Application of the weak-measurement technique to study atom-vacuum interactions

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Quantum weak measurement has attracted much interest recently [J. Dressel *et al.*, Rev. Mod. Phys. **86**, 307 (2014).] because it could amplify some weak signals and provide a technique to observe nonclassical phenomena. Here, we apply this technique to study the interaction between the free atoms and the vacuum in a cavity. Due to the gradient field in the vacuum cavity, the external orbital motions and the internal electronic states of atoms can be weakly coupled via the atom-field electric-dipole interaction. We show that, within the properly postselected internal states, the weak atom-vacuum interaction could generate a large change to the external motions of atoms due to the postselection-induced weak values.

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I. INTRODUCTION

The conception of quantum weak measurement was introduced by Aharonov, Albert, and Vaidman (AAV) in 1988 [1]. Their theory is based on the von Neumann measurement with very weak coupling between two quantum systems [2], for example, the weak spin-orbit coupling of electrons in the Stern-Gerlach (SG) device. A key feature of the weak measurement is that the observable quantity (acting as the pointer) is measured in a certain subensemble, for example, measuring the expectation value of the electrons' position with the postselected spin state $|f\rangle$. This measurement leads to an interesting result that the pointer has a shift proportional to the value

$$A_w = \frac{\langle f|\hat{A}|i\rangle}{\langle f|i\rangle}, \tag{1}$$

where $|i\rangle$ and $\hat{A}$ are, respectively, the initial state and the observable operator of the spin system. A_w is the so-called weak value. Compared to the strong measurement $\langle i|\hat{A}|i\rangle$, the weak value provides an improved approach to detect $\hat{A}$, and some interesting phenomena result.

Recently, the weak value has attracted much interest because it could be arranged to amplify some weak signals [3–8]. It is also used to study the foundational questions of quantum mechanics [9–13], such as Hardy's paradox [14], the Leggett-Garg inequality [15], Heisenberg's uncertainty relation [16], and the wave-particle correlation [17]. Regarding the physical implementations, most of the previous studies used the light both as the pointer and the measured system [18]. There are several interesting works implementing weak measurement using the condensed-matter system, e.g., the quantum dot [19], the superconducting phase qubit [20], and the semiconducting Aharonov-Bohm interferometer [21]. Recently, Ref. [22] studied the weak measurement of a cold-atom system based on the dynamics of spontaneous emission.

In this article, the weak measurement is applied to the system of atom-cavity interaction. In such a system, the cavity electrodynamics (cavity QED) have predicted many nonclassical phenomena such as the famous vacuum Rabi oscillation [23–25] and the vacuum Rabi splitting [26–28]. These effects concern the cavity-induced changes in the internal electron's states of atoms. Remarkably, it has been shown that the light in a cavity can significantly affect the atom's center-of-mass (c.m.) motions, for example, Kapitza-Dirac scattering [29–33]. This effect is due to the atom stimulated emitting and absorbing photon in the cavity (resulting in a momentum change in the atom). It can be found that a vacuum cavity can also generate a similar transverse effect of a neutral atom via the virtual excitation of a photon. Here, we propose a weak value amplification (WVA) setup to observe such an interesting nonclassical effect of vacuum. After the atom-cavity interaction, we perform a single-qubit operation on the two internal states of atoms and postselected on an internal state. Then, we obtain a weak value; its real and imaginary parts determine, respectively, the shifts of the average momentum and the position of the atoms' external motions. Consequently, the controllable weak value could be used to amplify the vacuum-induced transverse shifts of atoms. It is shown that the present WVA could offer some certain advantages for experimentally detecting the weak transverse effects of atoms.

Our paper is organized as follows. In Sec. II, we present the vacuum-induced weak coupling between the internal and external motions of free atoms. This coupling acts as a force to push the neutral atoms moving transversely. In Sec. III, we get the desirable weak value using the single-qubit operation and postselection and use it to amplify the transverse shifts of atoms. In Sec. IV, we discuss the physical meaning of WVA. Our conclusions are summarized in Sec. V.

II. THE VACUUM-INDUCED COUPLING BETWEEN THE INTERNAL AND EXTERNAL MOTIONS OF FREE ATOMS

Following the original work of AAV, we consider the weak-measurement experiment as shown in Fig. 1. The spatially coherent atoms, e.g., a released BEC [33], are injected into the equipment through a pinhole located around the point (0,0,0). This pinhole selects part of the matter wave, and thus, the positional uncertainty of the selected atoms is on the order of the size of the pinhole. Hence, one can use the typical Gaussian

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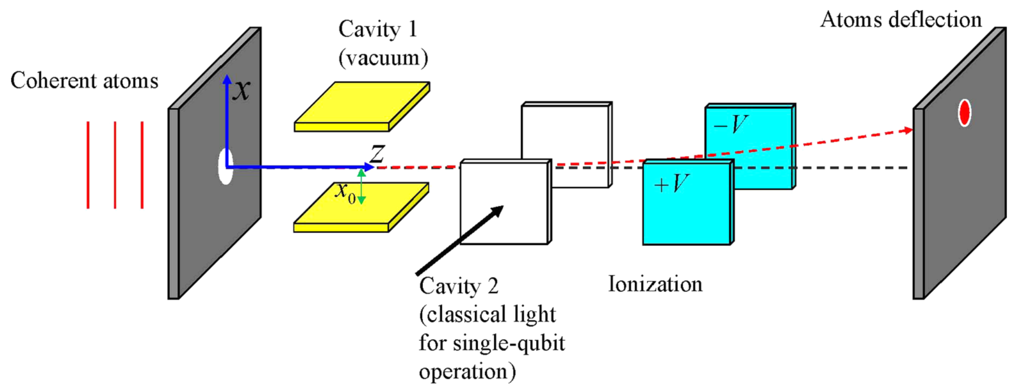

FIG. 1. (Color online) Sketch of the weak measurement process. The two-level atoms are prepared in a certain internal state $|S_i\rangle$ and pass through a pinhole with momentum along the z direction. The vacuum field (with the x-directional gradient) in cavity 1 generates weak coupling between the atoms' internal states and the external x-directional motions. Cavity 2, with classical light, resonantly excites atoms and generates the desirable single-qubit operation $\hat{U}$. The applied voltage $\pm V$ ionizes the atoms in the excited state (similar to the procedure in the experiments of the Haroche group [23–25]) and leaves the ground-state atoms to be detected. In the selected ensemble of ground states, the atoms have a shift (along the x direction) in the average position on the deposition plate. This shift can be described by the so-called weak value, which depends on the preselection $|S_i\rangle$ and the single-qubit operation $\hat{U}$.

wave packet to describe the spatially coherent atoms (after the pinhole). In the x direction, the Gaussian state reads

$$|G\rangle = \int_{-\infty}^{\infty} dx \phi(x)|x\rangle, \tag{2}$$

where $\phi(x) = \langle x|G\rangle = (2\pi\Delta^2)^{-1/4}\exp[-x^2/(4\Delta^2)]$ is the probability amplitude of the position eigenstate $|x\rangle$ and Δ describes the rms width of the wave packet. Of course, the state (2) can also be written as $|G\rangle = \int_{-\infty}^{\infty} dp\, \phi(p)|p\rangle$, with the momentum eigenstate $|p\rangle$ and the Gaussian function $\phi(p) = \langle p|G\rangle = [2\Delta^2/(\pi\hbar^2)]^{1/4}\exp(-\Delta^2 p^2/\hbar^2)$. For this Gaussian state, the expectation value of the position is $\langle x\rangle = 0$, and its uncertainty reads $\Delta = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$. The average momentum along the x direction is $\langle p\rangle = 0$, and its uncertainty reads $\Delta_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \hbar/(2\Delta)$. Physically, the uncertainty Δ (or Δ_p) determines the main distribution range of particles' positions (or momentums). Out of this range, the probability to find the particles is negligible. Below, we study the vacuum field (in cavity 1) induced change in the initial wave packet $\phi(x)$ within a very short duration (i.e., the free diffraction of the atom is negligible).

In cavity 1, the quantized field of a mode takes the form [34]

$$\vec{E} = \vec{\tau} E_0 \sin(kx + kx_0)(\hat{a}^\dagger + \hat{a}), \tag{3}$$

which excites the incoming atoms. Here, $\vec{\tau}$, E_0, and k are, respectively, the polarization vector, amplitude, and wave number of the standing wave (such as the first excited mode). $\hat{a}^\dagger$ and $\hat{a}$ are, respectively, the creation and annihilation operators of the corresponding cavity mode (with frequency ω_c). We consider the microwave excitation of the two-level Rydberg atoms. Although the orbit radius of Rydberg states is very large (about 10^3 a.u. [23–25]), it is far smaller than the wavelength of the microwave cavity (on the order of a centimeter). Therefore, in the atomic internal region the driving field (3) can be regarded as uniform. Performing the dipole approximation, the interaction between the atom and cavity field reads

$$\hat{H}_{\text{int}} = \hbar\Omega_0 \sin(kx + kx_0)(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x, \tag{4}$$

with the so-called Rabi frequency $\Omega_0 = E_0\mu/\hbar$ [34]. Here, $\hbar$ is the Planck constant divided by 2π, $\hat{\sigma}_x = |e\rangle\langle g| + |g\rangle\langle e|$ is the transition operator of the two-level atom with the ground state $|g\rangle$ and the exciting state $|e\rangle$, and μ is the transition matrix element of the two-level atom.

We consider $k\Delta \ll 1$ and $0 \ll kx_0 \ll \pi/2$; the Hamiltonian (4) can be approximately written as

$$\hat{H}_{\text{int}} = \hbar\Omega(x + x_c)(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x, \tag{5}$$

with the constants $\Omega = k\cos(kx_0)\Omega_0$ and $x_c = \tan(kx_0)/k$. Here, we have used the well-known trigonometric function $\sin(kx + kx_0) = \cos(kx_0)\sin(kx) + \cos(kx)\sin(kx_0)$ and neglected the high order of kx. Note that $k\Delta \ll 1$ means that the range of atomic motion in the x direction is much smaller than the wavelength of the cavity mode. The range of x depends on the initial uncertainty Δ and the wave-packet spread (i.e., the diffraction). As mentioned earlier, the diffraction of the atom is negligible as the duration of the cavity-atom interaction is very short, i.e., $t \ll m\Delta^2/\hbar$ (m is the mass of the atom). Thus, the value of x is on the order of its initial uncertainty Δ (e.g., 10 μm), which can be much smaller than the wavelength of the cavity mode (about 1 cm [25]).

With interaction (5), the total Hamiltonian of the system can be written as

$$\hat{H}_p = \frac{p^2}{2m} + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_c\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\Omega(\hat{x} + x_c)(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x \tag{6}$$

in the Hilbert space of momentum eigenstates. In this space, the position operator is given by $\hat{x} = i\hbar\partial/\partial p$. Physically, the first term on the right-hand side of Eq. (6) describes the c.m. motion of the free atom. The second term describes the two atomic internal levels (by the Pauli operator $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ and the transition frequency ω_a). The third term is the free

Hamiltonian of the cavity ground mode. The last term describes the coupling between the considered three degrees of freedom, i.e., a position-dependent Jaynes-Cummings interaction. In the rotating frame defined by $\hat{U}_1 = \exp[-ip^2t/(2m\hbar)]$, the Hamiltonian (6) can be written as

$$\hat{H}_p = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_c\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\Omega\left(\hat{x} + x_c + \frac{pt}{m}\right)(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x. \tag{7}$$

With such a transform, the free term $p^2/(2m)$ is eliminated. Considering the atom rapidly crosses the cavity (i.e., the effective interaction duration t is sufficiently short), there is an impulse atom-cavity interaction corresponding to the von Neumann measurement [1,2]. Thus, $pt/m \to 0$, and the Hamiltonian (7) reduces to

$$\hat{H}_p = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_c\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\Omega(\hat{x} + x_c)(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x. \tag{8}$$

Performing a unitary transformation of $\hat{U}_2 = \exp[-i\omega_c t(\hat{a}^\dagger\hat{a} + 1/2) - it\omega_a\hat{\sigma}_z/2]$, the Hamiltonian (8) further reduces to

$$\hat{H}_p = \hbar\Omega(\hat{x} + x_c)(\hat{a}^\dagger\hat{\sigma}_- e^{-i\delta t} + \hat{a}\hat{\sigma}_+ e^{i\delta t}), \tag{9}$$

with the detuning $\delta = \omega_a - \omega_c$ and the operators $\hat{\sigma}_- = |g\rangle\langle e|$ and $\hat{\sigma}_+ = |e\rangle\langle g|$. Here, the usual rotating-wave approximation is performed; that is, the terms relating to the sum frequency $\omega_a + \omega_c$ have been neglected.

The time-evolution operator for the Hamiltonian (9) can be given by the Dyson series:

$$\hat{U}_{\text{evol}} = 1 + \left(\frac{-i}{\hbar}\right)\int_0^t \hat{H}_p(t_1)dt_1 + \left(\frac{-i}{\hbar}\right)^2 \int_0^t \hat{H}_p(t_1) \times \int_0^{t_1} \hat{H}_p(t_2)dt_2 dt_1 + \cdots . \tag{10}$$

Under the conditions of large detuning, $\Omega \ll \delta$, the above time-evolution operator can be approximately written as

$$\hat{U}_{\text{evol}} \approx e^{-\frac{i}{\hbar}\hat{H}_{\text{eff}}t}, \tag{11}$$

with the effective Hamiltonian $\hat{H}_{\text{eff}} = (\hbar\Omega^2/\delta)(\hat{x} + x_c)^2(\hat{a}^\dagger\hat{a}\hat{\sigma}_z + |e\rangle\langle e|)$. Considering the cavity is in the vacuum state $|0\rangle$, i.e., $\hat{a}^\dagger\hat{a}|0\rangle = 0$, the effective Hamiltonian reduces to

$$\hat{H}_{\text{eff}} = \hbar g_0\left(\frac{1}{x_c}\hat{x} + 1\right)^2 |e\rangle\langle e|, \tag{12}$$

with $g_0 = (\Omega x_c)^2/\delta$. This Hamiltonian just describes a position-dependent vacuum Rabi splitting [35], and the parameter g_0 describes the coupling strength between the internal and external motions of the atom. Numerically, considering the wavelength $\lambda = 1$ cm of the cavity mode and the Rabi frequency $\Omega_0/2\pi = 10$ KHz [23], we have $\Omega x_c = \Omega_0 \sin(kx_0) \approx 2\pi \times 7$ KHz with $kx_0 = \pi/4$, and consequently, $g_0 \approx 2\pi \times 0.7$ KHz with $\Omega x_c/\delta = 0.1$. Of course, as detuning δ increased, the coupling strength g_0 decreased significantly.

III. THE WEAK-VALUE AMPLIFICATION

In the following, we will show that the vacuum-induced interaction (12) can generate a small shift to the initial wave packet $\phi(p)$, and this displacement can be amplified by using the weak-value technique. In momentum space, the evolved state of an atom can be written as $|\psi\rangle = \hat{U}_{\text{evol}}|G\rangle|S_i\rangle = \int_{-\infty}^{\infty} dp\psi|p\rangle$, with $|S_i\rangle$ being the initial state of the atomic qubit. We rewrite the initial Gaussian wave function as $\phi(p) = \phi(\tilde{p}) = (2\pi)^{-1/4}\Delta_p^{-1/2}\exp(-\tilde{p}^2)$ with the dimensionless number $\tilde{p} = p\Delta/\hbar$. Then, we have

$$\psi = e^{g_c|e\rangle\langle e|\frac{\partial}{\partial\tilde{p}}} e^{ig_c'|e\rangle\langle e|\frac{\partial^2}{\partial\tilde{p}^2}}\phi(\tilde{p})|i\rangle \tag{13}$$

by using the relation $\hat{x} = i\hbar\partial/\partial p = i\Delta\partial/\partial\tilde{p}$. Here, $g_c = 2g_0t(\Delta/x_c)$ and $g_c' = g_0t(\Delta/x_c)^2$ are the dimensionless coupling parameters, and $|i\rangle = \exp(-ig_0t|e\rangle\langle e|)|S_i\rangle$. Considering $\Delta \ll x_c$, i.e., $g_c' \ll g_c$, the state (13) can be approximately written as

$$\psi = e^{g_c|e\rangle\langle e|\frac{\partial}{\partial\tilde{p}}}\phi(\tilde{p})|i\rangle. \tag{14}$$

For simplicity, we redefine $|i\rangle = \alpha|g\rangle + \beta\exp(i\theta)|e\rangle$ as the initial internal state of the atoms (which can be prepared by the well-known single-qubit operations). Here, θ is the phase of the superposition state, and α and β are the superposition coefficients (real numbers) satisfying the normalized condition $\alpha^2 + \beta^2 = 1$. Immediately, we have the state evolution $\phi(p)|i\rangle \longrightarrow \alpha\phi(p)|g\rangle + \beta e^{i\theta}\phi(p + \hbar g_c/\Delta)|e\rangle$, and consequently, the expectation value of the atom's momentum reads

$$\langle p\rangle = -\beta^2\frac{\hbar g_c}{\Delta} = -2\beta^2 g_c\Delta_p. \tag{15}$$

This equation means that the vacuum in cavity 1 generates a transverse shift $\langle p\rangle - 0 = \langle p\rangle$ to the average momentum of atoms. Because $\beta^2 \leqslant 1$, the shift $\langle p\rangle \to 0$ for a very weak coupling of $g_c \to 0$. Furthermore, one can easily calculate the expectation value $\langle x\rangle = 0$ of the atomic position. These results indicate that the weak coupling g_c can generate significant changes neither on the observable $\langle p\rangle$ nor on $\langle x\rangle$.

We now use the weak-value technique to amplify the shifts $\langle p\rangle$ and $\langle x\rangle$. First, we perform a single-qubit operation $\hat{U} = \exp(-i\eta\hat{\sigma}_x)$ to the state (14) with the controllable parameter η. Alternatively, this single-qubit operation can be realized by the classical resonant light, as shown in Fig. 1. Consequently, we have the final state

$$\psi' = \hat{U}\psi = \hat{U}e^{g_c|e\rangle\langle e|\frac{\partial}{\partial\tilde{p}}}\phi(\tilde{p})|i\rangle = \hat{U}\left[1 + g_c(|e\rangle\langle e|)\frac{\partial}{\partial\tilde{p}} + \frac{g_c^2}{2}(|e\rangle\langle e|)^2\frac{\partial^2}{\partial\tilde{p}^2} + \cdots\right]\phi(\tilde{p})|i\rangle. \tag{16}$$

Second, we postselect an eigenstate of the atomic qubit, e.g., $|g\rangle$, and immediately, the external motion of atoms collapses on the wave function:

$$\psi'_w = \langle g|\psi'\rangle = \langle g|\hat{U}|i\rangle\left(1 + g_c A_w \frac{\partial}{\partial\tilde{p}} + \frac{g_c^2 A_w}{2}\frac{\partial^2}{\partial\tilde{p}^2} + \cdots\right)\phi(\tilde{p}), \tag{17}$$

with

$$A_w = \frac{\langle g|(\hat{U}|e\rangle\langle e|)|i\rangle}{\langle g|\hat{U}|i\rangle}. \tag{18}$$

Here, we have used the relation $(|e\rangle\langle e|)^n = |e\rangle\langle e|$, with $n = 1,2,3,\ldots$. A_w is our weak value, although it does not satisfy the standard definition of Eq. (1). This will be explained in Sec. IV. Physically, the postselection of $|g\rangle$ could be realized by the field ionization [23–25]. Since $|e\rangle$ and $|g\rangle$ have different ionization energies, the ionization is state selective. Suppose the atoms in only exciting state $|e\rangle$ are effectively ionized by the applied moderate electric field; then the exciting state atoms will be accelerated in the y direction and discarded. However, the ground-state atoms will arrive at the plate to be finally detected, as shown in Fig. 1.

Considering the weak interaction, i.e., $g_c \ll 1$ and $g_c^2|A_w| \ll 1$, the wave function (17) can be approximately written as

$$\psi_w = \frac{\psi'_w}{\langle g|\hat{U}|i\rangle} = \left(1 + g_c A_w \frac{\partial}{\partial\tilde{p}}\right)\phi(\tilde{p}) = \phi(p) - \frac{2g_c\Delta}{\hbar}\mathrm{Re}(A_w)p\phi(p) - i\frac{2g_c\Delta}{\hbar}\mathrm{Im}(A_w)p\phi(p). \tag{19}$$

Here, the high orders of g_c have been neglected, and $\mathrm{Re}(A_w)$ and $\mathrm{Im}(A_w)$ are, respectively, the real and imaginary parts of A_w. With this approximation, the probability for successfully postselecting $|g\rangle$ reads $P \approx |\langle g|\hat{U}|i\rangle|^2$. According to Eq. (19), we have the expectation value of momentum:

$$\langle\hat{p}\rangle_w = \int_{-\infty}^{\infty}\psi_w^* p\psi_w dp \approx -\hbar\frac{g_c}{\Delta}\mathrm{Re}(A_w) = -2g_c\Delta_p\mathrm{Re}(A_w). \tag{20}$$

This means that, within the postselected subensemble, the shift of average momentum $\langle p\rangle_w - 0 = \langle p\rangle_w$ is proportional to the real part of the weak value. On the other hand, in the position presentation, the wave function (19) reads

$$\phi_w = \int_{-\infty}^{\infty}\psi_w\langle x|p\rangle dp = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\phi(p)e^{ipx/\hbar}dp + \frac{1}{\sqrt{2\pi\hbar}}\frac{\hbar g_c A_w}{\Delta}\int_{-\infty}^{\infty}e^{ipx/\hbar}\frac{\partial\phi(p)}{\partial p}dp = \left(1 - i\frac{g_c A_w}{\Delta}x\right)\phi(x), \tag{21}$$

and consequently, the expectation value of positions reads

$$\langle x\rangle_w = \int_{-\infty}^{\infty}\phi_w^* x\phi_w dx \approx \frac{2g_c}{\Delta}\mathrm{Im}(A_w)\int_{-\infty}^{\infty}\phi(x)x^2\phi(x)dx = 2g_c\Delta\mathrm{Im}(A_w). \tag{22}$$

This indicates that, within the postselected subensemble, the shift of the average position $\langle x\rangle_w - 0 = \langle x\rangle_w$ is proportional to the imaginary part of the weak value.

Due to the single-qubit operations $\hat{U}|g\rangle = \cos(\eta)|g\rangle - i\sin(\eta)|e\rangle$ and $\hat{U}|e\rangle = \cos(\eta)|e\rangle - i\sin(\eta)|g\rangle$, our weak value reads

$$A_w = \frac{\langle g|(\hat{U}|e\rangle\langle e|)|i\rangle}{\langle g|\hat{U}|i\rangle} = \frac{1}{Ae^{i\vartheta}+1}, \tag{23}$$

with $A = \alpha\cos(\eta)/[\beta\sin(\eta)]$ and $\vartheta = (\pi/2) - \theta$. Consequently, we have

$$\mathrm{Re}(A_w) = \frac{1 + A\cos(\vartheta)}{A^2 + 2A\cos(\vartheta) + 1}, \tag{24}$$

$$\mathrm{Im}(A_w) = \frac{-A\sin(\vartheta)}{A^2 + 2A\cos(\vartheta) + 1}. \tag{25}$$

These values could be as large as we want if we properly adjust parameters A and ϑ. For example, if $\cos(\vartheta) = 1$ and $A \to -1$, then $\mathrm{Re}(A_w) = 1/(1 + A) \to \infty$. If $A = -\cos(\vartheta)$ and $\vartheta \to 0$, then $\mathrm{Im}(A_w) = \cot(\vartheta) \to \infty$. With these enlarged weak values, the weak interaction of g_c could significantly change the transverse c.m. motions of atoms via the basic equations

$$\frac{\langle p\rangle_w}{2\Delta_p} \approx -g_c\mathrm{Re}(A_w), \tag{26}$$

$$\frac{\langle x\rangle_w}{2\Delta} \approx g_c\mathrm{Im}(A_w). \tag{27}$$

We would like to emphasize that the shifts $\langle p\rangle_w$ and $\langle x\rangle_w$ cannot be infinitely amplified, as the weak values were obtained under the weak-interaction condition of $g_c^2|A_w| \ll 1$. That is, the amplified displacements of average position and momentum are limited in the regimes of $g_c\langle p\rangle_w/(2\Delta_p) \ll 1$ and $g_c\langle x\rangle_w/(2\Delta) \ll 1$, respectively. Hence, the present amplification effects are significant just for the weak interaction of $g_c \to 0$.

There is a cost to WVA. The probability $P \approx |\langle g|\hat{U}|i\rangle|^2$ for successfully postselecting $|g\rangle$ decreases rapidly with increasing $\mathrm{Re}(A_w)$ or $\mathrm{Im}(A_w)$, so that more significant amplification needs more atoms. In terms of metrology, the WVA may be suboptimal for parameter estimation since many atoms (information) were discarded [36–38]. However, in practical experimental systems the discarded atoms may also bring noises into the final detection. As pointed out in Refs. [39–44], the WVA can offer some certain technical advantages, for example, suppressing systematic errors [43] and avoiding detector saturation [44].

In the present system, it would be very difficult to precisely scan the position or momentum distribution of final atoms. Possibly, one can place two atom detectors (such as the hot-wire ionizers [33]) at the symmetrical positions x and $-x$ to estimate the transverse effects of atoms. In the unit time, the expected atom counting in the detectors is given by

$\bar{n}_1 = NP\int_{x-l/2}^{x+l/2} |\phi_w(x)|^2 dx$ and $\bar{n}_2 = NP\int_{-x-l/2}^{-x+l/2} |\phi_w(x)|^2 dx$, respectively. N is the total number of input atoms in the unit time, and $l < x$ is the atom-collecting region of detectors. According to $\bar{n}_1$ and $\bar{n}_2$, we have

$$\bar{s} = \frac{\bar{n}_1}{\bar{n}_2} - 1 = \frac{1 + 2g_c\mathrm{Im}(A_w)\frac{\bar{x}_l}{\Delta}}{1 - 2g_c\mathrm{Im}(A_w)\frac{\bar{x}_l}{\Delta}} - 1 \approx 4g_c\mathrm{Im}(A_w)\frac{\bar{x}_l}{\Delta}, \tag{28}$$

with $\bar{x}_l = \int_{x-l/2}^{x+l/2} x\phi^2(x)dx / \int_{x-l/2}^{x+l/2} \phi^2(x)dx$. Above, the high orders of g_c have been neglected, and $\bar{s}$ can be regarded as the signal of atom transverse shift. We note that $\bar{n}_1$, $\bar{n}_2$, and, consequently, $\bar{s}$ are the expectation values. In practice, the experimental results may take $n_i = \chi\bar{n}_i + \delta_i^s + \delta_i^r$ (with the index $i = 1,2$), and consequently, Eq. (28) is replaced by $s = (n_1/n_2) - 1$. χ is the detection efficiency of the atom detectors. There are two kinds of errors in measurements, namely, systematic error δ_i^s and random error δ_i^r. Certainly, the WVA does not offer advantages for suppressing the random error since the input atoms were reduced by the postselection [43]. However, it can be found that the WVA is very useful for suppressing the systematic error which is proportional to the number of atoms, i.e., $\delta_i^s = \delta_0\bar{n}_i$, with δ_0 being a small uncertainty coefficient. This systematic error arises perhaps because of the unsteady detection efficiency of the atom detector, the uncertain location of the detector, etc.

IV. DISCUSSION

Here, we give a brief discussion of the physical meaning of the WVA. In the original work of AAV [1], there are two SG devices. The first one is used to generate weak coupling between the spin and orbit of the electron, and the second one is arranged to perform the postselection of the electron's spin states. The present weak-measurement process is similar to that of AAV. Cavity 1 plays an atomic SG device to implement the coupling between the internal qubit and the external c.m. orbital motion of the atom. Cavity 2 acts as the second SG device of AAV for coherently manipulating the atoms. After cavity 1 the atom is in the state (14), which can be written as the standard form $\psi \approx \phi(p)|i\rangle - ig_c\hat{A}\hat{P}\phi(p)|i\rangle$, with $\hat{A} = |e\rangle\langle e|$ and $\hat{P} = i\hbar\partial/\partial p$. Using the orthonormal eigenstates $|g\rangle$ and $|e\rangle$ of the two-level atom, ψ can be further written as

$$\begin{aligned}\psi &= (|g\rangle\langle g| + |e\rangle\langle e|)\psi \\ &= \langle g|i\rangle\phi(p,A_g)|g\rangle + \langle e|i\rangle\phi(p,A_e)|e\rangle.\end{aligned} \tag{29}$$

Here, $A_g = \langle g|\hat{A}|i\rangle/\langle g|i\rangle$, $A_e = \langle e|\hat{A}|i\rangle/\langle e|i\rangle$, $\phi(p,A_g) = (1 - ig_cA_g\hat{P})\phi(p)$, and $\phi(p,A_e) = (1 - ig_cA_e\hat{P})\phi(p)$.

Obviously, Eq. (29) represses an entangled state. If the internal state $|g\rangle$ is measured, then the external motion of the atom collapses on the wave function $\phi(p,A_g)$; however, if the state $|e\rangle$ is measured, the atom collapses on $\phi(p,A_e)$. These measurements performed on the qubit are just the well-known projective measurements $\hat{P}_g = |g\rangle\langle g|$ and $\hat{P}_e = |e\rangle\langle e|$. The outcomes of A_g and A_e can be regarded as the weak values since they take the same form as Eq. (1). However, it can be found that $A_g = 0$ and $A_e = 1$ because $\hat{A} = |e\rangle\langle e|$, so that they cannot realize the desirable amplification functions, whatever the initial state $|i\rangle$ is. We note that A_g and A_e are both real. Hence, applying the projective measurements directly to the state (29) cannot yield the effect of positional shifts of atoms, as mentioned early.

Compared to the projective measurement, the weak measurement due to the postselection $\hat{P}_f = |f\rangle\langle f|$ is a more general conception because the state $|f\rangle$ is beyond the eigenstates of the system. For example, how can a coherent superposition of the eigenstates be realized? In AAV's proposal, the desired postselection is implemented by the second SG device. It couples the spin to the y-directional orbital motion of the electron (the third degree of freedom of the electron). Consequently, one can select the y-directional motions (via the strong measurement) to realize a postselection of the superposition state of the spin (see, e.g., Ref. [45], which discussed in detail AAV's idea). In recent optics experiments [18], the postselection is realized by a polarizer which is oriented at a certain angle and then selects the desirable superposition state of polarization of light.

Here, cavity 2 together with the ionization electrodes just realizes an operation $\hat{P}_f' = |g\rangle\langle g|\hat{U} = |g\rangle\langle f|$ to the state (29). The weak value (18) can be written as the standard form

$$A_w = \frac{\langle g|\hat{U}\hat{A}|i\rangle}{\langle g|\hat{U}|i\rangle} = \frac{\langle f|\hat{A}|i\rangle}{\langle f|i\rangle}, \tag{30}$$

with $\langle f| = \langle g|\hat{U}$. This weak value can be as large as we want, such as $\mathrm{Im}(A_w) \neq 0$. Physically, the present weak value can be regarded as an outcome of the coherent operation $\hat{U}$. It can be found that the standard postselection also implies coherent operations by writing $\hat{P}_f = |f\rangle\langle f| = \hat{R}|g\rangle\langle g|\hat{R}^\dagger$ with the unitary evolution operator $\hat{R}$ and the eigenstate $|g\rangle$ of any systems.

V. CONCLUSION

In this theoretical work, we have shown that a vacuum microwave cavity can shift the neutral atoms to move transversely. This nonclassical effect is due to the vacuum-induced coupling between the internal and external motions of free atoms, i.e., a position-dependent vacuum Rabi splitting. We have further shown that the present effect could be amplified by the weak-value technique. After the atom-cavity coupling, we performed a single-qubit rotation on the atomic internal states and consequently postselected an internal eigenstate (strong measurement). Then, we obtained a weak value which was used to amplify the vacuum-induced shift of the average position or momentum of atoms. Technically, the present WVA could offer advantages in practical experiment systems for observing the weak transverse effect of atoms, such as suppressing the systematic error of detectors. Physically, our WVA is a quantum-mechanical effect due to the necessary single-qubit operation. Finally, we hope the present studies will encourage further studies on weak measurements and cavity QED.

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