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Frequency-offset separated oscillatory fields

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A frequency-offset separated-oscillatory-field technique is presented. The technique is a modification of the Ramsey method of separated oscillatory fields [Phys. Rev. **76**, 996 (1949)], in which the frequencies of the two separated oscillatory fields are slightly offset from each other, so that the relative phase of the two fields varies continuously with time. With this technique, the detection signal oscillates in time at the offset frequency, and the resonance frequency is obtained by using a simple straight-line fit of the phase of this signal. The technique has the advantages of being insensitive to the frequency response of the experimental system, of being sensitive only to noise at the offset frequency, and of allowing systematic effects to be more cleanly resolved due to the simple lineshape.

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I. INTRODUCTION

The Ramsey technique [1,2] of separated oscillatory fields (SOFs) has been widely used for performing precision measurements (see, e.g., Refs. [3–15]). When using this method, an atom (or other system) with resonant frequency f_0 is exposed to two pulses of oscillating field with frequency f separated by a time T, as shown in Fig. 1. The technique leads to SOF signals with a lineshape [16]

$$S_{\rm SOF}(f) = S_{\rm ni}(f) + S_{\rm i}(f)\cos\left[2\pi(f-f_0)T + \Delta\phi\right], \quad (1)$$

where the cosine term, with its envelope function $S_{\rm i}(f)$, comes from interference between quantum-mechanical amplitudes driven by the two separated fields, and $S_{\rm ni}(f)$ does not involve interference. Both $S_{\rm i}(f)$ and $S_{\rm ni}(f)$ are similar to the lineshape that would result if only one pulse were used. Examples of SOF lineshapes are shown in Fig. 2 for a transition from a stable state to another stable state [Fig. 2(a)], and a transition from a stable state to a state with radiative decay [Fig. 2(b)]. In the latter case, the linewidth of $\frac{1}{2T}$ is less than the natural width.

We present a modified SOF technique in which the frequencies of the two separated fields are slightly offset from each other. In this work we show that this frequency-offset separated-oscillatory-field (FOSOF) technique allows us to determine the resonant line center by using a straight-line fit and discuss the advantages of using the FOSOF technique. Primary among these advantages is that the technique is insensitive to the frequency response of the experimental system (for example, the carrier-frequency dependence of the intensity of the electromagnetic wave). A second advantage is the possibility of a better signal-to-noise ratio, since the signal obtained with this technique is only sensitive to noise at the offset frequency, and this offset can be set to a frequency where the noise spectral density is low. The simple FOSOF lineshape also allows systematic effects to be more cleanly resolved than they can be with the SOF lineshape.

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II. FREQUENCY-OFFSET SEPARATED-OSCILLATORY-FIELD TECHNIQUE

The FOSOF driving fields are given by

$$F = \begin{cases} F_1\cos\left[2\pi\left(f-\frac{\delta f}{2}\right)t+\phi_1\right] & \text{if } 0<t<D \\ F_2\cos\left[2\pi\left(f+\frac{\delta f}{2}\right)t+\phi_2\right] & \text{if } T<t<T+D \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

as shown in Fig. 1(a). The frequencies of the two separated fields differ by the small offset frequency δf ($\delta f \ll 1/T, 1/D$), and this offset causes the relative phases of the two fields [$\Delta\phi$ of Eq. (1)] to vary linearly in time as

$$\Delta\phi = \phi_{21} + 2\pi\delta f t, \quad (3)$$

where ϕ_{21} is the phase difference $\phi_2 - \phi_1$.

The effect of this time-dependent phase is that the two fields continuously cycle between being in phase and out of phase, and the observed FOSOF signal [Eq. (1)] varies sinusoidally in time at the small offset frequency δf as

$$S_{\rm FOSOF}(f,t) = S_{\rm ni}(f) + S_{\rm i}(f)\cos[2\pi\delta f t + \phi_{21} + 2\pi(f-f_0)T]. \quad (4)$$

This FOSOF signal is shown as a function of time t in Fig. 3 for three of the frequencies of Fig. 2(b). The amplitude of the sinusoid is the range indicated by the gray lines in Fig. 2(b), and $S_{\rm ni}(f)$ is the central value of this range at each value of f. The phase of the sinusoid varies with f, as shown by the three curves of Fig. 3. Also shown in Fig. 3 is a reference signal, which determines the relative phase ϕ_{21} of the two fields. This reference can be derived, for example, by mixing the two frequency-offset fields, and it oscillates between zero (when the two fields are out of phase by π) and a maximum (when the two fields are in phase). The $f = f_0$ FOSOF signal is in phase with this reference but is shifted in phase by $\Delta\theta$ for $f \neq f_0$, as shown in Fig. 3. This $\Delta\theta$ can be determined experimentally by recording both the FOSOF and reference signals versus time (the solid and dashed lines in the figure), with their phase difference determined by using least-squares fits of these signals to sinusoidal functions.

From Eq. (4), the phase shift is $\Delta\theta = 2\pi T(f - f_0)$, which is illustrated by the straight lines in Fig. 4. In the FOSOF technique, the resonance frequency f_0 is experimentally

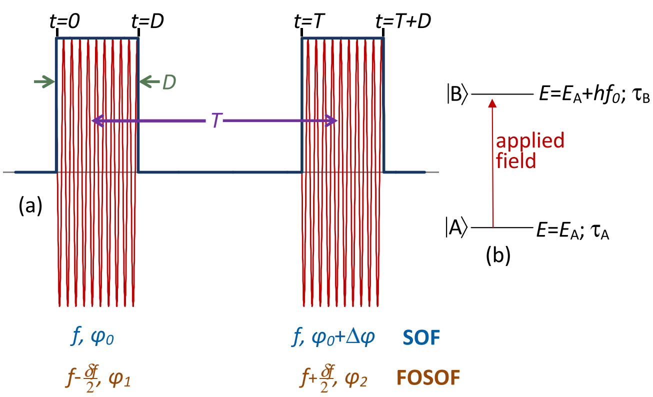

FIG. 1. (Color online) (a) Timing and (b) energy-level diagram for SOF and FOSOF experiments, along with the frequencies and phases of the two separated fields.

determined by fitting the measured phase shifts, $\Delta\theta$ (measured at a set of frequencies), to this linear relationship.

III. CANCELLATION OF PHASE IMPERFECTIONS

For both the SOF and FOSOF techniques, precise knowledge of the relative phase of the two separated fields is critical. To determine the line center to one part in N of the $\frac{1}{2T}$ linewidth of Fig. 2, it is necessary to know the relative phase to an accuracy of π/N.

Very precise relative-phase determinations can often be difficult to achieve because of the different phase response,

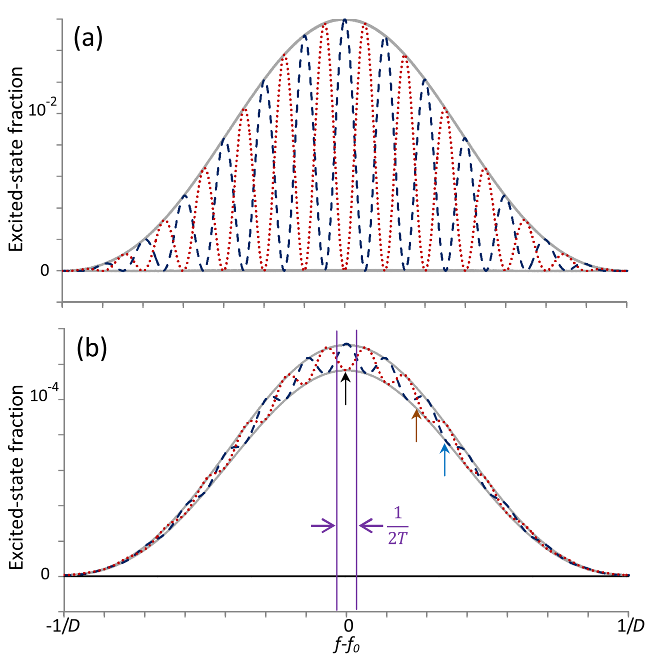

FIG. 2. (Color online) Typical SOF lineshapes, with $D = T/7$ [Fig. 1(a)]. The dotted and dashed curves give the population in a state $|B\rangle$ for atoms starting in a stable state $|A\rangle$ [Fig. 1(b)] for the cases of the two pulses having relative phases and 0 and π, respectively. For panel (a), $|B\rangle$ is also stable, and for panel (b) it has a lifetime of $\tau = T/7$. The width of the SOF interference is $\frac{1}{2T}$ in both cases.

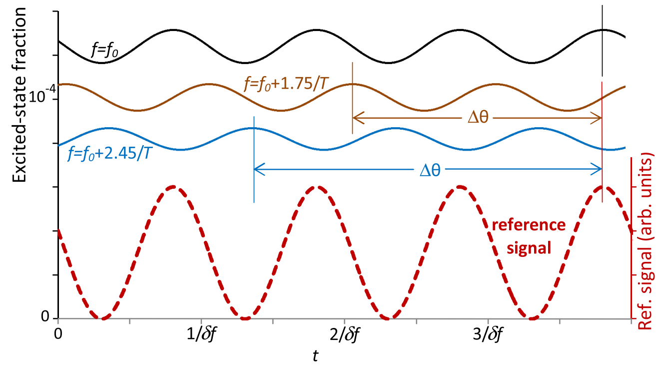

FIG. 3. (Color online) Fraction of population in the excited state for the FOSOF technique for the three values of f indicated by arrows in Fig. 2(b). The population varies sinusoidally in time (at the small offset frequency δf), and the phase of this sinusoid differs for different values of f. The relative phase of the two separated fields, ϕ_{21}, is obtained from a reference signal, which can, e.g., be generated by a mixer. The phase of the sinusoidally varying population agrees with the phase of this reference signal for $f = f_0$, but differs by the values of $\Delta\theta$ shown for $f \neq f_0$.

electrical length, and interfering reflected waves that might be present in the networks that produce the two separated fields. Thus, in general, $\phi_{21} = \phi_2 - \phi_1$ of Fig. 1(a) and Eq. (4) will depend on frequency. Furthermore, the finite bandwidths of detectors and other electrical filters in the apparatus can add phase shifts to the measured sinusoidal atomic signal.

We denote the measured FOSOF atomic signal as

$$S_{\text{meas}}(f,t) = S_{\text{ni}}(f) + S_{\text{i}}(f)\cos[2\pi\,\delta f\,t + \phi_{21}(f) + 2\pi(f - f_0)T + \psi(f)], \quad (5)$$

where the frequency dependence of ϕ_{12} has been included, and $\psi(f)$ is the phase shift added by the instrumental response of the detection system.

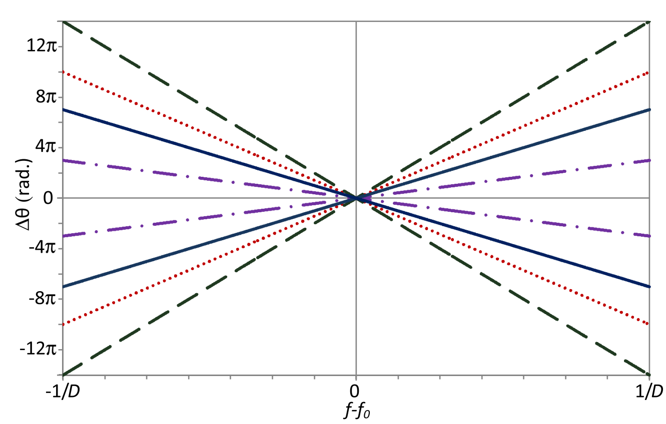

FIG. 4. (Color online) Phase shifts $\Delta\theta$ of the FOSOF signal versus f for for both positive (lines with positive slope), and negative (lines with negative slopes) offset frequencies δf. The solid line corresponds to a separation time $T = 7D$, as in Figs. 2(b) and 3. The dash-dot, dotted, and dashed lines correspond to $T = 3D$, $10D$, and $14D$, respectively. The FOSOF technique can determine the resonant frequency f_0 from the intercept of any one of these lines.

The FOSOF measurement also requires a reference signal oscillating at the offset frequency (for example, from a mixer, as in Fig. 3). The reference signal can also acquire a frequency-dependent phase shift due to electrical length mismatches or multiple reflections in the reference-generation system.

We denote the reference signal as

$$R_{\mathrm{ref}}(f,t) = R_0 \cos[2\pi\ \delta f\ t + \phi_{21}(f) + \xi(f)], \tag{6}$$

where $\xi(f)$ is the phase shift added by imperfections in the reference-generation system.

The measured phase difference between the atomic and reference signals is

$$\Delta\Theta = 2\pi(f - f_0)T + \psi(f) - \xi(f), \tag{7}$$

which differs from the ideal $\Delta\theta = 2\pi(f - f_0)T$ due to the $\psi(f)$ and $\xi(f)$ phase errors.

We note that repeating measurements with positive and negative values of δf (as in Fig. 4) does not lead to a cancellation of these phase errors. This is evident from Eq. (7), in which δf does not appear, or by considering the phase difference between the signals of Eqs. (5) and (6) with $\delta f \to -\delta f$.

These phase errors can be canceled by using a second measurement in which the atoms encounter the separated oscillatory fields in the reversed order, keeping the same sign of δf. The measured atomic signal for an order-reversed configuration [with $\phi_{21}(f)$ unchanged by the reversal] is

$$\widetilde{S}_{\mathrm{meas}}(f,t) = S_{\mathrm{ni}}(f) + S_{\mathrm{i}}(f)\cos[-2\pi\delta f t - \phi_{21}(f) + 2\pi(f - f_0)T - \psi(f)]. \tag{8}$$

If this is accomplished without perturbing the reference signal as well, then the phase difference between atomic and reference signals in the order-reversed configuration is

$$\Delta\widetilde{\Theta} = -2\pi(f - f_0)T + \psi(f) - \xi(f). \tag{9}$$

The true atomic phase is extracted from the difference

$$\Delta\Theta - \Delta\widetilde{\Theta} = 4\pi(f - f_0)T. \tag{10}$$

An extreme example of such a phase error, $\psi(f) - \xi(f)$, is shown as a dash-dot curve in Fig. 5. The phase error shifts the $\Delta\theta$ determination for both the regular and the order-reversed configuration, as shown by the dotted curves in Fig. 5. However, the difference of these two dotted curves perfectly cancels the phase errors and recovers the atomic signal, which is shown by the straight-line solid curve. The intercept of this solid line exactly matches the resonant frequency f_0.

Keeping ϕ_{21} unchanged when reversing the order in which the atoms encounter the separated oscillatory fields is not trivial, since the phases ϕ_1 and ϕ_2 of Eq. (2) could easily change when performing this reversal. However, we have developed two methods which leave these phases unchanged.

For a precision microwave FOSOF measurement of the $n = 2$ Lamb shift of hydrogen, we use a beam of metastable hydrogen atoms passing through spatially separated oscillatory field regions. We perform the order reversal by physically rotating the entire rigidly connected microwave system (field regions, microwave components, and microwave generators) by 180 degrees, so that the atomic beam encounters the two regions in the reverse order. Since the microwave system is unchanged under this rotation, the phases ϕ_1 and ϕ_2 remain unaffected.

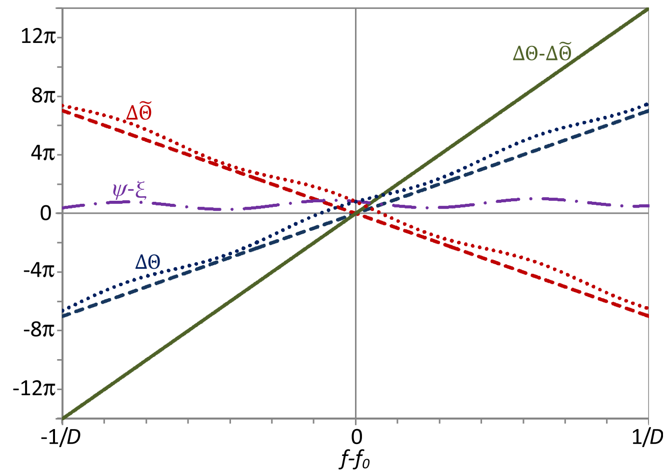

FIG. 5. (Color online) Phase $\Delta\Theta$ of the observed FOSOF signal versus f for for both perfect phase measurements (dashed lines) and for measurements with imperfect phase determinations (dotted curves). For illustration purposes, a large phase imperfection, $\psi(f) - \xi(f)$, is used (dot-dash curve). The solid line, which has twice the slope of the dashed lines, is the difference, $\Delta\Theta - \Delta\widetilde{\Theta}$ of Eq. (10), between the positive-slope dashed line and the negative-slope dashed line. The difference between the dotted lines also exactly equals the solid line, showing that phase errors exactly cancel by taking the $\Delta\Theta - \Delta\widetilde{\Theta}$ difference.

For a precision FOSOF measurement of the helium $n = 2$ triplet P fine structure, we use microwave pulses to generate temporally separated fields. We create a train of microwave pulses, with the frequency of consecutive pulses alternating between $f - \frac{\delta f}{2}$ and $f + \frac{\delta f}{2}$. Each FOSOF measurement selects two of these pulses by exciting the atoms up to the 2^3P state before one of the pulses and detecting the number of atoms that undergo the microwave transition after the next pulse. For this measurement, the reversal is achieved by changing the timing of the laser excitation and detection (without changing the microwave pulse train). For obtaining $\Delta\Theta$, the excitation to 2^3P occurs before an $f - \frac{\delta f}{2}$ microwave pulse and the detection occurs after the following pulse, which has a frequency of $f + \frac{\delta f}{2}$. For obtaining $\Delta\widetilde{\Theta}$, the excitation occurs before an $f + \frac{\delta f}{2}$ pulse, and the detection occurs after the next pulse, which has a frequency of $f - \frac{\delta f}{2}$. Again, the microwave system is left untouched by the reversal and therefore ϕ_1 and ϕ_2 are unaffected.

IV. ADVANTAGES OF FREQUENCY-OFFSET SEPARATED-OSCILLATORY-FIELD TECHNIQUE

A. Insensitivity to experimental frequency response

For SOF measurements, the variation of the intensity of the driving fields as a function of frequency can be a major concern. Such variations can be caused by the frequency response of the frequency generator, or of components used to deliver the power to the atoms. They can also be caused by interference between reflected waves, and by buildup cavities, if these are used. The frequency-dependent variations cause F_1

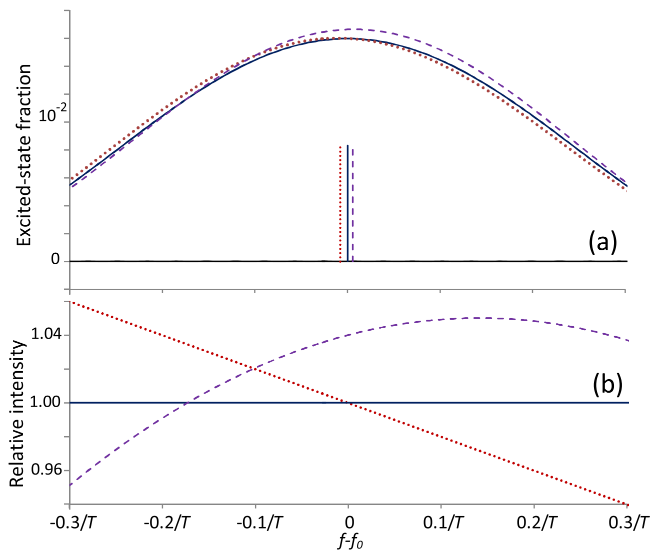

FIG. 6. (Color online) (a) The SOF lineshape is distorted if the intensity of the field driving it varies with frequency, as shown in panel (b). Here the central lobe of Fig. 2(a) is shown for constant intensity (solid lines), for a linearly varying intensity (dotted lines), and for a quadratically varying intensity (such as may be caused by a buildup cavity, dashed lines). The vertical lines in panel (a) indicate the resonance center, as determined from the half-maximum points, and these show that for precision measurements using SOF lineshapes, significant shifts of the line center can be caused by intensity variations.

and F_2 of Eq. (2) to vary with frequency, and this distorts the SOF lineshape, as shown, for example, in Fig. 6. In precision measurements, the resulting shift of the lineshape is often a limiting factor for determining the line center.

The SOF technique is influenced by variations of intensity versus frequency because the measured quantity is a product [Eq. (1)] of an interference envelope (which depends on field intensity) and an interference cosine (which depends on the phases of the fields). Therefore, field intensity variations can mimic phase variations in the SOF signal.

The FOSOF technique is immune to intensity changes versus frequency since it measures the phase rather than the amplitude of the interference signal. This immunity to intensity variations is the key strength of the FOSOF technique. The FOSOF technique measures the phase of the interference cosine directly and therefore focuses on the essential quantity of interest for SOF measurements.

B. Insensitivity to low-frequency noise

The statistical uncertainty in the line-center determination is similar for the SOF and FOSOF techniques. That is, a SOF measurement that maps out the lineshape (such as that of Fig. 2) and a FOSOF measurement that uses the same data-collection time to map out the linear phase variation (such as in Fig. 4) lead to similar statistical uncertainties in the determined line centers.

However, a second advantage of the FOSOF technique is insensitivity to ubiquitous low-frequency drifts and noise in the experimental measurement. Only noise at the offset frequency δf contributes to the uncertainty in the extracted phase shift $\Delta\theta$. The offset frequency can be chosen to be in a quiet region of the noise spectrum, thus allowing for a better signal-to-noise ratio when using the FOSOF technique.

C. Simple lineshape for resolving systematic effects

A major advantage of the FOSOF technique is its simple straight-line lineshape. For precision measurements, examining the deviation from the ideal lineshape is often an important tool for measuring or eliminating a systematic effect. For SOF measurements, such a deviation can be very difficult to observe. This difficulty arises because the SOF lineshape is often fit to several parameters [usually including at least a width, center, and amplitude parameter for the envelope function $S_{\rm i}(f)$, and a phase and oscillation rate for the interference cosine, as well as possible parameters for the noninterference term $S_{\rm ni}(f)$]. Adjustment of such a large number of parameters can absorb the lineshape deviation, making the precise form of the original deviation inextricable from the residuals of the fit. Also, any imperfections in the execution of the SOF measurement could lead to additional distortions of the lineshape, which would mask the deviation caused by the systematic effect. These imperfections could include variation of field intensities versus frequency, atoms that do not experience the full SOF sequence, and averaging over unknown velocity distributions of the atoms.

The straight-line lineshape of FOSOF requires only a two-parameter fit, which can often be further constrained by knowledge of T. As described in Sec. IV A, distortions which result from variation of the intensity of the driving fields are also eliminated. Remaining distortions (for example distortions due to overlap or interference [17] from a neighboring resonance) are much more easily identifiable as deviations from the expected straight line.

V. RELATIONSHIP TO OTHER WORK

The FOSOF method builds upon the phase-variation technique described by Klein *et al.* [18]. For the phase-variation technique, the relative phase between the oscillatory field regions is stepped by discrete amounts (using, e.g., phase shifters or added electrical lengths), which could introduce correlated field-intensity variations. In contrast, the FOSOF method uses a continuous phase variation generated by the offset frequency δf, which changes the phase smoothly from 0 to 2π without field-intensity variations. The FOSOF varies the phase on a timescale of $1/\delta f$, which can easily be set to a much shorter time than would be possible for variation by using phase shifters or electrical length, thus avoiding experimental drifts of signals and low-frequency noise.

Traditionally, high-precision SOF experiments, especially experiments with decaying states, where the interference signals are small compared with the noninterference background $S_{\rm ni}(f)$ [as in Fig. 2(b)], have used a phase-switching technique to extract the interference signal. The relative phase of the separated fields is switched between 0 and π, and the difference between the 0 and π signals [in the ideal case, twice the interference signal: $2S_{\rm i}(f)\cos[2\pi(f-f_0)T]$] is obtained.

This technique is even more susceptible to systematics due to frequency-dependent field-intensity variation effects such as in Sec. IV A. Introducing an accurate π relative phase shift between the oscillatory field regions, without correlated frequency-dependent intensity changes, is difficult in practice. Thus, the 0 and π signals [such as, for example, those in Fig. 2(b)] will be distorted differently by field intensity variations, and the distortions will show up prominently in the much smaller difference between the two signals.

The phase-switching technique can, in principle, switch rapidly to get around slow drifts and improve signal-to-noise ratios. However, the systematics due to correlated intensity changes often become worse with rapid switching. In comparison, smooth and rapid changes of relative phase are automatically achieved with the frequency offset in the FOSOF method.

A two-frequency SOF technique (2FSOF) was described by Jarvis *et al.* [19] and Garvey *et al.* [20], where the offset frequency essentially averages continuously over the Ramsey fringes, and the envelope of the oscillations $S_i(f)$ is measured. This allows spectroscopic measurements to be made without the need to accurately determine phase offsets, albeit with a loss of resolution. This 2FSOF technique is still susceptible to line-center shifts resulting from distortions of the interference signal envelope, caused by intensity-variation effects of the sort that are discussed in Sec. IV A. In comparison, the phase shifts measured in the FOSOF method are independent of field-intensity variations, while retaining the full resolution of the Ramsey SOF method.

VI. CONCLUSIONS

We have described a FOSOF technique for high-precision spectroscopic measurements, which obtains the resonance line centers by using information contained in the phase of the atomic signal. The FOSOF technique is unaffected by systematic effects due to frequency-dependent field-intensity variations of the separated oscillatory fields. The freedom to place the offset frequency in a low-noise spectral neighborhood improves the signal-to-noise ratio of measurements, and the simple straight-line lineshape allows any remaining systematic effects to be more cleanly resolved.

We expect that this technique can be beneficially used to improve the precision and accuracy of separated-oscillatory-fields measurements. We are presently using the FOSOF technique for precision measurements of $n = 2$ hydrogen and helium fine structure in our laboratory.

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