Quantum state transfer in cavity electro-optic modulators

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We study the problem of quantum state transfer from the optical cavity field to the microwave resonator in cavity electro-optic modulators. We give the analytical expression of the transfer fidelities of Gaussian states and some non-Gaussian states. We discuss the influences of the electro-optic coupling strength and the temperature of the environment on the fidelity of the state transfer. We show that it is possible to achieve high transfer fidelity for Gaussian states and some non-Gaussian states. We find that the transfer fidelity of the non-Gaussian state produced by performing the photon-addition operation, the photon-subtraction operation, or the photon-subtraction-then-addition operation on a Gaussian state is lower than that of the corresponding Gaussian state.

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I. INTRODUCTION

It is well known that high fidelity transfer of quantum states is a necessary prerequisite for large-scale quantum information processing. Hence, the problems of quantum state transfer have recently been the object of extensive studies. For microwave photons and optical photons, each has its own advantages. Microwave photons can be easily controlled in superconducting circuits [1], while optical photons are ideal candidates for distributing quantum information. Thus the quantum state transfer between optical and microwave photons allows one to make use of their advantages [2]. Recently, considerable theoretical and experimental work has been devoted to the investigation of the quantum state transfer between the optical mode and the microwave mode using a hybrid device based on their common interaction with a nanomechanical resonator [3–10]. Reference [9] experimentally demonstrates bidirectional conversion between microwave and optical light with conversion efficiencies of $\sim$10%. The conversion efficiency is mainly limited by the thermal noise of the mechanical oscillator. Reference [10] compares the three different schemes (double swap, adiabatic passage, and hybrid scheme) used in the quantum state transfer from an optical cavity to a microwave cavity in detail. They show that the effect of the thermal noise of the mechanical oscillator on the fidelity of the quantum state transfer in the adiabatic scheme is lower than those in the other two schemes.

On the other hand, there is considerable interest in cavity electro-optic modulators [11–20] because they can directly couple an optical cavity mode to a microwave resonator in nonlinear electro-optic materials such as lithium niobate based on the linear electro-optic effect, where the higher-order electro-optic effects are weak compared with the linear electro-optic effect so that they can be neglected [18]. It has been shown that a cavity electro-optic system is analogy to an optomechancial system [21]. In addition, it has been theoretically shown that frequency conversion between microwave and optical domains with 100% efficiency can be achieved in a cavity electro-optic system in the absence of the thermal noise of the environment [22]. It has been experimentally shown that a photon-number conversion efficiency of 5×10^{-4}% can be achieved in a lithium niobate whispering-gallery-mode electro-optic modulator [15]. And Ref. [15] points out that it is possible to achieve a unity conversion efficiency using a whispering gallery resonator with the quality factor $Q \approx 4 \times 10^8$. In the future, the conversion efficiency can be greatly improved with the rapid progress in the electro-optic technologies [12,14,16]. Here we study the quantum state transfer in a cavity electro-optic modulator. We consider the transfer of Gaussian states (a coherent state and a squeezed coherent state) and the transfer of non-Gaussian states such as a Fock state, a qubit state, a single-photon-added coherent state, a single-photon-subtracted squeezed vacuum state, and a photon-subtracted-then-added squeezed vacuum state. The last three kinds of the non-Gaussian states are generated through the de-Gaussification procedure which is realized through adding photons, subtracting photons, or subtracting then adding photons on a Gaussian state. We analyze the dependence of the transfer fidelities of Gaussian states and some non-Gaussian states on the electro-optic coupling strength and the temperature of the environment. We find that the high fidelity of transferring Gaussian states and several non-Gaussian states from the optical mode to the microwave mode is achievable in such a system. Interestingly, we find that the fidelity of the coherent state transferred directly from the optical mode to the microwave mode is comparable to that achieved by the adiabatic approach in an optomechanical system by the aid of a mechanical mode [10]. Moreover, we find that the transfer fidelity of the non-Gaussian state obtained by the de-Gaussification operation is lower than that of the corresponding Gaussian state.

The paper is organized as follows. In Sec. II, we introduce the model, give the quantum Langevin equations. In Sec. III, we consider the quantum state transfer from the optical mode to the microwave mode when the quantum noises are not taken into account. In Sec. IV, we investigate the quantum state transfer with quantum noises, and analyze how the transfer fidelities of Gaussian states and several non-Gaussian states are affected by the effective electro-optic coupling strength and the temperature of the environment. We compare the fidelities of different initial states of the optical mode to be transferred. Finally, we conclude our work in the last section.

II. MODEL

We consider a cavity electro-optic modulator (EOM) as shown in Fig. 1 [21,22]. The EOM consists of a nonlinear electro-optic medium such as lithium niobate [11–17]. The

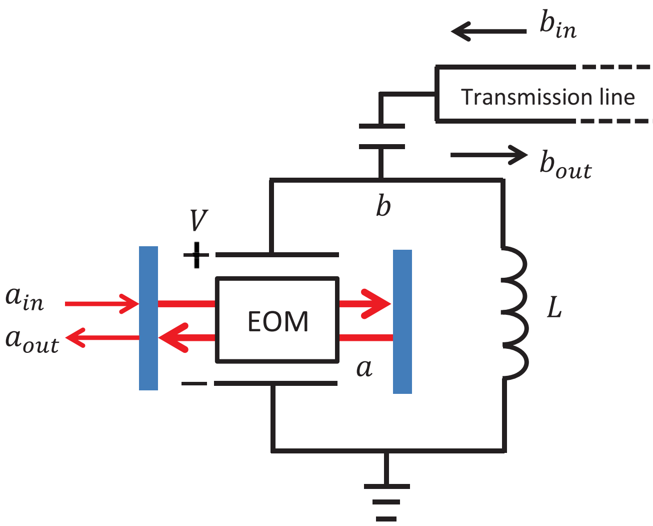

FIG. 1. (Color online) Schematic of a cavity electro-optic modulator. EOM denotes an electro-optic modulator in which the optical cavity mode a is coupled to the microwave resonator b.

optical cavity field with frequency ω_a is driven by an external laser field with frequency ω_l. A voltage V is applied across the medium, which is perpendicular to the propagation direction of the cavity field, thus the EOM is a transverse modulator. The electric field established in the medium by the application of the voltage leads to a change in the refractive index of the medium, inducing a phase shift on the cavity field propagating through the medium. If the variance in the refractive index is linearly proportional to the electric field, the interaction between the electric field and the light field is known as the linear electro-optic effect or the Pockels effect [18–20], which is a second-order nonlinear effect. Here the voltage V corresponds to the mechanical position in the optomechanical system, and the modulator is treated as a capacitor in a single-mode microwave resonator b with frequency ω_b [21]. The electro-optic coupling strength between the optical field and the microwave resonator can be defined as [21]

$$g \equiv \frac{\omega_a n^3 r l}{c\tau d}\sqrt{\frac{\hbar\omega_b}{2C}}, \tag{1}$$

which is determined by the optical refractive index n of the electro-optic medium, the electro-optic coefficient r in the units of m/V, the length l of the medium, the speed c of light in vacuum, the optical round trip time τ, the thickness d of the medium, and the capacitance C of the microwave resonator. If the microwave frequency ω_b is close to the spacing $\Delta\omega$ between the optical modes, and $\Delta\omega - \omega_b$ is much larger than the cavity linewidth γ_a, we can restrict our attention to three optical modes with frequencies ω_a, $\omega_a - \omega_b$, $\omega_a + \omega_b$ [12,13]. Furthermore, if we transform the three optical modes and the microwave mode to frames rotating at their respective resonance frequencies, make rotating wave approximation, and let the external laser field be red detuned with respect to the optical mode with frequency ω_a by the microwave frequency ω_b ($\omega_l = \omega_a - \omega_b$), the system is approximately equivalent to a beam splitter, i.e., the optical mode with frequency ω_a exchanges energy with the microwave mode with frequency ω_b [21]. The equations of motion are given by

$$\begin{aligned}\frac{da}{dt} &= ig\alpha b - \frac{\gamma_a}{2}a + \sqrt{\gamma_a}a_{\rm in},\\ \frac{db}{dt} &= ig\alpha^* a - \frac{\gamma_b}{2}b + \sqrt{\gamma_b}b_{\rm in},\end{aligned} \tag{2}$$

where a and b are the annihilation operators of the optical and microwave modes, respectively, α is the steady-state cavity amplitude, γ_b is the damping rate of the microwave resonator, and $a_{\rm in}$ and $b_{\rm in}$ are input quantum noise operators with zero mean values; they have the following nonzero time-domain correlation functions:

$$\begin{aligned}\langle a_{\rm in}^\dagger(t)a_{\rm in}(t')\rangle &= N(\omega_a)\delta(t-t'),\\ \langle a_{\rm in}(t)a_{\rm in}^\dagger(t')\rangle &= [N(\omega_a)+1]\delta(t-t'),\\ \langle b_{\rm in}^\dagger(t)b_{\rm in}(t')\rangle &= N(\omega_b)\delta(t-t'),\\ \langle b_{\rm in}(t)b_{\rm in}^\dagger(t')\rangle &= [N(\omega_b)+1]\delta(t-t'),\end{aligned} \tag{3}$$

where $N(\omega) = \frac{1}{e^{\hbar\omega/(k_B T)}-1}$ is the average number of thermal photons in the optical mode and the microwave mode, k_B is the Boltzmann constant, and T is the temperature of the environment. At optical frequency ω_a, one has $\hbar\omega_a/(k_B T) \gg 1$, thus $N(\omega_a) \simeq 0$. For simplicity, we write $N(\omega_a)$ and $N(\omega_b)$ as N_a and N_b in the following.

III. QUANTUM STATE TRANSFER IN THE ABSENCE OF QUANTUM NOISES

First we consider the case that the input noise $a_{\rm in}$ of the optical field and the input noise $b_{\rm in}$ of the microwave resonator are neglected. The optical mode and the microwave mode at time t can be derived from Eq. (2), which yield

$$\begin{aligned}a(t) &= e^{pt}[\cos(qt)a(0) + i\sin(qt)b(0)],\\ b(t) &= e^{pt}[i\sin(qt)a(0) + \cos(qt)b(0)],\end{aligned} \tag{4}$$

where $p = -\frac{1}{4}(\gamma_a+\gamma_b)$, $q = \sqrt{G^2 - \frac{(\gamma_a-\gamma_b)^2}{16}}$, and $G = g|\alpha|$. When $t = \pi/(2q)$, one has

$$\begin{aligned}a(t=\pi/(2q)) &= ie^{\pi p/(2q)}b(0),\\ b(t=\pi/(2q)) &= ie^{\pi p/(2q)}a(0).\end{aligned} \tag{5}$$

It is seen that the quantum state transfer can be achieved in this system, the transferring efficiency is determined by the effective coupling strength G and the decay rates γ_a, γ_b. For the special case $\gamma_a = \gamma_b = 0$, we have

$$\begin{aligned}a(t=\pi/(2G)) &= ib(0),\\ b(t=\pi/(2G)) &= ia(0),\end{aligned} \tag{6}$$

which indicates the perfect quantum state transfer between the two modes a and b, i.e., the state of one mode at time $t = \pi/(2G)$ is exactly the same as the initial state of the other mode except the phase difference i.

IV. QUANTUM STATE TRANSFER IN THE PRESENCE OF QUANTUM NOISES

In this section, we present our result for a real physical system taking into account the optical noise $a_{\rm in}$ and the microwave noise $b_{\rm in}$. We discuss the transfer of Gaussian states (a coherent state and a squeezed coherent state) and the transfer of non-Gaussian states (a Fock state, a qubit state, a single-photon-added coherent state, a single-photon-subtracted squeezed vacuum state, and a photon-subtracted-then-added squeezed vacuum state) from the optical mode to the microwave mode.

From Eq. (2), we obtain the final states of the optical cavity mode and the microwave mode, which yield

$$\begin{aligned} a(t) &= e^{pt}[\cos(qt)a(0) + i\sin(qt)b(0)] \\ &\quad + \sqrt{\gamma_a}\int_0^t d\tau\, e^{p\tau}\cos(q\tau)a_{\rm in}(t-\tau) \\ &\quad + i\sqrt{\gamma_b}\int_0^t d\tau\, e^{p\tau}\sin(q\tau)b_{\rm in}(t-\tau), \\ b(t) &= e^{pt}[i\sin(qt)a(0) + \cos(qt)b(0)] \\ &\quad + i\sqrt{\gamma_a}\int_0^t d\tau\, e^{p\tau}\sin(q\tau)a_{\rm in}(t-\tau) \\ &\quad + \sqrt{\gamma_b}\int_0^t d\tau\, e^{p\tau}\cos(q\tau)b_{\rm in}(t-\tau). \end{aligned} \tag{7}$$

We define $\tilde{a}(t) = a(t)/i$ and $\tilde{b}(t) = b(t)/i$ to remove the phase i of the states $a(t)$ and $b(t)$. The amplitude and phase quadratures of the fluctuations of the optical mode and the microwave mode are defined as $x_a(t) = \frac{1}{\sqrt{2}}[\tilde{a}(t) + \tilde{a}(t)^\dagger]$, $y_a(t) = \frac{1}{\sqrt{2}i}[\tilde{a}(t) - \tilde{a}(t)^\dagger]$, $x_b(t) = \frac{1}{\sqrt{2}}[\tilde{b}(t) + \tilde{b}(t)^\dagger]$, and $y_b(t) = \frac{1}{\sqrt{2}i}[\tilde{b}(t) - \tilde{b}(t)^\dagger]$, respectively.

A. The transfer of Gaussian states

The efficiency of the quantum state transfer can be quantified by the fidelity. The Ulhmann fidelity [23] is defined as

$$F = [\mathrm{Tr}(\sqrt{\sqrt{\rho_i}\rho_f\sqrt{\rho_i}})]^2, \tag{8}$$

where ρ_i and ρ_f are the reduced density matrices for the initial and final states, respectively. The fidelity is unity when the two states are the same, and zero when they are totally different. If both the states ρ_i and ρ_f are pure or one of them is pure while the other is mixed, the fidelity in Eq. (8) reduces to $F = \mathrm{Tr}(\rho_i\rho_f)$, and it can be represented by an overlap between the Wigner functions of the initial and final states in the phase space, $F = \pi\int d^2\beta W_i(\beta)W_f(\beta)$. First we focus on the case in which the initial state of the optical mode to be transferred is a pure Gaussian state, then the fidelity of the state transfer between two Gaussian quantum states can be simplified as [24]

$$F = \frac{1}{\sqrt{\det\frac{\mathbf{V}_i+\mathbf{V}_f}{2}}}\exp[-(\mu_i-\mu_f)^T(\mathbf{V}_i+\mathbf{V}_f)^{-1}(\mu_i-\mu_f)], \tag{9}$$

where $\mathbf{V}_i$ and $\mathbf{V}_f$ have the form,

$$\mathbf{V} = \begin{pmatrix} 2\sigma_{xx} & 2\sigma_{yx} \\ 2\sigma_{yx} & 2\sigma_{yy} \end{pmatrix}. \tag{10}$$

Here σ_{xx}, σ_{yx}, and σ_{yy} are the elements of the covariance matrix. The covariance of two operators X and Y is defined as

$$\sigma_{XY}(t) = \frac{\langle X(t)Y(t)\rangle + \langle Y(t)X(t)\rangle}{2} - \langle X(t)\rangle\langle Y(t)\rangle. \tag{11}$$

And $\mu_i = \binom{\langle x_i\rangle}{\langle y_i\rangle}$ and $\mu_f = \binom{\langle x_f\rangle}{\langle y_f\rangle}$ are the expectation values of the quadratures of the optical field and the microwave resonator, respectively.

Suppose that the optical mode a to be transferred is initially prepared in a single-mode squeezed coherent state given by $|\alpha,r\rangle = D(\alpha)S(r)|0\rangle$ where $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ is the unitary displacement operator with amplitude $\alpha = |\alpha|e^{i\phi}$, and $S(r) = \exp[r(a^2 - a^{\dagger 2})/2]$ is the unitary single-mode squeezing operator with the real squeezing parameter r, then the expectation values of the quadratures and the covariance matrix of the initial state of the optical mode a are

$$\mu_i = \begin{pmatrix} \sqrt{2}|\alpha|\cos\phi \\ \sqrt{2}|\alpha|\sin\phi \end{pmatrix}, \tag{12}$$

$$\mathbf{V}_i = \begin{pmatrix} e^{-2r} & 0 \\ 0 & e^{2r} \end{pmatrix}. \tag{13}$$

Moreover, we assume that the initial state of the microwave mode b is in the thermal state at the temperature T of the environment. The expectation values of the quadratures and the covariance matrix of the final state of the microwave resonator b at time t can be calculated by Eq. (7). They are

$$\mu_f = \begin{pmatrix} \sqrt{2}e^{pt}\sin(qt)|\alpha|\cos\phi \\ \sqrt{2}e^{pt}\sin(qt)|\alpha|\sin\phi \end{pmatrix}, \tag{14}$$

$$\mathbf{V}_f = \begin{pmatrix} u(r,t) & 0 \\ 0 & v(r,t) \end{pmatrix}, \tag{15}$$

where

$$\begin{aligned} u(r,t) &= \Lambda(t) + e^{2pt}\sin^2(qt)e^{-2r} + 2e^{2pt}\cos^2(qt) \\ &\quad \times(N_b + 0.5), \\ v(r,t) &= \Lambda(t) + e^{2pt}\sin^2(qt)e^{2r} + 2e^{2pt}\cos^2(qt) \\ &\quad \times(N_b + 0.5), \\ \Lambda(t) &= -\frac{\gamma_a}{4}(2N_a+1)\left[\frac{e^{2(p+iq)t}-1}{2(p+iq)} - \frac{e^{2pt}-1}{p} \right. \\ &\quad \left. + \frac{e^{2(p-iq)t}-1}{2(p-iq)}\right] + \frac{\gamma_b}{4}(2N_b+1) \\ &\quad \times\left[\frac{e^{2(p+iq)t}-1}{2(p+iq)} + \frac{e^{2pt}-1}{p} + \frac{e^{2(p-iq)t}-1}{2(p-iq)}\right]. \end{aligned} \tag{16}$$

After some calculations, we obtain the fidelity,

$$\begin{aligned} F &= \frac{2}{\sqrt{E_1E_2}}\exp\left\{-2|\alpha|^2[e^{pt}\sin(qt)-1]^2 \right. \\ &\quad \left. \times\frac{\cos^2(\phi)E_2 + \sin^2(\phi)E_1}{E_1E_2}\right\}, \end{aligned} \tag{17}$$

where $E_1 = e^{-2r} + u(r,t)$, $E_2 = e^{2r} + v(r,t)$. We choose the values of the parameters which are similar to those used in the recent experiment [12]: the wavelength of the input

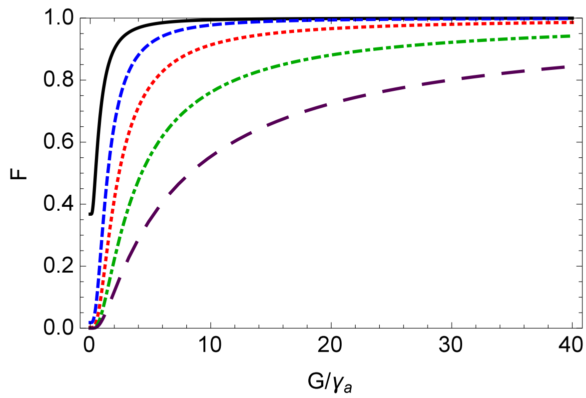

FIG. 2. (Color online) The state transfer fidelity F from an optical cavity field to a microwave resonator as a function of the coupling strength G/γ_a for the temperature of the environment $T = 0$ K at the time $t = \pi/(2q)$. The curves from top to bottom correspond to the initial state of the optical cavity field in $|\alpha = 1, r = 0\rangle$, $|\alpha = 2, r = 0\rangle$, $|\alpha = 2, r = 0.5\rangle$, $|\alpha = 2, r = 1\rangle$, and $|\alpha = 2, r = 1.5\rangle$, respectively.

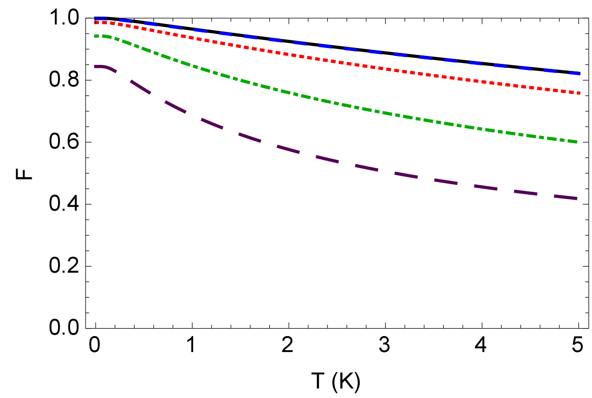

FIG. 3. (Color online) The state transfer fidelity F from an optical cavity field to a microwave resonator as a function of the temperature T (K) of the environment for the coupling strength $G = 40\gamma_a$ at the time $t = \pi/(2q)$. The curves from top to bottom correspond to the initial state of the optical cavity field in $|\alpha = 1, r = 0\rangle$, $|\alpha = 2, r = 0\rangle$, $|\alpha = 2, r = 0.5\rangle$, $|\alpha = 2, r = 1\rangle$, and $|\alpha = 2, r = 1.5\rangle$, respectively.

laser $\lambda = 2\pi c/\omega_a = 1550$ nm, and the resonance frequency of the microwave mode $\omega_b = 2\pi \times 9$ GHz. We assume that the optical decay rate γ_a and the microwave damping rate γ_b are equal.

We first consider the influence of the coupling strength on the state transfer fidelity. We fix the time $t = \pi/(2q)$ and the temperature of the environment $T = 0$ K. Figure 2 plots the fidelity F of the quantum state transfer from the optical field to the microwave resonator against the coupling strength G/γ_a when the initial state of the optical mode a is a coherent state with $\alpha = 1,2$ and $r = 0$, and a squeezed coherent state with $\alpha = 2$ and $r = 0.5,1,1.5$. For the coherent state transfer, with increasing the coupling strength G, the fidelity is rapidly increased to unity. For a smaller coherent amplitude α, the fidelity arrives at unity at a lower coupling strength G. For the squeezed coherent state transfer, with increasing the coupling strength G, the fidelity is increased. For a given coupling strength G, increasing the squeezing parameter r leads to a decrease in the transfer fidelity of the squeezed coherent state. When $G = 40\gamma_a$, $\alpha = 2$, and $r = 0.5$, 1, 1.5, the fidelities are 99%, 94%, 84%, respectively.

We now show the effect of the temperature of the environment on the state transfer fidelity. We fix the time $t = \pi/(2q)$ and the coupling strength $G = 40\gamma_a$. Figure 3 plots the fidelity F of the quantum state transfer from the optical field to the microwave resonator against the temperature T (K) of the environment when the initial state of the optical mode a is a coherent state with $\alpha = 1$, 2 and $r = 0$, and a squeezed coherent state with $\alpha = 2$ and $r = 0.5$, 1, 1.5. For the transfer of the coherent state with different amplitudes, the fidelities are decreased with the temperature T of the environment at the same rate. It is noted that the transfer fidelity of the coherent state is close to the result as shown in Fig. 6(a) in Ref. [10] by using adiabatic scheme. Moreover, it is found that the detrimental effect of the temperature of the environment on the fidelity of the squeezed coherent state transfer is larger than that on the fidelity of the coherent state. The reason is that the coherent state is the closest analog to a classical light field, but the squeezed coherent state is a nonclassical state, which is very fragile to the thermal environment. Furthermore, increasing the squeezing parameter r, the fidelity of the squeezed coherent state is more sensitive to the change of the temperature of the environment.

B. The transfer of some non-Gaussian states

In the previous subsection, we present the transfer of the Gaussian states. However, it is well known that non-Gaussian states are necessary in quantum information processing and quantum metrology due to their nonclassical features such as the negativity of the Wigner functions [25]. In this subsection, we discuss the transfer of a non-Gaussian state from the cavity mode a to the microwave mode b. For simplicity, we assume that the decay rate γ_a of the cavity mode is equal to the damping rate γ_b of the microwave resonator. Moreover, we assume that the initial state of the microwave mode b is in the thermal state at the temperature T of the environment. The fidelity F of quantum state transfer can be evaluated by the overlap between two Wigner functions,

$$F = \pi \int d^2\beta W_a(\beta) W_b(\beta, t = \pi/(2q)), \tag{18}$$

where $W_a(\beta)$ is the Wigner function of the initial state of the cavity mode a, and $W_b(\beta, t = \pi/(2q))$ is the Wigner function of the state of the microwave mode b at time $t = \pi/(2q)$. The Wigner function of the microwave mode b at $t = \pi/(2q)$ is given by

$$W_b(\beta, t) = \frac{1}{\pi^2} \int d^2\xi_b \chi_b(\xi_b, t) \exp[-(\xi_b \beta^* - \xi_b^* \beta)], \tag{19}$$

where $\chi_b(\xi_b, t)$ is the characteristic function of the state of the microwave resonator, which can be obtained from the characteristic function of the total optical-microwave state [26],

$$\chi_b(\xi_b, t) = \chi_{a,b}(\xi_a = 0, \xi_b, t). \tag{20}$$

The time evolution of the full characteristic function is defined as

$$\chi_{a,b}(\xi_a,\xi_b,t) = \mathrm{Tr}[\rho_{a,b}(t)D_a(\xi_a)D_b(\xi_b)], \quad (21)$$

where $\rho_{a,b}(t)$ is the density operator of the system; $D_a(\xi_a) = \exp(\xi_a a^\dagger - \xi_a^* a)$ and $D_b(\xi_b) = \exp(\xi_b b^\dagger - \xi_b^* b)$ are the displacement operators. If the optical field is initially prepared in Gaussian states such as a coherent state and a squeezed coherent state, the full characteristic function of the system [26] reads

$$\chi_{a,b}(\xi_a,\xi_b,t) = \exp[-\tfrac{1}{2}\xi^T\sigma(t)\xi + i\xi^T d(t)], \quad (22)$$

where $d(t)$ are the displacement vector $d(t) = \langle\Upsilon(t)\rangle$, $\Upsilon(t) = (x_a(t),y_a(t),x_b(t),y_b(t))^T$, the canonical commutation relations are collected in a matrix ϵ,

$$[\Upsilon(t)_i,\Upsilon(t)_j] = i\epsilon_{ij}, \quad \epsilon = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad (23)$$

$\sigma(t)$ is the 4×4 covariance matrix with elements defined as $\sigma(t)_{ij} = \frac{\langle\Upsilon(t)_i\Upsilon(t)_j\rangle + \langle\Upsilon(t)_j\Upsilon(t)_i\rangle}{2} - \langle\Upsilon(t)_i\rangle\langle\Upsilon(t)_j\rangle$, and

$$\xi = \sqrt{2}\epsilon \begin{pmatrix} \mathrm{Re}(\xi_a) \\ \mathrm{Im}(\xi_a) \\ \mathrm{Re}(\xi_b) \\ \mathrm{Im}(\xi_b) \end{pmatrix}. \quad (24)$$

For various non-Gaussian states we consider below, the characteristic functions of the total optical-microwave state can be derived from Eq. (22), the characteristic functions and the Wigner functions of the state of the microcave mode b at time $t = \pi/(2q)$ are presented in Appendix.

1. Fock state transfer

As a first example, we discuss the transfer of a Fock state from the optical mode to the microwave resonator. Fock states are useful in quantum communication [27] and enhancing the phase sensitivity of interferometers [28]. If the cavity mode a to be transferred is in a single-photon Fock state, which can be expressed in terms of coherent states,

$$|1\rangle = \frac{\partial}{\partial\alpha}\left(e^{|\alpha|^2/2}|\alpha\rangle\right)_{\alpha=0}, \quad (25)$$

the characteristic function of the total optical-microwave state $\chi_{a,b}(\xi_a,\xi_b,t)$ can be written as [26]

$$\chi_{a,b}(\xi_a,\xi_b,t) = \frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}\left(e^{|\alpha|^2}\mathcal{A}\right)_{\alpha=0}, \quad (26)$$

in which $\mathcal{A} = \exp[-\frac{1}{2}\xi^T\sigma(t)\xi + i\xi^T d(t)]$ is the full characteristic function of the system when the optical field is initially in a coherent state. The Wigner function of the initial state of the cavity mode a in the single-photon Fock state is found to be

$$W_a(\beta) = \frac{2}{\pi}(4|\beta|^2 - 1)\exp(-2|\beta|^2). \quad (27)$$

After some calculations, we arrive at the following expression for the transfer fidelity of the single-photon Fock state,

$$F = \frac{2h^2}{s}\left[2 - \frac{1}{s} - 2\Gamma\left(1 - \frac{4h}{s}\right)\right], \quad (28)$$

where

$$s = \frac{1}{2}u(0,t = \pi/(2q)),$$
$$\Gamma = \frac{\exp(\pi p/q)}{s}, \quad (29)$$
$$h = \frac{1}{2 + \frac{1}{s}}.$$

If the initial state of the cavity mode a to be transferred is prepared in the two-photon Fock state $|2\rangle$, which can be expressed in terms of coherent states,

$$|2\rangle = \frac{1}{\sqrt{2!}}\frac{\partial^2}{\partial\alpha^2}\left(e^{|\alpha|^2/2}|\alpha\rangle\right)_{\alpha=0}, \quad (30)$$

the corresponding characteristic function of the total optical-microwave state $\chi_{a,b}(\xi_a,\xi_b,t)$ is

$$\chi_{a,b}(\xi_a,\xi_b,t) = \frac{1}{2!}\frac{\partial^2}{\partial\alpha^2}\frac{\partial^2}{\partial\alpha^{*2}}\left(e^{|\alpha|^2}\mathcal{A}\right)_{\alpha=0}. \quad (31)$$

The Wigner function of the initial state of the cavity mode a in the two-photon Fock state $|2\rangle$ is found to be

$$W_a(\beta) = \frac{2}{\pi}(1 - 8|\beta|^2 + 8|\beta|^4)\exp(-2|\beta|^2). \quad (32)$$

Then we find that the state transfer fidelity for the two-photon Fock state $|2\rangle$ is given by

$$F = \frac{2h^3}{s}\left[\left(2 - \frac{1}{s}\right)(1-\Gamma)^2 + 2\frac{h}{s}\left(20 - \frac{12}{s} + \frac{1}{s^2}\right)\times\Gamma(1-\Gamma) + \frac{h^2}{s^2}\left(52 - \frac{20}{s} + \frac{1}{s^2}\right)\Gamma^2\right]. \quad (33)$$

2. Qubit state transfer

As a second example, we consider the transfer of a qubit state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ from the optical mode to the microwave resonator. The qubit state, a linear superposition of the two Fock states $|0\rangle$ and $|1\rangle$, is a fundamental unit of quantum information. In this case, the characteristic function of the total optical-microwave state $\chi_{a,b}(\xi_a,\xi_b,t)$ becomes

$$\chi_{a,b}(\xi_a,\xi_b,t) = \frac{1}{2}\left\{\mathcal{A}_{\alpha=0} + \frac{\partial}{\partial\alpha}\mathcal{A}_{\alpha=0} + \frac{\partial}{\partial\alpha^*}\mathcal{A}_{\alpha=0} + \frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}\left(e^{|\alpha|^2}\mathcal{A}\right)_{\alpha=0}\right\}. \quad (34)$$

The Wigner function of the initial state of the cavity mode a in the qubit state is found to be

$$W_a(\beta) = \frac{2}{\pi}(\beta + \beta^* + 2|\beta|^2)\exp(-2|\beta|^2). \quad (35)$$

Then the transfer fidelity of the qubit state is given by

$$F = \frac{2h^2}{s}\left(\frac{2h\Gamma}{s} + 2 - \Gamma + \sqrt{\frac{\Gamma}{s}}\right). \quad (36)$$

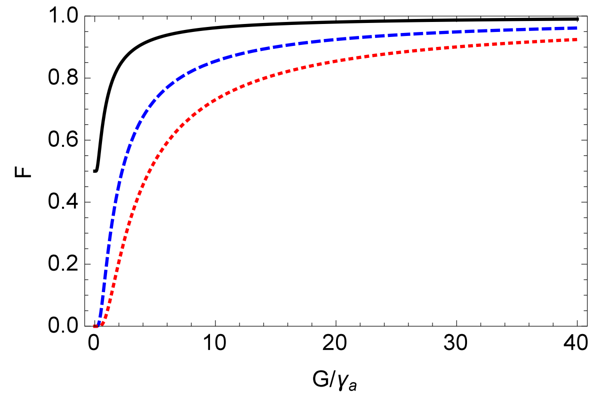

FIG. 4. (Color online) The transfer fidelities F of a single-photon Fock state $|1\rangle$, a two-photon Fock state $|2\rangle$, and a qubit state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ as a function of the coupling strength G/γ_a for the temperature of the environment $T = 0$ K. The black solid curve corresponds to the qubit state, the blue dashed curve corresponds to the Fock state $|1\rangle$, and the red dotted curve corresponds to the Fock state $|2\rangle$.

The transfer fidelities of a single-photon Fock state $|1\rangle$, a two-photon Fock state $|2\rangle$, and a qubit state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ as a function of the coupling strength G for different coherent amplitudes α at $T = 0$ K are plotted in Fig. 4. Note that the fidelity increases with increasing the coupling strength G. For a given coupling strength G, it is seen that the transfer fidelity of the qubit state is higher than that of the single-photon Fock state, and the transfer fidelity of the single-photon Fock state is higher than that of the two-photon Fock state $|2\rangle$. When $G = 40\gamma_a$, the transfer fidelities of $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|1\rangle$, and $|2\rangle$ are 99%, 96%, 92%, respectively.

3. Single-photon-added coherent state transfer

As a third example, we show the transfer of a single-photon-added coherent state from the optical mode to the microwave resonator. The single-photon-added coherent state is obtained by the action of the photon creation operator on a coherent state, i.e., $|\alpha,1\rangle = \frac{a^\dagger|\alpha\rangle}{\sqrt{1+|\alpha|^2}}$. It has been theoretically proposed [29] and experimentally generated [30,31]. It can be used to study the quantum-classical transition [30]. Non-Gaussianity and nonclassicality of a single-photon-added coherent state have been demonstrated experimentally in Ref. [32]. The single-photon-added coherent state can be written as

$$|\alpha,1\rangle = \frac{1}{\sqrt{1+|\alpha|^2}} e^{-|\alpha|^2/2} \frac{\partial}{\partial\alpha} e^{|\alpha|^2/2}|\alpha\rangle, \tag{37}$$

thus the characteristic function of the total optical-microwave state $\chi_{a,b}(\xi_a,\xi_b,t)$ has the form,

$$\chi_{a,b}(\xi_a,\xi_b,t) = \frac{1}{(1+|\alpha|^2)} e^{-|\alpha|^2} \frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}\left(e^{|\alpha|^2}\mathcal{A}\right). \tag{38}$$

The Wigner function of the initial state of the cavity mode a in the single-photon-added coherent state is found to be

$$W_a(\beta) = \frac{2}{\pi(1+|\alpha|^2)}(|2\beta-\alpha|^2 - 1)\exp(-2|\beta-\alpha|^2). \tag{39}$$

For simplicity, we assume that α is real. We find that the transfer fidelity of the single-photon-added coherent state $|\alpha,1\rangle$ is given by

$$\begin{aligned} F = {} & \frac{2I_1}{s(1+\alpha^2)^2}\Bigg\{(\alpha^2-1)\left[I_2 + I_3I_4 + \frac{\Gamma}{s}\left(I_4^2+h\right)\right] \\ & + 4\Bigg[I_2\left(I_4^2+h\right) + I_3I_4\left(I_4^2+2h\right) \\ & + \frac{\Gamma}{s}\left(\left(I_4^2+h\right)\left(I_4^2+2h\right) + I_4^2h\right)\Bigg] \\ & - 4\alpha\left[I_2I_4 + I_3\left(I_4^2+\frac{h}{2}\right) + \frac{\Gamma}{s}I_4\left(I_4^2+2h\right)\right]\Bigg\} \\ & \times \exp[-\alpha^2(2+\Gamma)], \end{aligned} \tag{40}$$

where

$$\begin{aligned} I_1 &= h\exp\left[\alpha^2 h\left(2+\sqrt{\frac{\Gamma}{s}}\right)^2\right], \\ I_2 &= \alpha^2(1-\Gamma)^2 + 1 - \Gamma, \\ I_3 &= 2\alpha\sqrt{\frac{\Gamma}{s}}(1-\Gamma), \\ I_4 &= \alpha h\left(2+\sqrt{\frac{\Gamma}{s}}\right). \end{aligned} \tag{41}$$

When $\alpha = 0$, the single-photon-added coherent state is simply a single-photon Fock state $|1\rangle$, thus the fidelity given by Eq. (40) is equal to Eq. (28). When $\alpha = 1$, the transfer fidelity of the state $|\alpha,1\rangle$ at $T = 0$ K ($s = \frac{1}{2}$, $h = \frac{1}{4}$) can be written as

$$\begin{aligned} F = {} & \frac{\Gamma}{8}\left\{1 + \left(\frac{\Gamma}{2}-2\right)\left[\left(\sqrt{\frac{\Gamma}{2}}-1\right)^2 - 3\right]\right\} \\ & \times \exp\left[-\frac{1}{4}(2-\sqrt{2\Gamma})^2\right], \end{aligned} \tag{42}$$

where $\Gamma = 2\exp(-\frac{\pi\gamma_a}{2G})$. For the ideal case $\gamma_a = \gamma_b = 0$, $\Gamma = 2$, the transfer fidelity F of the state $|\alpha,1\rangle$ will be unity.

The transfer fidelity F of a single-photon-added coherent state $|\alpha,1\rangle$ with $\alpha = 1$ as a function of the coupling strength G at $T = 0$ K is plotted in Fig. 5. For comparison, we also plot the transfer fidelities of the coherent state $|\alpha\rangle$ with $\alpha = 1$ and the single-photon Fock state $|1\rangle$ as a function of the coupling strength G at $T = 0$ K. One can see that the transfer fidelity of $|\alpha,1\rangle$ increases with increasing the coupling strength G. When $G = 40\gamma_a$, the transfer fidelity for $|\alpha,1\rangle$ with $\alpha = 1$ is 99%. Note that the transfer fidelity of a single-photon-added coherent state $|\alpha,1\rangle$ is lower than that of the coherent state $|\alpha\rangle$ for the same value of α, but it is higher than that of the single-photon Fock state $|1\rangle$. The reason is that the coherent state is nearest to the classical field, and the single-photon-added coherent state is a nonclassical state, and its nonclassicality is smaller than that of the single-photon Fock state, because the volume of the negative part of the Wigner function of the single-photon-added coherent state is less than that of the single-photon Fock state, which can be seen from Fig. 6.

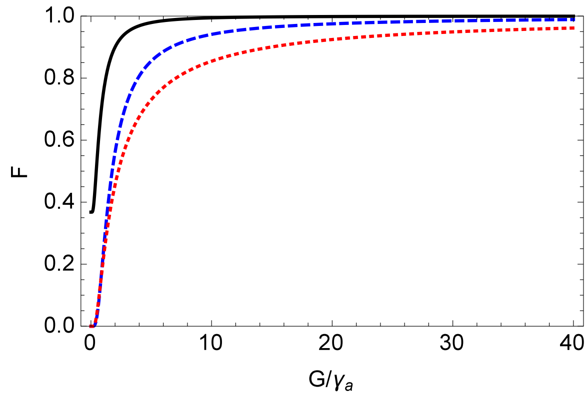

FIG. 5. (Color online) The transfer fidelity F of a single-photon-added coherent state as a function of the coupling strength G/γ_a for the temperature of the environment $T = 0$ K. The black solid curve corresponds to $|\alpha\rangle$ with $\alpha = 1$, the blue dashed curve corresponds to $|\alpha,1\rangle$ with $\alpha = 1$, and the red dotted curve corresponds to $|1\rangle$.

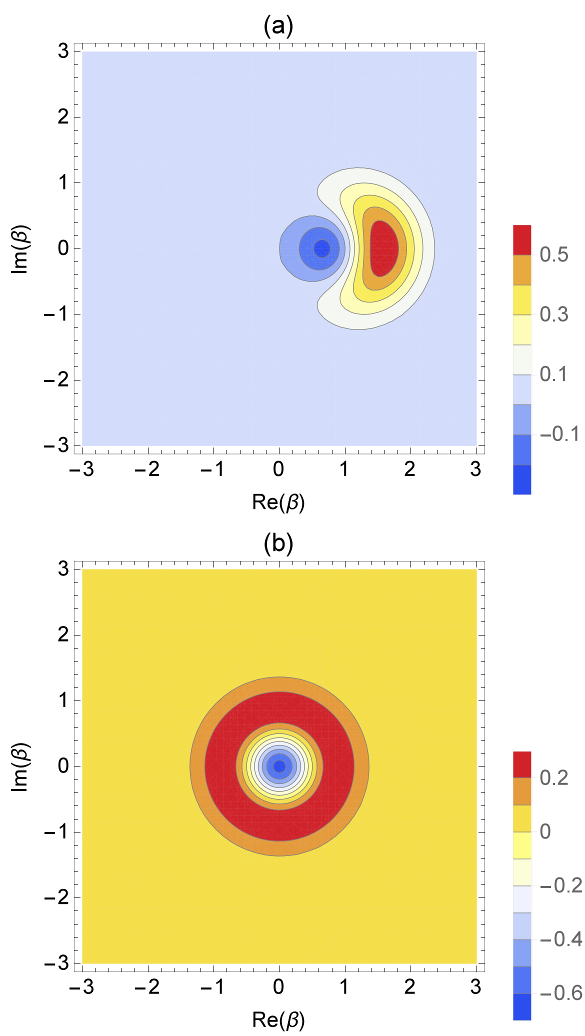

FIG. 6. (Color online) (a) The Wigner function $W_a(\beta)$ of the initial state of the optical mode in the single-photon-added coherent state $|\alpha,1\rangle$ with $\alpha = 1$. (b) The Wigner function $W_a(\beta)$ of the initial state of the optical mode in the single-photon Fock state $|1\rangle$.

4. Single-photon-subtracted squeezed vacuum state transfer

As a fourth example, we study the transfer of a single-photon-subtracted squeezed vacuum state from the optical mode to the microwave resonator. The single-photon-subtracted squeezed vacuum state can be obtained theoretically by operating the photon annihilation operator on a squeezed vacuum state $S(r)|0\rangle$, i.e., $\frac{1}{\sinh(r)}aS(r)|0\rangle$ with $\frac{1}{\sinh(r)}$ being the normalization constant [33,34]. When the squeezing parameter r takes small values, these states approximate the Schrödinger kitten states, i.e., superpositions of coherent states with low amplitudes [35,36], which can be used in quantum information protocols [37]. The experimental generation of this kind of state has been reported [38,39]. Using the unitary transformation,

$$S^\dagger(r)aS(r) = a\cosh(r) - a^\dagger \sinh(r), \tag{43}$$

we obtain $\frac{1}{\sinh(r)}aS(r)|0\rangle = -S(r)|1\rangle$, thus a single-photon-subtracted squeezed vacuum state is a squeezed single-photon Fock state with a phase π. Hence the characteristic function of the total optical-microwave state $\chi_{a,b}(\xi_a,\xi_b,t)$ has the form,

$$\chi_{a,b}(\xi_a,\xi_b,t) = \frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}\left(e^{|\alpha|^2}\Theta\right)_{\alpha=0}, \tag{44}$$

where $\Theta = \exp[-\frac{1}{2}\xi^T\sigma(t)\xi + i\xi^T d(t)]$ is the full characteristic function of the system when the initial state of the optical field is a squeezed coherent state. The Wigner function of the initial state of the cavity mode a in the single-photon-subtracted squeezed vacuum state is found to be

$$W_a(\beta) = \frac{2}{\pi}\{4[e^{2r}\mathrm{Re}^2(\beta) + e^{-2r}\mathrm{Im}^2(\beta)] - 1\} \times \exp\{-2[e^{2r}\mathrm{Re}^2(\beta) + e^{-2r}\mathrm{Im}^2(\beta)]\}. \tag{45}$$

The transfer fidelity of the single-photon-subtracted squeezed vacuum state has the expression,

$$F = \frac{2}{\sqrt{\bar{u}\bar{v}\bar{U}\bar{V}}}\left\{-\bar{A} + (-\bar{B} + 4\bar{A}e^{2r})\frac{1}{2\bar{U}} + (-\bar{C} + 4\bar{A}e^{-2r})\frac{1}{2\bar{V}} + (\bar{B}e^{-2r} + \bar{C}e^{2r})\frac{1}{\bar{U}\bar{V}} + \frac{3\bar{B}e^{2r}}{\bar{U}^2} + \frac{3\bar{C}e^{-2r}}{\bar{V}^2}\right\}, \tag{46}$$

where

$$\bar{u} = \frac{1}{2}u(r,t=\pi/(2q)), \quad \bar{v} = \frac{1}{2}v(r,t=\pi/(2q)),$$
$$\bar{U} = 2e^{2r} + \frac{1}{\bar{u}}, \quad \bar{V} = 2e^{-2r} + \frac{1}{\bar{v}},$$
$$\bar{A} = 1 - \frac{e^{(\pi p/q)}e^{-2r}}{2\bar{u}} - \frac{e^{(\pi p/q)}e^{2r}}{2\bar{v}},$$
$$\bar{B} = \frac{e^{(\pi p/q)}e^{-2r}}{\bar{u}^2}, \quad \bar{C} = \frac{e^{(\pi p/q)}e^{2r}}{\bar{v}^2}. \tag{47}$$

When $r = 0$, the single-photon-subtracted squeezed vacuum state reduces to the single-photon Fock state $|1\rangle$ with a phase π. In this case, we find that the fidelity given by Eq. (46) is equal to Eq. (28).

The transfer fidelities F of a single-photon-subtracted squeezed vacuum state as a function of the coupling strength G

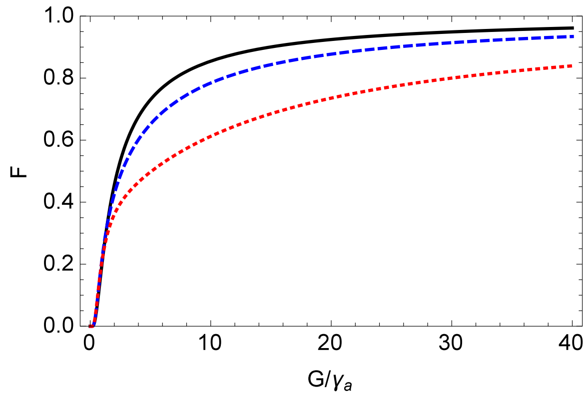

FIG. 7. (Color online) The transfer fidelities F of a single-photon-subtracted squeezed vacuum state as a function of the coupling strength G/γ_a for different squeezing parameters r and the temperature of the environment $T = 0$ K. The curves from top to bottom correspond to $r = 0, 0.5, 1$, respectively.

for different squeezing parameters r and $T = 0$ K is plotted in Fig. 7. For a given squeezing parameter r, the transfer fidelity F increases with the coupling strength G. For a given coupling strength G, the bigger the squeezing parameter r is, the lower the transfer fidelity is. Note that the transfer fidelity of a single-photon-subtracted squeezed vacuum ($r \neq 0$) is lower than that of a single-photon Fock state ($r = 0$). When $G = 40\gamma_a$, $r = 0.5, 1$, the transfer fidelities are 93%, 84%, respectively.

5. Photon-subtracted-then-added squeezed vacuum state transfer

As a last non-Gaussian example, we investigate the transfer of a photon-subtracted-then-added squeezed vacuum state from the optical mode to the microwave resonator. The photon-subtracted-then-added squeezed vacuum state is generated by first subtracting a single photon to a squeezed vacuum state and then another photon is subsequently added [40], i.e.,

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}_{a^\dagger a}}} a^\dagger a S(r)|0\rangle. \tag{48}$$

It has been shown that a single photon subtraction and subsequent addition with a squeezed vacuum state can lead to the generation of the approximate squeezed superpositions of coherent states [40,41], which can be used for the proof-of-principle experiments such as quantum teleportation or single qubit gates [42]. Using Eq. (43) and $S^\dagger(r)a^\dagger S(r) = a^\dagger \cosh(r) - a\sinh(r)$, the photon-subtracted-then-added squeezed vacuum state can be written as

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}_{a^\dagger a}}} S(r)[\sinh(r)|0\rangle - \sqrt{2}\cosh(r)|2\rangle], \tag{49}$$

in which $\mathcal{N}_{a^\dagger a} = \sinh^2(r) + 2\cosh^2(r)$ is the normalization factor. Hence, the photon-subtracted-then-added squeezed vacuum state is a quantum superposition state of the squeezed vacuum state $S(r)|0\rangle$ and the squeezed Fock state $S(r)|2\rangle$. For a photon-subtracted-then-added squeezed vacuum state, the characteristic function of the total optical-microwave state $\chi_{a,b}(\xi_a,\xi_b,t)$ becomes

$$\chi_{a,b}(\xi_a,\xi_b,t) = \frac{1}{\mathcal{N}_{a^\dagger a}}\bigg\{\sinh^2(r)\Theta_{\alpha=0} - \sqrt{2}\cosh(r)\sinh(r)\frac{1}{\sqrt{2!}}\frac{\partial^2}{\partial\alpha^2}\Theta_{\alpha=0} - \sqrt{2}\cosh(r)\sinh(r)\frac{1}{\sqrt{2!}}\frac{\partial^2}{\partial\alpha^{*2}}\Theta_{\alpha=0} + 2\cosh^2(r)\frac{1}{2!}\frac{\partial^2}{\partial\alpha^2}\frac{\partial^2}{\partial\alpha^{*2}}\left(e^{|\alpha|^2}\Theta\right)_{\alpha=0}\bigg\}. \tag{50}$$

The Wigner function of the initial state of the cavity mode a in a photon-subtracted-then-added squeezed vacuum state is found to be

$$W_a(\beta) = \frac{2}{\pi\mathcal{N}_{a^\dagger a}}[\mathcal{J}_1 + \mathcal{J}_2\mathrm{Re}^2(\beta) + \mathcal{J}_3\mathrm{Im}^2(\beta) + \mathcal{J}_4\mathrm{Re}^2(\beta)\mathrm{Im}^2(\beta) + \mathcal{J}_5\mathrm{Re}^4(\beta) + \mathcal{J}_6\mathrm{Im}^4(\beta)] \times \exp\{-2[e^{2r}\mathrm{Re}^2(\beta) + e^{-2r}\mathrm{Im}^2(\beta)]\}, \tag{51}$$

where

$$\mathcal{J}_1 = \sinh^2(r) + 2\cosh^2(r), \quad \mathcal{J}_2 = -4e^{2r}[\sinh(2r) + 4\cosh^2(r)], \quad \mathcal{J}_3 = 4e^{-2r}[\sinh(2r) - 4\cosh^2(r)], \quad \mathcal{J}_4 = 32\cosh^2(r), \quad \mathcal{J}_5 = 16e^{4r}\cosh^2(r), \quad \mathcal{J}_6 = 16e^{-4r}\cosh^2(r). \tag{52}$$

Then the transfer fidelity of a photon-subtracted-then-added squeezed vacuum state is obtained as

$$F = \frac{2}{\mathcal{N}^2_{a^\dagger a}\sqrt{\bar{u}\bar{v}\bar{U}\bar{V}}}\bigg\{\mathcal{J}_1\mathcal{K}_1 + \frac{1}{2\bar{U}}(\mathcal{J}_1\mathcal{K}_2 + \mathcal{J}_2\mathcal{K}_1) + \frac{1}{2\bar{V}}(\mathcal{J}_1\mathcal{K}_3 + \mathcal{J}_3\mathcal{K}_1) + \frac{1}{4\bar{U}\bar{V}}(\mathcal{J}_1\mathcal{K}_4 + \mathcal{J}_4\mathcal{K}_1 + \mathcal{J}_2\mathcal{K}_3 + \mathcal{J}_3\mathcal{K}_2) + \frac{3}{4\bar{U}^2}(\mathcal{J}_1\mathcal{K}_5 + \mathcal{J}_5\mathcal{K}_1 + \mathcal{J}_2\mathcal{K}_2) + \frac{3}{4\bar{V}^2}(\mathcal{J}_1\mathcal{K}_6 + \mathcal{J}_6\mathcal{K}_1 + \mathcal{J}_3\mathcal{K}_3) + \frac{3}{8\bar{U}^2\bar{V}}(\mathcal{J}_2\mathcal{K}_4 + \mathcal{J}_4\mathcal{K}_2 + \mathcal{J}_3\mathcal{K}_5 + \mathcal{J}_5\mathcal{K}_3) + \frac{3}{8\bar{U}\bar{V}^2}(\mathcal{J}_2\mathcal{K}_6 + \mathcal{J}_6\mathcal{K}_2 + \mathcal{J}_3\mathcal{K}_4 + \mathcal{J}_4\mathcal{K}_3) + \frac{9}{16\bar{U}^2\bar{V}^2}(\mathcal{J}_4\mathcal{K}_4 + \mathcal{J}_5\mathcal{K}_6 + \mathcal{J}_6\mathcal{K}_5) + \frac{15}{16\bar{U}^3\bar{V}}(\mathcal{J}_4\mathcal{K}_5 + \mathcal{J}_5\mathcal{K}_4) + \frac{15}{16\bar{U}\bar{V}^3}(\mathcal{J}_4\mathcal{K}_6 + \mathcal{J}_6\mathcal{K}_4) + \frac{15}{8\bar{U}^3}(\mathcal{J}_2\mathcal{K}_5 + \mathcal{J}_5\mathcal{K}_2) + \frac{15}{8\bar{V}^3}(\mathcal{J}_3\mathcal{K}_6 + \mathcal{J}_6\mathcal{K}_3) + \frac{105}{16\bar{U}^4}\mathcal{J}_5\mathcal{K}_5 + \frac{105}{16\bar{V}^4}\mathcal{J}_6\mathcal{K}_6\bigg\}, \tag{53}$$

where

$$\begin{aligned}\mathcal{K}_1 &= n_1 + \frac{n_2}{2\bar{v}} + \frac{n_3}{2\bar{u}} + \frac{3n_4}{4\bar{v}^2} + \frac{n_5}{4\bar{u}\bar{v}} + \frac{3n_6}{4\bar{u}^2},\\ \mathcal{K}_2 &= -\frac{n_3}{\bar{u}^2} - \frac{n_5}{2\bar{u}^2\bar{v}} - \frac{3n_6}{\bar{u}^3},\\ \mathcal{K}_3 &= -\frac{n_2}{\bar{v}^2} - \frac{n_5}{2\bar{u}\bar{v}^2} - \frac{3n_4}{\bar{v}^3},\\ \mathcal{K}_4 &= \frac{n_5}{\bar{u}^2\bar{v}^2},\\ \mathcal{K}_5 &= \frac{n_6}{\bar{u}^4},\\ \mathcal{K}_6 &= \frac{n_4}{\bar{v}^4},\end{aligned} \tag{54}$$

and

$$\begin{aligned}n_1 &= \sinh^2(r) + 2\cosh^2(r),\\ n_2 &= -e^{2r}[\sinh(2r) + 4\cosh^2(r)]\exp(\pi p/q),\\ n_3 &= e^{-2r}[\sinh(2r) - 4\cosh^2(r)]\exp(\pi p/q),\\ n_4 &= e^{4r}\cosh^2(r)\exp(2\pi p/q),\\ n_5 &= 2\cosh^2(r)\exp(2\pi p/q),\\ n_6 &= e^{-4r}\cosh^2(r)\exp(2\pi p/q).\end{aligned} \tag{55}$$

When $r = 0$, the photon-subtracted-then-added squeezed vacuum state reduces to the two-photon Fock state $|2\rangle$ with a phase π. In this case, we find that the fidelity given by Eq. (53) is equal to Eq. (33).

Figure 8 shows the transfer fidelities F of a photon-subtracted-then-added squeezed vacuum state as a function of the coupling strength G for different squeezing parameters r and $T = 0$ K. For a given squeezing parameter r, the transfer fidelity increases as the coupling strength G is increased. For a given coupling strength G, the transfer fidelity gets better as the squeezing parameter r becomes smaller. Note that the transfer fidelity of a photon-subtracted-then-added squeezed vacuum state ($r \neq 0$) is lower than that of a two-photon Fock state $|2\rangle$ ($r = 0$). When $G = 40\gamma_a$ and $r = 0.5$, 1, the transfer fidelities are 88%, 75%, respectively.

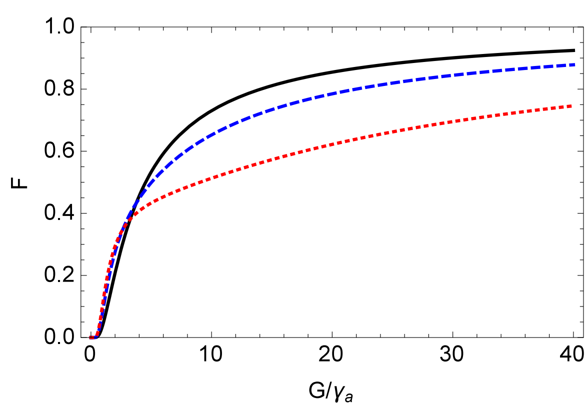

FIG. 8. (Color online) The transfer fidelities F of a photon-subtracted-then-added squeezed vacuum state as a function of the coupling strength G/γ_a for different squeezing parameters r and the temperature of the environment $T = 0$ K. The curves from top to bottom correspond to $r = 0$, 0.5, 1, respectively.

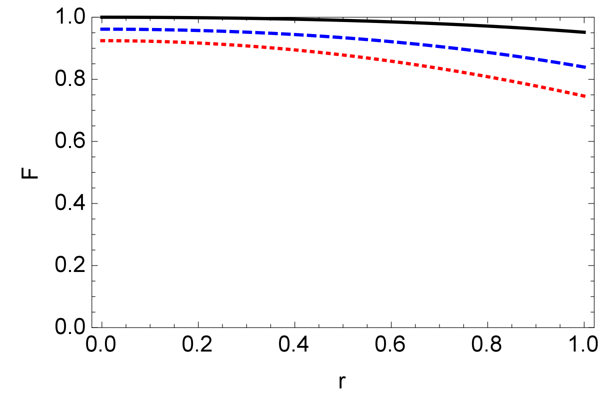

FIG. 9. (Color online) The transfer fidelities F of a single-photon-subtracted squeezed vacuum state and a photon-subtracted-then-added squeezed vacuum state as a function of the squeezing parameter r for $G = 40\gamma_a$ and $T = 0$ K. The black solid curve is for a squeezed vacuum state, the blue dashed curve is for a single-photon-subtracted squeezed vacuum state, and the red dotted curve is for a photon-subtracted-then-added squeezed vacuum state.

Figure 9 shows the transfer fidelities F of a single-photon-subtracted squeezed vacuum state and a photon-subtracted-then-added squeezed vacuum state as a function of the squeezing parameter r for $G = 40\gamma_a$ and $T = 0$ K. For comparison, we also plot the transfer fidelity of a squeezed vacuum state as a function of the squeezing parameter r for $G = 40\gamma_a$ and $T = 0$ K. For a fixed value of the squeezed parameter r, we find that both the transfer fidelity of the single-photon-subtracted squeezed vacuum and the transfer fidelity of the photon-subtracted-then-added squeezed vacuum state are lower than that of the squeezed vacuum state, and the transfer fidelity of the photon-subtracted-then-added squeezed vacuum state is the lowest.

We further investigate how the temperature T of the environment affects the transfer fidelities F of various non-Gaussian states. Figure 10 shows the transfer fidelities F of a single-photon Fock state $|1\rangle$, a two-photon Fock state $|2\rangle$, and a qubit state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, a single-photon-added coherent state with $\alpha = 1$, a single-photon-subtracted squeezed vacuum state with $r = 0.5$, and a photon-subtracted-then-added squeezed vacuum state with $r = 0.5$ as a function of the temperature T (K) of the environment for the coupling strength $G = 40\gamma_a$. Due to the detrimental effect of the temperature T of the environment, the fidelity decreases with increasing the temperature T of the environment. It is seen that the curve for the qubit state overlaps that for the single-photon-added coherent state with $\alpha = 1$. Note that the transfer fidelity of the single-photon Fock state degrades faster than that of the qubit state but slower than that of the two-photon Fock state $|2\rangle$. For the same amount of squeezing, the transfer fidelity of a single-photon-subtracted squeezed vacuum state decreases with the temperature T more slowly than that of a photon-subtracted-then-added squeezed vacuum state. Additionally, for a fixed value of T, the transfer fidelity of the single-photon-subtracted squeezed vacuum state is lower than that of the single-photon Fock state, and the transfer fidelity of the photon-subtracted-then-added squeezed vacuum state is lower than that of the two-photon Fock state $|2\rangle$.

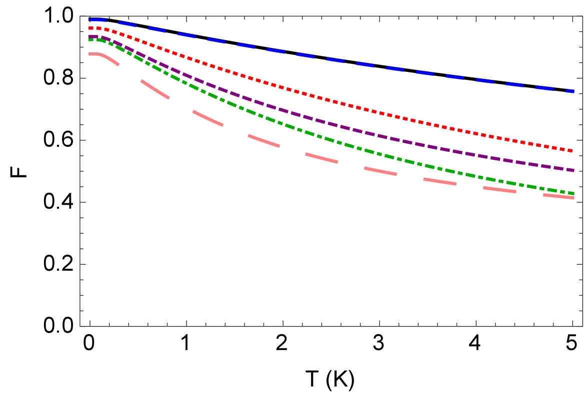

FIG. 10. (Color online) The transfer fidelities F of a single-photon Fock state (red dotted), a two-photon Fock state $|2\rangle$ (green dot dashed), a qubit state (black solid), a single-photon-added coherent state with $\alpha = 1$ (blue middle dashed), a single-photon-subtracted squeezed vacuum state with $r = 0.5$ (purple short dashed), and a photon-subtracted-then-added squeezed vacuum state with $r = 0.5$ (pink long dashed) as a function of the temperature T (K) of the environment for the coupling strength $G = 40\gamma_a$.

V. CONCLUSIONS

In summary, we have investigated the quantum state transfer from the optical mode to the microwave mode in the cavity electro-optic modulator. In the absence of the noises, we show that the quantum state transfer is performed perfectly if the decay rates of the optical mode and the microwave mode are equal to zero. In the presence of the noises, the fidelity of the quantum state transfer depends on the effective coupling strength and the temperature of the environment. The results show that the high fidelity state transfer for Gaussian states and some non-Gaussian states is possible to achieve if the effective coupling strength is strong enough and the temperature of the environment is low enough. We also make a comparison between the fidelities of different initial states of the optical mode to be transferred. We find that the transfer fidelity of the non-Gaussian state obtained by the de-Gaussification procedure is lower than that of the corresponding Gaussian state.

Finally, we briefly discuss experimental feasibility. In the cavity electro-optic modulator demonstrated experimentally by Ilchenko *et al.* [12], the electro-optic coupling strength is $g \sim 2\pi \times 20$ Hz, the optical decay rate and the microwave resonator's damping rate are $\gamma_a = \gamma_b = 2\pi \times 40$ MHz, the external laser power is 2 mW, and the ratio G/γ_a is on the order of 10^{-3} only [21]. It has been pointed out that there is plenty of room for improvement in the parameters [21]. For example, the electro-optic coupling strength could be improved to $g \sim 2\pi \times 5$ kHz, and the optical decay rate and the microwave damping rate could be reduced to $2\pi \times 0.2$ MHz; then $G/\gamma_a = 40$ is possible to be reached, thus the high fidelity of quantum state transfer could be achieved in the cavity electro-optic modulator. Moreover, all the initial states of the optical field considered above are pure. However, in a real experiment, the optical field is initially prepared in a mixed state which is nearly pure. In this case, the final state of the microwave resonator is also a mixed state. The fidelity between the two mixed states ρ_i and ρ_f can be calculated by using Eq. (8). The mixed state to be transferred is described by the density matrix,

$$\rho_i = \sum_{j=1}^{M} \mathcal{P}_j |\psi_j\rangle\langle\psi_j|, \tag{56}$$

where $\mathcal{P}_j$ is the probability of the mixed state ρ_i to be in the pure state $|\psi_j\rangle$, $0 < \mathcal{P}_j < 1$, and $\sum_{j=1}^{M} \mathcal{P}_j = 1$. We assume that $\mathcal{P}_1$ is close to 1, thus $\sum_{j=2}^{M} \mathcal{P}_j$ is close to zero. Hence, this mixed state ρ_i is near the pure state $|\psi_1\rangle$. The transfer fidelity of the mixed state ρ_i depends on all the coefficients $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_M$ and all the pure states $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_M\rangle$. The transfer fidelity of the mixed state ρ_i might be lower or higher than or equal to that of the pure state $|\psi_1\rangle$. Hence the cavity electro-optic modulator provides an alternative efficient method to transfer quantum states from the optical mode to the microwave mode.

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APPENDIX: THE CHARACTERISTIC FUNCTIONS AND THE WIGNER FUNCTIONS OF THE STATE OF THE MICROWAVE RESONATOR AT TIME $t = \frac{\pi}{2q}$

From Eqs. (18) and (19), it is seen that if we want to obtain the fidelities F of transferring different non-Gaussian states from the optical mode to the microwave mode, we first need to calculate the characteristic functions and the Wigner functions of the state of the microwave resonator at $t = \frac{\pi}{2q}$, which are shown in the following.

When the optical field is initially in a coherent state $|\alpha\rangle$, the characteristic function of the state of the microwave resonator at $t = \frac{\pi}{2q}$ can be obtained by using Eqs. (20) and (22), which yields

$$\chi_b\left(\xi_b, t = \frac{\pi}{2q}\right) = \chi_{a,b}\left(\xi_a = 0, \xi_b, t = \frac{\pi}{2q}\right)$$
$$= \exp[-s|\xi_b|^2 + (\xi_b\alpha^* - \xi_b^*\alpha)\mathcal{B}], \tag{A1}$$

where $\mathcal{B} = \exp[\pi p/(2q)]$.

When the optical field is initially in a single-photon Fock state $|1\rangle$, the characteristic function of the state of the microwave resonator at $t = \frac{\pi}{2q}$ can be obtained by using Eqs. (20), (26), and (A1), which yields

$$\chi_b\left(\xi_b, t = \frac{\pi}{2q}\right) = (1 - |\xi_b|^2\mathcal{B}^2)\exp(-s|\xi_b|^2). \tag{A2}$$

Substituting Eq. (A2) into Eq. (19), we find the Wigner function of the state of the microwave resonator at $t = \frac{\pi}{2q}$,

$$W_b\left(\beta, t = \frac{\pi}{2q}\right) = \frac{1}{\pi s}\left[1 - \frac{\mathcal{B}^2}{s}\left(1 - \frac{|\beta|^2}{s}\right)\right]\exp\left(-\frac{|\beta|^2}{s}\right). \tag{A3}$$

When the optical field is initially in a two-photon Fock state $|2\rangle$, the characteristic function of the state of the microwave resonator at $t = \frac{\pi}{2q}$ can be obtained by using Eqs. (20), (31), and (A1), which yields

$$\chi_b\left(\xi_b,t=\frac{\pi}{2q}\right)=\frac{1}{2}(2-4|\xi_b|^2\mathcal{B}^2+|\xi_b|^4\mathcal{B}^4)\exp(-s|\xi_b|^2). \tag{A4}$$

Then we substitute Eq. (A4) into Eq. (19) and derive the Wigner function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ as

$$W_b\left(\beta,t=\frac{\pi}{2q}\right)=\frac{1}{\pi s}\left[1-\frac{2\mathcal{B}^2}{s}\left(1-\frac{|\beta|^2}{s}\right)+\left(1-\frac{2|\beta|^2}{s}+\frac{|\beta|^4}{2s^2}\right)\frac{\mathcal{B}^4}{s^2}\right]\times\exp\left(-\frac{|\beta|^2}{s}\right). \tag{A5}$$

When the optical field is initially in a qubit state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, the characteristic function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be obtained by using Eqs. (20), (34), and (A1), which yields

$$\chi_b\left(\xi_b,t=\frac{\pi}{2q}\right)=\frac{1}{2}[2+2i\mathcal{B}\mathrm{Im}(\xi_b)-|\xi_b|^2\mathcal{B}^2]\times\exp(-s|\xi_b|^2). \tag{A6}$$

From Eq. (19), the Wigner function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ is found to be

$$W_b\left(\beta,t=\frac{\pi}{2q}\right)=\frac{1}{2\pi s}\left[2+\frac{2\mathcal{B}}{s}\mathrm{Re}(\beta)-\frac{\mathcal{B}^2}{s}\left(1-\frac{|\beta|^2}{s}\right)\right]\times\exp\left(-\frac{|\beta|^2}{s}\right). \tag{A7}$$

When the optical field is initially in a single-photon-added coherent state, the characteristic function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be obtained by using Eqs. (20), (38), and (A1), which yields

$$\chi_b\left(\xi_b,t=\frac{\pi}{2q}\right)=\frac{1}{1+|\alpha|^2}[1+|\alpha|^2+(\xi_b\alpha^*-\xi_b^*\alpha)\mathcal{B}-|\xi_b|^2\mathcal{B}^2]\times\exp[-s|\xi_b|^2+(\xi_b\alpha^*-\xi_b^*\alpha)\mathcal{B}]. \tag{A8}$$

Then the Wigner function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be found by using Eq. (19), which reads

$$W_b\left(\beta,t=\frac{\pi}{2q}\right)=\frac{1}{\pi s(1+|\alpha|^2)}\left[1+|\alpha|^2+\frac{\mathcal{B}}{s}(\alpha\beta^*+\alpha^*\beta-2|\alpha|^2\mathcal{B})-\frac{\mathcal{B}^2}{s}\left(1-\frac{1}{s}|\beta-\alpha\mathcal{B}|^2\right)\right]\exp\left(-\frac{1}{s}|\beta-\alpha\mathcal{B}|^2\right). \tag{A9}$$

When the optical field is initially in a squeezed coherent state $|\alpha,r\rangle$, the characteristic function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be obtained by using Eqs. (20) and (22), which yields

$$\chi_b\left(\xi_b,t=\frac{\pi}{2q}\right)=\exp\{-[\bar{v}\mathrm{Re}^2(\xi_b)+\bar{u}\mathrm{Im}^2(\xi_b)]+(\xi_b\bar{\alpha}^*-\xi_b^*\bar{\alpha})\mathcal{B}\}, \tag{A10}$$

where $\bar{\alpha}=\alpha\cosh(r)-\alpha^*\sinh(r)$.

When the optical field is initially in a single-photon-subtracted squeezed vacuum state, the characteristic function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be obtained by using Eqs. (20), (44), and (A10), which yields

$$\chi_b\left(\xi_b,t=\frac{\pi}{2q}\right)=\{1-\mathcal{B}^2[e^{2r}\mathrm{Re}^2(\xi_b)+e^{-2r}\mathrm{Im}^2(\xi_b)]\}\times\exp[-\bar{v}\mathrm{Re}^2(\xi_b)-\bar{u}\mathrm{Im}^2(\xi_b)]. \tag{A11}$$

Then the Wigner function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be found by using Eq. (19), which has the form,

$$W_b\left(\beta,t=\frac{\pi}{2q}\right)=\frac{1}{\pi\sqrt{\bar{u}\bar{v}}}\left\{1-\mathcal{B}^2\left[\frac{e^{2r}}{\bar{v}}\left(\frac{1}{2}-\frac{\mathrm{Im}^2(\beta)}{\bar{v}}\right)+\frac{e^{-2r}}{\bar{u}}\left(\frac{1}{2}-\frac{\mathrm{Re}^2(\beta)}{\bar{u}}\right)\right]\right\}\times\exp\left[-\frac{\mathrm{Re}^2(\beta)}{\bar{u}}-\frac{\mathrm{Im}^2(\beta)}{\bar{v}}\right]. \tag{A12}$$

When the optical field is initially in a photon-subtracted-then-added squeezed vacuum state, the characteristic function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be obtained by using Eqs. (20), (50), and (A10), which yields

$$\chi_b\left(\xi_b,t=\frac{\pi}{2q}\right)=\frac{1}{\mathcal{N}_{a^\dagger a}}[n_1+n_2\mathrm{Re}^2(\xi_b)+n_3\mathrm{Im}^2(\xi_b)+n_4\mathrm{Re}^4(\xi_b)+n_5\mathrm{Re}^2(\xi_b)\mathrm{Im}^2(\xi_b)+n_6\mathrm{Im}^4(\xi_b)]\times\exp[-\bar{v}\mathrm{Re}^2(\xi_b)-\bar{u}\mathrm{Im}^2(\xi_b)]. \tag{A13}$$

Then the Wigner function of the state of the microwave resonator at $t=\frac{\pi}{2q}$ can be found by using Eq. (19), which reads

$$W_b\left(\beta,t=\frac{\pi}{2q}\right)=\frac{1}{\pi\mathcal{N}_{a^\dagger a}\sqrt{\bar{u}\bar{v}}}[\mathcal{K}_1+\mathcal{K}_2\mathrm{Re}^2(\beta)+\mathcal{K}_3\mathrm{Im}^2(\beta)+\mathcal{K}_4\mathrm{Re}^2(\beta)\mathrm{Im}^2(\beta)+\mathcal{K}_5\mathrm{Re}^4(\beta)+\mathcal{K}_6\mathrm{Im}^4(\beta)]\times\exp\left[-\frac{\mathrm{Re}^2(\beta)}{\bar{u}}-\frac{\mathrm{Im}^2(\beta)}{\bar{v}}\right]. \tag{A14}$$

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