One-pion-exchange effect in the energy spectrum of muonic hydrogen

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In this work, the effects from one-pion exchange in ep and μp bound states by two-photon coupling are discussed. We at first calculate the effective couplings of a pion with a lepton (electron or muon) by two-photon coupling and the corresponding effective nonrelativistic potential. Then we calculate its corrections to the hyperfine structure of $2S$ and $2P$ states. We find that the corrections to the hyperfine structures of electronic hydrogen's $2S$ and $2P$ states and muonic hydrogen's $2P$ state are small and can be neglected, while the correction to the hyperfine structure of muonic hydrogen's $2S$ state $\Delta E^{2S}_{\rm HFS}(F=1,\mu p)$ is about 0.0028 meV. And after some further discussion we suggest that the similar exchange of a scalar meson such as σ between μp by two-photon coupling may give a much larger contribution to the Lamb shift of muonic hydrogen.

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I. INTRODUCTION

The precise measurement of the Lamb shift in muonic hydrogen was first reported in [1], which gave the proton size $r_p = 0.8418$ fm, and this result was confirmed by later measurement [2], while CODAE2010 [3] gave $r_p = 0.878$ fm from the ep scattering data and electronic hydrogen data. If the latter value is taken to estimate the Lamb shift of muonic hydrogen, there is a discrepancy of about 0.316 meV between the theoretical and experimental result [4]. Due to this discrepancy, many new theoretical discussions and calculation on the energy spectrum of muonic hydrogen are given in the literature [5,6].

In this paper, we consider a new correction due to one-pion exchange between the electron or muon and the proton by two-photon coupling, which is shown in Fig. 1(a). Such effects are usually neglected in the literature since the exchange of a pion results in a short distance potential due to its mass (compared with the size of electronic hydrogen), which is not important to the energy spectrum of electronic hydrogen. For muonic hydrogen, the muon is much closer to the proton than the electron is in the electronic hydrogen, which means that the energy spectrum of muonic hydrogen is more sensitive to a short distance potential than the electronic hydrogen case. Since the masses of the muon and pion are of the same order, physically it is natural to expect that the effects from pion exchange by two-photon coupling may play a role in muonic hydrogen. In the literature, similar effects from two-pion exchange between muon and proton are discussed in the frame of chiral perturbative theory [6].

II. BASIC FORMULA

The effective coupling of π^0 with 2γ can be written as

$$L^{\rm int}_{\pi^0\gamma\gamma} = g_{\pi^0\gamma\gamma}\epsilon^{\mu\nu\rho\omega}\phi_{\pi^0}F_{\mu\nu}F_{\rho\omega}, \tag{1}$$

where $\epsilon^{\mu\nu\rho\omega}$ is the four-dimensional Levi-Civita antisymmetric tensor with $\epsilon^{0123}=1$ and the corresponding vertex is written as

$$\Gamma^{\mu\rho}_{\pi^0\gamma\gamma} = -4ig_{\pi^0\gamma\gamma}\epsilon^{\mu\nu\rho\omega}q_{1\nu}q_{2\omega}, \tag{2}$$

where q_1 and q_2 are the incoming momenta of the two photons with the Lorentz indexes μ and ρ, respectively.

With this coupling, the decay width of π^0 to 2γ is expressed as

$$\Gamma^{th}_{\pi^0\to2\gamma} = \frac{M_\pi^3}{\pi}g^2_{\pi^0\gamma\gamma}, \tag{3}$$

with M_π the mass of π^0. Combining this theoretical expression with the experimental data $\Gamma^{\rm ex}_{\pi^0\to2\gamma} = 7.7$ eV, we can get $g_{\pi^0\gamma\gamma} = 3.14\times10^{-3}$ GeV^{-1}, where the choice of the sign of the coupling is consistent with the calculation of chiral anomaly.

When the two photons are off-shell, the form factors are needed to describe the behavior. Here we add the following form factors to the vertex to describe the behavior as [7]

$$\Gamma^{\mu\rho,\rm full}_{\pi^0\gamma\gamma} = -4ig_{\pi^0\gamma\gamma}\epsilon^{\mu\nu\rho\omega}q_{1\nu}q_{2\omega}\frac{-\Lambda^2}{q_1^2-\Lambda^2}\frac{-\Lambda^2}{q_2^2-\Lambda^2}. \tag{4}$$

Such a form is similar to the dipole form used in Ref. [8] with similar parameter $\Lambda_{\pi^0} = 0.776$ GeV fitted from the experimental data. In our calculation, we take $\Lambda = 0.77$ GeV as a typical example to show the results and take $\Lambda = 0.6$–1 GeV to show the sensitivity to the parameter Λ.

With this coupling, the amplitude of the lepton part shown in Fig. 1(a) can be calculated directly. And after the integration of the momentum in the loop, the effective vertices between electron or muon and pion can be written as

$$iM_{l\to l\pi^0} = 2\int\frac{d^4k}{(2\pi)^4}\overline{u}(p_3,m_l)(-ie\gamma^{\bar\rho})\frac{i(\not k+M_l)}{k^2-M_l^2+i\epsilon}$$
$$\times(-ie\gamma^{\bar\mu})u(p_1,m_l)\frac{-ig_{\bar\mu\mu}}{q_1^2+i\epsilon}\frac{-ig_{\bar\rho\rho}}{q_2^2+i\epsilon}\Gamma^{\mu\rho,\rm full}_{\pi^0\gamma\gamma}$$

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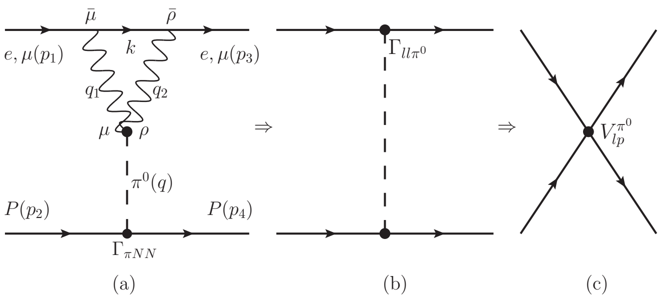

FIG. 1. (a) The one-pion exchange between lepton and proton by two-photon coupling. (b) The effective direct one-pion exchange between lepton and proton. (c) The effective nonrelativistic potential due to the one-pion exchange.

$$\equiv -\frac{f_{ll\pi^0}(Q^2)}{M_\pi}\overline{u}(p_3,m_l)\gamma_5(\not{p}_3-\not{p}_1)u(p_1,m_l)$$
$$\equiv \overline{u}(p_3,m_l)\Gamma_{ll\pi^0}u(p_1,m_l), \quad (5)$$

where the index l refers to the lepton (electron or muon); p_3, k, p_1, q_1, and q_2 are the corresponding momenta shown in Fig. 1(a); $Q^2 \equiv -q^2 \equiv -(p_3-p_1)^2$; and the factor 2 is due to the symmetry between the Feynman diagram shown in Fig. 1(a) and its crossed diagram. With Eq. (5), the effective coupling $\Gamma_{ll\pi^0}$ can be directly used to describe the one-pion-exchange interaction between lepton and proton shown as Fig. 1(b). Since we are interested in the corrections to the energy shifts of muonic hydrogen and electronic hydrogen, the behavior of the couplings $f_{ll\pi^0}(Q^2)$ in the nonrelativistic approximation is important for such corrections, and so we expand the effective couplings at $Q^2=0$:

$$f_{ll\pi^0}(Q^2\approx 0)\approx\left[c_{1,l}+c_{2,l}\frac{Q}{M_\pi}+c_{3,l}\left(\frac{Q}{M_\pi}\right)^2\right]g_{\pi^0\gamma\gamma}. \quad (6)$$

In the practical calculation, the parameters $c_{1,l}$ can be written down analytically (see the Appendix); the parameters $c_{3,l}$ can also be calculated rigorously by expanding the original expression Eq. (5) and the parameters $c_{2,l}$ can be calculated by LoopTools [9] after the integration. The final numerical results for the parameters are presented in Table I and Fig. 2.

From Eq. (6) and Table I, we see that for the ep and μp system we can safely apply the nonrelativistic approximation and neglect the last two terms of $f_{ll\pi^0}$, since the typical energy scales of muonic hydrogen and electronic hydrogen are much smaller than M_π. From Fig. 2 we see that the effective coupling $c_{1,\mu}$ is not very sensitive to the parameter Λ, which means that we can control the uncertainty of the contribution in a good way.

TABLE I. The numerical results for the effective couplings $c_{i,l}$ with $\Lambda = 0.77$ GeV as input.

	$c_{1,l}$ (GeV)	$c_{2,l}$ (GeV)	$c_{3,l}$ (GeV)
$l=\mu$	0.00371	−0.00199	3.27×10^{-6}
$l=e$	0.0136	−0.409	3.21×10^{-6}

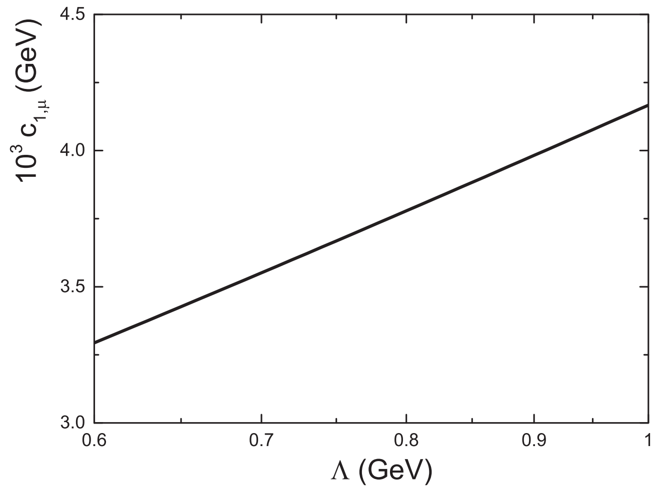

FIG. 2. The dependence of $c_{1,\mu}$ on the parameter Λ.

For the effective interaction of π^0 and proton at low Q^2, we have the following [10]:

$$L_{\pi NN}^{\rm int} = -\frac{f_{\pi NN}}{M_\pi}\overline{\psi}\gamma_5\gamma_\mu\psi\partial^\mu\phi_\pi, \quad (7)$$

with $f_{\pi NN}\approx 1$, and the corresponding effective vertex is written as

$$\Gamma_{\pi NN} = -\frac{f_{\pi NN}}{M_\pi}\gamma_5(\not{p}_f-\not{p}_i), \quad (8)$$

with p_i and p_f the corresponding momenta of the initial and final proton.

Using these effective interactions, we can use the quasipotential method by matching the amplitude from the effective interactions in quantum field theory [shown as Fig. 1(b)] and that from the effective nonrelativistic potential in quantum mechanics [shown as Fig. 1(c)] similar to the case of pion exchange between two nucleons. And we can get the following effective potential in coordinate space:

$$V_{lp}^{\pi^0}(r) = \frac{c_{1,l}g_{\pi^0\gamma\gamma}f_{\pi NN}}{12\pi}\left\{\left[(3\hat{r}\cdot\vec{\sigma}_1\hat{r}\cdot\vec{\sigma}_2-\vec{\sigma}_1\cdot\vec{\sigma}_2)\times\left(1+\frac{3}{M_\pi r}+\frac{3}{M_\pi^2r^2}\right)+\vec{\sigma}_1\cdot\vec{\sigma}_2\right]\times\frac{e^{-M_\pi r}}{r}-\frac{4\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\delta(r)\right\}, \quad (9)$$

where σ_1 and σ_2 are the spin matrices of the lepton and proton.

From Eq. (9), we see that there are only contributions to the hyperfine structure of muonic hydrogen and electronic hydrogen, the energy levels of which are shown in Fig. 3. Using the properties of the Pauli matrix, we have the matrix elements for the spin-related parts of Eq. (9) as shown in Table II.

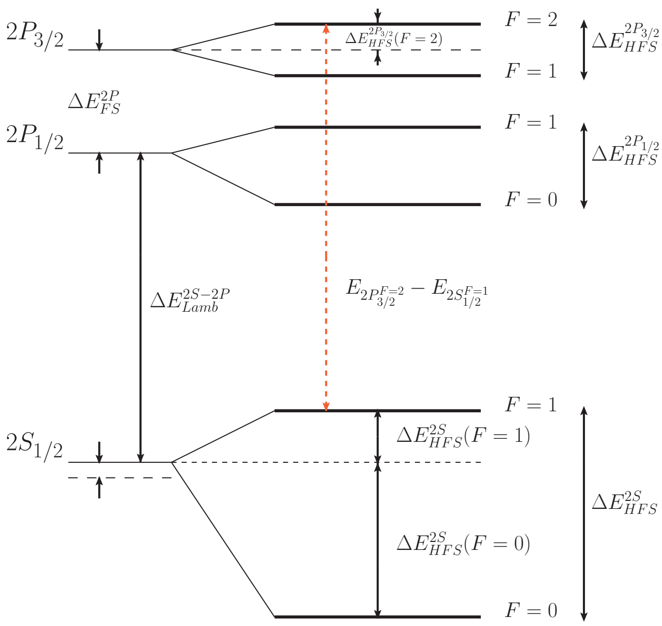

FIG. 3. (Color online) The $2S$ and $2P$ energy levels of muonic hydrogen.

III. NUMERICAL RESULTS, DISCUSSION, AND CONCLUSION

The corrections to the energy spectrum can be calculated directly using perturbation theory and the final numerical results are presented in Table III.

From Table III, we see that the correction from one-pion exchange is about 0.0028 meV for $\Delta E^{2S}_{\rm HFS}(F=1,\mu p)$. The other contributions are very small and can be neglected. In the literature, the full two-photon-exchange correction to the Lamb shift is usually taken as 0.0333 meV [11]. The new contribution of 0.0028 meV is about 8% of this correction and about 0.9% of the current discrepancy (0.316 meV) [4]. At first glance, one may conclude that such a mechanism may give no help to explain the current puzzle of proton size and can be neglected. We should note that the small contribution of 0.0028 meV is mainly due to the small coupling of $\pi^0\gamma\gamma$ due to the chiral anomaly and not only because of the mass of the pion. To show this property, we present the dependence of the correction on M_π in Fig. 4 by fixing the couplings $g_{\pi_0\gamma\gamma}$ and $f_{\pi NN}$. From Fig. 4, we see that the correction at $M_\pi = 500$ MeV is about 10% of that at $M_\pi = 135$ MeV. On the other hand, due to the anomaly of the triangle diagram, we know the couplings of 0^{-+} mesons with 2γ are much smaller than other mesons such as the scalar resonance. For example, in the s channel, the similar coupling of σ with 2γ is much larger than that of π^0 with 2γ [12], and its coupling to the nucleon is also about 13 times larger than $f_{\pi NN}$ [10]. However, the coupling in the s channel may be quite different from the t channel for σ, while naively we can expect the ratio is of a similar order and such enhancement will result in larger corrections even if the mass of a scalar resonance is as large as 500 MeV. The precise and reliable estimation of such a contribution is still far away due to the difficulty of precise estimation of the couplings and its Q^2 dependence in the t channel. In the literature, the possibility to solve the proton puzzle by introducing some new physical particles was discussed [13], while from our calculation we see that by the two-photon coupling the exchange of a meson also gives a new contribution to the energy spectrum of muonic hydrogen and may play a similar role as the introduced new physical particles.

TABLE II. The general relations of hyperfine splitting for the spin-related parts.

	$3(\sigma_v\cdot\hat{\mathbf{r}})(\sigma_p\cdot\hat{\mathbf{r}})$	$\sigma_p\cdot\sigma_v$
$2S_{1/2}^{f=0}$	-3	-3
$2S_{1/2}^{f=1}$	1	1
$2P_{3/2}^{f=1}$	-1	$-\frac{5}{3}$
$2P_{3/2}^{f=2}$	$\frac{3}{5}$	1

TABLE III. The corrections to the hyperfine structure from one-pion-exchange effects.

	$\Delta E^{2S}_{\rm HFS}(F=1,lp)$ (meV)	$\Delta E^{2P_{3/2}}_{\rm HFS}(F=2,lp)$ (meV)
$l=\mu$	0.0028	1.95×10^{-8}
$l=e$	1.61×10^{-9}	3.30×10^{-19}

In summary, we suggest that one-pion exchange in muonic hydrogen with two-photon coupling may give a new correction to the energy spectrum, and find that the contribution to $\Delta E^{2S}_{\rm HFS}(F=1,\mu p)$ is about 0.0028 meV, which is still much too small to explain the proton puzzle. The result hints that a similar mechanism may give enhanced corrections to the energy spectrum of muonic hydrogen, especially the exchange of a scalar resonance such as σ by two-photon coupling.

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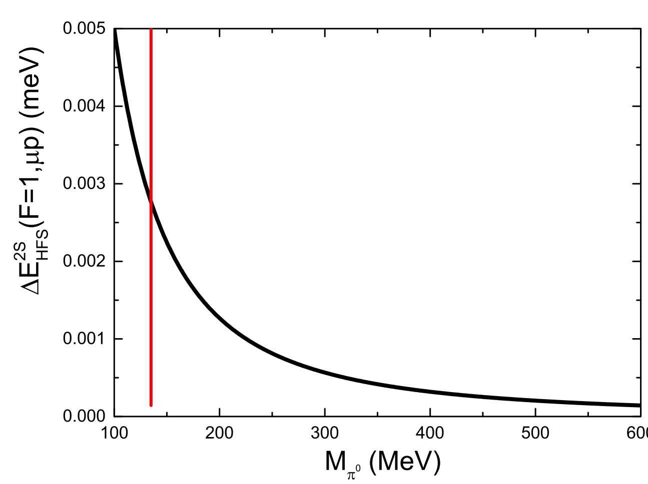

FIG. 4. (Color online) The mass dependence of the contribution to $\Delta E^{2S}_{\rm HFS}(F=1,\mu p)$.

APPENDIX

The analytical expression for the effective coupling $c_{1,l}$ is written as

$$c_{1,l} = \frac{9\Lambda\left[\lambda\Lambda^4 + 2\Lambda m_l^2(\Lambda_{\text{eff}} - \lambda\Lambda) + 16 d m_l^4 + 2\Lambda^3\Lambda_{\text{eff}}\log\left(\frac{m_l}{\Lambda}\right)\right]}{54\,800\pi m_l^4 \Lambda_{\text{eff}}}, \tag{A1}$$

where

$$\Lambda_{\text{eff}} \equiv \sqrt{\Lambda^2 - 4m_l^2}, \qquad \lambda \equiv \log\left(\frac{\Lambda^2 + \Lambda_{\text{eff}}\Lambda - 2m_l^2}{2m_l^2}\right), \tag{A2}$$

with m_l the mass of the lepton.

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