All-optical simulations of nonclassical noise-induced effects in quantum optomechanics

Kimin Park,[*] Petr Marek, and Radim Filip
Department of Optics, Palacký University, 17. listopadu 1192/12, 77146 Olomouc, Czech Republic
(Received 16 July 2015; published 9 September 2015)

The key feature of optomechanics is that its basic interactions are inherently nonlinear. This feature allows coherent and nonclassical effects such as squeezing on the mechanical mode induced by chaotic incoherent light. Since this effect is too challenging to observe directly in the membrane-in-the-middle arrangement of quantum optomechanics, we analyze an analogical high-order nonlinear effect and propose to simulate it all-optically. Specifically, we exploit hybrid quantum optics based on single photon states and homodyne detection to conditionally construct a simulator for noise-driven nonclassical effects. We show that this simulation can confirm the presence of squeezing caused by nonlinear coupling pumped by a noisy light. Our proposal opens the possibilities to emulate nonclassical effects challenging on the natural experimental platforms.

DOI: 10.1103/PhysRevA.92.033813 PACS number(s): 42.50.Pq, 42.50.Dv, 42.50.Ex, 37.30.+i

I. INTRODUCTION

Quantum optics is a mature field of research that has been proven to be an excellent experimental platform both for modern quantum information technologies and fundamental tests of quantum physics. One of the strong aspects of light is its ability to be coupled to other physical systems [1]. Quantum states of light were coupled to individual atoms or ions [2,3], solid state systems [4–6], collective spin of an atomic cloud [7], or a vibrational mode of a mechanical oscillator [8]. On one hand, this ability allows the light to serve as an effective mediator and interface between these quantum systems, possibly at a large distances [9]. On the other hand, it helps to cover one of its main contemporary drawbacks—the lack of various high-order nonlinear operations for quantum states of light. For a long time, the only achievable high-order nonlinear transformations of quantum states of light came from the interaction with discrete levels of atoms [10]. Recently, quantum optomechanics has opened another principal possibility.

Quantum optomechanics studies the effect of coupling between light and mechanical oscillators [8]. This coupling is realized via a radiation pressure on a mechanical object, in which the intensity of the light causes a change in mechanical momentum of the oscillator.

This is a naturally occurring coupling that exhibits an exact cubic nonlinearity which is missing at quantum level in all-optical systems. An interesting aspect of the coupling is that it allows conversion of an incoherent quantity—energy, present also in thermal noise—into a coherent effect such as the displacement. Unfortunately, this effect has not been observed experimentally since the coupling is not strong enough for a straightforward application. Furthermore, the coupling becomes linear and the coherent effect driven by a noise vanishes in the process of enhancing the coupling strength by a high-intensity coherent light with a well-defined phase.

Observing the noise-induced coherent effects by simulating the coupling on a different experimental platform is therefore important to overcome these limitations. In this paper we propose a feasible all-optical simulation of an optomechanical coupling that enables noise-induced nonclassical effects. More explicitly, we simulate the noise-induced nonclassical effects such as nonclassical squeezing induced by a thermal or a Poissonian noise. This emulates the effects of a more challenging membrane-in-the-middle configuration of quantum optomechanics [11–15]. This coupling bears a similarity to a single mode cubic nonlinearity [16], of which the first steps towards realization were made by observing the nonlinear cubic state [17]. However, we are interested not only in feasible physical realization of the nonlinear coupling but also in observing its dynamical effects. To this end we are taking advantage of the recently proposed method of the conditional simulation of a short-time motion of a particle in an arbitrary nonlinear potential [18]. This method employs a sequence of elementary X gates implemented with the standard tools of quantum optics—single photon sources, linear optical elements, and homodyne detections. This method can also be extended to realize multimode non-Gaussian entangling operations [19].

The paper is organized as follows. In Sec. II, we analyze the occurrence and the origin of noise-induced squeezing at mechanical oscillator by the target optomechanical Hamiltonian. In Sec. III, we show that this effect can be simulated optically by the first-order expansion of the evolution operator and compare how close this simulation is to the ideal cases. In Sec. IV we propose a feasible experimental scheme, and in Sec. V we conclude.

II. NOISE-INDUCED SQUEEZING

Our main interest lies in operations that allow for coherent nonclassical effects (squeezing, in particular) induced by incoherent quantum states, such as a thermal light and a mixture of coherent states. At the same time, we are only interested in operations we can simulate experimentally. The traditional optomechanical interaction Hamiltonian $\hat{H}_{\rm trad} = \hbar\kappa_1 \hat{n}_L \hat{X}_M$, where the subscripts L and M stand for optical and mechanical modes, is not considered as it does not lead to the desired squeezing of the mechanical mode. We set $\hbar = 1$ for simplicity below. The interaction Hamiltonian found in the membrane-in-the-middle configuration with $\hat{H}_{\rm mem} = \hbar\kappa_2 \hat{n}_L \hat{X}_M^2$ does lead to the desired effect, but simulating it is not feasible as the number operator $\hat{n}_L = (\hat{X}_L^2 + \hat{P}_L^2 - 1)/2$ contains both of the quadrature operators and is therefore incompatible with the techniques of Refs. [18,19]. Note that

[*]park@optics.upol.cz

$\kappa_{1,2}$ are influenced by actual experimental parameters such as the cavity decay rate and strength of optomechanical coupling [11–15]. The dependence on number operator can be removed by using a light with high intensity which leads to the linearized interaction with $\hat{H} \propto \hat{X}_L \hat{X}_M^2$, but this linearization also removes the possibility of the operation being driven by an optical noise.

We can instead consider a coupling described by the interaction Hamiltonian

$$\hat{H}_{\text{target}} = \chi \hat{X}_L^2 \hat{X}_M^2, \tag{1}$$

where the strength of the coupling will be set as $\chi = 1$ for the simplicity of description. This Hamiltonian is only a function of position operators and thus can be simulated efficiently. The coupling itself has a coherent phase reference for the optical mode, but if we assume the state of light to be incoherent and phase insensitive, the interaction behaves in a qualitatively identical way to the naturally occurring optomechanical coupling. Furthermore, in the weak-interaction regime with interaction time $t \ll 1$, with which we will be mainly concerned during the simulation, the two operations are exactly identical. The defining feature of operation (1) is the ability to squeeze the mechanical mode. A straightforward way to see this effect is to apply the evolution operator to the joint optomechanical system in a separable input state $\rho^{\text{in}} = \rho_L^{\text{in}} \otimes \rho_M^{\text{in}}$ followed by a measurement in position quadrature X_L in light mode. For a measured value $x_L = q$, the output state in mechanical mode is $\rho_M^{\text{out}} = e^{-itq^2\hat{X}_M^2} \rho_M^{\text{in}} e^{itq^2\hat{X}_M^2}$, irrespectively of the initial state of the light mode (which determines only the probability of success).

We do not need to rely on the measurement to achieve squeezing, and the state of the mechanical mode can be squeezed even without any measurement on the light, as the squeezing is an inherent part of the operation represented by the square of the respective quadrature operator. In this case, the nature and the amount of the squeezing is determined by the initial state of the optical mode. The reduced state of the mechanical mode is then given as

$$\rho_M^{\text{out}} = \iint dq dq' e^{-itq^2\hat{X}_M^2} \rho_M e^{itq'^2\hat{X}_M^2} \rho_L(q,q'), \tag{2}$$

where $\rho_L(q,q') = \langle q|\rho_L^{\text{in}}|q'\rangle$ denotes the reduced density matrix of the optical mode in the position representation. It is immediately apparent that a squeezing operation (accompanied by possible rotations in phase space) is implemented when the light is in a position eigenstate $\rho_L^{\text{in}} = |q''\rangle_L\langle q''|$ which reduces the output state to the previous one. For an arbitrary input state of light, the situation is slightly more complex to analyze.

The squeezing effect is seen in the covariance matrix of the output state in the mechanical mode. The matrix elements can be obtained as $\Sigma_{ij} = (\langle \xi_i\xi_j + \xi_j\xi_i\rangle)/2 - \langle\xi_i\rangle\langle\xi_j\rangle$, where $\xi = (\hat{X}_M, \hat{P}_M)$ represents the vector of quadrature operators. An easy way to analyze this squeezing effect is by working in Heisenberg representation where the operators are expressed by transformation relations

$$\begin{aligned} &\hat{X}_L \to \hat{X}_L, && \hat{X}_M \to \hat{X}_M, \\ &\hat{P}_L \to \hat{P}_L + 2t\hat{X}_L\hat{X}_M^2, && \hat{P}_M \to \hat{P}_M + 2t\hat{X}_M\hat{X}_L^2. \end{aligned} \tag{3}$$

In terms of moments of the initial state, the covariance matrix elements are therefore expressed as

$$\begin{aligned} \Sigma_{11} &= \text{var}(\hat{X}_M), \\ \Sigma_{12} &= \text{cov}(\hat{X}_M,\hat{P}_M) + 2t\langle\hat{X}_L^2\rangle\text{var}(\hat{X}_M), \\ \Sigma_{22} &= \text{var}(\hat{P}_M) + 2t\langle\hat{X}_L^2\rangle\text{cov}(\hat{X}_M,\hat{P}_M) \\ &\quad + 4t^2\langle\hat{X}_L^2\rangle^2\text{var}(\hat{X}_M) + 4t^2\text{var}(\hat{X}_L^2)\langle\hat{X}_M^2\rangle, \end{aligned} \tag{4}$$

where $\text{var}(\hat{Q}) = \langle\hat{Q}^2\rangle - \langle\hat{Q}\rangle^2$ and $\text{cov}(\hat{X},\hat{P}) = (\langle\hat{X}\hat{P} + \hat{P}\hat{X}\rangle)/2 - \langle\hat{X}\rangle\langle\hat{P}\rangle$ represent the variance and the covariance of the respective quadrature operators. As expected, when the mode of light is in a state with strong coherence (either a high-amplitude coherent state or a displaced quadrature eigenstate), $\langle\hat{X}_L^4\rangle = \langle\hat{X}_L^2\rangle^2 = \langle\hat{X}_L\rangle^4$ and the whole process becomes again a parametric squeezing process with Hamiltonian $H = \langle\hat{X}_L\rangle^2\hat{X}_M^2$ as the last term of Σ_{22} in (4) representing the fluctuations added in the P_M quadrature vanishes. However, a remarkable property of the Hamiltonian (1) is that the squeezing can also appear when the light mode is in a state without a classical phase reference for which $\langle X_L\rangle = 0$. A natural example of such a state is a thermal state, but (4) shows that the optimal state for squeezing would have the smallest added noise for the minimal value of X_L-quadrature excess kurtosis $K = \langle X_L^4\rangle/\langle X_L^2\rangle^2 - 3$, the characterization of flatness of the distribution, for a given energy $\bar{n} = 2\langle X_L^2\rangle - 1$ which depends only on $\text{var}(X_L)$ for states without classical coherence. The minimal value of kurtosis is $K_{\min} = -2$ leading to $\langle X_L^4\rangle = \langle X_L^2\rangle^2$ or $\text{var}(X_L^2) = 0$. Such a state would allow us to achieve a perfect Gaussian squeezing operation without any added noise in P_L quadrature, even though the state of light describes essentially a white noise with a flat distribution. Such states are impossible to prepare for continuous variable systems described by an infinite dimensional Hilbert space. Fortunately, even when the kurtosis is not minimal a low kurtosis still is beneficial. We can see in (4) that the excess kurtosis manifests as additive fluctuations in quadrature P_M and in that quadrature alone.

In the following analysis two kinds of noisy states in the light mode are considered explicitly. The first one is the thermal state of light with a Bose-Einstein photon statistics, whose density matrix in Fock basis is given by $\rho_{\text{th}} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}|n\rangle\langle n|$ with the mean energy $\bar{n}$. It is the prime example of a noisy Gaussian light, and its kurtosis is $K = 0$ for all values of $\bar{n}$. The second kind of state is a classical phase-insensitive mixture of coherent states with a positive mean amplitude α and density matrix

$$\rho_{\text{coh}}[\alpha] = \frac{1}{2\pi}\int_0^{2\pi} d\phi |\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}|. \tag{5}$$

This state has Poissonian photon statistics and its kurtosis is

$$K = \frac{6\bar{n}^2 + 12\bar{n} + 3}{4\bar{n}^2 + 4\bar{n} + 1} - 3, \tag{6}$$

where $\bar{n} = \alpha^2$. This is the lowest kurtosis achievable by a phase-insensitive classical state with a given energy $\bar{n}$. It does not reach the lower bound of -2 but approaches $-3/2$ in the limit of $\bar{n} \to \infty$. See the Appendix for further details.

III. APPROXIMATING THE OPERATION

The ideal unitary operator of $e^{itX_L^2X_M^2}$ is not feasible even with the current state-of-the-art technologies, but with the help of quantum optics we can realize an approximation by a finite expansion similarly as in Refs. [18,19]. This finite approximation is given by a first-order Taylor expansion of the ideal unitary operator or the conditional transformation described by the operator

$$\hat{O} = \hat{1} - it\hat{X}_L^2\hat{X}_M^2. \tag{7}$$

We note that the parameter t is a value chosen to simulate the interaction time. The transformed state is then given by

$$\rho^{\text{out}} = \frac{\hat{O}\rho^{\text{in}}\hat{O}^\dagger}{\text{Tr}[\hat{O}\rho^{\text{in}}\hat{O}^\dagger]}. \tag{8}$$

We again look at the resulting covariance matrix to analyze the squeezing in this state. However, the probabilistic nature of the operation $\hat{O}$ is making the covariance matrix elements' dependence on the initial state highly complex. Therefore, for the sake of simplicity, we shall consider the mechanical mode to be initially in a state with zero mean values of the quadrature operators [20] which include the ground state and the thermal states. The covariance matrix elements in these cases are

$$\begin{aligned}
\Sigma_{11} &= \frac{1}{N}\left[\text{var}(X_M) + t^2\langle X_L^4\rangle\langle X_M^6\rangle\right],\\
\Sigma_{12} &= \frac{1}{N}\left[\text{cov}(X_M,P_M) + 2t\langle X_L^2\rangle\langle X_M^2\rangle\right.\\
&\quad \left. + \frac{t^2}{2}\langle X_L^2\rangle\langle X_M^2(X_MP_M + P_MX_M)X_M^2\rangle\right], \qquad (9)\\
\Sigma_{22} &= \frac{1}{N}\left[\text{var}(P_M) + 2t\langle X_L^2\rangle\text{cov}(X_M,P_M)\right.\\
&\quad \left. + t^2\langle X_L^4\rangle\langle X_M^2P_M^2X_M^2\rangle\right],
\end{aligned}$$

where $N = 1 + t^2\langle X_L^4\rangle\langle X_M^4\rangle$ is the normalization coefficient. We can see that the effect is close to ideal squeezing for the minimal kurtosis state for short interaction time $t \ll 1$, and any excess kurtosis manifests as additive noise in quadratures X_M and P_M.

Let us assess the quality of this finite approximation. The finite nature of the approximation makes it suitable only for states with weak excitation, so we begin with the mechanical mode initially in a vacuum state. The mechanical mode is expected to be approximately in a squeezed state expressed as $|0\rangle + t/2\sqrt{2}|2\rangle$ after the operation $\hat{O}$ with a suitable phase shift $U(\phi) = e^{i(\phi+\pi/2)\hat{n}}$, where t is considered small. For a realistic noisy state of the light mode, the mechanical mode ends up in a mixed squeezed state of the form as in (2), so we need to compare the density matrices in the Fock representation $\rho_{ij} = \langle i|\rho_M^{\text{out}}|j\rangle$. Note that the presence of off-diagonal elements is a necessary condition for squeezing, since any mixture of the Fock states cannot have either off-diagonal term or squeezing. The most confirmative evidence for squeezing in weakly excited states is the presence of off-diagonal terms ρ_{20} and the absence of the terms ρ_{11}. The comparison of the Fock distributions between ideal operation and the approximate one in Fig. 1 shows a good qualitative match and the absence of the all terms with an odd number of photons. For a quantitative assessment, the standardized matrix elements defined as

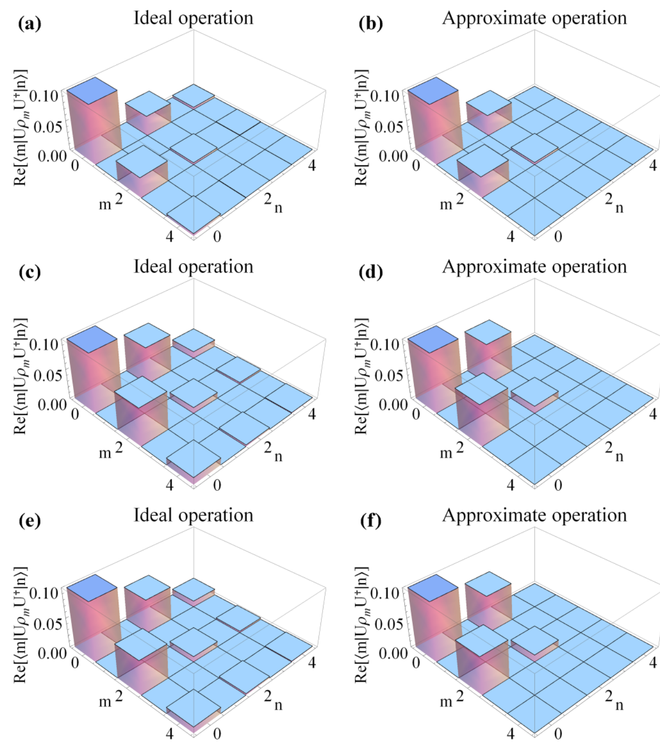

FIG. 1. (Color online) Reduced Fock representation in mechanical mode $\langle m|U(\phi)\rho_MU^\dagger(\phi)|n\rangle$ at $t = 0.1$ after phase rotations $U(\phi) = e^{i(\phi+\pi/2)\hat{n}}$ for [(a) and (b)] vacuum light, [(c) and (d)] phase-randomized coherent light with $\alpha = \sqrt{0.4}$, and [(e) and (f)] thermal light with $\bar{n} = 0.4$. The output states in mechanical mode for [(a), (c), and (e)] ideal and [(b), (d), and (f)] approximate operation are shown to be close to the state $|0\rangle + \frac{it}{2}\hat{X}^2|0\rangle \approx |0\rangle + \frac{it}{2\sqrt{2}}|2\rangle$ for small values of $t \ll 1$. The phase rotation applied after ideal operation is $\phi = 0.717$ and 0.823 after approximate operation for vacuum light and 0.675 and 0.85 for phase-randomized coherent light and thermal light. The output state from vacuum light and from thermal light with $\bar{n} = 0.4$ are close to the squeezed state of $S[r = -0.0501]|0\rangle$ and $S[r = -0.0905]|0\rangle$, respectively. The vacuum components are all suppressed for visualization.

$$R_{20} = \frac{\rho_{20}}{\sqrt{\rho_{00}\rho_{22}}} \tag{10}$$

can be compared which, for both modes initially in the vacuum state, are given as $R_{20}^{\text{id}} = 0.588$ for the ideal interaction and $R_{20}^{\text{ap}} = 0.578$ for the approximate one at $t = 0.1$ with the superscript id(ap) stands for ideal (approximate) case. For light in a thermal state with $\overline{n} = 0.4$ the values are $R_{20}^{\text{id}} = 0.599$ and $R_{20}^{\text{ap}} = 0.580$, while for the light in the subnormal kurtosis mixture of phase randomized coherent states with $\overline{n} = 0.4$ they are $R_{20}^{\text{id}} = 0.637$ and $R_{20}^{\text{ap}} = 0.623$. This result shows a good agreement between the ideal and the approximate case, although a stronger squeezing effect comes from the optimal kurtosis state. The off-diagonal terms for Fock states higher than $|2\rangle$ are missing in approximations, as they can not be simulated by the first-order expansion in (7), but they do not affect the squeezing significantly for small $\bar{n}$.

Another aspect of comparison relies on the phonon structure of the resulting state of the mechanical mode. Since the output states are squeezed, the phonons should exhibit a behavior akin to photon bunching. This bunching can be observed by looking at the second-order autocorrelation function $g^{(2)}(0) = \langle a^{\dagger 2}a^2\rangle/\langle a^\dagger a\rangle^2$. For classical coherent states we have $g^{(2)}(0) = 1$, while classical thermal states exhibit $g^{(2)}(0) = 2$ as long as their energy is nonzero. On the other hand, the nonclassical squeezed states possess an autocorrelation function that can reach a value arbitrarily larger than $g^{(2)}(0) = 2$ when the state is pure enough. For a large squeezing, the $g^{(2)}(0)$ converges to 3 regardless of the thermal noise. In our scenario with the optical and the mechanical mode initially in a vacuum state, the ideal interaction at $t = 0.1$ creates state with $g^{(2)}(0) = 161$, while the approximate one leads to $g^{(2)}(0) = 133$ at the same moment. These high numbers confirm the presence of the bunching effect on phonons as well as the high purity of the state.

The squeezing imparted on the mechanical mode generally grows with the mean energy of the optical mode regardless of the actual input states and interaction time and the dependence on them is quantified as following. The obtained squeezing is best characterized in terms of the minimal and maximal squeezed variances that are denoted by V_- and V_+ defined respectively as

$$V_\mp = \tfrac{1}{2}\left[\Sigma_{11} + \Sigma_{22} \mp \sqrt{(\Sigma_{11} - \Sigma_{22})^2 + 4\Sigma_{12}^2}\right], \tag{11}$$

where the covariance matrix elements are from (4) for the ideal case and from (9) for the approximate one. The explicit form of the least variance in the mechanical mode after an ideal interaction with a thermal light is given as

$$V_-^{\text{th}} = \frac{3t^2(2\bar{n}+1)^2 + 2 - \sqrt{9t^4(2\bar{n}+1)^4 + 4t^2(2\bar{n}+1)^2}}{2} \tag{12}$$

and that after the ideal interaction with a phase-randomized coherent light with help of (4) and (A2) as

$$V_-^{\text{coh}} = \tfrac{1}{4}[3t^2(2\bar{n}^2 + 4\bar{n} + 1) + 2 - \sqrt{9t^4(2\bar{n}^2 + 4\bar{n} + 1)^2 + 4t^2(2\bar{n}+1)^2}]. \tag{13}$$

The explicit forms for the approximate operation are calculated equivalently but are more complex than for the ideal operation. In Fig. 2, these minimal variances are plotted relative to the initial mean energy of the light mode and the interaction time. Generally, both the mean energy $\bar{n}$ and the duration of the interaction t increase the squeezing manifested by the initial decreasing of V_-'s. This noise-driven generation of squeezing is the main result of our simulation, which confirms the presence of the nonlinear effects in the approximative method. The phase-insensitive mixture of coherent states does indeed produce a higher squeezing than the corresponding thermal state with the same number of photons due to the kurtosis effect. The validity of the approximation holds for optical mode energies up to $\bar{n} \approx 1$ for the mechanical mode initially in a vacuum state at a very short interaction time $t = 0.05$. At double the time $t = 0.1$, the approximation holds only up to $\bar{n} \approx 0.2$. The longest time that can be faithfully approximated is around $t \approx 0.15$ when both modes are initially in a vacuum state. The operation still performs a squeezing even when

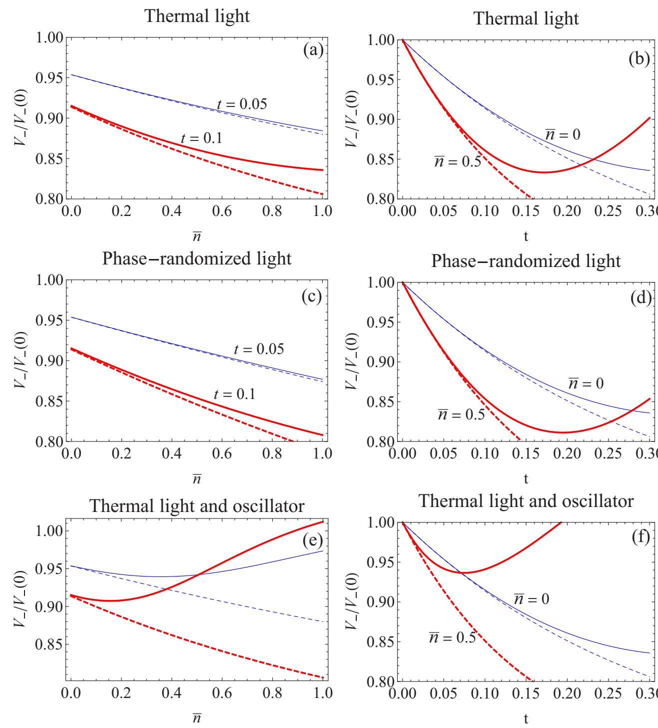

FIG. 2. (Color online) The noise reduction expressed as the ratio of the initial and squeezed minimal variances $V_-/V_-(0)$ relative to the initial mean energy $\bar{n}$ in the light mode [(a), (c) and (d)] and interaction time t [(b), (d), and (f)]. Solid curves follow the approximation, while dashed curves represent the ideal unitary interaction. Each set of lines, differentiated by color, refers to a different parameter, as given explicitly in the figures. In panels (a) and (b) the mechanical mode is initially in a vacuum state, while the optical mode is in a thermal state with mean energy $\bar{n}$. In panels (c) and (d) the mechanical mode is initially in a vacuum state while the optical mode is in a phase-insensitive mixture of coherent states. In panels (e) and (f) both the modes are initially in a thermal state with mean energy $\bar{n}$.

the approximation is not perfect, witnessed by the largest noise reduction of −0.45 dB achieved at $t \approx 0.35$. When both modes are plugged initially with the same thermal states at equilibrium, both the approximation and the squeezing itself break down much more rapidly. For example, the largest value of the mean energy for squeezing below the shot noise is $\bar{n} \approx 0.08$. By comparing Figs. 2(a) and 2(c) for thermal lights and Figs. 2(b) and 2(d) for phase-randomized coherent lights we can also see that amplitude fluctuations in thermal light do not significantly affect the observed squeezing effects.

Now an important question is how large squeezing we may get either ideally or approximately. To answer this question we find the minimal value of the least variances $V_{\min} = \min_t[V_-(t)]$ at any given mean energy $\bar{n}$ of the light mode over the entire domain of t. Figure 3 shows the dependence of $V_{\min}$ on the average energy $\bar{n}$ for both the thermal light and the phase-insensitive mixture of coherent light with the mechanical mode initially in a vacuum state. Immediately we see that the minimal achievable variance for the thermal lights is not dependent on average photon number in input state of light. As a consequence, any thermal state (including the vacuum) can generate at most 1.76 dB of squeezing

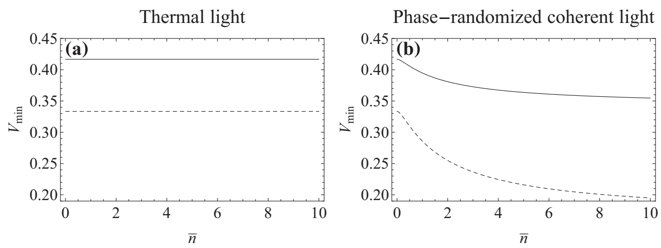

FIG. 3. Optimum least variance $V_{\min}$ relative to t drawn against the average photon number $\bar{n}$ for (a) thermal light and (b) phase-randomized coherent light. Solid lines represent the finite approximation while the dashed lines denote the ideal case. The coherent light can induce a larger squeezing due to a smaller kurtosis.

($V_{\min} = 1/3$). This follows from relations (4) and (9), where t always appears with the same power as $\langle X_L^2\rangle$ or $\langle X_L^4\rangle^{1/2}$ for both ideal and approximate operation, so we can always find the same $V_{\min}$ at the instance when $\bar{n}t$ is large enough. The situation differs for the phase-insensitive mixture of coherent states, which in the limit of infinite energy the least variance approaches the optimal value of 1/6 equivalent to 4.7 dB of squeezing by the ideal operation. The minimum least variance is given as $V_{\min} = \frac{\bar{n}^2+4\bar{n}+1}{6\bar{n}^2+12\bar{n}+3}$ and we reach the aforementioned value of 1/6 in the limit $\bar{n} \to \infty$. Let us stress here that our aim is not to generate large squeezing but to demonstrate the generation of squeezing driven by a noise.

We note that the mechanical squeezing by vacuum states in the optical mode is counterintuitive for the naturally occuring Hamiltonian $H_{\text{mem}} = \hat{n}_L\hat{X}_M^2$. The difference between this natural Hamiltonian and the simulating approximation $H_{\text{target}} = \hat{X}_L^2\hat{X}_M^2$ when the light is in a thermal state is a constant offset term of 1/2 arising from the operator relation $\hat{n}_L = (\hat{X}_L^2 + \hat{P}_L^2 - 1)/2$. This relation can be translated for a thermal light into an relation in averages as $\langle\hat{n}_L\rangle = (\langle\hat{X}_L^2\rangle + \langle\hat{P}_L^2\rangle - 1)/2 = \langle\hat{X}_L^2\rangle - 1/2$. The last offset term 1/2 is the cause of the squeezing by vacuum light for the target Hamiltonian while no squeezing can be expected in the natural optomechanics setup from the vacuum state of light.

The main limiting factor to the achievable squeezing is the impurity of the generated state in mechanical mode arising from an entanglement between the light and the mechanical modes over the course of the interaction. For the initial ground state in mechanical mode, the impurity can be quantified by the deviation of the uncertainty product V_-V_+ from its value of 1/4 for pure Gaussian states or $\Delta = V_-V_+ - 1/4$. We can relate this impurity with the noise term in Σ_{22} [or $\text{var}(X_L^2)$] as follows. From (11) the product of variances are given as the determinant of the covariance matrix $V_-V_+ = \Sigma_{11}\Sigma_{22} - \Sigma_{12}^2$. As the covariance matrix can be decomposed into the parametric squeezing σ_{ij} and a noise c as $\begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{12} & \sigma_{22}+c\end{pmatrix}$, the impurity is given simply as $\Delta = c\sigma_{11}$. Considering σ_{11} is fixed in our case, we can safely say that the variance $c = 2t^2\text{var}(X_L^2)$ is the direct cause of the increase in uncertainty. The impurities relative to the duration of the interaction and the energy of the light mode is shown in Fig. 4, and we notice that it monotonously increases by more input photons and interaction time regardless of the initial states in light mode and the thermal light imposes the noise effect more strongly. Now as $\text{var}(X_L^2) = (1 + 4\bar{n} + 4\bar{n}^2)/2$ for the thermal state and $\text{var}(X_L^2) = (1 + 4\bar{n} + \bar{n}^2)/2$ for the coherent state are related to the kurtosis, the larger noise effect by a thermal state is explained clearly. The impurity of the state is faithfully reflect by the approximation for small parameters $\bar{n} < 0.5$ and $t < 0.1$.

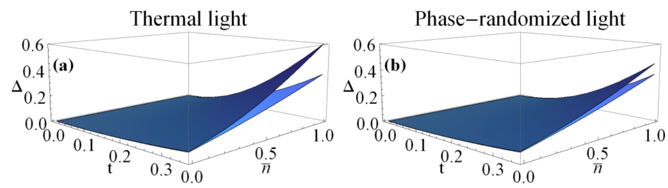

FIG. 4. (Color online) Impurities for (a) thermal light and (b) phase-randomized light. Curves for (white) ideal and (gray) approximate operations are compared. We notice that the thermal light shows a larger impurity due to a larger kurtosis.

IV. EXPERIMENTAL SIMULATION OF THE COUPLING

A practical scheme for experimental implementation is necessary to study the effect of the nonlinear optomechanical coupling in the all-optical regime. The full unitary operation is beyond the capabilities of contemporary state-of-the-art experiments, but the first-order approximation in (7) can be implemented with X gates introduced in Ref. [18]. This method is based on employing nonclassical ancillas together with weakly reflective beam splitters and projective homodyne detections. For example, if the ancillary mode in a superposition of zero and one photon $c_0|0\rangle + c_1|1\rangle$ is mixed at the beam splitters with the signal and then measured by the homodyne detection, a successful measurement heralds realization of an operation $c_0\hat{1} + c_1\hat{X}$.

The nonlinear coupling (7) can be decomposed into a sequence of two mode operations

$$\hat{O} = (1 + (-it)^{1/2}\hat{X}_1\hat{X}_2)(1 - (-it)^{1/2}\hat{X}_1\hat{X}_2). \tag{14}$$

The individual nonlocal operations jointly described by $1 + c\hat{X}_1\hat{X}_2$ can be realized by use of the above-mentioned method when starting from a two-mode ancillary state $|00\rangle + c'|11\rangle$ to implement two local X gates on the two modes of the signal, as illustrated in Fig. 5(a). Realistic beam splitters with transmissivity T implement an operation

$$\exp\left[-(1-T^2)\left(a_1^2 + a_2^2\right)\right] \times T^{\hat{n}_1+\hat{n}_2}(1 + 2c'(1-T^2)/T^2\hat{X}_1\hat{X}_2), \tag{15}$$

which approximates $\hat{O}$ in the limit of $T \to 1$ with a suitable resource state with $c' = \pm(-it)^{1/2}T^2/[2(1-T^2)]$.

We also have a reliable way of preparing the required resource state. One way of preparation employs a commonly used two-mode squeezed vacuum state subject to quantum scissors, but there exists a more experimentally feasible path. We can start from a pair of single photons each generated by spontaneous parametric down-conversion and heralded by a single-photon measurement of the idler as in Fig. 5(b). Each of these single photons is then split on a beam splitter with transmissivity T' and two of the four resulting modes are mixed together on another balanced beam splitter. These interacting modes are then measured by a pair of homodyne detectors and

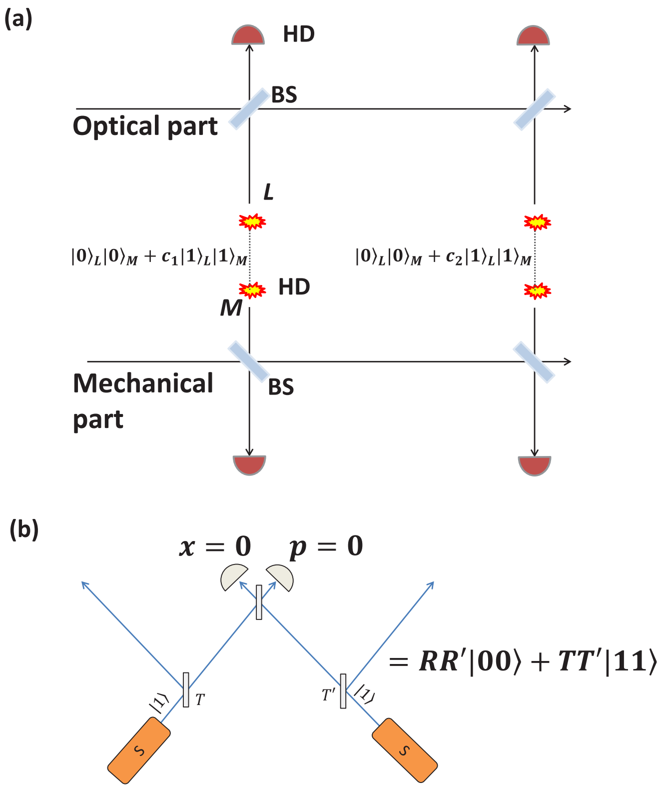

FIG. 5. (Color online) (a) Our scheme for achieving an first-order approximative operation in (7). BS, beam splitter; HD, homodyne detection. (b) A scheme for generating the required two-photon entangled ancillas from two separate single photons.

when these measurements yield values $x_1 = 0$ and $p_2 = 0$, the remaining two modes are prepared in the desired state:

$$\begin{aligned}
&\langle\mu|U_{\mathrm{BS}}(T'|10\rangle + R'|01\rangle)_{12}(R'|10\rangle + T'|01\rangle)_{34}\\
&\quad= \langle\mu|\{T'^2|1\rangle_1(|10\rangle + |01\rangle)_{23}|0\rangle_4 + T'^2|1\rangle_1|00\rangle_{23}|1\rangle_4\\
&\qquad + RT'|0\rangle_1(|10\rangle - |01\rangle)_{23}|1\rangle_4\\
&\qquad + RR'|0\rangle_1(|20\rangle - |02\rangle)_{23}|0\rangle_4\}\\
&\quad\propto T'^2|11\rangle_{14} + R'^2|00\rangle_{14},
\end{aligned} \tag{16}$$

where $\langle\mu| = \langle x=0|_2\langle p=0|_3$ represents the joint measurement and $U_{\mathrm{BS}} = 2^{-\hat{a}^\dagger\hat{a}/2}e^{-\sqrt{2}\hat{b}^\dagger\hat{a}}e^{\sqrt{2}\hat{b}\hat{a}^\dagger}2^{\hat{b}^\dagger\hat{b}/2}$ stands for the unitary operator of the balanced beam splitter.

If single-photon ancillas are not perfect and represented by a mixed state $p|1\rangle\langle 1| + (1-p)|0\rangle\langle 0|$ with a probability $0 \leqslant p \leqslant 1$, the operation performed by this scheme would end up as a mixture of several operations, only one of which corresponding to the perfect single-photon ancillas applies the operation $1 + it\hat{X}_L^2\hat{X}_M^2$, the source of the desired squeezing. We find the minimum value $p_{\min} = 0.95$ above which our scheme achieves a squeezing below shot noise level $V_-(t) \leqslant 0.5$ at some range of parameters t.

V. CONCLUSION

Many interesting nonclassical effects arise due to high-order nonlinear interactions in regimes which are still challenging to reach directly. Quantum optics has been pioneering in testing physical effects at a proof-of-principle level. In this paper we have proposed an all-optical method for simulating a special kind of highly nonlinear optomechanical coupling driven purely by a noise of light. The coupling we have considered is peculiar in that it allows us to generate a strong nonclassical coherent effect, a squeezing, from a noisy optical pump. We have analyzed the effects and the limits both for the ideal operation and the feasible finite approximation that can be readily realized. This work is the first step towards understanding nonlinear optomechanical couplings that go well beyond the traditional linearized model.

ACKNOWLEDGMENTS

We acknowledge financial support from Grant No. GA14-36681G of the Czech Science Foundation. R.F. acknowledges the support of EU FP7 BRISQ2 (Grant No. 308803) co-financed by Grant No. 7E13032 of Czech Ministry of Education, Youth and Sports.

APPENDIX: OPTIMAL KURTOSIS STATES

In the following we are going to search for a completely noisy quantum state that has the optimal possible value of kurtosis for its energy. By completely noisy quantum state we consider a state that is classical but without a determined phase. Such a state can always be expressed as a classical mixture of phase-randomized coherent states:

$$\frac{1}{2\pi}\sum_k c_k\int_0^{2\pi} d\phi|\alpha_k e^{i\phi}\rangle\langle\alpha_k e^{i\phi}|, \tag{A1}$$

where c_k are probabilities for which $\sum_k c_k = 1$ and $|\alpha_k\rangle$ denotes coherent states with discrete amplitudes α_k's. We have chosen the discrete mixture for the sake of clarity, but the argument can be straightforwardly generalized for a continuous mixture. The second and fourth quadrature moments for state (A1) can be found as

$$\begin{aligned}
\langle X^2\rangle &= \frac{2\sum_k c_k|\alpha_k|^2 + 1}{2},\\
\langle X^4\rangle &= \frac{6\sum_k c_k|\alpha_k|^4 + 12\sum_k c_k|\alpha_k|^2 + 3}{4}.
\end{aligned} \tag{A2}$$

Therefore we can minimize the kurtosis by minimizing the term $\sum c_k|\alpha_k|^4$ while keeping the energy $E = \sum_k c_k|\alpha_k|^2$ fixed. If we take values $|\alpha_k|^2$ as belonging to a random variable Y with probabilities given by the coefficients c_k, then we can express the term we are interested in as

$$\sum_k c_k|\alpha_k|^4 = \langle Y^2\rangle = \langle Y\rangle^2 + \mathrm{var}(Y). \tag{A3}$$

From here it can be easily seen that the kurtosis is minimized when the mixture (A1) is composed only of states with the same amplitude, because then and only then will the term $\mathrm{var}(Y)$ be zero. The optimal kurtosis is thus obtained for a phase-randomized mixture of coherent states of the form $\rho_L = \int_0^{2\pi} d\phi|\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}|$ with an amplitude $\alpha^2 = \bar{n}$. For this state, the kurtosis is

$$K = \frac{6\alpha^4 + 12\alpha^2 + 3}{4\alpha^4 + 4\alpha^2 + 1} - 3. \tag{A4}$$

[1] G. S. Agrawal, *Quantum Optics* (Cambridge University Press, Cambridge, 2013).
[2] S. Haroche, Rev. Mod. Phys. **85**, 1083 (2013).
[3] D. J. Wineland, Rev. Mod. Phys. **85**, 1103 (2013).
[4] K. De Greve, L. Yu, P. L. McMahon, J. S. Pelc, Ch. M. Natarajan, N. Y. Kim, E. Abe, S. Maier, Ch. Schneider, M. Kamp, S. Höfling, R. H. Hadfield, A. Forchel, M. M. Fejer, and Y. Yamamoto, Nature **491**, 421 (2012).
[5] W. B. Gao, P. Fallahi, E. Togan, J. Miguel-Sanchez, and A. Imamoglu, Nature **491**, 426 (2012).
[6] W. Pfaff, B. Hensen, H. Bernien, S. B. van Dam, M. S. Blok, T. H. Taminiau, M. J. Tiggelman, R. N. Schouten, M. Markham, D. J. Twitchen, and R. Hanson, Science **345**, 532 (2014).
[7] K. Hammerer, A. S. Sörensen, and E. S. Polzik, Rev. Mod. Phys. **82**, 1041 (2010).
[8] *Cavity Optomechanics*, edited by M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt (Springer-Verlag, Berlin, 2014).
[9] H. J. Kimble, Nature **453**, 1023 (2008).
[10] S. Haroche and J.-M. Raimond, *Exploring the Quantum: Atoms, Cavities, and Photons* (Oxford University Press, New York, 2006).
[11] A. M. Jayich, J. C. Sankey, B. M. Zwickl, C. Yang, J. D. Thompson, S. M. Girvin, A. A. Clerk, F. Marquardt, and J. G. E. Harris, New J. Phys. **10**, 095008 (2008).
[12] A. Nunnenkamp, K. Borkje, J. G. E. Harris, and S. M. Girvin, Phys. Rev. A **82**, 021806(R) (2010).
[13] J. C. Sankey, C. Yang, B. M. Zwickl, A. M. Jayich, and J. G. E. Harris, Nat. Phys. **6**, 707 (2010).
[14] J.-Q. Liao and F. Nori, Sci. Rep. **4**, 6302 (2014).
[15] D. Lee, M. Underwood, D. Mason, A. B. Shkarin, S. W. Hoch, and J. G. E. Harris, Nat. Commun. **6**, 6232 (2014).
[16] M. Yukawa, K. Miyata, H. Yonezawa, P. Marek, R. Filip, and A. Furusawa, Phys. Rev. A **88**, 053816 (2013).
[17] M. Yukawa, K. Miyata, T. Mizuta, H. Yonezawa, P. Marek, R. Filip, and A. Furusawa, Opt. Express **21**, 5529 (2013).
[18] K. Park, P. Marek, and R. Filip, Phys. Rev. A **90**, 013804 (2014).
[19] K. Park, P. Marek, and R. Filip, Phys. Rev. A **91**, 033814 (2015).
[20] See the appendix for the full form.