Bright solitons of the nonautonomous cubic-quintic nonlinear Schrödinger equation with sign-reversal nonlinearity

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In New J. Phys. **16**, 053048 (2014), Wang *et al.* have experimentally demonstrated the formation of bright solitons with repulsive nonlinearity and compared their results numerically by making use of the modified nonlinear Schrödinger equation (NLSE) with constant coefficients. We have extended their study by making the coefficients of NLSE time dependent, as they represent more realistic scenarios and are helpful in understanding the physics of the system in a better way. We have then analytically presented the bright soliton solutions with repulsive nonlinearity for the variable coefficient NLSE. These results have been obtained for the small variation of the time-dependent localized cubic and quintic nonlinearities. Additionally, we have discussed other possibilities which may support the formation of bright solitons in yttrium iron garnet thin film strips.

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I. INTRODUCTION

The term *soliton* refers to a nonlinear pulse or wave packet which travels without spreading. There are two types of envelope solitons, called dark and bright solitons, which propagate in the nonlinear dispersive media. The fundamental cause of their generation is modulation instability (MI). The basic soliton formation is described in nonlinear systems by various nonlinear evolution equations and one-dimensional cubic nonlinear Schrödinger equation (NLSE) is one of those fundamental equations.

The nature of the soliton is governed by the two factors, namely, the dispersion D and the nonlinear N parameters. The D signifies the curvature of the frequency versus wave number dispersion and N represents the change in the carrier frequency with signal amplitude. In general the sign of the product of the dispersion and the nonlinearity parameter determines which kind of soliton is exhibited by the nonlinear medium. It has been shown that when $DN < 0$, one has attractive nonlinearity and yields the formation of bright soliton, while for $DN > 0$, one has repulsive nonlinearity and admits dark soliton formation in a nonlinear dispersive media [1,2]. The concept of cubic NLSE has been extended as the terms such as self-steepening, self-frequency shift, Raman scattering, potential, and higher order dispersion terms have been added to it in order to describe the pulse propagation in various nonlinear systems, as in optical fibers [3], Bose-Einstein condensates (BECs) [4], negative-index materials [5], etc. These nonlinear equations are referred as modified NLSE or sometimes higher order nonlinear Schrödinger equation (HNLSE). In Ref. [6] the MI of HNLSE has been studied, and interestingly, the existence of MI has been demonstrated in the normal group velocity dispersion (GVD) regime in the presence of non-Kerr quintic nonlinearity. As discussed, with MI being the precursor of soliton generation, the analysis done in Ref. [6] indicates the possibility of observing solitons in the normal GVD regime, contrary to their usual observation in anomalous GVD regime. Hong obtained the optical solitary wave solutions for HNLSE by including non-Kerr terms [7]. Additionally, the band gap structure and the properties of the lattice solitons in the presence of periodic potential have been investigated by employing HNLSE [8]. Besides the discussed HNLSE in one dimension (1D), the coupled NLSE equations including cubic-quintic nonlinearity have been thoroughly explored in various contexts. In Ref. [9] the integrability aspect of the coupled cubic-quintic NLSE has been studied and the role of quintic nonlinearity on the ultra-short-pulse propagation in a non-Kerr media has been revealed. Qi *et al.* have presented the soliton interactions for the coupled system that governs the pulse propagation in twin-core nonlinear optical fibers and wave guides by the applying Darboux transformation [10]. Very recently, the coupled cubic-quintic NLSE model in two dimensions (2D) has been analyzed to understand the drag forces in fluids and in BECs [11]. To describe more realistic phenomena, the inhomogeneities present in the nonlinear systems have also been considered and the governing model equation becomes variable-coefficient NLSE (vcNLSE) and is also known as generalized NLSE (GNLSE). A great deal of research took place on vcNLSE and GNLSE in different fields. Serkin *et al.* have thoroughly explored the NLSE with time-dependent coefficients and revealed the interesting dynamical properties of solitons in BECs [12–14]. Additionally, vcNLSE has also been employed to study the controllable behavior of breathers and rogue waves in BECs [15–20]. The variable coefficient NLSE has not only been used in BECs but has also been exploited in the context of nonlinear optical fibers and wave guides [21–30].

Recently, there has been a renewed interest in studying the solitons in magnetic yttrium iron garnet (YIG) film strips. A number of experimental studies took place in this regard and they have successfully demonstrated the emergence of solitons in YIG films [31–35]. They have been reported for various magnetic field and propagation combinations such as surface wave [36], forward-volume [37], and backward-volume wave [31] configurations. In addition to experimental works, analytical solutions [38] and numerical simulations have also supported the occurrence of solitons in YIG films [33,35,39]. The occurrence of the dark solitons in YIG films has been observed with repulsive [31] and attractive [33] nonlinearity.

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The existence of bright solitons with attractive nonlinearity in YIG films was shown in 2005 [33]. Very recently, Wang *et al.* have demonstrated the formation of bright solitons with repulsive nonlinearity [39]. They have reproduced the experimental results through numerical simulations by using constant-coefficient modified NLSE. We are extending it and are considering modified NLSE with variable coefficients as it may be the starting point for a more realistic system, mentioned above. The modified NLSE with time-dependent coefficients is given as

$$i\psi_t - \frac{D(t)}{2}\psi_{xx} + N(t)|\psi|^2\psi + S(t)|\psi|^4\psi + i v_g(t)\psi_x + iG(t)\psi = 0, \quad (1)$$

where x and t are the normalized dimensionless variables. Parameter $D(t)$ denotes the dispersion coefficient, and $N(t)$ and $S(t)$ represent the nonlinear cubic and quintic interactions. The term proportional to $v_g(t)$ is associated with the group velocity and $G(t)$ describes the gain or loss term. The variants of Eq. (1) have been employed to study the pulse propagation in different nonlinear media [12,40–42]. In the absence of a gain term, the model has been used to study the optical solitons for the different types of nonlinearity such as parabolic, dual power, etc. [43]. On replacing the variable t with z and for $v_g = 0$ Eq. (1) has been applied to investigate the propagation dynamics of chirped and chirp-free self-similar solitary waves in a soliton control system [44]. For $S(t) = v_g(t) = 0$, the model equation has been exploited to understand the dynamics of topological optical solitons [45,46]. The aim of this paper is to present the exact bright soliton solution for the vcNLSE given by Eq. (1) for the repulsive cubic nonlinearity. The exact analytical solutions will be obtained by using a technique that involves the mapping of the variable coefficient modified NLSE to the constant coefficient ϕ^6 field equation through the appropriate choice of ansatz. Similar methodology was proposed by Serkin *et al.* to find a number of the novel stable soliton management regimes for the variable coefficient cubic-quintic NLSE [47].

The paper is organized as follows: In Sec. II we discuss the methodology that has been employed to get soliton solution. In Sec. III we present the exact form of analytical solutions and reveal the role of various parameters on the soliton profile. Concluding remarks and the future applications are given in Sec. IV.

II. METHODOLOGY

We are interested to find the exact bright soliton solution for Eq. (1). To do so we are choosing the ansatz

$$\psi(x,t) = \rho(t)\exp[i\theta(x,t)]\phi[\eta(x,t)] \quad (2)$$

with the phase $\theta(x,t)$ as

$$\theta(x,t) = bx + c(t), \quad (3)$$

where b is an arbitrary constant. The $\rho(t)$, $\theta(x,t)$, $\phi[\eta(x,t)]$, and $c(t)$ are real functions.

On substituting Eq. (2) along with Eq. (3) in Eq. (1), it reduces to

$$\phi_{\eta\eta} + \delta\phi + \beta\phi^3 + \gamma\phi^5 = 0 \quad (4)$$

for the following conditions:

$$\eta(x,t) = k_1 x + k_2(t), \quad (5)$$

$$N(t) = -\frac{\beta D(t)k_1^2}{2\rho^2}, \quad (6)$$

$$S(t) = -\frac{\gamma D(t)k_1^2}{2\rho^4}, \quad (7)$$

$$G(t) = -\frac{\rho_t}{\rho}, \quad (8)$$

$$v_g(t) = -\frac{k_{2t}}{k_1} - D(t)b, \quad (9)$$

$$c_t = \frac{\delta D(t)k_1^2}{2} + \frac{1}{2}Db^2 - \frac{k_{2t}}{k_1}b. \quad (10)$$

δ, β, and γ are arbitrary constants while k_1 and $k_2(t)$ are associated with the width of the localized solution and its center of mass location. Equation (1) admits a class of solutions depending upon the nature of the cubic ($N(t)$) and quintic ($S(t)$) nonlinearities. Since Eq. (1) has been reduced to Eq. (4) for the set of conditions given by Eqs. (5)–(10) and Eq. (4) can be mapped onto ϕ^6 field equation, which is well known to admit bright soliton, dark soliton, kink, double kink, and bell-shaped solutions [48–50], one can obtain a variety of solutions for Eq. (1). Being motivated by the experimental evidence on the existence of bright solitons with repulsive nonlinearity in YIG thin films [39] and in Bose-Einstein condensate [51], in this paper we analytically obtain the bright soliton solutions with repulsive nonlinearity.

III. EXACT ANALYTICAL SOLUTIONS

A. Bright soliton solution with repulsive cubic nonlinearity and attractive quintic nonlinearity

As discussed, in order to obtain the analytical solutions for Eq. (1), we have mapped it to Eq. (4) and introduced the three arbitrary constants β, γ, and δ. It is clear from Eqs. (6) and (7) that β and γ are associated with the cubic and quintic nonlinearities, respectively. In this work we are looking for bright solitons with repulsive cubic nonlinearity and attractive quintic nonlinearity so it automatically fixes their sign as $\beta < 0$ and $\gamma > 0$ for the negative sign of dispersion. δ can be positive or negative. Now we present the bright soliton solutions corresponding to the cases $\delta > 0$, $\delta < 0$, and $\delta = 0$.

Case I. For $\delta > 0$, Eq. (1) admits bright soliton of the following form:

$$\psi(x,t) = \rho(t)p\sqrt{1 + \text{sech}[q\eta]}\exp[i\theta(x,t)], \quad (11)$$

where $p^2 = -\frac{8\delta}{5\beta}$, $q^2 = \frac{4\delta}{5}$, and $\gamma = \frac{15\beta^2}{64\delta}$ with $\gamma > 0$ and $\beta < 0$. The phases θ and η are given by Eqs. (3) and (5), respectively. We have plotted the intensity profile of bright soliton and the corresponding gain and nonlinear parameters in Figs. 1 and 2 for the two cases. The two different choices have been made through the different forms of dispersion parameter $D(t)$. Clearly, the variations of the nonlinear terms $N(t)$ and $S(t)$ are very small and exist only in a small time window, while they get saturated at larger times. On comparing Figs. 1 and 2, we infer that under the influence of the same gain $G(t)$ the variation of the nonlinear terms can be controlled or tuned

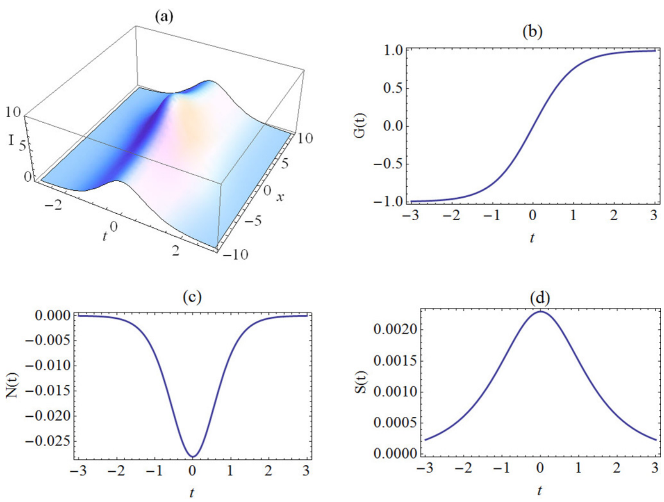

FIG. 1. (Color online) (a) Intensity profile of bright soliton, (b) gain profile, (c) profile of cubic nonlinearity, and (d) profile of quintic nonlinearity. The other parameters are $\beta = -0.35, \delta = 1$, $\rho = \text{sech}[t]$, $D = -\text{sech}^5[t]$, $k_1 = 0.4$, $k_2 = t$, $b = 1$.

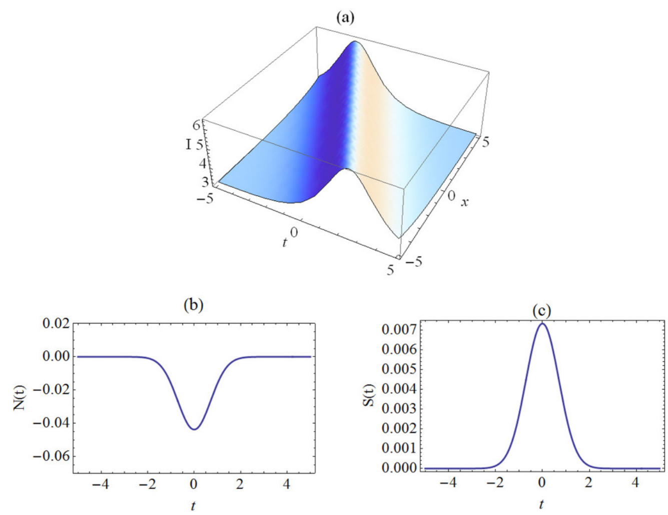

FIG. 3. (Color online) (a) Intensity profile of bright soliton, (b) profile of cubic nonlinearity, and (c) profile of quintic nonlinearity. The other parameters are $\beta = -0.5, \delta = 1$, $\rho = 1$, $D = -\exp[-t^2]$, $k_1 = 0.5$, $k_2 = t$, $b = 1$.

through $D(t)$. Thus one can observe the bright soliton with repulsive nonlinearity by suitably tuning these parameters.

It is worth mentioning that if we choose ρ to be constant, then Eq. (8) reveals that it leads to the vanishing of the gain term. The corresponding intensity plot of the bright soliton and the profiles of the nonlinear terms have been plotted in Fig. 3. We can conclude that the absence of gain term not only decreases the amplitude of the bright soliton but also causes it to appear on a constant background in contrast to Figs. 1 and 2. Moreover, switching off the gain term leads to the modifications of the profiles of nonlinear terms. In addition to ρ, if we make the dispersion term constant, then Eq. (1) will reduce to the NLSE given in Ref. [39] with the gain term absent.

Case II. For $\delta < 0$, Eq. (1) exhibits the other kind of bright soliton which is given as

$$\psi(x,t) = \rho(t)\frac{P\,\text{sech}[Q\eta]}{\sqrt{1 - R\tanh^2[Q\eta]}}\exp[i\theta(x,t)], \qquad (12)$$

where $Q^2 = -\delta$, $P^2 = \frac{2(1+R)\delta}{\beta}$, and $\gamma = \frac{3\beta^2 R}{4\delta(1+R)^2}$ with $\beta < 0$, $\gamma > 0$, and $R < -1$. η is given by Eq. (5) and the phase θ can be obtained by using Eq. (3). The intensity of bright soliton and the profile of the gain and the nonlinear terms are shown in Fig. 4. It is interesting to mention that the solution given in

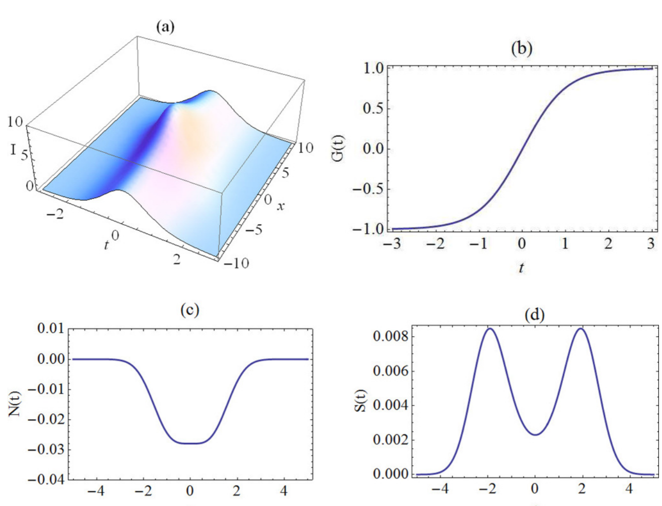

FIG. 2. (Color online) (a) Intensity profile of bright soliton, (b) gain profile, (c) profile of cubic nonlinearity, and (d) profile of quintic nonlinearity. The other parameters are $\beta = -0.35, \delta = 1$, $\rho = \text{sech}[t]$, $D = -\exp[-t^2]$, $k_1 = 0.4$, $k_2 = t$, $b = 1$.

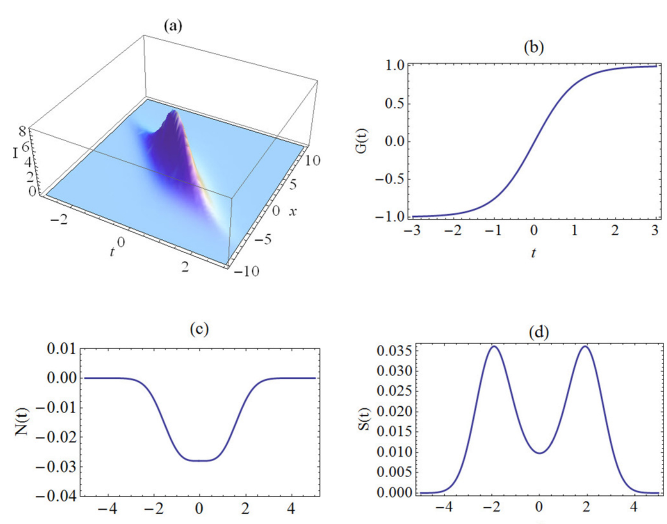

FIG. 4. (Color online) (a) Intensity profile of bright soliton, (b) gain profile, (c) profile of cubic nonlinearity, and (d) profile of quintic nonlinearity. The other parameters are $\beta = -0.35, \delta = -1.5$, $\rho = \text{sech}[t]$, $D = -\exp[-t^2]$, $k_1 = 0.4$, $k_2 = t$, $b = 1$.

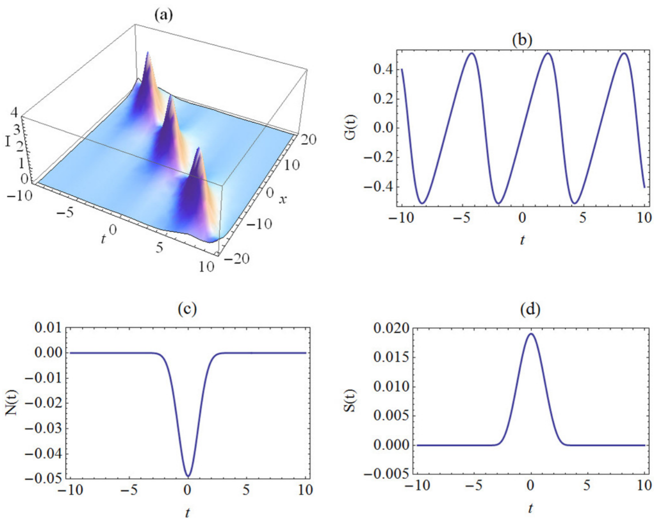

FIG. 5. (Color online) (a) Intensity profile of bright soliton, (b) gain profile, (c) profile of cubic nonlinearity, and (d) profile of quintic nonlinearity. The other parameters are $\beta = -1, \gamma = 1$, $\rho = 1.1 + 0.5\cos[t]$, $D = -\exp[-t^2]$, $k_1 = 0.5$, $k_2 = t$, $b = 1$.

Eq. (12) also exists for the following cases:

$$0 < R < 1, \quad \delta < 0, \quad \beta > 0, \quad \gamma < 0, \tag{13}$$

$$-1 < R < 0, \quad \delta < 0, \quad \beta > 0, \quad \gamma > 0. \tag{14}$$

The set of conditions mentioned in Eq. (13) refers to the case of attractive cubic nonlinearity and repulsive quintic nonlinearity while the conditions given in Eq. (14) depict the case where both nonlinearities are attractive in nature. Thus by suitably choosing the value of R one can have bright solitons for the different combinations of the nonlinear terms. We have discussed the existence of bright solitons for the cases when δ is positive and negative. For the sake of completeness we also report the form of the analytical solution permitted by Eq. (1) when $\delta = 0$.

Case III. For $\delta = 0$, Eq. (1) possesses the following algebraic solution for the repulsive cubic nonlinearity and attractive quintic nonlinearity

$$\psi(x,t) = \rho(t)\frac{1}{\sqrt{A + B\eta^2}}\exp[i\theta(x,t)], \tag{15}$$

where $A = -\frac{2\gamma}{3\beta}$, $B = -\frac{\beta}{2}$, with $\gamma > 0$ and $\beta < 0$. η is given by Eq. (5). The exact form of the phase θ can be worked out by solving Eq. (3). The intensity profile and the other parameters like $G(t)$, $N(t)$, and $S(t)$ have been plotted in Fig. 5. The profile of the nonlinear terms is similar to that in Fig. 1 but instead of a single soliton [Fig. 1(a)] we get periodic emergence of solitons due to the presence of periodic gain.

B. Bright soliton in the absence of cubic nonlinearity

If we exclude the cubic nonlinearity term $N(t)$ by putting $\beta = 0$, then Eq. (1) possesses the bright soliton solution of the form

$$\psi(x,t) = \rho(t)p_1\sqrt{\text{sech}[q_1\eta]}\exp[i\theta(x,t)], \tag{16}$$

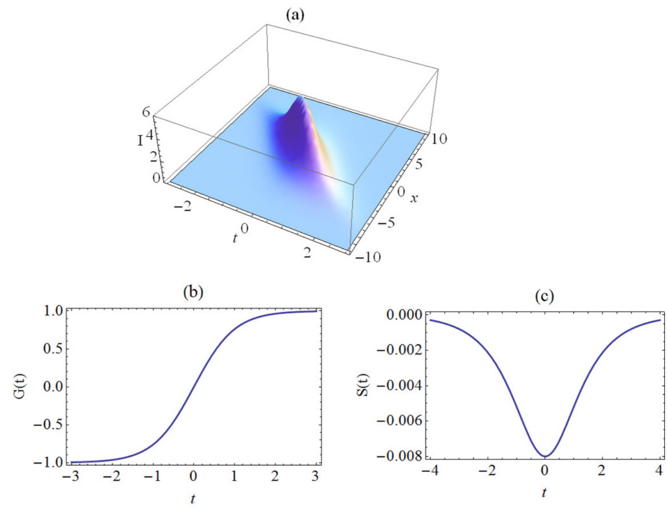

FIG. 6. (Color online) (a) Intensity profile of bright soliton, (b) gain profile, and (c) profile of quintic nonlinearity. The other parameters are $\gamma = 0.1, \delta = -1$, $\rho = \text{sech}[t]$, $D = \text{sech}^5[t]$, $k_1 = 0.4$, $k_2 = t$, $b = 1$.

where $p_1 = (-\frac{3\delta}{\gamma})^{1/4}$, $q_1^2 = -4\delta$, with $\delta < 0$ and $\gamma > 0$. η can be obtained from Eq. (5). The exact form of the phases θ and η can be worked out by using Eqs. (3) and (5), respectively.

In Fig. 6(a) we have plotted the intensity of bright soliton. The profiles of the gain and the quintic nonlinearity have been shown in Figs. 6(b) and 6(c), respectively. Clearly, here the sign of the dispersion term $D(t)$ is positive and the variation of quintic nonlinearity $S(t)$ is negative, which reflects the case of attractive nonlinearity. So here we have demonstrated that the bright soliton can exist even in the absence of cubic nonlinearity, provided the system is under the influence of attractive quintic nonlinearity.

C. Bright soliton in the absence of quintic nonlinearity

If we eliminate the quintic term by putting $\gamma = 0$ Eq. (1) still supports the bright soliton solution of the form

$$\psi(x,t) = \rho(t)p_2\text{sech}[\sqrt{q_2}\eta]\exp[i\theta(x,t)], \tag{17}$$

where $p_2 = \sqrt{\frac{-2\delta}{\beta}}$, $q_2 = \sqrt{-\delta}$ with $\delta < 0$ and $\beta > 0$. Here, η is given by Eq. (5). The exact form of the phase θ can be worked out by using Eq. (3). The intensity of the bright soliton is shown in Fig. 7(a). The gain and the nonlinearity parameter are drawn in Figs. 7(b) and 7(c), respectively. We have found that the periodic gain yields the periodic occurrence of bright solitons on a constant background. Again, the existence of the bright soliton in the absence of quintic nonlinearity has been supported by attractive cubic nonlinearity.

We have found that if one of the nonlinear terms, either cubic or quintic, is switched off, then the bright soliton exists only for attractive nonlinearity. On the other hand, one can observe the bright soliton with repulsive nonlinearity only if both nonlinear terms, cubic and quintic, are simultaneously present. This is happening because the quintic term $(S(t)|\psi|^4\psi)$ overcomes the cubic term $(N(t)|\psi|^2\psi)$, which in turn leads to a repulsive to attractive nonlinear transition

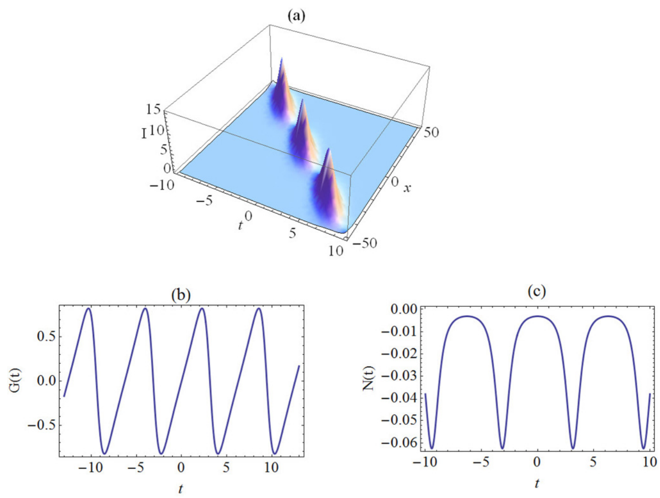

FIG. 7. (Color online) (a) Intensity profile of bright soliton, (b) gain profile, and (c) profile of cubic nonlinearity. The other parameters are $\beta = 0.5, \delta = -1$, $\rho = 1.1 + 0.7\cos[t]$, $D = 1$, $k_1 = 0.2$, $k_2 = t$, $b = 1$.

and the formation of a bright soliton. As the variation of the coefficients is very small, our analytical results support the formation of bright solitons with repulsive nonlinearity in magnetic thin films [39]. Additionally, we have also predicted the other possible combinations of the cubic and quintic nonlinear terms that can support bright solitons.

IV. CONCLUSION

We have shown the existence of bright solitons for the variable coefficient modified NLSE. The exact solutions have been obtained by choosing an ansatz which maps the vcNLSE to the ϕ^6 field equation for certain conditions among the equation parameters. The explicit forms of the solutions have been presented by borrowing the solutions of the ϕ^6 field equation and inserting them in the chosen ansatz. In principle, our results are valid for any well-defined functional form of the distributive parameters and their specific forms can be chosen depending upon the system under study. In this work we have considered the localized parameters whose variation with time is very small so that they can be mapped to study the solitons in YIG thin films. Our analytical results support the formation of bright solitons with repulsive nonlinearity, which has recently been demonstrated experimentally in Ref. [39].

We have analytically revealed the role of nonlinear parameters on the formation of bright solitons in YIG thin films and their occurrence in the same have already been supported experimentally [39]. The next possibility is to study the applications of solitons in optical nonlinear logic gates in YIG thin films. Recently, a number of experimental works have shown utility of spatial solitons in constructing optical logic gates in different nonlinear media such as in azobenzene liquid crystalline cells [52], nematic liquid crystals [53], ^{85}Rb vapors [54], etc. Additionally, in Ref. [55] the existence of solitons has been demonstrated by taking into account the fifth-seventh (focusing-defocusing) refractive nonlinearities. The applications of solitons in logic gates have not only been studied experimentally but also have been explored numerically by using NLSE [56–59]. In order to discuss the soliton logic gates in YIG films analytically, we need to consider the soliton interactions. To incorporate that we require two-soliton solutions for Eq. (1). This work is currently in progress.

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