations shows that in these cases the results will be qualitatively the same as described above. Thus, the proposed method is quite general and can be tuned to meet requirement and restrictions of a particular experiment.

VI. CONCLUSION

To conclude, we propose a model of smooth-shaped single π pulse which can be applied to realistic disordered qubit ensembles coupled to a transmission line. Such a π pulse provides an effective suppression of the inhomogeneous broadening and can be used as qubit synchronization technique. Our findings can serve as a complementary method to the disorder-suppression techniques reported in Ref. [11], where the sequences of rectangular pulses were used to increase the efficiency in the excitation of qubits within a certain frequency range. Similar technique can be effectively applied to create $\pi/2$ pulses to prepare entangled states of the inhomogeneously broadened qubits.

ACKNOWLEDGMENTS

The authors thank Alexey V. Ustinov and Walter V. Pogosov for discussions.

[1] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).

[2] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).

[3] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov Jr., Yu. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Science **327**, 840 (2010).

[4] L. DiCarlo, J. M. Chow, J. M. Gambetta, L. S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **460**, 240 (2009).

[5] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).

[6] P. D. Nation, J. R. Johansson, M. P. Blencowe, and F. Nori, Rev. Mod. Phys. **84**, 1 (2012).

[7] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008).

[8] T. P. Orlando, J. E. Mooij, L. Tian, C. H. van der Wal, L. S. Levitov, S. Lloyd, and J. J. Mazo, Phys. Rev. B **60**, 15398 (1999).

[9] J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999).

[10] M. V. G. Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A. S. Zibrov, P. R. Hemmer, and M. D. Lukin, Science **316**, 1312 (2007).

[11] S. Putz, D. O. Krimer, R. Amsüss, A. Valookaran, T. Nöbauer, J. Schmiedmayer, S. Rotter, and J. Majer, Nat. Phys. **10**, 720 (2014).

[12] K. Sandner, H. Ritsch, R. Amsüss, Ch. Koller, T. Nöbauer, S. Putz, J. Schmiedmayer, and J. Majer, Phys. Rev. A **85**, 053806 (2012).

[13] J. J. L. Morton, A. M. Tyryshkin, R. M. Brown, S. Shankar, B. W. Lovett, A. Ardavan, T. Schenkel, E. E. Haller, J. W. Ager, and S. A. Lyon, Nature (London) **455**, 1085 (2008).

[14] D. I. Schuster, A. P. Sears, E. Ginossar, L. DiCarlo, L. Frunzio, J. J. L. Morton, H. Wu, G. A. D. Briggs, B. B. Buckley, D. D. Awschalom, and R. J. Schoelkopf, Phys. Rev. Lett. **105**, 140501 (2010).

[15] P. Macha, G. Oelsner, J.-M. Reiner, M. Marthaler, S. André, G. Schön, U. Hübner, H.-G. Meyer, E. Il'ichev, and A. V. Ustinov, Nat. Commun. **5**, 5146 (2014).

[16] A. L. Rakhmanov, A. M. Zagoskin, S. Savel'ev, and F. Nori, Phys. Rev. B **77**, 144507 (2008).

[17] P. A. Volkov and M. V. Fistul, Phys. Rev. B **89**, 054507 (2014).

[18] N. I. Zheludev and Y. S. Kivshar, Nat. Mater. **11**, 917 (2012).

[19] D. S. Shapiro, P. Macha, A. N. Rubtsov, and A. V. Ustinov, Photonics **2**, 449 (2015).

[20] T. Brandes, Phys. Rep. **408**, 315 (2005).

[21] L. J. Zou, D. Marcos, S. Diehl, S. Putz, J. Schmiedmayer, J. Majer, and P. Rabl, Phys. Rev. Lett. **113**, 023603 (2014).

[22] P. L. Stanwix, L. M. Pham, J. R. Maze, D. Le Sage, T. K. Yeung, P. Cappellaro, P. R. Hemmer, A. Yacoby, M. D. Lukin, and R. L. Walsworth, Phys. Rev. B **82**, 201201 (2010).

[23] J. H. Wesenberg, A. Ardavan, G. A. D. Briggs, J. J. L. Morton, R. J. Schoelkopf, D. I. Schuster, and K. Mølmer, Phys. Rev. Lett. **103**, 070502 (2009).

[24] J. H. Wesenberg, Z. Kurucz, and K. Mølmer, Phys. Rev. A **83**, 023826 (2011).

[25] B. Julsgaard and K. Mølmer, Phys. Rev. A **88**, 062324 (2013).

[26] C. Grezes, B. Julsgaard, Y. Kubo, M. Stern, T. Umeda, J. Isoya, H. Sumiya, H. Abe, S. Onoda, T. Ohshima, V. Jacques, J. Esteve, D. Vion, D. Esteve, K. Mølmer, and P. Bertet, Phys. Rev. X **4**, 021049 (2014).

[27] H. Wu, R. E. George, J. H. Wesenberg, K. Mølmer, D. I. Schuster, R. J. Schoelkopf, K. M. Itoh, A. Ardavan, J. J. L. Morton, and G. A. D. Briggs, Phys. Rev. Lett. **105**, 140503 (2010).

[28] N. W. Carlson, L. J. Rothberg, A. G. Yodh, W. R. Babbitt, and T. W. Mossberg, Opt. Lett. **8**, 483 (1983).

[29] H. Lin, T. Wang, and T. W. Mossberg, Opt. Lett. **20**, 1658 (1995).

[30] M. Afzelius, C. Simon, H. de Riedmatten, and N. Gisin, Phys. Rev. A **79**, 052329 (2009).

[31] A. A. Rangelov, N. V. Vitanov, L. P. Yatsenko, B. W. Shore, T. Halfmann, and K. Bergmann, Phys. Rev. A **72**, 053403 (2005).

[32] J.-H. Schönfeldt, J. Twamley, and S. Rebić, Phys. Rev. A **80**, 043401 (2009).

[33] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).

[34] J. S. Hodges, N. Y. Yao, D. Maclaurin, C. Rastogi, M. D. Lukin, and D. Englund, Phys. Rev. A **87**, 032118 (2013).