Tunneling time in attosecond experiments and the time-energy uncertainty relation

Ossama Kullie*

Theoretical Physics, Institute for Physics, Department of Mathematics and Natural Science, University of Kassel, Kassel, Germany

(Received 12 May 2015; published 24 November 2015)

In this work we present a theoretical model of the tunneling time and the tunneling process (in attosecond experiment for the He atom). Our model is supported with physical reasoning leading to a relation which performs an excellent estimation for the tunneling time in attosecond and strong-field experiments, where we address the important case of the He atom. Our tunneling time estimation is found by utilizing the time-energy uncertainty relation and represents a quantum clock. The tunneling time is also featured as the time of passage through the barrier similar to Einstein's *photon-box Gedanken experiment*. Our work tackles an important case study for the theory of time in quantum mechanics and is very promising for the search for a (general) time operator in quantum mechanics. The work can be seen as a fundamental step in dealing with the tunneling time in strong-field and ultrafast science and is appealing for more elaborate treatments using quantum wave-packet dynamics and especially for complex atoms and molecules.

DOI: 10.1103/PhysRevA.92.052118 PACS number(s): 03.65.Xp, 42.50.Md, 42.50.Hz, 33.20.Xx

I. INTRODUCTION

A comprehensive theory of time measurement in quantum mechanics is missing to date [1] (Chap. 3). Often it is said that time plays a role essentially different from the role of the position in quantum mechanics. In contrast, Hilgevoord [2] argued that there is nothing in the formalism of the quantum mechanics that forces us to treat time and position differently. Observables such as position, velocity, etc., both in classical mechanics as well as in quantum mechanics, are relative observables, and one never measures the absolute position of a particle, but the distance in between the particle and some other object [2,3]. Indeed, there are many attempts to consider time as a dynamical intrinsic or an observable time called event time. Hilgevoord concluded [2] that, when looking to a time operator, a distinction must be made between universal time coordinate t, a c-number like a space coordinate, and the dynamical time variable of a physical system situated in space-time, i.e., clocks. Busch [4] argued that the conundrum of the time-energy uncertainty relation (TEUR) in quantum mechanics is related in the first place to the fact that time is identified as a parameter in Schrödinger equation (SEQ). He classified three types of time in quantum mechanics: external time (referred to as parametric or laboratory time), intrinsic or dynamical time, and observable time. External time measurements are carried out with clocks that are not dynamically connected with the object studied in the experiment and usually called parametric time. The intrinsic or dynamical time is measured in terms of the physical system undergoing, dynamically, a change, where every dynamical variable marks the passage of time. We will see that this is important for our time invention where the energy serves as the dynamical variable in question, which enables a quantitative measure for the length of the time interval of the tunneling or the tunneling time (T-time) in strong-field experiments. The third type of time according to Busch is the observable time or event time, for example, the time of arrival of the decay products at a detector.

In the history of the quantum mechanics, the earliest attempt, which causes one of the most impressive debates, is Einstein's *photon-box Gedanken experiment (GE)* [5] or the Bohr-Einstein weighing *photon-box GE* [3] (and the references therein). A photon is allowed to escape from a box through a hole, which is closed and opened temporarily by a shutter. The period of time is determined by a clock, which is part of the box system, which means that the time is intrinsic and dynamically connected with the system under consideration. The total mass of the box before and after a photon passes is measured. Bohr showed that the process of weighting introduces a quantum uncertainty (in the gravitational field), leading to an uncertainty in time τ, which is the time needed to pass out of the box and is usually called the time of passage [4], in accordance with the TEUR, Eq. (1) below. Aharonov and Reznik [3] offer a similar interpretation, that the weighing leads, due to the backreaction of the system underlying a perturbation (energy measurement), to an uncertainty in the time of the internal clock relative to the external time [3]. Hence, for quantum systems it is important to observe the time from within the system or using an internal clock. Busch [4] presented an argument which makes no assumptions concerning the method of measurement and is simply based only on a version of quantum clock uncertainty relation as follows: If the energy of the escaping photons is determined with an accuracy δE from the difference of energy before and after the opening period of the shutter, then these energies must be defined within an uncertainty δE; i.e., the box energy uncertainty ΔE must satisfy $\Delta E \leqslant \delta E$. Then the clock uncertainty allows us to conclude that the box system needs at least a time $t_0 = \frac{\hbar}{\Delta E}$ in order to evolve from the initial state, "shutter closed," to the orthogonal state, "shutter open." Accordingly, the time interval within which a photon can pass through the shutter is indeterminate by an amount $\Delta T = t_0$. This leads to Bohr's TEUR $\Delta T \delta E \approx \hbar$ [1] (Chap. 3).

In this work we use similar ideas. We define the T-time as the time delay caused by the barrier (denoted $\tau_{T,d}$) and it is suggested to be similar to the time of passage through the barrier (and escaping at the exit of the barrier). The (quantum) particle (an electron) undergoing this process spends a time that is the time needed from the moment of entering the barrier region (i.e., the classically forbidden region) to the moment of

*kullie@uni-kassel.de

escaping the barrier in the tunneling direction. In addition, we suggest a time interval needed to reach the entrance of the barrier (denoted $\tau_{T,i}$) after it is shaken off by the laser field at its initial position x_i. $\tau_{T,d}$ is similar to the traversal time used in the context of the tunneling approaches [6–8] or the Feynman path integral (FPI) approach [9–11] (and [1], Chap. 7). However, our approach differs conceptually from other approaches, particularly because it is not probabilistic (statistical). For example, in contrast to the FPI, we do not make any assumption about paths inside the barrier, while, as is well known, the FPI approach is based on all paths starting at the entrance of the barrier at $t = 0$ and ending at the exit of the barrier at time $t = \tau$, which defines a time duration τ and is identified as the traversal time through the barrier and assumed to be equal to the delay time measured by the experiment [12]. A second type of T-time that we invent is what we call the symmetrical T-time or the total T-time (denoted $\tau_{T,\mathrm{sym}}$). We will see that that can be easily calculated from the symmetry property of the T-time but then later we find that $\tau_{T,\mathrm{sym}} = \tau_{T,i} + \tau_{T,d}$, which is the time accounted from the moment of starting the interaction process, where the electron gets a shakeoff, responds, and jumps up to the tunneling "entrance" point, taking the (opposite) orientation of the field, passes the barrier region, and overcomes the barrier at the "exit" point (the tunneling) and escapes to the continuum. The key issue of the present work is (in the words of Busch [1], Chap. 3) a case study, the T-time in attosecond experiments and ultrafast science, derived by utilizing the TEUR:

$$\Delta T \Delta E \geqslant \frac{\hbar}{2}. \tag{1}$$

In Sec. II we present our theoretical model, in Sec. III we offer a convincing physical reasoning for our theoretical model, in Sec. IV we discuss our result with a comparison to the experiment, and finally we give a conclusion to our work.

II. THEORY AND MODEL

In this section we suggest a way to approximate the T-time in attosecond experiments based on simple mathematical and quantum mechanical rules. We first prepare in Sec. II A the basic material needed to introduce our model. The outline of the model is given in Sec. II B and the T-time model is presented in Sec. II C.

A. Preview

Our starting point is a model of Augst *et al.* [13,14], where the appearance (or the threshold) intensity of a laser pulse for the ionization of the noble gases is predicted. The appearance intensity is defined [13] as the intensity at which a small number of ions is produced. In this model (in atomic units) the effective potential of the atom-laser system is given by

$$V_{\mathrm{eff}}(x) = V(x) - xF = -\frac{Z_{\mathrm{eff}}}{x} - xF, \tag{2}$$

where $F = F_m$ is the field strength at the maximum of the laser pulse (in this work, in all our formulas F stands for F_m) and Z_{eff} is the effective nuclear charge that can be found by treating the (active) electron orbital as hydrogenlike, similar to the well-known single-active-electron (SAE) model [15,16]. The

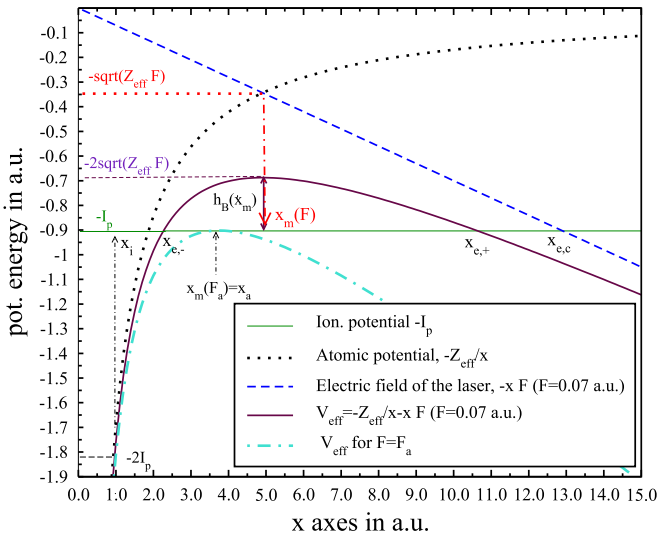

FIG. 1. (Color online) Graphic display of the potential curves, the barrier width and the two inner and outer points $x_{e,\pm} = (I_p \pm \delta_z)/2F$, the "classical exit" point $x_{e,c} = I_p/F$, and $x_m(F) = \sqrt{Z_{\mathrm{eff}/F}}$, the position at the maximum of the barrier height [note $x_a = x_m(F = F_a)$]; see text.

choice of Z_{eff} is easily recognized for a multielectron system and well-known in atomic, molecular, and plasma physics [14,17–19]. Many authors use the quantum defect [20], which is very similar to the effective charge approximation. We take a one-dimensional model along the x axis as justified by Klaiber and Yakaboylu *et al.* [21,22]. Augst *et al.* [13] calculated the position of the barrier maximum x_m by setting $\partial V_{\mathrm{eff}}(x)/\partial x = 0 \Rightarrow x_a = x_m(F_a) = (\sqrt{Z_{\mathrm{eff}}/F_a})$ and by equating $V_{\mathrm{eff}}(x_m)$ to the ionization potential $V_{\mathrm{eff}}(x_a) = -I_p$ (compare Fig. 1, the lower green curve). They found an expression for the atomic-field strength F_a,

$$-\frac{Z_{\mathrm{eff}}}{x_a} - x_a F = -I_p \Rightarrow F_a = \frac{I_p^2}{4Z_{\mathrm{eff}}}, \tag{3}$$

and the appearance intensity $I_a = F_a^2$. Now we take this idea and relate our argumentation to this model for $F \leqslant F_a$, i.e., for the tunnel ionization in the regime of the well-known Keldysh [23] parameter $\gamma_K < 1$. It is easy to see from Fig. 1 that the tunnel exit (denoted $x_{e,+}$) obeys $x_{e,+}(F) \geqslant x_m(F)$, where the equality is valid for $F = F_a$. In this regime, when $F < F_a$ the energy of the tunneling electron is not sufficient to reach the top of the barrier (as for F_a), suggesting an energy uncertainty, which is determined by the energy that the electron needs to overcome the barrier and appears in the continuum at the exit point. It appears with zero velocity at the exit point $x_{e,+}$ according to the strong-field approximation (SFA) due to Keldysh-Faisal-Reiss theory [23–25]. Indeed, the barrier height at a position x is given by (compare Fig. 1)

$$\overline{h_B(x)} = |h_B(x)| = |E - V_{\mathrm{eff}}(x)| = \left| -I_p + \frac{Z_{\mathrm{eff}}}{x} + xF \right|, \tag{4}$$

where $\overline{h_B(x)}$ is equal to the difference between the ionization potential and effective potential $V_{\mathrm{eff}}(x)$ of the system (atom + laser) at the position x, where $E = -I_p$ is the binding energy

of the electron before interacting with the laser. Note that we can also get x_m and the maximum $h_B(x_m)$ from the derivative of Eq. (4), $\partial h_B(x)/\partial x = 0$. An immediately arising question is as follows: What about its maximum $h_B(x_m)$ and how this maximum is related to the energy uncertainty when the electron passes the barrier region and is shifted to the continuum or a "quasi" energy level? First, in light of the work of Augst *et al.* and the derivation of $x_a = x_m(F_a)$ and Eq. (3), it turns out (compare Fig. 1) that the maximum of the barrier height $h_B(x_m)$ for arbitrary field strength lies at $x_m(F) = \sqrt{Z_{\rm eff}/F}$; see the Appendix. Indeed, Eq. (3) can be generalized as the following, for a field strength $F \leqslant F_a$ we get

$$F \leqslant \frac{I_p^2}{4Z_{\rm eff}} \Rightarrow \delta_z^2 = I_p^2 - 4Z_{\rm eff}F \geqslant 0. \tag{5}$$

The equality $\delta_z = 0$ is valid for $F = F_a$. We will see that $\delta_z = \delta_z(F) = \sqrt{I_p^2 - 4Z_{\rm eff}F}$ is a key quantity; it controls the tunneling process and determines the "time delay" $\tau_{T,d}$ caused by the barrier and the total or the symmetrical T-time $\tau_{T,\rm sym}$, Sec. II C. From Fig. 1 we see the barrier height at x_m:

$$h_B(x_m) = |-I_p - V_{\rm eff}(x_m)| = |-I_p + \sqrt{4Z_{\rm eff}F}|. \tag{6}$$

This is the maximum of the barrier height, and by setting $h_B(x_m) = 0$ one obtains $F_a = I_p^2/(4Z_{\rm eff})$, which is equivalent to the setting $V_{\rm eff}(x_m) = -Ip$ as done by Augst *et al.* [13] and can be easily verified from Eqs. (4) and (6). Note, in contrast to a statical barrier, that the barrier height and the barrier width are interdependent (both depend on F), and we have $h_B(x_m) = 0 \Leftrightarrow d_B = 0$, where d_B is the barrier width, which is the length between the entrance and the exit points of the barrier region along the tunneling direction, the x axis, determined by the major axis of the laser field F at its maximum. The "entrance" and "exit" points of the tunneling region, $x_{e,-}$ and $x_{e,+}$, respectively, are determined by the vanishing barrier height $h_B(x) = 0$ (compare Fig. 1) and are the crossing points of $V_{\rm eff}(x)$ with the $-I_p$ line. Using Eq. (4) and setting $h_B(x) = 0$ leads to

$$x_{e,\pm} = \frac{I_p \pm \delta_z}{2F} \Rightarrow d_B = x_{e,+} - x_{e,-} = \frac{\delta_z}{F}$$
$$\text{and } x_{e,+} = x_{e,c} - x_{e,-}, \tag{7}$$

where $x_{e,c}$ is called the "classical exit" point, the intersection of the electric field line $-x\,F$ with the ionization potential of the electron $-Ip$ line; hence, $x_{e,c} = \frac{I_p}{F}$, and $\delta_z = \delta_z(F)$ is given in Eq. (5). We emphasize the dependence of δ_z on $Z_{\rm eff}$ [14,19]. Note the origin of the axes is at 0.

B. Outline of the model

The (tunneling-) ionization happens according to SFA with zero kinetic energy at the exit point x_e. Our idea is that the uncertainty in the energy can be quantitatively discerned from the atomic potential energy at the exit point $\Delta E \sim |V(x_e)| = |-\frac{Z_{\rm eff}}{x_e}|$ for arbitrary field strength $F \leqslant F_a$. Then, when the electron is moving in the x direction [21,22] (the major axis orientation of the laser field) and overcomes the barrier, its kinetic energy is decreasing; at the same time it moves upwards on the potential energy scale, losing potential energy ($\frac{-Z_{\rm eff}}{x}$ getting smaller in absolute value). The change happens simultaneously in the potential and the kinetic energy while tunneling [one can imagine while tunneling that the electron is staying at the level $-Ip$ on the energy scale ($-Ip$ line in Fig. 1) until reaching the exit point, which possibly defines a quasienergy state; in a similar way a metastable state is defined (see the Appendix)] until its kinetic energy becomes zero at the exit point, although its (atomic) potential energy is $\frac{-Z_{\rm eff}}{x_e} \neq 0$. This can be also gathered from the analysis of the short-range Yukawa and long-range Coulomb potentials given by Torlina *et al.* [26]. Their conclusion, supported with *ab initio* numerical tests, is that for long-range potentials ionization is not yet completed at the "moment" the electron exits the tunneling barrier in contrast to the usual assumption that ionization is completed once the electron emerges from the barrier. Indeed, it is not difficult to see that the electric field of the laser pulse shifts the electron far from the nucleus (mainly along the x-axis direction [21,22]), reaching the exit point with zero velocity; i.e., the electron is forced by the electric field to take and move (mainly or approximately) along a preferred direction and, most importantly, to reduce its kinetic energy to zero at the exit point x_e (the field interacts only kinematically with the electron since there is no photon absorption), where it is still underlying the attraction of the atomic potential $V(x_e) = -\frac{Z_{\rm eff}}{x_e}$ that defines the uncertainty in the energy and acts as a shutter, open-closed, like in the photon-box GE with an uncertainty proportional to $\Delta E \sim |V(x_e)|$. We will see in Sec. IV that the result is very convincing.

It is straightforward to show, when setting $\Delta E \sim |V(x_{e,c})|$, that the classical exit point $x_{e,c}$ (see the Appendix) leads to incorrect T-time (i.e., it fails to predict T-time measured by the experiment). Thus, it is important to use a correct exit point. From Fig. 1, one can see that $x_{e,c}$ is far from the "correct" exit point $x_{e,+}$; see Eq. (7). Therefore, to use the classical exit point, $x_{e,c}$, to determine the T-time will never give a correct answer. Here we can indicate a failure of the Keldysh time (see the Appendix), which results primarily from its inadequate definition. If we recall the definition of the Keldysh T-time, the time it takes a classical electron (with an average velocity) to cross a static barrier of a length l [27], where $l \equiv d_B$. For $h_B(x_m) = 0 \Rightarrow l = 0$ we get the Keldysh T-time $\tau_k = 0$ (meaning the ejection of an electron happen instantaneously at $F = F_a$) because the barrier width vanishes for $F \to F_a$, $|x_{e,+} - x_{e,-}| = 0$. However, we know, at the appearance (or the threshold) intensity ($I_a = F_a^2$) the ionization time is equal to $\frac{1}{I_p}$ (in a.u.) [28] (Chap. 8) and is not (and cannot be) zero. This seems natural because the energy gap that has been overcome is I_p, and as we will see in the next section this follows immediately from our model. As a consequence, Keldysh time represents a laboratory clock (parametric or external time), whereas in our following T-time model and time relation(s) the time is dynamically connected to the system (to observe the time form within the system and consider the quantum nature of the particle); thus, it represents a quantum clock.

C. Tunneling time

Our goal now is to find an expression to calculate the T-time, and what we need is the correct exit point, where many approximations exist. The most used one in the literature can

be obtained from Eq. (4) and given in Eq. (7). As seen in Fig. 1, $x_{e,-}$ is the inner crossing point, the entrance point, and $x_{e,+}$ is the outer crossing point, the exit point. Physically, it is argued that the electron escapes the barrier at $x_{e,+}$, when it moves in the direction $x_{e,-} \rightarrow x_{e,+}$, and vice versus for the opposite direction. We will see that this presents a useful symmetry property of the tunneling process when deriving the T-time. For the atomic-field strength F_a, $\delta_z = 0$ and we have $x_{e,+} = x_{e,-} = x_a$. A first arising question in our model is, what happens in the limit of the appearance intensity, i.e., for $F \rightarrow F_a = I_p^2/(4Z_{\text{eff}})$, where the electron is shifted from the ground state $E_0 = -I_p$ to $E_f \approx 0$ appearing at $x_{e,-} = x_a = x_{e,+}$ with zero velocity. Its energy uncertainty (since there is no photon absorption) is then (according to our model) $\Delta E(F_a) \sim |-\frac{Z_{\text{eff}}}{x_a}| = \frac{I_p}{2}$. One sees that for atomic-field strength (F_a) the electron is heavily disturbed but appearing not far from the nucleus at $x_a = x_m(F_a) = \sqrt{\frac{Z_{\text{eff}}}{F_a}} = \frac{2Z_{\text{eff}}}{I_p}$ with an ionization time that follows immediately from TEUR in Eq. (1),

$$\tau_a = \frac{1}{2|\Delta E(F_a)|} = \frac{1}{I_p}, \tag{8}$$

as it should be for the ionization process at the F_a [28] (Chap. 8). However, we will see later that F_a is a special case because $x_{e,-} = x_a = x_{e,+}$ [a double solution of $h_B(x) = 0$] and the limit case $\frac{1}{I_p}$ for $F \rightarrow F_a$ is a sum of two equal terms $\frac{1}{2I_p}$; see further below and Sec. III A. The special case F_a showed that our model is meaningful and that atomic potential energy of the electron at the tunnel exit $x_{e,+}$ (instead of the gravitational potential in the Einstein-Bohr GE) amounts to calculating the uncertainty of the energy in the tunneling process, and hence the T-time, by virtue of Eq. (1), which leads to an excellent result, as we see in Sec. IV.

We turn now to the general case $F < F_a$, where we can calculate the uncertainty in the energy $\Delta E(x_{e,+})$ by using the exit point $x_{e,+}$ (in the direction $x_{e,-} \rightarrow x_{e,+}$) and from this the T-time. From Eqs. (7) and (1) and according to our model, we get

$$\Delta E(x_{e,+}) = \left|\frac{-Z_{\text{eff}}}{x_{e,+}}\right| = \frac{Z_{\text{eff}} 2F}{(I_p + \delta_z)} = \frac{(I_p - \delta_z)}{2}, \tag{9}$$

$$\tau_{T,\text{unsy}} = \frac{1}{2}\frac{1}{\Delta E} = \frac{1}{(I_p - \delta_z)}, \tag{10}$$

which we call the unsymmetrical T-time $\tau_{T,\text{unsy}}$. One can easily show that a T-time resulting from using the classical exit point $x_{e,c}$ is the first order of Eq. (10) [see the Appendix, Eq. (A3)]. We show now that a factor (1/2) is missing, which can be recovered by a symmetry consideration. What about the inner point $x_{e,-}$? We could assume, due to the δ_z symmetry between $x_{e,+}$ and $x_{e,-}$, that the electron enters the barrier backwards from $x_{e,+}$ entrance to $x_{e,-}$ exit with an uncertainty (according to our model) $\Delta E(x_{e,-}) = |\frac{-Z_{\text{eff}}}{x_{e,-}}|$, which leads to [compare Eqs. (9) and (10)]

$$\frac{1}{2\Delta E(x_{e,-})} = \frac{(I_p - \delta_z)}{4Z_{\text{eff}} F} = \frac{1}{(I_p + \delta_z)}. \tag{11}$$

This symmetry is deduced in a way similar to how the Aharonov-Bohm time operator [29] is defined for a free particle $\hat{T} = \frac{1}{2}(\hat{x}\,\hat{p}^{-1} + \hat{p}^{-1}\hat{x})$ or in more elaborate and detailed treatment (the so-called bilinear form) given by Olkhovsky and Recami [30]. Such operators (given by Aharonov-Bohm or Olkhovsky) have the property of being maximally symmetric in the case of the continuous energy spectra and the property of quasi-self-adjoint [31] operators in the case of the discrete energy spectra [30] (and the references therein); they are the next-best thing to self-adjoined operators and satisfy the conjugate relation with the Hamiltonian and therefore implies an ordinary TEUR [30], [1] (Chap. 1). We use this property, i.e., we assume that the maximally symmetric (or almost self-adjoint [30]) property holds for the T-time (for more details, see [30]), which leads to using Eqs. (10) and (11) in a simple relation for what we call the symmetrical (or total) T-time given by

$$\tau_{T,\text{sym}} = \tau_{T,+} + \tau_{T,-} = \frac{1}{2}\left(\frac{1}{\Delta E^-} + \frac{1}{\Delta E^+}\right) = \frac{1}{2}\left[\frac{1}{(Ip + \delta_z)} + \frac{1}{(Ip - \delta_z)}\right] = \frac{I_p}{4Z_{\text{eff}} F}, \tag{12}$$

where we defined $\tau_{T,\pm} = 1/(2\Delta E^{\mp}) = [2(Ip \pm \delta_z)]^{-1}$ or $(1/2)\Delta E^{\pm} = \Delta E(x_{e,\pm})$. Relation (12) has again (clearly because $\delta_z = 0$) the correct limit for atomic-field strength [compare Eq. (8) and the discussion after it]:

$$\tau_{T,\text{sym}}(F \rightarrow F_a) = \frac{1}{2Ip} + \frac{1}{2Ip} = \frac{I_p}{4\,Z_{\text{eff}}\frac{Ip^2}{4\,Z_{\text{eff}}}} = \frac{1}{Ip}.$$

Note that the limit $F \rightarrow F_a$ gives $x_{e,+} = x_{e,-} = x_m(F_a) = x_a$, which means that the two points coincide at the top of the barrier. The question is whether this means a symmetry break of the tunneling process, so that the "tunneling" becomes a "real" ionization (or an ejection) process at the appearance intensity $I_a = F_a^2$, and δ_z becomes imaginary for superatomic-field strength $F > F_a$ (whereas $F < F_a$ is called the subatomic-field strength); a further discussion is given in Sec. IV.

A further question concerns whether the time to reach the entrance of the barrier and overcome and escape the barrier at the exit, which is an intrinsic or dynamical type of time to be measured by a quantum clock, becomes a classical, external (or parametric) time after the tunneling due to the break of some symmetry property, so that (only) under such a symmetry break does a quantum clock (the internal time) coincide with a laboratory clock (the external time), as is the case for a free particle (continuous energy spectra).

In Sec. IV we see that Eq. (12) (especially $\tau_{T,-} = \frac{1}{2}\frac{1}{(Ip-\delta_z)}$) gives an excellent agreement with the experimental result of [32]. In the next section, Sec. III, we discuss our model following a physical reasoning.

III. PHYSICAL REASONING

A. Tunneling time and a model of a shutter

The theoretical mathematical model developed in Sec. II can be supported and derived by physical arguments; the only difference is that we try to figure a physical insight that helps us to put physics in mathematical relations. From Fig. 1 and

Eq. (7) we get [$d_B(F)$ is the barrier width]

$$d_B(F) = x_{e,+} - x_{e,-} = \frac{(I_p + \delta_z)}{2F} - \frac{(I_p - \delta_z)}{2F} = \frac{\delta_z}{F}. \quad (13)$$

From this it follows $\delta_z = (x_{e,+} - x_{e,-})\,F$. Because, for atomic field strength $F = F_a \Rightarrow \delta_z = 0$, $d_B = 0$, we can interpret δ_z as the field's (kinetic) energy exerting in the tunneling process between the entrance (or the "inner") point and the "outer" exit point. The uncertainty in the energy as the electron moves to $x_{e,+}$, i.e., when tunneling, overcomes the barrier region and escapes at the exit to the continuum, is then $\Delta E^+ = \mathrm{abs}(-I_p + \delta_z)$. That means the barrier itself causes a time delay relative to the atomic-field strength F_a ($\delta_z = 0$). The T-time is then obtained from Eq. (1),

$$\tau_{T,d} \equiv \frac{1}{2\Delta E^+} = \frac{1}{2\,(I_p - \delta_z)} \,(\text{for } F \leqslant F_a), \quad (14)$$

which we call $\tau_{T,d}$ [$=\tau_-$, compare Eq. (12)], meaning that that is the delay in the time relative to the atomic field (more details in [34]) or the time duration (time interval) to pass the the barrier region (in the direction $x_{e,-} \to x_{e,+}$) and escape at the exit point $x_{e,+}$ to the continuum. The term $\tau_{T,d}(F \to F_a) = \frac{1}{2I_p}$ at the limit $F = F_a$ accounts for turning off the wave packet (or shakeoff step, as discussed in Secs. III B and III C) at the "entrance-exit" point x_a to escape to the continuum, which indicates that the shakeoff step or turning at $x_{e,+}$ to the continuum for $F < F_a$ happens with a time delay as given in Eq. (14). Expanding $\delta_z = Ip\sqrt{1 - (4Z_{\mathrm{eff}}F/Ip^2)}$ and taking the first order, we get Eq. (12), $\tau \approx \frac{I_p}{4Z_{\mathrm{eff}}F} = \tau_{T,\mathrm{sym}}$. It means using the symmetrization gives a linearized time duration (linearized T-time).

Now we argue that this picture fits well in the GE of Einstein and the (intrinsic) time $\tau_{T,d}$ [Eq. (14)] or that the second term in Eq. (12) is rather the time of passage, where the shutter open-closed time interval (generated in the internal time [3]) is related by the virtue of Eq. (1) to the uncertainty $(1/2)\Delta E^+ = \Delta E(x_{e,+}) = V(x_{e,+})$, which acts as a shutter [note that we recovered in Eqs. (12) and (14) the factor (1/2) missing in Eq. (10)]. We think also that the attosecond experiment, with the help of our model, represents a realization of the photon-box GE (with the electron as a particle instead of the photon) with an uncertainty being determined from the (Coulomb) atomic potential due to the electron being disturbed by the field F, instead of being disturbed by the weighting process and, as a result, an uncertainty in the gravitational potential [3], as shown by the famous proof of Bohr to the uncertainty (or indeterminacy) of time in the photon-box GE [3,4].

B. Total time ($F = F_a$)

At the moment, one can obtain the total time, i.e., including the time to reach the entrance of the barrier $x_{e,-}$, by adding the term $1/(2I_p)$ to Eq. (14),

$$\tau_{T,\mathrm{sym}} \approx \tau_{T,t} = \frac{1}{2}\left(\frac{1}{I_p} + \frac{1}{I_p - \delta_z}\right), \quad (15)$$

where the index t is used only to distinguish between different notations. Nevertheless, the term $1/(2I_p)$ that we added is exact only when $F = F_a$ or $x_{e,-}(F_a) = x_a = x_{e,+}(F_a)$, since the time to reach the entrance-exit point x_a follows from the uncertainty (the response or jump-up to the $-I_p$-level at entrance-exit point)

$$\Delta E = |\Delta V_{(x_i,x_a)}| = I_p \Rightarrow \tau_{(x_i,x_a)} = \frac{1}{2I_p}, \quad (16)$$

where x_i is the initial point. It can be viewed as the response time of the electron to the field, that is, the electron received a kick by the field, and is polarized along the field direction, while (jumping up [35]) moving from x_i to the entrance-exit point x_a to the continuum, $x_i \to x_{e,-}(= x_a = x_{e,+}) \to \infty$. In this case [$h_B(F = F_a) = 0$, $\delta_z = 0$], the most probable "tunneling" mechanism is that the laser field distorts the electron, shakes it up (moving from x_i to x_a), and shakes it off (moving to the continuum) at a (total) time given in Eqs. (12) or (15) $\tau = \frac{1}{2}(\frac{1}{I_p} + \frac{1}{I_p}) = \frac{1}{I_p}$. In this model, for $F = F_a$ the (illustrative) two steps are not strictly separated, whereas, for $F < F_a$ they are well separated due to the barrier $d_B(F) > d_B(F_a) = 0$, as we discuss in the next section.

C. Total time for subatomic field $F < F_a$

However, $x_a = x_{e,-} = x_{e,+}$ is the maximum entrance point (the most-far-right-lying entrance point, see Fig. 1), the electron is less disturbed for $F < F_a$ and moved to a point $x_{e,-} < x_a$ that is closer to the initial point x_i [36], this shortens the time to reach the entrance point, and we expect that the response of the electron to a small field strength F is weaker than that to a stronger field $F \to F_a$. The time reduction in $\tau_{(x_i,x_{e,-})}$ for F compared to $\tau_{(x_i,x_a)} = 1/(2I_p)$ for F_a [Eq. (16)] is a factor depending on δ_z [see discussion after Eq. (13)], because the kinetic energy experiences a change proportional to $(x_{e,-} - x_i)\,F < (x_a - x_i)\,F_a$. A (weaker) field $F < F_a$ is not sufficient to compensate for the kinetic energy at the (shakeup) step; instead, the electron is at the entrance $x_{e,-} < x_a$ with a velocity that is sufficient to enter the barrier region, overcomes the barrier, and reaches the exit point $x_{e,+}$ with zero velocity. Here we indicate another failure of the Keldysh time (see the last passage of Sec. II B); that is, the electron does not enter the barrier region (or start the tunneling) with the initial velocity $\sqrt{2I_p}$ suggested by many authors [35]. Now we give the following relation for the time needed to reach the entrance point $x_{e,-}$ and show an explanation further below:

$$\tau_{T,i} \equiv \tau_{(x_i,x_{e,-})} = \frac{1}{2(I_p + \delta_z)} \equiv \frac{1}{2\Delta E^-}. \quad (17)$$

The factor δ_z [comparing to Eq. (16) for F_a] in the denominator results from two parts. Indeed, we follow [3] in that the uncertainty is a result of the different reactions or responses of the electron to different field strengths. The first part comes from the difference of moving along the x axis, i.e., the difference in shifting the electron to x_a with F_a or to $x_{e,-} < x_a$ for $F < F_a$, which leads to $\Delta_1 = x_a F_a - x_{e,-}F = \frac{I_P}{2} - \frac{(I_p - \delta)}{2} = \frac{\delta_z}{2}$. The other part can be deduced from the change on the vertical (potential energy) scale. When the electron receives a kick, changing its potential (on a vertical scale), its atomic potential experiences different changes between $V(x_i)$ and $V(x)$ for $x = x_a$ or $x_{e,-} < x_a$. However, this is a result of different responses (on energy scale), while the electron is forced to follow an orientation along the (opposite) field direction at x_a or at $x_{e,-}$.

Therefore, this part can be approximated from the difference $\Delta V(x)$ in the atomic potential at the different entrance points, which gives $\Delta_2 = V(x_a) - V(x_{e,-}) = -\sqrt{Z_{\rm eff} F_a} + \frac{2Z_{\rm eff} F}{(I_p - \delta_z)} = -\frac{I_p}{2} + \frac{(I_p+\delta_z)}{2} = \frac{\delta_z}{2}$. We are led to a difference equal to $\Delta_1 + \Delta_2 = \delta_z$ between reaching the entrance point $x_{e,-}$ (for F) relative to the entrance-exit point x_a (for F_a), leading to an energy uncertainty $\Delta E = \text{abs}(-I_p - \delta_z) = (I_b + \delta)$, for $F \leqslant F_a$ and with a time $\tau_{T,i}(F) = \frac{1}{2(I_p+\delta_z)}$ for arbitrary subatomic-field strength F; hence, one obtains Eq. (17) instead of Eq. (16). We have explained so far Eqs. (14) and (17); in doing so we explained the physical meaning of the symmetry consideration done above [see Eq. (12) and the discussion before] and we obtain from Eqs. (14) and (17) the result obtained in Eq. (12),

$$\tau_{T,\rm sym} = \tau_{T,i} + \tau_{T,d} \equiv \tau_{T,+} + \tau_{T,-} = \frac{1}{2}\left[\frac{1}{I_p + \delta_z} + \frac{1}{(I_p - \delta_z)}\right] = \frac{I_p}{4Z_{\rm eff} F}, \quad (18)$$

where the first term $\tau_{T,i} = \tau_{T,+}$ corresponds to the first step, where the electron is shaken up and moved to the entrance $x_{e,-}$ (or x_a for F_a) that takes the times $1/(2E^-) = [2(I_p + \delta_z)]^{-1}$. The second term $\tau_{T,d} = \tau_{T,-}$ corresponds to the actual T-time, or the time it takes the electron to move from $x_{e,-}$ to $x_{e,+}$, overcome the barrier, and be shaken off to the continuum, with a time delay $1/(2E^+) = [2(I_p - \delta_z)]^{-1}$ due to the barrier relative to the atomic-field strength F_a, where $\delta_z = 0$ and $\tau_{T,d} = (2I_p)^{-1}$.

For $F = F_a$, as already mentioned, the second or shakeoff step immediately follows the first or shakeup step and the two steps are not strictly separated. For $F < F_a$ the two steps of the tunneling process are well separated. They happen with opposite directions at the time scale; the first step is less time consuming since F causes a smaller disturbance relative to F_a, and the the electron does not move far from its initial position for a small F, $x_{e,-} < x_a$, whereas the second step happens with a time delay, $x_{e,+} > x_a$, relative to the ionization at atomic-field strength. So far, our theoretical model is assisted with an explanation through a physical reasoning. In the following we show and discuss our result for the He atom with a comparison to the experiment [32,33,37].

IV. RESULT AND DISCUSSION

In Fig. 2 we show the results of Eq. (10), the unsymmetrical $\tau_{T,\rm unsy}$, and Eq. (12), the symmetrical T-time $\tau_{T,\rm sym}$. The results for $\tau_{T,d}$ Eq. (14) and again the symmetrical (or total) T-time $\tau_{T,\rm sym}$ of Eq. (18) are shown in Fig. 3. Note that Eq. (18) is identical with Eq. (12), whereas Eq. (14) is the second term of Eq. (18) or (12), which is the actual T-time, i.e., the time needed to pass the barrier region ($x_{e,-} \to x_{e,+}$) and escape at $x_{e,+}$ to the continuum, and most likely that is the T-time measured in the experiment, as explained in the next paragraph. The results are for the He atom in a comparison with the experimental result of [32,33,37]. The experimental data and the error bars in the figure were kindly sent by Landsman [37]. We plotted the relations (10), (12), (14), and (18) at the values of the field strength at the maximum of the elliptically polarized laser pulse ($\lambda = 735$, elliptical parameter $\varepsilon = 0.87$, $F = F_0/\sqrt{1+\varepsilon^2}$) used by the experiment exactly as given in [37].

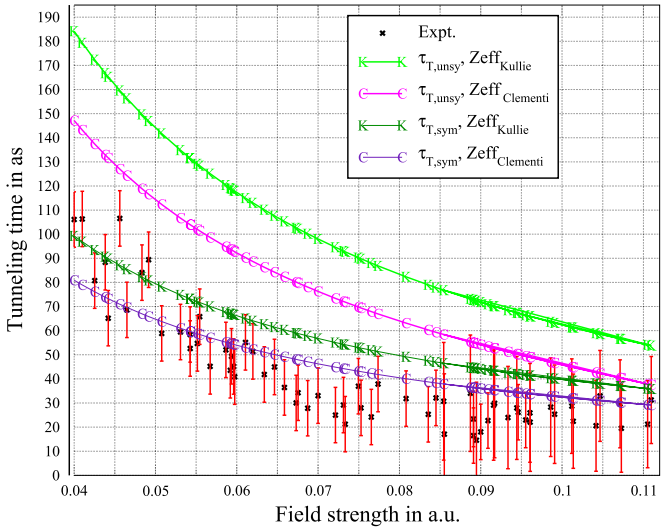

FIG. 2. (Color online) T-time $\tau_{T,\rm unsy}$ [Eq. (10)] and $\tau_{T,\rm sym}$ [Eq. (12)] for two $Z_{\rm eff}$ models [41] and [42]. Time is in attosecond units vs laser-field strength in atomic units, corresponding to the tunneling ionization of the He atom in strong field (compare Fig. 3). Experimental values were kindly sent by Landsman [37].

The experiment. Concerning the experiment by the group of Keller *et al.* [37], the time delay due the barrier is measured indirectly, $t_T^{\rm expt} = \frac{(\theta_m - \theta_C)}{\omega}$, where ω is the laser frequency, θ_m the angular offset of the center of angular distribution, and θ_C a correction due to the Coulomb force of the ion calculated by classical-trajectory simulation [32,33,37–39]. Unfortunately, in this experiment the beginning of the interaction between the laser field and the bound electron cannot be directly observed or exactly determined.

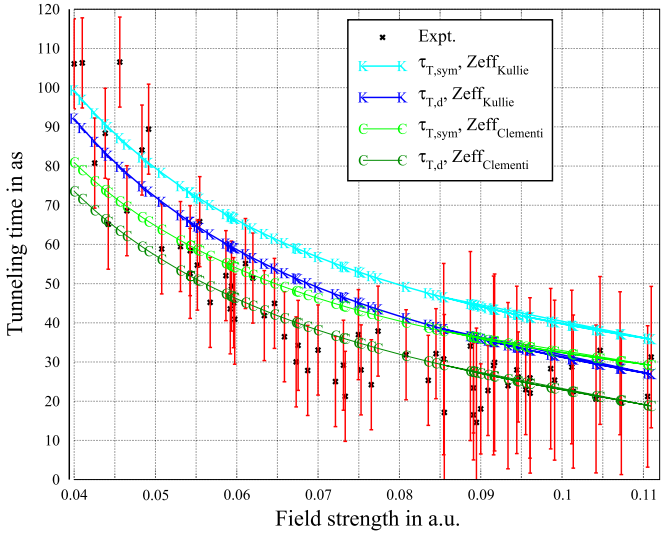

FIG. 3. (Color online) T-time $\tau_{T,d}$ [Eq. (14)] and $\tau_{T,\rm sym}$ [Eq. (18)] for two different $Z_{\rm eff}$ models as in Fig. 2. Note that Eq. (18) is identical to Eq. (12) [see Fig. 2], with units as in Fig. 2. T-time corresponds to the tunneling ionization of the He atom in strong field, in excellent agreement with the experimental result [32,33,37]. Experimental values as in Fig. 2.

The time zero $t_0^{\rm expt}$ is one of the assumptions of the model, which is used to interpret the data. Consequently, in the experiment it is not possible to distinguish between the instant of the interaction $t_0 = 0$ and the instant when the electron enters the barrier region, say $t_{T,l}$ [40]. The assumption implies that $t_0^{\rm expt}$, the moment when the laser-field points along the major axis, is consistent with the time "zero-time-calibration," say $t_0^{\rm simul}$, of a model based on classical-trajectory simulation to interpret the data $t_0^{\rm simul} = t_0^{\rm expt}$. It is likely (in our opinion and according to our communication [40]) that $t_0^{\rm expt}$ (the moment when the laser field points along the major axis, as assumed in the experiment) differs from the instant of (starting) the interaction with the laser field $t_0 = 0$. On the other hand, it is difficult to identify $t_0^{\rm expt}$ with $t_{T,i} = \tau_{T,i}$, which is the instant of the orientation along the laser-field direction (in our model the moment of entering the barrier region).

However, arguing that the probability of tunneling is highest when the barrier is shortest, corresponding to $t_0^{\rm expt}$ [40], suggests that the data from the measurement are comparable to the time delay caused by the barrier (the time spent in the classical forbidden region [40]), which means that $t_T^{\rm expt}$ corresponds to or (approximately) equals the actual T-time delay of our model, $t_T^{\rm expt} \approx \tau_{T,d}$.

Discussion and comparison to the experiment. In Fig. 2, the upper two curves are the unsymmetrical T-time $\tau_{T,\rm unsy}$, Eq. (10), for two different models of the effective nuclear charge $Z_{\rm eff}$ that the tunneling electron experiences during the tunneling process. Accordingly, the lower two curves are the symmetrical T-time, $\tau_{T,\rm sym}$ [Eq. (12)], for the two different models of $Z_{\rm eff}$. We mention that Eq. (15) (not plotted) gives a closer result to Eqs. (12) and (18). The two different effective charge models are from Kullie [41], with $Z_{\rm eff,K} = 1.375$, and Clementi and Raimondi [42], with $Z_{\rm eff,C} = 1.6875$. In Fig. 2 we see that our $\tau_{T,\rm unsy}$ is not close to the experimental data, whereas $\tau_{T,\rm sym}$ is close for both models of the $Z_{\rm eff}$. However, we notice (see discussion further below) that the first term in Eqs. (12) and (18) is much smaller than the second $\tau_{T,i} < \tau_{T,d}$ for small F (relative to F_a).

Concerning $Z_{\rm eff}$, we see for a small field strength $F \lesssim 0.05$ that $\tau_{T,\rm sym}$, with $Z_{\rm eff,K}$, is closer to the experimental data (and especially for $\tau_{T,d}$, see discussion of Fig. 3 below). The reason is that the $Z_{\rm eff,K}$ model is a H-atom-like model, based on the assumption that the first electron of the He atom occupies the $1s$ orbital [with probability density $|\Psi(r)|^2$], which screens the nuclear charge and the second electron is treated as an active electron or a "valence" electron [41] similar to that done in the SAE approximation. This is a good approximation when the tunneling electron moves far from the left atomic core (He^+) or when the barrier width is large ($x_{e,+} > 15$ a.u.), hence the better agreement, and possibly this is important for smaller field strength in the region, where $\gamma_K \approx 1$. In the range of larger field strength, multielectron effects are expected and the model $Z_{\rm eff,C}$ based on the Hartree-Fock calculation is more reliable, where the electron moves not far from the left atomic core (small barrier width) and hence the better agreement in this region.

Now we look to Fig. 3, where $\tau_{T,\rm sym}$ Eq. (18) and $\tau_{T,d}$ Eq. (14) are shown. Equation (18) is the same as Eq. (12) (shown in Fig. 2). For the $\tau_{T,d}$ we see an excellent agreement with the experiment. As already discussed, $\tau_{T,d}$ corresponds to the T-time measured in the experiment, that is, the time (interval) needed to pass the barrier (the classically forbidden) region between the entrance to the exit point and escape to the continuum with a shakeoff, or between the instant of orientation at $x_{e,-}$ and the instant of ionization at $x_{e,+}$, which corresponds to the time spent in the classically forbidden region. Concerning $Z_{\rm eff}$ in Fig. 3, we readily see for $\tau_{T,d}$ the same behavior as in Fig. 2 for $\tau_{T,\rm sym}$. For small $F \lesssim 0.055$ a.u., $Z_{{\rm eff},K}$ gives better agreement with the experiment, whereas for larger field strength $Z_{{\rm eff},C}$ is more reliable, where multielectron effects are expected due to the decreasing width of the barrier and the tunneling electron is closer to the first one when it tunnels through the barrier region. It is likely that a model depending on the x coordinate $Z_{\rm eff}(x; x_{e,+})$ will achieve a better agreement that smoothly fits the two regions. Moreover, when taking $Z_{{\rm eff},C}$ we get $F_a = 0.115$ a.u., which is a good estimation in regard to the experimental data and the trends of the curves in Fig. 3.

In Fig. 3 we see that the difference between the total or symmetrical T-time $\tau_{T,\rm sym}$ and the (actual) T-time $\tau_{T,d}$ is small, because the second term $\tau_{T,d}$ in Eq. (18) incorporates the time delay caused by the barrier and is the main time contribution of the tunneling process for a large barrier, whereas the first part $\tau_{T,i}$, is due to the shakeup of the electron by the field moving it from its initial position to the entrance $x_{e,-}$, which is small for a small F. For large field strength the two parts become closer because the barrier width decreases $\delta_z/F = (x_{e,+} - x_{e,-}) \to 0$ and for the appearance intensity ($\delta_z = 0$) they become equal.

In Fig. 4 we plot our result $\tau_{T,d}$ Eq. (14) together with the FPI and Larmor Clock (LC) results of [37] (data were kindly sent by Landsman and Hofmann). In Fig. 4, the FPI is in a good agreement with our result, and the difference between the two results is smaller than the experimental error bars. Indeed, we expect that the FPI would agree better for large field strength $F > 0.055$ a.u. with the lower curve ($Z_{{\rm eff},C}$, green).

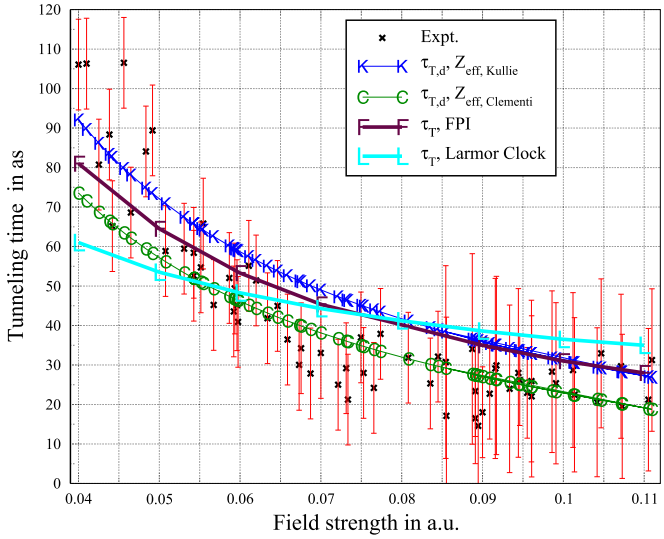

FIG. 4. (Color online) T-time $\tau_{T,d}$ Eq. (14) for two different $Z_{\rm eff}$ models as in Fig. 2 together with the FPI and Larmor clock results [37] and the experimental result [32,33]. Experimental values, the FPI, and Larmor result were kindly sent by Landsman and Hofmann [37].

For small field strength, FPI is more or less close to both curves (green, blue), but our upper curve ($Z_{\mathrm{eff},K}$, blue) tends to be in a better agreement with the experimental data. An important point is that our model and result(s) estimate a real T-time (time delay or time interval) of a single particle similar to the LC time; it is not distributive or that of an ensemble (although indeterminately in regard to the uncertainty relation) and we make no assumption about the path of the particle inside the barrier, whereas the FPI treatment is probabilistic and/or distributive and makes use of all possible (classical) paths inside the barrier that have a traversal tunneling time $\tau = \tau_0$. Furthermore, Landsman *et al.* [37] uses the time τ_0, which is determined by the measurement to coarse grain the FPI distribution of the T-times to achieve the desired results, whereas Sokolovski [11], [1] (Chap. 7) claims (in regard to his FPI description) that no real time is associated with the tunneling. We think that the two views (our result and the FPI of Landsman *et al.* [37]) are rather complementary, as it is usual in quantum mechanics: wave-particle or individual (single particle)-statistical (distributive), etc. The result of the LC should, in principle, agree better with our result, but the data of Landsman *et al.* [12,37,38] show that the agreement is good only for $F \approx 0.05$–0.1 a.u.; hence, the LC values are inferior for $F < 0.05$ a.u. The same holds for $F > 0.1$ a.u. Although the difference to our result here is smaller than the error bars, the trends of the LC curve for a large field strength looks somewhat too flat. In general, the LC curve is flat compared to the other curves and tends to disagree with the experimental data for a small field strength $F < 0.05$ a.u.

In Fig. 5 we show the tunneling time $\tau_{T,d}$ versus the barrier width $d_B(F)$. $\tau_{T,d}$ shows a linear dependence on the barrier width $d_B(F)$ in the region $F = 0.04$–0.11 a.u. and a limit $1/(2I_p)$ at $d_B(F_a) = 0$. The other limit for very large barrier width ($F \to 0, \delta_z \to I_p$) is $\approx \frac{I_p}{4Z_{\mathrm{eff}}F} = \tau_{T,\mathrm{sym}}$, which is straightforward because, for very large barrier, $\tau_{T,d} \gg \tau_{T,i} \Rightarrow \tau_{T,d} \approx \tau_{T,\mathrm{sym}}$. We note, as seen in Fig. 5, that the time spent by a particle (a photon) to traverse the same barrier width with the speed of light is much smaller than the T-time of an electron in the He atom.

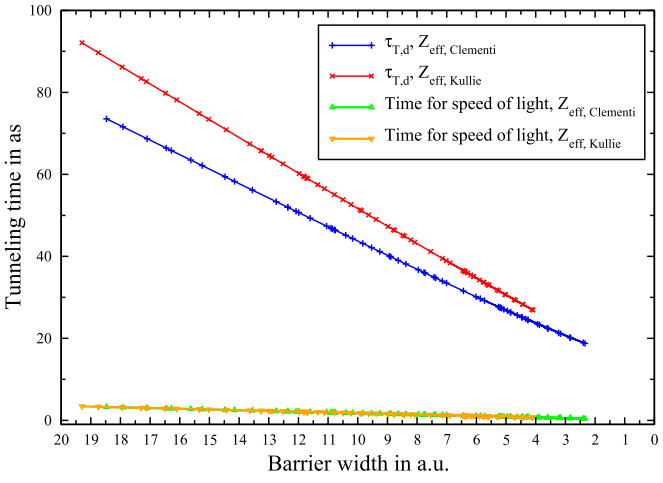

FIG. 5. (Color online) T-time $\tau_{T,d}$ [Eq. (14)] vs barrier width $d_B(F)$ [Eq. (13)] for two different Z_{eff} models as in Fig. 2. The lines at the bottom of the figure show the time spent by a particle (a photon) traversing the same barrier at the speed of light.

Further discussion. At the limit $F = F_a$ of the subatomic-field strength the tunneling process is out and an ionization process called "above the barrier decay" is beginning [20]. For superatomic-field strength $F > F_a$, δ_z becomes imaginary (and so the crossing points, compare Eq. (7), but still a real $x_m = \sqrt{Z_{\mathrm{eff}}/F}$), which indicates that the real part $\frac{1}{2I_p}$ of $\tau_{T,d}$ or $\tau_{T,i}$, is the limit for a "real" time tunnel-ionization process. Indeed, in this case the atomic potential is heavily disturbed and the imaginary part of the time $\tau_{T,d}$ is then due to the release or the escape of the electron [at $x_m(F)$] from a lower energy level than $-Ip$ (and possibly escaping with a high velocity), where the ionization happens mainly by a shakeoff step [28] (Chap. 9). Here we see the clear difference between the quantum mechanical and the classical clocks [3,29]. Classically, we can make the interaction time with the system arbitrarily small, the real part of the time can be made arbitrary small, and an imaginary part is absent. In quantum mechanics the tunneling-ionization time has a real part limit $\tau_{T,d} = 1/(2I_p)$; an imaginary part arises when the field strength is larger than the atomic-field strength F_a, in terms $\tau_{T,i}$ and $\tau_{T,d}$.

However, in our treatment, although $\tau_{T,i}$ and $\tau_{T,d}$ both have an imaginary part when $F > F_a$, we get a real total or symmetrical T-time $\tau_{T,\mathrm{sym}} = \frac{I_p}{4Z_{\mathrm{eff}}F}$ for ionization processes with an arbitrary field strength. It becomes very small for very large field strengths and probably loses its validity in this regime, suspecting a break of some symmetry, nonlinear effects arise, and the interaction becomes physically a different character. It certainly also loses its validity in the multiphoton regime, i.e., for large Keldysh parameter $\gamma \gg 1$, where $F \ll F_a$. It is apparent from $\tau_{T,\mathrm{sym}}$ [Eqs. (12) and (18)] that the T-time has no imaginary part when the symmetry of the time is considered, i.e., when assuming the maximally symmetrical (or quasi-self-adjoint) property discussed in detail by Olkhovsky *et al.* [30]. It is now the question to what extent the above relation $\frac{I_p}{4Z_{\mathrm{eff}}F}$ preserves its validity for $F > F_a$ (or for small $F \ll F_a$, where $\gamma_K \gg 1$), where or what is or are the limit(s) of its validity? A break of some symmetry for $F \geqslant F_a$ (or $F \ll F_a$) can probably give a hint to answer this question. Finally, we mention that for $F > F_a$ or intensities $I > I_a$ Stark-shift, relativistic, and nonlinear effects become large, the perturbation theory breaks down (which is valid for small parameter $\xi = \frac{F}{F_a}$ [28]), and several regions appear at intensities larger than the appearance intensity I_a, such as the critical I_c and the saturation I_s intensities, $I_s > I_c > I_a$, where multiple ionization occurs [28] (Chaps. 7 and 9).

V. CONCLUSION

We presented in this work an analysis for the tunneling time and the tunneling process in attosecond experiments and found an accurate and simple relation to calculate the tunneling time for the important case of the He atom, where reliable experimental data is available. Our result (especially the T-time $\tau_{T,d}$) was shown to be in excellent agreement with the experiment [32,33,37] and with the FPI treatment of [37], although for small field strengths our result of $Z_{\mathrm{eff},K}$ tends to agree better with the experiment. Note that in Figs. 2–5 we use for the evaluation of our result the same values of the

field strengths used by the experiment, i.e., the field strength at the maximum; see [12,37,38]. The T-time in our treatment is dynamical or intrinsic type of time and represents a quantum clock, i.e., to observe the time form within the system and consider the quantum nature of the (bound) particle, in contrast to the classical Keldysh time which is external (or parametric), where we indicated two of its failures to treat the T-time in our (study) case. Further investigation and more details will be given in [34].

Further, we suggest a model of a shutter in the tunneling process in attosecond experiment, and we think the experiment together with our tunneling model (Secs. II B, II C, and III A) offers a realization of the Bohr-Einstein's photon-box GE, with the electron as a particle instead of the photon and with the uncertainty being determined from the (Coulomb) atomic potential instead of the gravitational potential. Our treatment suggests that a symmetry (maximally symmetrical or quasi-self-adjoint) [30] assumption to calculate the T-time is important and gives a hint to the search for a time operator in the tunneling process and maybe for a general time operator in quantum mechanics. Our result uses two models of the effective charge $Z_{\rm eff}$ of the left core He^+ that the tunneling electron experiences. The $Z_{\rm eff,K} = 1.375$ of Kullie [41], which is based on a model similar to that of the SAE, is better for a small field strength $F \lesssim 0.055$ a.u. [barrier width $d_B(F) > 14$ a.u.], whereas $Z_{\rm eff,C} = 1.6875$ of Clementi *et al.* [42] is more reliable for larger field strengths because it is based on the Hartree-Fock calculation, and that is justified when the multielectron effects are not negligible.

ACKNOWLEDGMENTS

The author is indebted to the anonymous referees for valuable comments and useful suggestions, which helped to improve the manuscript and to avoid some confusing expressions. I am grateful to A. S. Landsmann and C. Hofmann for sending the experimental, Feynman Path Integral and Larmor clock data, and C. Hofmann and C. Cirelli for their kindly communication. I also would like to thank Professor Martin Garcia from Theoretical Physics of the Institute of Physics at the University of Kassel for his support.

APPENDIX

(1) *The maximum position of the barrier,* $x_m(F) = \sqrt{Z_{\rm eff}/F}$. This follows immediately from the fact that x_m is determined by the maximum of the effective potential energy for arbitrary field strength, and that is the intersection point of the two potentials, $V(x) = -\frac{Z_{\rm eff}}{x}$ and $-x\,F$ (see Fig. 1); then

$$\frac{-Z_{\rm eff}}{x} = -x \Rightarrow x_m(F) = \sqrt{Z_{\rm eff}/F}. \tag{A1}$$

Otherwise (to both sides), one of the two potentials ($-\frac{Z_{\rm eff}}{x}$ or $-xF$) slopes down more quickly than the other slopes up [which can be easily gathered from Fig. 1 and Eq. (6)], leading to $V_{\rm eff}(x) < V_{\rm eff}(x_m)$ for $x \neq x_m$.

(2) *The classical exit point* $x_{e,c}$. $x_{e,c}$ is determined by neglecting the atomic potential [35],

$$v_e^2 - v_0^2 = 0 - 2I_p = -2F(x_e - x_0) \Rightarrow x_{e,c} = \frac{I_p}{F}, \tag{A2}$$

where $x_e \approx (x_e - x_0), x_0 \approx 0$ is the initial point of the electron, and assuming that the electron moves along the x axis direction [21,22].

(3) *The "classical" T-time,* $\tau_{T,c}$. We show that the first order of Eq. (10) is equal to the T-time $\tau_{T,c}$, which results from using $x_{e,c}$, the classical exit point, to calculate the uncertainty ΔE form $\Delta V(x_{e,c})$. Expanding Eq. (10) in terms of $\eta = (\frac{4F}{I_p^2})$, we get immediately that the first order equals $\tau_{T,c}$, the T-time at the classical exit point $x_{e,c}$; then

$$O^1(\tau_{T,\rm unsy}) = \frac{I_p}{2F} = \frac{1}{2}\frac{1}{\Delta E_c} = \tau_{T,c}, \tag{A3}$$

where $\Delta E_c = |\frac{-1}{x_{e,c}}|$ (classically, $Z_{\rm eff} = 1$).

(4) *The Keldysh T-time,* τ_k. The calculation of Keldysh T-time is based on the assumption that $l \approx x_{e,c}$, where $x_{e,c}$ is given in Eq. (A2), l is defined as length or the width of the barrier, and the average velocity $\overline{v}$ of the electron to pass the barrier is classically determined, $\overline{v} = (v_f - v_0)/2 = \sqrt{2I_p}/2$, where $v_f = v_e$ is the velocity at the exit point,

$$\tau_k = \frac{l}{\overline{v}} = \frac{x_{e,c}}{\overline{v}} = \frac{I_p}{F}\frac{2}{\sqrt{2I_p}} = \frac{\sqrt{2I_p}}{F}. \tag{A4}$$

(5) *Delay time and lifetime.* Some authors define or claim that the T-time (in an attosecond experiment) is the lifetime of the electron in a metastable state or the time for which the electron detained in the barrier or the well. This speciously seems to be similar to the delay time caused by the barrier that we calculated in our model, which we do and cannot define it as the lifetime because the concept of the lifetime, which is borrowed from the atomic physics in the perturbation regime, is rather misleading in the regime of strong-field or attosecond science. The electron does not "occupy" a metastable state and become detained in the well, or "wait," until the barrier is opened, but it moves along a preferred direction far from the nucleus and escapes to the continuum or to a "quasi" energy level. That means the delay time in our model is a different concept from the lifetime of a metastable state. Indeed, Orlando *et al.* [27] used the later concept, the Mandelstam-Tamm relation, which is usually (and almost exclusively) used to calculate the lifetime. Their result was in disagreement with the experimental finding of Eckle *et al.* [33,37].

[1] G. Muga, R. S. Mayato, and I. Egusquiza (eds.), in *Time in Quantum Mechanics*, Lecture Notes in Physics 734 (Springer-Verlag, Berlin, 2008), Vol. I.
[2] J. Hilgevoord, Am. J. Phys. **70**, 301 (2002).
[3] Y. Aharonov and B. Reznik, Phys. Rev. Lett. **84**, 1368 (2000).
[4] P. Busch, Found. Phys. **20**, 1 (1990); **20**, 33 (1990).
[5] L. F. Cooke, Am. J. Phys. **48**, 142 (1980).
[6] E. H. Huge and Støvneng, Rev. Mod. Phys. **61**, 917 (1989).
[7] R. Landauer and T. Martin, Rev. Mod. Phys. **66**, 217 (1994).

[8] C. A. A. de Carvalho and H. M. Nussenzveig, Phys. Rep. **364**, 83 (2002).
[9] N. Yamada, Phys. Rev. Lett. **93**, 170401 (2004).
[10] D. Sokolovski, S. Brouard, and J. N. L. Connor, Phys. Rev. A **50**, 1240 (1994).
[11] D. Sokolovski, S. Brouard, and J. N. L. Connor, Phys. Rev. A **42**, 6512 (1990).
[12] A. S. Landsman and U. Keller, J. Phys. B **47**, 204024 (2014).
[13] S. Augst, D. Strickland, D. D. Meyerhofer, S. L. Chin, and J. H. Eberly, Phys. Rev. Lett. **63**, 2212 (1989).
[14] S. Augst, D. D. Meyerhofer, D. Strickland, and S. L. Chin, J. Opt. Soc. Am. B **8**, 858 (1991).
[15] H. G. Muller, Phys. Rev. A **60**, 1341 (1999).
[16] X. M. Tong and C. Lin, J. Phys. B **38**, 2593 (2005).
[17] D. Schlüter, Z. Phys. D **6**, 249 (1987).
[18] R. Lange and D. Schlüter, J. Quant. Spectrosc. Radiat. Transfer **48**, 153 (1992).
[19] I. Dreissigacker and M. Lein, Chem. Phys. **414**, 69 (2013).
[20] N. B. Delone and V. P. Krainov, Phys. Usp. **41**, 469 (1998).
[21] M. Klaiber, E. Yakaboylu, H. Bauke, K. Z. Hatsagortsyan, and C. H. Keitel, Phys. Rev. Lett. **110**, 153004 (2013).
[22] E. Yakaboylu, M. Klaiber, H. Bauke, K. Z. Hatsagortsyan, and C. H. Keitel, Phys. Rev. A **88**, 063421 (2013).
[23] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1945 (1964) [English translation: Soviet Phys. JETP **20**, 1307 (1965)].
[24] F. H. M. Faisal, J. Phys. B **6**, L89 (1973).
[25] H. R. Reiss, Phys. Rev. A **22**, 1786 (1980).
[26] L. Torlina, F. Morales, J. Kaushal, H. G. Muller, I. Ivanov, A. Kheifets, A. Zielinski, A. Scrinzi, M. Ivanov, and O. Smirnova, arXiv:1402.5620.
[27] G. Orlando, C. R. McDonald, N. H. Protik, and T. Brabec, Phys. Rev. A **89**, 014102 (2014); G Orlando, C. R. McDonald, N. H. Protik, G. Vampa, and T. Brabec, J. Phys. B **47**, 204002 (2014).
[28] N. B. Delone and V. P. Krainov, *Multiphoton Processes in Atoms*, 2nd ed. (Springer-Verlag Berlin, 2000).
[29] Y. Aharonov and D. Bohm, Phys. Rev. **122**, 1649 (1961).
[30] V. S. Olkhovsky, ARPN J. Sc. T. **4**, 110 (2014); Adv. Math. Phys. **2009**, 1 (2009); Vladislav S. Olkhovsky, E. Recami, and J. Jakiel, Phys. Rep. **398**, 133 (2004); V. S. Olkhovsky and E. Recami, Lett. Nuovo Cimento **4**, 1165 (1970).
[31] It can be chosen as an almost self-adjoint operator with practically almost any degree of the accuracy [30].
[32] P. Eckle, A. N. Pfeiffer, C. Cirelli, A. Staudte, R. Dörner, H. G. Muller, M. Büttiker, and U. Keller, Science **322**, 1525 (2008).
[33] P. Eckle, M. Smolarski, F. Schlup, J. Biegert, A. Staudte, M. Schöffler, H. G. Muller, R. Dörner, and U. Keller, Nat. Phys. **4**, 565 (2008).
[34] O. Kullie, (unpublished).
[35] M. Y. Ivanov, M. Spanner, and O. Smirnova, J. Mod. Opt. **52**, 165 (2005).
[36] Note that for a small field strength $F \ll F_a$ the electron absorbs one or more photons and moves vertically on the energy scale emerging at exit point very close to the initial point x_i on the coordinate scale.
[37] A. S. Landsman, M. Weger, J. Maurer, R. Boge, A. Ludwig, S. Heuser, C. Cirelli, L. Gallmann, and U. Keller, Optica **1**, 343 (2014).
[38] A. S. Landsman and U. Keller, Phys. Rep. **547**, 1 (2015).
[39] C. Hofmann, A. S. Landsman, C. Cirelli, A. N. Pfeiffer, and U. Keller, J. Phys. B **46**, 125601 (2013).
[40] C. Hofmann and C. Cirelli (private communication).
[41] O. Kullie, Diploma thesis, Master's degree diploma (M. S.), Christian-Albrecht-University of Kiel, Germany, 1997; O. Kullie and D. Schlüter, (unpublished).
[42] E. Clementi and D. L. Raimondi, J. Chem. Phys. **38**, 2686 (1963).