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Neutral kaons as an open quantum system in a second quantization approach

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We have shown that it is possible to formulate the consistent and probability-preserving description of the *CP*-symmetry-violating evolution of a system of decaying particles. This has been done within the framework of quantum mechanics of open systems. This approach allows the description of both the exponential decay and flavor oscillations. We have solved explicitly the Kossakowski-Lindblad master equation for a system of particles with violated *CP* symmetry and found the evolution of *any* observable bilinear in creation and annihilation operators. The choice of a concrete observable can be done by the proper choice of initial conditions for the system of differential equations. We have calculated the evolution as well as mean values of the observables most interesting from the physical point of view, and we have found their lowest order difference with the *CP*-preserved values.

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I. INTRODUCTION

In recent years Bell inequalities [1] were tested in systems of correlated neutral K [2,3] or B mesons [4]. However, the quantum mechanical analysis of such systems encounters an important difficulty, namely, the irreversibility of time evolution of unstable particles. Only the complete system consisting of the decaying particle as well as the decay products undergoes unitary evolution, which is actually described by quantum field theory. But in Einstein-Podolsky-Rosen (EPR) correlation experiments [5], it is more useful to consider the decaying particles only, neglecting the evolution of decay products. In the usual approach one introduces a non-Hermitian Hamiltonian, as it was done in the classical works of Weisskopf and Wigner [6]. However, the non-Hermitian Hamiltonian does not provide an unambiguous way of calculating the probability of finding the system consisting of a few such particles in a given state after the measurement (such an approach causes also other ambiguities, see, e.g., [7] for a discussion). Indeed, using this approach, we must reinterpret quantum mechanical results with the help of probability theory, especially if we would like to analyze correlation experiments with unstable particles, namely, neutral K or B mesons [8]. From the quantum mechanical point of view, this means that unit trace and positivity of the density matrix for the system under consideration must be preserved [9]. Fortunately, it is possible to resolve the above-mentioned problems using the open quantum systems theory [10]. The idea that unstable particles can be treated as open quantum systems was proposed first by Alicki [11] and was developed by various authors in different contexts (see, e.g., [12,13]). Recently, this approach was successfully applied to the system of particles with flavor oscillations (like in the case of neutral kaons or B mesons) [14–16], and it has been used successfully in the description of EPR correlations and evolution of entanglement in $K^0\overline{K^0}$ systems [17,18].

In this theory the evolution of a quantum system is described by the master equation [19], which can be treated as the replacement for either von Neumann or Heisenberg equations, for the picture of quantum mechanics in use. Here, we follow the approach presented in [16], where systems with an arbitrary number of particles were described with the use of the second quantization formalism, which seems to be the most natural language for a system with varying number of particles.

The paper is organized as follows. In Sec. II we introduce the notation and conventions used through the paper. In Sec. III we introduce the master equation for a system of neutral kaons with violated *CP* symmetry and we find the evolution of the observables in the Heisenberg picture, and, in Sect. IV, we analyze the evolution of the total number of particles and strangeness observable as well as the number of each of neutral kaon flavor. Finally, we conclude the paper in Sec. V.

II. PRELIMINARIA

We start with introducing some notations, conventions, and definitions used throughout the paper, as well as give experimental values of some parameters important in the description of the systems of K^0 and $\overline{K^0}$.

We define one-particle states $|K^0\rangle$ and $|\overline{K^0}\rangle$ as

$$|K^0\rangle = a^\dagger\,|0\rangle, \tag{1a}$$

$$|\overline{K^0}\rangle = b^\dagger\,|0\rangle, \tag{1b}$$

where a and b fulfills the usual canonical commutation relations

$$[a,a^\dagger] = [b,b^\dagger] = 1, \tag{2a}$$

$$[a,b] = [a^\dagger,b^\dagger] = 0. \tag{2b}$$

States $|K^0\rangle$ and $|\overline{K^0}\rangle$ are orthonormal, i.e.,

$$\langle K^0|K^0\rangle = \langle\overline{K^0}|\overline{K^0}\rangle = 1, \tag{3a}$$

$$\langle K^0|\overline{K^0}\rangle = 0, \tag{3b}$$

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and they are eigenstates of the strangeness operator $S = a^\dagger a - b^\dagger b$:

$$S\,|K^0\rangle = |K^0\rangle\,, \tag{4a}$$

$$S\,|\overline{K^0}\rangle = -\,|\overline{K^0}\rangle\,. \tag{4b}$$

For our convenience we will use the following short-hand notation for multiparticle states:

$$|\#K^0 = n, \#\overline{K^0} = \bar{n}\rangle \equiv |n,\bar{n}\rangle. \tag{5}$$

Therefore

$$|n,\bar{n}\rangle = \frac{1}{\sqrt{n!\bar{n}!}}(a^\dagger)^n (b^\dagger)^{\bar{n}}\,|0\rangle, \tag{6}$$

and

$$S\,|n,\bar{n}\rangle = (n-\bar{n})\,|n,\bar{n}\rangle\,. \tag{7}$$

However, the states $|K^0\rangle$ and $|\overline{K^0}\rangle$ are not eigenstates of time evolution. Actually, under time evolution the "well behaving" states are $|K_S^0\rangle$ and $|K_L^0\rangle$, defined as follows:

$$|K_S^0\rangle = p\,|K^0\rangle + q\,|\overline{K^0}\rangle\,, \tag{8a}$$

$$|K_L^0\rangle = p\,|K^0\rangle - q\,|\overline{K^0}\rangle\,, \tag{8b}$$

with mean lifetimes $\tau_S = 0.8954 \times 10^{-10}$ s and $\tau_L = 5.116 \times 10^{-8}$ s, respectively [20]. Moreover, if $p \neq q$ then CP symmetry is violated. Indeed,

$$\frac{p}{q} = \frac{1+\tilde{\epsilon}}{1-\tilde{\epsilon}}, \tag{9}$$

with

$$|p|^2 + |q|^2 = 1, \tag{10a}$$

$$|p|^2 - |q|^2 = A_L, \tag{10b}$$

and $A_L = 2\mathrm{Re}(\epsilon)/(1+|\epsilon|^2)$ is a measure of violation of CP symmetry (hereafter we use the so-called Wu-Yang phase convention [21] in which $\tilde{\epsilon} = \epsilon$). Experimentally obtained values are $A_L = 0.332\%$ and $|\epsilon| = 2.228 \times 10^{-3}$ [20]. Notice also that the basis (8) is not orthogonal since

$$\langle K_S^0 | K_L^0\rangle = A_L. \tag{11}$$

III. MASTER EQUATION

The aim of this section is to find the evolution of the multiparticle system of K^0 and $\overline{K^0}$. We know that the evolution of a single neutral kaon can be described as an open system obeying the Kossakowski-Lindblad master equation [15]. In this case the one-particle Hamiltonian for this master equation was written in the base (8) and reads

$$\begin{aligned} H = \frac{1}{1-A_L^2}\bigg\{ & m_S\,|K_S^0\rangle\langle K_S^0| + m_L\,|K_L^0\rangle\langle K_L^0| \\ & - A_L\bigg[\left(m - \frac{i}{4}\Delta\Gamma\right)|K_S^0\rangle\langle K_L^0| \\ & + \left(m + \frac{i}{4}\Delta\Gamma\right)|K_L^0\rangle\langle K_S^0|\bigg]\bigg\}, \end{aligned} \tag{12a}$$

while Lindbladians are of the form

$$\begin{aligned} L_1 = {} & \frac{1}{1-A_L^2}\sqrt{\Gamma_S - A_L^2\frac{\Gamma^2 + \Delta m^2}{\Gamma_L}} \\ & \times \left(|0\rangle\langle K_S^0| - A_L\,|0\rangle\langle K_L^0|\right), \end{aligned} \tag{12b}$$

$$\begin{aligned} L_2 = {} & \frac{1}{1-A_L^2}\bigg[\left(\sqrt{\Gamma_L} - A_L^2\frac{\Gamma - i\Delta m}{\sqrt{\Gamma_L}}\right)|0\rangle\langle K_L^0| \\ & - A_L\left(\sqrt{\Gamma_L} - \frac{\Gamma - i\Delta m}{\sqrt{\Gamma_L}}\right)|0\rangle\langle K_S^0|\bigg], \end{aligned} \tag{12c}$$

and $K \equiv -(L_1^\dagger L_1 + L_2^\dagger L_2)/2$, where $m = (m_S + m_L)/2 = 497.614\,\mathrm{MeV}/c^2$ is the K^0 mean mass, $\Delta m = m_L - m_S = 0.5293 \times 10^{10}\,\hbar/\mathrm{s}$ [20], $\Gamma = (\Gamma_S + \Gamma_L)/2$, and $\Delta\Gamma = \Gamma_S - \Gamma_L$.

The state of a single-particle system under the evolution given by the master equation with operators (12) is in general a mixed state of a single particle and vacuum (see [15]). After the projection of this state on the single-particle sector one gets the state obtained by the non-trace-preserving equations, considered usually in the literature (see, e.g., [3]).

A. Master equation in second quantization formalism

Now, let us go to the description of a multiparticle system. We achieve this by the replacement of intertwining operators with the appropriate combinations of annihilation and creation operators, as was done in [16]. In the base (1) this Hamiltonian and these Lindbladians take the form

$$\begin{aligned} H = {} & m(a^\dagger a + b^\dagger b) - \frac{pq^*}{1-A_L^2}\left(\Delta m + \tfrac{i}{2}A_L\Delta\Gamma\right)a^\dagger b \\ & - \frac{qp^*}{1-A_L^2}\left(\Delta m - \tfrac{i}{2}A_L\Delta\Gamma\right)b^\dagger a, \end{aligned} \tag{13a}$$

and

$$L_1 = \sqrt{\Gamma_S - A_L^2\frac{\Gamma^2 + \Delta m^2}{\Gamma_L}}\left(\frac{p^*}{1+A_L}a + \frac{q^*}{1-A_L}b\right), \tag{13b}$$

$$\begin{aligned} L_2 = {} & \frac{p^*}{1+A_L}\left(\sqrt{\Gamma_L} + A_L\frac{\Gamma - i\Delta m}{\sqrt{\Gamma_L}}\right)a \\ & - \frac{q^*}{1-A_L}\left(\sqrt{\Gamma_L} - A_L\frac{\Gamma - i\Delta m}{\sqrt{\Gamma_L}}\right)b, \end{aligned} \tag{13c}$$

$$\begin{aligned} K = {} & -\frac{1}{2}\Gamma(a^\dagger a + b^\dagger b) - \frac{pq^*}{1-A_L^2}\left(\frac{1}{2}\Delta\Gamma - iA_L\Delta m\right)a^\dagger b \\ & - \frac{qp^*}{1-A_L^2}\left(\frac{1}{2}\Delta\Gamma + iA_L\Delta m\right)b^\dagger a. \end{aligned} \tag{13d}$$

Let us consider a single-particle state described by a density matrix of the form

$$\begin{aligned} \rho = {} & p_1\,|K^0\rangle\langle K^0| + p_2\,|\overline{K^0}\rangle\langle\overline{K^0}| + w\,|K^0\rangle\langle\overline{K^0}| \\ & + w^*\,|\overline{K^0}\rangle\langle K^0| + (1 - p_1 - p_2)\,|0\rangle\langle 0| \end{aligned} \tag{14}$$

with $0 \leqslant p_1 \leqslant 1$, $0 \leqslant p_2 \leqslant 1$, $0 \leqslant p_1 + p_2 \leqslant 1$, and $|w|^2 \leqslant p_1 p_2$. The evolution of this state in the Schrödinger picture is governed by the master equation

$$\partial_t \rho(t) = -i[H,\rho(t)] + \{K,\rho(t)\} + \sum_i L_i \rho(t) L_i^\dagger. \tag{15}$$

By a straightforward calculation, using relation (8) between the bases $\{|K_S^0\rangle, |K_L^0\rangle\}$ and $\{|K^0\rangle, |\overline{K^0}\rangle\}$, one can check that the choices of Eqs. (12) and (13) for operators appearing in Eq. (15) lead to the same evolution equation for $\rho(t)$.

Moreover, the evolution of the two-particle system obtained from Eqs. (13) and (15) lead to the evolution of the two-particle state obtained by taking a symmetrized (the particles are indistinguishable) tensor product of single-particle evolution (see [17]).

Now, consider the Kossakowski-Lindblad master equation for the evolution of observable $\Omega(t)$ in the Heisenberg picture

$$\partial_t \Omega(t) = \mathcal{L}[\Omega(t)], \tag{16}$$

where

$$\mathcal{L}[\Omega(t)] = i[H,\Omega(t)] + \frac{1}{2}\sum_i \{[L_i^\dagger,\Omega(t)]L_i + L_i^\dagger[\Omega(t),L_i]\}, \tag{17}$$

and we choose H and L_i ($i = 1,2$) in the form (13). We assume that $\Omega(t)$ can be written as a bilinear form in annihilation and creation operators

$$\Omega(t) = \omega_{aa}(t)a^\dagger a + \omega_{ab}(t)a^\dagger b + \omega_{ba}(t)b^\dagger a + \omega_{bb}(t)b^\dagger b, \tag{18}$$

with the condition $\omega_{ba}(t) = \omega_{ab}^*(t)$ to guarantee that $\Omega(t)$ is Hermitian.

B. General solution

Here we want to find the evolution of any observable $\Omega(t)$ which is bilinear in annihilation and creation operators. To achieve this aim, we begin with the following observation:

$$\mathcal{L}[a^\dagger a] = -\Gamma a^\dagger a - \frac{pq^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)a^\dagger b - \frac{qp^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)b^\dagger a, \tag{19a}$$

$$\mathcal{L}[a^\dagger b] = -\Gamma a^\dagger b - \frac{qp^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)a^\dagger a - \frac{qp^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)b^\dagger b, \tag{19b}$$

$$\mathcal{L}[b^\dagger a] = -\Gamma b^\dagger a - \frac{pq^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)a^\dagger a - \frac{pq^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)b^\dagger b, \tag{19c}$$

$$\mathcal{L}[b^\dagger b] = -\Gamma b^\dagger b - \frac{pq^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)a^\dagger b - \frac{qp^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)b^\dagger a. \tag{19d}$$

We see that if $\Omega(t)$ is of the form (18) at a given moment of time t_0, it must preserve this form for all the time $t \geqslant t_0$.

If we take into account the linearity of $\Omega(t)$ and linear independence of operators $a^\dagger a$, $a^\dagger b$, $b^\dagger a$, and $b^\dagger b$, we get the following system of first-order differential equations for ω:

$$\dot{\omega}_{aa}(t) = -\Gamma\omega_{aa}(t) - \frac{qp^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)\omega_{ab}(t) - \frac{pq^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)\omega_{ba}(t), \tag{20a}$$

$$\dot{\omega}_{ab}(t) = -\frac{pq^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)\omega_{aa}(t) - \Gamma\omega_{ab}(t) - \frac{pq^*}{1+A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)\omega_{bb}(t), \tag{20b}$$

$$\dot{\omega}_{ba}(t) = -\frac{qp^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)\omega_{aa}(t) - \Gamma\omega_{ba}(t) - \frac{pq^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)\omega_{bb}(t), \tag{20c}$$

$$\dot{\omega}_{bb}(t) = -\frac{qp^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma + i\Delta m\right)\omega_{ab}(t) - \frac{pq^*}{1-A_L}\left(\tfrac{1}{2}\Delta\Gamma - i\Delta m\right)\omega_{ba}(t) - \Gamma\omega_{bb}(t). \tag{20d}$$

Using straightforward methods we can find that the general solution of Eq. (20) is of the form

$$\begin{aligned}\omega_{aa}(t) = \frac{e^{-\Gamma t}}{2}\Big\{&\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + \cos(\Delta m t)\right]\omega_{aa}(0) - \frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) - i\sin(\Delta m t)\right]\omega_{ab}(0)\\ &- \frac{1-A_L}{1+A_L}\frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + i\sin(\Delta m t)\right]\omega_{ba}(0) + \frac{1-A_L}{1+A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right]\omega_{bb}(0)\Big\},\end{aligned} \tag{21a}$$

$$\begin{aligned}\omega_{ab}(t) = \frac{e^{-\Gamma t}}{2}\Big\{&-\frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) - i\sin(\Delta m t)\right]\omega_{aa}(0) + \left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + \cos(\Delta m t)\right]\omega_{ab}(0)\\ &+ \frac{1-A_L}{1+A_L}\left(\frac{p}{q}\right)^2\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right]\omega_{ba}(0) - \frac{1-A_L}{1+A_L}\frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + i\sin(\Delta m t)\right]\omega_{bb}(0)\Big\},\end{aligned} \tag{21b}$$

$$\omega_{ba}(t) = \frac{e^{-\Gamma t}}{2}\left\{-\frac{1+A_L}{1-A_L}\frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + i\sin(\Delta m t)\right]\omega_{aa}(0) + \frac{1+A_L}{1-A_L}\left(\frac{q}{p}\right)^2\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right]\omega_{ab}(0)\right.$$
$$\left. + \left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + \cos(\Delta m t)\right]\omega_{ba}(0) - \frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) - i\sin(\Delta m t)\right]\omega_{bb}(0)\right\}, \tag{21c}$$

$$\omega_{bb}(t) = \frac{e^{-\Gamma t}}{2}\left\{\frac{1+A_L}{1-A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right]\omega_{aa}(0) - \frac{1+A_L}{1-A_L}\frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + i\sin(\Delta m t)\right]\omega_{ab}(0)\right.$$
$$\left. - \frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) - i\sin(\Delta m t)\right]\omega_{ba}(0) + \left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + \cos(\Delta m t)\right]\omega_{bb}(0)\right\}. \tag{21d}$$

The Hermicity of $\Omega(t)$ is preserved (i.e., the condition $\omega_{ab}(0) = \omega_{ba}^*(0)$ implies $\omega_{ab}(t) = \omega_{ba}^*(t)$ for all the time $t \geqslant 0$), since from Eqs. (9) and (10) we have

$$\left(\frac{p}{q}\right)^* = \frac{1+A_L}{1-A_L}\frac{q}{p}. \tag{22}$$

Now let us observe that the different choices of initial conditions for ω allow us to get the time evolution of different physically interesting observables of the form (18).

IV. EVOLUTION OF OBSERVABLES AND THEIR AVERAGES

Now we are prepared to find the evolution of some physically interesting observables, as well as the time dependence of their mean values. We begin with the total number of particles and strangeness observables, next we analyze the numbers of each neutral kaon flavor.

A. Total number of particles and strangeness

For the total number of particles observable N, we have $N(0) = a^\dagger a + b^\dagger b$, so $\omega_{aa}(0) = \omega_{bb}(0) = 1$ and $\omega_{ab}(0) = \omega_{ba}(0) = 0$, and finally we get

$$N(t) = e^{-\Gamma t}\left\{\frac{1}{1+A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + A_L\cos(\Delta m t)\right]a^\dagger a\right.$$
$$- \frac{1}{1+A_L}\frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) - iA_L\sin(\Delta m t)\right]a^\dagger b$$
$$- \frac{1}{1-A_L}\frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + iA_L\sin(\Delta m t)\right]b^\dagger a$$
$$\left. + \frac{1}{1-A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - A_L\cos(\Delta m t)\right]b^\dagger b\right\}. \tag{23}$$

The expectation value of $N(t)$ in the state $|n,\bar{n}\rangle$ is

$$\langle N(t)\rangle = \frac{e^{-\Gamma t}}{1-A_L^2}\left\{\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - A_L^2\cos(\Delta m t)\right](n+\bar{n})\right.$$
$$\left. - A_L\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right](n-\bar{n})\right\} \tag{24}$$

and is depicted in Fig. 1. If we take $A_L = 0$ in Eq. (24), we obtain the case with preserved CP symmetry, for which we recover the usual result for mean number of particles

$$\langle N(t)\rangle_{\mathrm{CP}} = \tfrac{1}{2}\left(e^{-\Gamma_S t} + e^{-\Gamma_L t}\right)(n+\bar{n}). \tag{25}$$

The leading part of the difference between CP-violated and CP-preserved values is

$$\langle N(t)\rangle - \langle N(t)\rangle_{\mathrm{CP}}$$
$$= -A_L\left[\tfrac{1}{2}\left(e^{-\Gamma_S t} + e^{-\Gamma_L t}\right) - e^{-\Gamma t}\cos(\Delta m t)\right]$$
$$\times (n-\bar{n}) + O\left(A_L^2\right), \tag{26}$$

and is shown in Fig. 1, too.

For strangeness S we have $S(0) = a^\dagger a - b^\dagger b$, so $\omega_{aa}(0) = -\omega_{bb}(0) = 1$, $\omega_{ab}(0) = \omega_{ba}(0) = 0$, and

$$S(t) = e^{-\Gamma t}\left\{\frac{1}{1+A_L}\left[A_L\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + \cos(\Delta m t)\right]a^\dagger a\right.$$
$$- \frac{1}{1+A_L}\frac{p}{q}\left[A_L\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) - i\sin(\Delta m t)\right]a^\dagger b$$
$$- \frac{1}{1-A_L}\frac{q}{p}\left[A_L\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + i\sin(\Delta m t)\right]b^\dagger a$$
$$\left. + \frac{1}{1-A_L}\left[A_L\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right]b^\dagger b\right\}. \tag{27}$$

The expectation value of $S(t)$ in the state $|n,\bar{n}\rangle$ is therefore

$$\langle S(t)\rangle = \frac{e^{-\Gamma t}}{1-A_L^2}\left\{\left[\cos(\Delta m t) - A_L^2\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)\right](n-\bar{n})\right.$$
$$\left. + A_L\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) - \cos(\Delta m t)\right](n+\bar{n})\right\} \tag{28}$$

and is shown in Fig. 2.

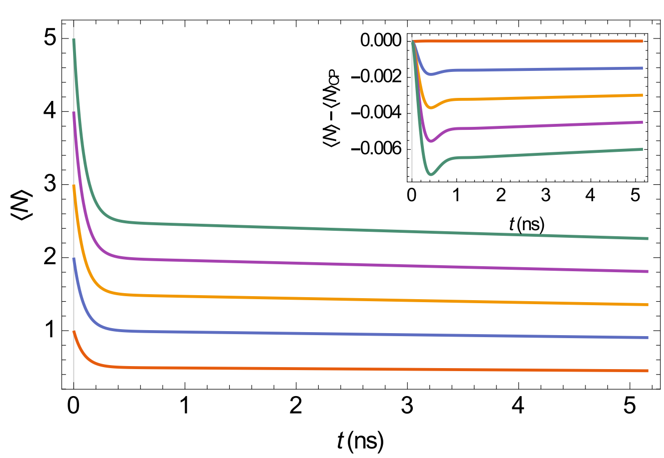

FIG. 1. (Color online) Mean number of particles vs time for $n + \bar{n} = 1,\ldots,5$ (bottom-up) and the difference between $\langle N(t)\rangle$ for CP-violated and CP-preserved case vs time for $n + \bar{n} = 4$ and $n - \bar{n} = 0,\ldots,4$ (top-down).

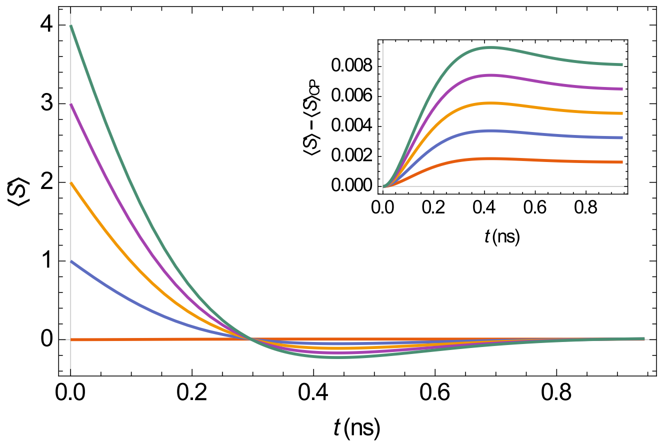

FIG. 2. (Color online) Mean strangeness vs time for $n+\bar{n}=4$ and $n-\bar{n}=0,\dots,4$ (bottom-up) with the strangeness oscillation phenomenon explicitly visible when $n-\bar{n}\neq 0$ and the difference between $\langle S(t)\rangle$ for CP-violated and CP-preserved cases vs time for $n+\bar{n}=1,\dots,5$ and $n-\bar{n}=0$ (bottom-up).

As previously, when $A_L=0$ we get the CP-preserved value

$$\langle S(t)\rangle_{\mathrm{CP}} = e^{-\Gamma t}\cos(\Delta m t)(n-\bar{n}), \tag{29}$$

so the leading term of the difference between CP-violated and CP-preserved values is

$$\langle S(t)\rangle - \langle S(t)\rangle_{\mathrm{CP}} = A_L\left[\tfrac{1}{2}\left(e^{-\Gamma_S t}+e^{-\Gamma_L t}\right) - e^{-\Gamma t}\cos(\Delta m t)\right] \times (n+\bar{n}) + O\left(A_L^2\right) \tag{30}$$

and is shown also in Fig. 2.

B. Number of K^0 and $\overline{K^0}$

For the numbers of K^0 and $\overline{K^0}$ we have $N_{K^0}(0)=a^\dagger a$ and $N_{\overline{K^0}}(0)=b^\dagger b$, so

$$\begin{aligned} N_{K^0}(t) = \frac{e^{-\Gamma t}}{2}\bigg\{&\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)+\cos(\Delta m t)\right]a^\dagger a \\ &-\frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right)-i\sin(\Delta m t)\right]a^\dagger b \\ &-\frac{1+A_L}{1-A_L}\frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right)+i\sin(\Delta m t)\right]b^\dagger a \\ &+\frac{1+A_L}{1-A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)-\cos(\Delta m t)\right]b^\dagger b\bigg\}, \end{aligned} \tag{31a}$$

and

$$\begin{aligned} N_{\overline{K^0}}(t) = \frac{e^{-\Gamma t}}{2}\bigg\{&\frac{1-A_L}{1+A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)-\cos(\Delta m t)\right]a^\dagger a \\ &-\frac{1-A_L}{1+A_L}\frac{p}{q}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right)+i\sin(\Delta m t)\right]a^\dagger b \\ &-\frac{q}{p}\left[\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right)-i\sin(\Delta m t)\right]b^\dagger a \\ &+\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)+\cos(\Delta m t)\right]b^\dagger b\bigg\}. \end{aligned} \tag{31b}$$

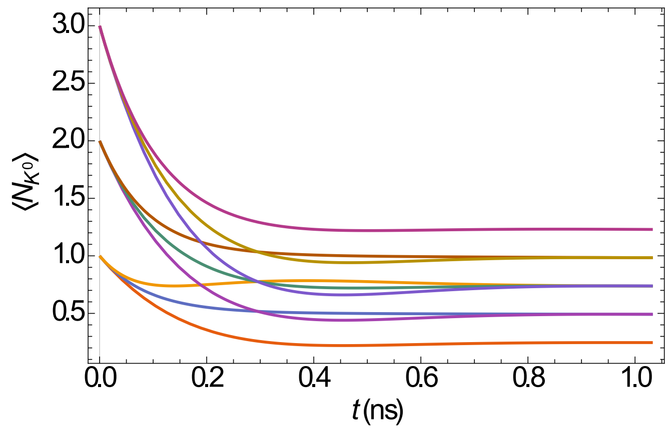

FIG. 3. (Color online) Mean number of K^0 vs time for $n=1,2,3$ (bundles bottom-up) and $\bar{n}=0,1,2$ (bottom-up inside bundles).

Their mean values are

$$\begin{aligned} \langle N_{K^0}(t)\rangle = \frac{e^{-\Gamma t}}{2}\bigg\{&\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)+\cos(\Delta m t)\right]n \\ &+\frac{1+A_L}{1-A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)-\cos(\Delta m t)\right]\bar{n}\bigg\} \end{aligned} \tag{32a}$$

and

$$\begin{aligned} \langle N_{\overline{K^0}}(t)\rangle = \frac{e^{-\Gamma t}}{2}\bigg\{&\frac{1-A_L}{1+A_L}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)-\cos(\Delta m t)\right]n \\ &+\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right)+\cos(\Delta m t)\right]\bar{n}\bigg\}, \end{aligned} \tag{32b}$$

respectively. We show these time evolution in the Figs. 3 and 4. Notice that after a suitable period of time (approx. 8 ns) different initial states can give the same mean number of K^0 or $\overline{K^0}$.

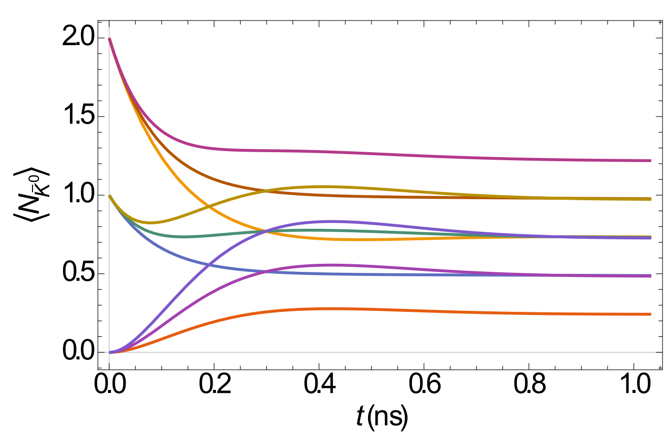

FIG. 4. (Color online) Mean number of $\overline{K^0}$ vs time for $n=1,2,3$ (bottom-up inside bundles) and $\bar{n}=0,1,2$ (bundles bottom-up).

If we put $A_L = 0$, we get again *CP*-preserved values

$$\langle N_{K^0}(t)\rangle_{\rm CP} = \frac{e^{-\Gamma_S t}+e^{-\Gamma_L t}}{2}\frac{n+\bar{n}}{2} + e^{-\Gamma t}\cos(\Delta m t)\frac{n-\bar{n}}{2} \tag{33a}$$

and

$$\langle N_{\overline{K^0}}(t)\rangle_{\rm CP} = \frac{e^{-\Gamma_S t}+e^{-\Gamma_L t}}{2}\frac{n+\bar{n}}{2} - e^{-\Gamma t}\cos(\Delta m t)\frac{n-\bar{n}}{2}. \tag{33b}$$

The leading term of the differences for the *CP*-violated and *CP*-preserved values are

$$\langle N_{K^0}(t)\rangle - \langle N_{K^0}(t)\rangle_{\rm CP} = A_L\left[\tfrac{1}{2}\left(e^{-\Gamma_S t}+e^{-\Gamma_L t}\right) - e^{-\Gamma t}\cos(\Delta m t)\right]\bar{n} + O\left(A_L^2\right), \tag{34a}$$

$$\langle N_{\overline{K^0}}(t)\rangle - \langle N_{\overline{K^0}}(t)\rangle_{\rm CP} = -A_L\left[\tfrac{1}{2}\left(e^{-\Gamma_S t}+e^{-\Gamma_L t}\right) - e^{-\Gamma t}\cos(\Delta m t)\right]n + O\left(A_L^2\right). \tag{34b}$$

C. Number of K_S^0 and K_L^0

Let us return to the short and long living states of neutral kaon, $|K_S^0\rangle$ and $|K_L^0\rangle$, respectively. If we define another two pairs of annihilation and creation operators, such that

$$\left|K_S^0\right\rangle = c_S^\dagger\left|0\right\rangle, \tag{35a}$$

$$\left|K_L^0\right\rangle = c_L^\dagger\left|0\right\rangle, \tag{35b}$$

we can find, with the use of Eq. (8), that

$$c_S = p^*a + q^*b, \tag{36a}$$

$$c_L = p^*a - q^*b. \tag{36b}$$

The operators c_S and c_L fulfill almost the usual canonical commutation relations with the following exception:

$$[c_S, c_L^\dagger] = [c_L, c_S^\dagger] = A_L. \tag{37}$$

This reflects the fact that $|K_S^0\rangle$ and $|K_L^0\rangle$ are not orthogonal, see Eq. (11).

Now we define multiparticle states for K_S^0 and K_L^0 as

$$\left|\#K_S^0 = n\right\rangle \equiv |n_S\rangle = \frac{(c_S^\dagger)^n}{\sqrt{n!}}\,|0\rangle, \tag{38a}$$

$$\left|\#K_L^0 = n\right\rangle \equiv |n_L\rangle = \frac{(c_L^\dagger)^n}{\sqrt{n!}}\,|0\rangle. \tag{38b}$$

These states can be expressed in terms of the sates $|n,k\rangle$ as follows

$$|n_S\rangle = \sum_{k=0}^{n}\sqrt{\binom{n}{k}}p^{n-k}q^k\,|n-k,k\rangle, \tag{39a}$$

$$|n_L\rangle = \sum_{k=0}^{n}(-1)^k\sqrt{\binom{n}{k}}p^{n-k}q^k\,|n-k,k\rangle. \tag{39b}$$

For the number of K_S^0, $N_{K_S^0}(0) = c_S^\dagger c_S = \frac{1+A_L}{2}a^\dagger a + \frac{1-A_L}{2}\frac{p}{q}a^\dagger b + \frac{1+A_L}{2}\frac{q}{p}b^\dagger a + \frac{1-A_L}{2}b^\dagger b$, so

$$N_{K_S^0}(t) = \frac{e^{-\Gamma t}}{2}\left\{\frac{1}{1+A_L}\left[e^{-\frac{1}{2}\Delta\Gamma t} + A_L^2 e^{\frac{1}{2}\Delta\Gamma t} + 2A_L\cos(\Delta m t)\right]a^\dagger a + \frac{1}{1+A_L}\frac{p}{q}\left[e^{-\frac{1}{2}\Delta\Gamma t} - A_L^2 e^{\frac{1}{2}\Delta\Gamma t} + 2iA_L\sin(\Delta m t)\right]a^\dagger b\right.$$
$$\left. + \frac{1}{1-A_L}\frac{q}{p}\left[e^{-\frac{1}{2}\Delta\Gamma t} - A_L^2 e^{\frac{1}{2}\Delta\Gamma t} - 2iA_L\sin(\Delta m t)\right]b^\dagger a + \frac{1}{1-A_L}\left[e^{-\frac{1}{2}\Delta\Gamma t} + A_L^2 e^{\frac{1}{2}\Delta\Gamma t} - 2A_L\cos(\Delta m t)\right]b^\dagger b\right\}. \tag{40}$$

Similarly, for the number of K_L^0, $N_{K_L^0}(0) = c_L^\dagger c_L = \frac{1+A_L}{2}a^\dagger a - \frac{1-A_L}{2}\frac{p}{q}a^\dagger b - \frac{1+A_L}{2}\frac{q}{p}b^\dagger a + \frac{1-A_L}{2}b^\dagger b$, and

$$N_{K_L^0}(t) = \frac{e^{-\Gamma t}}{2}\left\{\frac{1}{1+A_L}\left[e^{\frac{1}{2}\Delta\Gamma t} + A_L^2 e^{-\frac{1}{2}\Delta\Gamma t} + 2A_L\cos(\Delta m t)\right]a^\dagger a - \frac{1}{1+A_L}\frac{p}{q}\left[e^{\frac{1}{2}\Delta\Gamma t} - A_L^2 e^{-\frac{1}{2}\Delta\Gamma t} - 2iA_L\sin(\Delta m t)\right]a^\dagger b\right.$$
$$\left. - \frac{1}{1-A_L}\frac{q}{p}\left[e^{\frac{1}{2}\Delta\Gamma t} - A_L^2 e^{-\frac{1}{2}\Delta\Gamma t} + 2iA_L\sin(\Delta m t)\right]b^\dagger a + \frac{1}{1-A_L}\left[e^{\frac{1}{2}\Delta\Gamma t} + A_L^2 e^{-\frac{1}{2}\Delta\Gamma t} - 2A_L\cos(\Delta m t)\right]b^\dagger b\right\}. \tag{41}$$

Now we are prepared to find the expectation values of $N_{K_S^0}(t)$ and $N_{K_L^0}(t)$ in the states $|n_S\rangle$ and $|n_L\rangle$, respectively. First, observe that

$$a^\dagger b\,|n_S\rangle = \frac{q}{p}a^\dagger a\,|n_S\rangle, \quad a^\dagger b\,|n_L\rangle = -\frac{q}{p}a^\dagger a\,|n_L\rangle, \tag{42a}$$

$$b^\dagger a\,|n_S\rangle = \frac{p}{q}b^\dagger b\,|n_S\rangle, \quad b^\dagger a\,|n_L\rangle = -\frac{p}{q}b^\dagger b\,|n_L\rangle. \tag{42b}$$

Thus

$$N_{K_S^0}(t)\,|n_S\rangle = \frac{e^{-\Gamma t}}{1-A_L^2}\big[\big(e^{-\frac{1}{2}\Delta\Gamma t} - A_L^2 e^{i\Delta m t}\big)N(0) - A_L\big(e^{-\frac{1}{2}\Delta\Gamma t} - e^{i\Delta m t}\big)S(0)\big]\,|n_S\rangle \tag{43a}$$

and

$$N_{K_L^0}(t)\,|n_L\rangle = \frac{e^{-\Gamma t}}{1-A_L^2}\big[\big(e^{\frac{1}{2}\Delta\Gamma t} - A_L^2 e^{-i\Delta m t}\big)N(0) - A_L\big(e^{\frac{1}{2}\Delta\Gamma t} - e^{-i\Delta m t}\big)S(0)\big]\,|n_L\rangle. \tag{43b}$$

Of course $|n_S\rangle$ and $|n_L\rangle$ are eigenvectors of $N(0)$:

$$N(0)\,|n_S\rangle = n\,|n_S\rangle, \tag{44a}$$

$$N(0)\,|n_L\rangle = n\,|n_L\rangle, \tag{44b}$$

but they are not eigenvectors of $S(0)$ since

$$S(0)\,|n_S\rangle = n\,|n_S\rangle - 2\sum_{k=0}^{n} k\sqrt{\binom{n}{k}}p^{n-k}q^k\,|n-k,k\rangle, \tag{45a}$$

$$S(0)\,|n_L\rangle = n\,|n_L\rangle - 2\sum_{k=0}^{n}(-1)^k k\sqrt{\binom{n}{k}}p^{n-k}q^k\,|n-k,k\rangle. \tag{45b}$$

Notice that in view of Eq. (45) the states $|n_S\rangle$ and $|n_L\rangle$ are *not* eigenvectors of $N_{K_S^0}$ and $N_{K_L^0}$, respectively, as one can naïvely expect. However, the expectation values of $S(0)$ in the states $|n_S\rangle$ and $|n_L\rangle$ are well established and can be calculated as

$$\langle n_S|\,S(0)\,|n_S\rangle = A_L n, \tag{46a}$$

$$\langle n_L|\,S(0)\,|n_L\rangle = A_L n, \tag{46b}$$

because

$$\sum_{k=0}^{n} k\binom{n}{k}|p|^{2(n-k)}|q|^{2k} = n|q|^2(|p|^2+|q|^2)^{n-1} = n|q|^2. \tag{47}$$

In conclusion, we get that the mean number of K_S^0 and K_L^0 in the states $|n_S\rangle$ and $|n_L\rangle$, respectively, evolves in time according to the Geiger-Nutall law

$$\langle n_S|\,N_{K_S^0}(t)\,|n_S\rangle = ne^{-\Gamma_S t}, \tag{48a}$$

$$\langle n_L|\,N_{K_L^0}(t)\,|n_L\rangle = ne^{-\Gamma_L t}. \tag{48b}$$

The states $|n_S\rangle$ and $|n_L\rangle$ are not orthogonal, thus we expect nonzero expectation values of $N_{K_S^0}(t)$ and $N_{K_L^0}$ in the states $|n_L\rangle$ and $|n_S\rangle$, respectively. Indeed,

$$N_{K_S^0}(t)\,|n_L\rangle = \frac{e^{-\Gamma t}A_L}{1-A_L^2}\big[\big(e^{-i\Delta m t} - A_L^2 e^{\frac{1}{2}\Delta\Gamma t}\big)S(0) - A_L\big(e^{-i\Delta m t} - e^{\frac{1}{2}\Delta\Gamma t}\big)N(0)\big]\,|n_L\rangle \tag{49a}$$

and

$$N_{K_L^0}(t)\,|n_S\rangle = \frac{e^{-\Gamma t}A_L}{1-A_L^2}\big[\big(e^{i\Delta m t} - A_L^2 e^{-\frac{1}{2}\Delta\Gamma t}\big)S(0) - A_L\big(e^{i\Delta m t} - e^{-\frac{1}{2}\Delta\Gamma t}\big)N(0)\big]\,|n_S\rangle, \tag{49b}$$

thus,

$$\langle n_L|\,N_{K_S^0}(t)\,|n_L\rangle = nA_L^2 e^{-\Gamma_L t}, \tag{50a}$$

$$\langle n_S|\,N_{K_L^0}(t)\,|n_S\rangle = nA_L^2 e^{-\Gamma_S t}. \tag{50b}$$

So, finally we see that we can detect a fraction of order A_L^2 of the other flavor in multiparticle short- or long-lived neutral kaon states.

V. CONCLUSIONS

We have shown that it is possible to formulate the consistent and probability-preserving description of the *CP*-symmetry-violating evolution of a system containing any number of decaying particles. This has been done within the framework of quantum mechanics of open systems based on the approach developed in [15,16]. To achieve this aim we have considered master equations built up from creation and annihilation operators which generate dynamical semigroups that can describe the exponential decay and flavor oscillations for a system of many particles. It should be noted that this dynamical semigroup was used for the description of the entire system of unstable particles, and not only for decoherence effects, as was done in, e.g., [9,14].

We have used the fact that the decay of the particle can be regarded as a Markov process (the probability of decay of a particle is constant, so it does not depend on its history) and we have solved explicitly the Kossakowski-Lindblad master equation for a system of particles with violated *CP* symmetry. To avoid working with infinite numbers of density matrix elements we performed calculations in the Heisenberg picture of quantum mechanics. This allowed us to deal with a low-dimensional system of linear differential equations for evolution of *any* observable bilinear in creation and annihilation operators. The choice of a concrete observable is then reduced to the proper choice of initial conditions for the system of differential equations. To show explicitly the effectiveness of the introduced approach we have calculated the evolution of the observables which are most interesting from the physical point of view. We have also found the evolution of mean values of these observables as well as their lowest order difference with the *CP*-preserved values.

It seems to us that the presented analysis of time evolution of neutral kaons could be applied to the description of EPR states. Of course one must construct such a state by labeling annihilation and creation operators either by a discrete index, as done in quantum optics, or by a continuous parameter, as done in quantum field theory (beware that in the latter case annihilation and creation operators are actually operator valued distributions).

Moreover, we think that our approach would be interesting and helpful also in the analysis of mechanical systems and electric circuits. This follows from the fact that there is a strong analogy with nonreversible classical mechanics and quantum systems with *CP* violation [22].

Finally, we would like to point out that all the presented results are also valid for neutral *B* mesons after appropriate change of notation and values of physical quantities. This follows from the fact that *B* mesons evolve according to the same scheme as kaons.

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