Observing the Casimir-Lifshitz force out of thermal equilibrium

Giuseppe Bimonte
Dipartimento di Fisica, Università di Napoli Federico II, Complesso Universitario MSA, Via Cintia, I-80126 Napoli, Italy and INFN Sezione di Napoli, I-80126 Napoli, Italy
(Received 2 April 2015; published 11 September 2015)

The thermal Casimir-Lifshitz force between two bodies held at different temperatures displays striking features that are absent in systems in thermal equilibrium. The manifestation of this force has been observed so far only in Bose-Einstein condensates close to a heated substrate, but never between two macroscopic bodies. Observation of the thermal Casimir-Lifshitz force out of thermal equilibrium with conventional Casimir setups is very difficult because for experimentally accessible separations the thermal force is small compared to the zero-temperature quantum Casimir force unless prohibitively large temperature differences among the plates are considered. We describe an apparatus that allows for direct observation of the thermal force out of equilibrium for submicron separations and for moderate temperature differences between the plates.

DOI: 10.1103/PhysRevA.92.032116 PACS number(s): 12.20.−m, 03.70.+k, 42.25.Fx

I. INTRODUCTION

Casimir-Lifshitz forces [1,2], i.e., dispersion forces between polarizable bodies originating from quantum and thermal fluctuations of the electromagnetic (em) field, play an important role in different fields of science (physics, biology, and chemistry) and in technology. The first comprehensive theory of dispersion forces was developed in the 1950s by Lifshitz [3], on the basis of Rytov's theory of electromagnetic fluctuations [4,5]. Still today, Lifshitz theory is routinely used to interpret experiments on dispersion forces.

In its original formulation, Lifshitz theory dealt with two material slabs in thermal equilibrium. Recently, the theory has been generalized by Antezza *et al.* [6–8] to situations out of equilibrium, in which the interacting bodies may have different temperatures. The study of the thermal component of the Casimir-Lifshitz force has attracted much interest in recent years. Observing the thermal force is very difficult, as it becomes visible only at distances of the order of the thermal wavelength $\lambda_T = \hbar c/k_B T$ (about 7 μm at room temperature). At such large distances both the quantum Casimir force and the thermal force are very small and thus very difficult to measure. On the other hand, for smaller distances the thermal force is masked by the much stronger $T = 0$ quantum component of the Casimir-Lifshitz force and therefore it is difficult to separate it unambiguously. As of now, only two experiments have observed the thermal Casimir-Lifshitz force. The first one is the experiment by Obrecht *et al.* [9], which observed the thermal Casimir-Polder force between an ultracold atomic cloud placed at a distance of a few microns from a dielectric substrate. In order to enhance the thermal force, the measurement was done out of thermal equilibrium by heating the substrate and was found to be in agreement with the theory developed in [6]. The second experiment by Sushkov *et al.* [10] observed the equilibrium thermal Casimir force between a large Au sphere and a Au plate, in the wide range of separations from 0.7 to 7.3 μm. The theoretical interpretation of the experiment by Sushkov *et al.* is controversial [11] because of the presence in the signal of a ten times larger force of unclear origin that was attributed to large electrostatic patches on the gold surfaces.

Out of thermal equilibrium the Casimir-Lifshitz force displays remarkable features that disappear when the system is brought in a state of thermal equilibrium [6–8]. These features originate from a peculiar contribution $\bar{F}^{(\mathrm{neq})}(T_1,T_2)$ to the nonequilibrium force, which is antisymmetric under an exchange of the body temperatures T_1 and T_2. The presence of such a term, first pointed out in [6] for the case of a polarizable small particle in front of a flat dielectric surface and then in [7] for two plane-parallel slabs, was later shown to be a general feature of the nonequilibrium force between two bodies of any shape and composition [12–15]. Being antisymmetric in the body temperatures, this term can have either sign and it can be harnessed to tune the force in both strength and sign [16] and to realize self-propelling systems [17]. This thermal force enjoys more striking features: It vanishes identically for two bodies with identical scattering matrices [8,12–15] and it is nonadditive in the limit where one of the two bodies is a rarefied gas [6,7]. In view of its unique features, it would be clearly of great interest to observe the effect of this term in the Casimir force between two macroscopic bodies held at different temperatures. So far this has been an impossible task, because in order to observe this term by current Casimir setups it would be necessary to achieve a large temperature difference between the plates (hundreds of degrees) and to go to separations of several microns. Both things are very difficult to realize in practice.

In this paper we describe an apparatus that should allow for a direct observation of the antisymmetric component of the nonequilibrium thermal Casimir-Lifshitz force at submicron distances, with small temperature differences between the plates. The scheme is based on a differential force measurement of the Casimir force between a gold-coated sphere and two dissimilar sectors of a flat surface, one made of gold and the other of silicon. The crucial feature of the proposed setup is the presence of a uniform gold overlayer covering the plate, which filters out the otherwise dominant $T = 0$ component of the Casimir force. Due to the filtering property of the gold layer, the signal in our setup originates entirely from the thermal component of the Casimir force, thus making its observation unambiguous. The scheme described in this paper is similar to setups recently proposed by the author [18–21] to probe the equilibrium thermal Casimir force between magnetodielectric or superconducting surfaces.

The plan of the paper is as follows. In Sec. II we describe our setup and explain its workings. In Sec. III we present our computations of the differential Casimir force. We summarize in Sec. IV.

II. SETUP

The setup, schematically shown in Fig. 1, consists of a gold sphere of radius R at temperature T_2 placed at a (minimum) distance a from a planar slab at temperature T_1, divided into two regions made of gold and of (high-resistivity) silicon, respectively. The key feature of the apparatus is the gold overlayer of thickness w, covering both the gold and the silicon regions of the plate. For any fixed sphere-plate separation a, we consider measuring the difference

$$\Delta F(T_1,T_2) = F_{\rm Si}(T_1,T_2) - F_{\rm Au}(T_1,T_2) \tag{1}$$

between the values $F_{\rm Au}(T_1,T_2)$ and $F_{\rm Si}(T_1,T_2)$ of the (normal) Casimir force on the sphere (negative forces correspond to attraction towards the plate) that is obtained when the tip of the sphere is respectively above a point q deep in the Au region and a point p deep in the Si region.[1] The principle behind this differential measurement can be easily explained. One considers that em quantum fluctuations contributing to the $T = 0$ Casimir force have characteristic frequencies of the order of $\omega_c = c/2a = 5 \times 10^{14}$ rad/s, for a separation of 300 nm. Photons with this frequency have a penetration depth δ_0 in Au of 20 nm or so. On the other hand, inspection of the spectrum of the thermal Casimir-Lifshitz force [and in particular of the antisymmetric contribution $\bar{F}^{(\rm neq)}(T_1,T_2)$ that is our main interest] reveals that the important photon frequencies are smaller than $0.05(k_B T/\hbar) \simeq 2 \times 10^{12}$ rad/s for temperatures T around 300 K. The penetration depth δ_T of these thermal photons in Au is around 160 nm. Therefore, if the thickness w of the gold overlayer is chosen such that

$$\delta_0 \ll w \ll \delta_T, \tag{2}$$

it is clear that the overlayer filters out from the signal ΔF the uninteresting $T = 0$ component of the Casimir force, which would otherwise mask the much weaker thermal force. In contrast, low-frequency thermal photons contributing to $\bar{F}^{(\rm neq)}(T_1,T_2)$, being able to traverse the Au overlayer, are sensitive to the different optical properties of the Au-Si substrates. In our computations we took $w = 100$ nm and we found that for a large sphere radius $R \gg a$ the signal $\Delta F(a)$ is essentially equal (as it will be better explained in the remainder of the paper) to the antisymmetric component $\bar{F}^{(\rm neq)}_{\rm Si}(T_1,T_2)$ of the thermal force that is obtained when the sphere is above the Si sector:

$$\Delta F(T_1,T_2) \simeq \bar{F}^{(\rm neq)}_{\rm Si}(T_1,T_2). \tag{3}$$

This shows that by our measurement scheme it is possible to directly observe the nonequilibrium thermal force discovered by Antezza *et al.* Another important virtue of the proposed scheme is that it is immune by design from the problem of electrostatic patch forces that represent a major difficulty in conventional Casimir absolute force measurements [22–27]. This is so because electrostatic forces originating from patches on the exposed surface of the gold overlayer are on average independent of the position of the sphere tip above the Au overlayer and therefore cancel out from ΔF. Possible electrostatic patches existing at the Si-Au interface are also harmless, because the resulting electrostatic fields are screened out by the gold overlayer and thus cannot reach the gold sphere. In this regard our scheme is similar to recent differential experiments where the effect of the Casimir force is minimized (so-called "Casimir-less" experiments) [28,29] searching for non-Newtonian gravity in the submicron range, which also utilized Au overlayers of thicknesses similar to ours to screen out both electrostatic and Casimir forces. It has been estimated recently [30] that for separations larger than 200 nm random fluctuations of the patch potential from point to point on the surface of the Au overlayer imply a limit of 0.1 fN on the sensitivity of the apparatus of López and co-workers.

[1]Out of thermal equilibrium, the force difference $\Delta F(T_1,T_2)$ includes in general an uninteresting distance-independent contribution originating from the thermal radiation of the environment [8], which we subtracted from the signal.

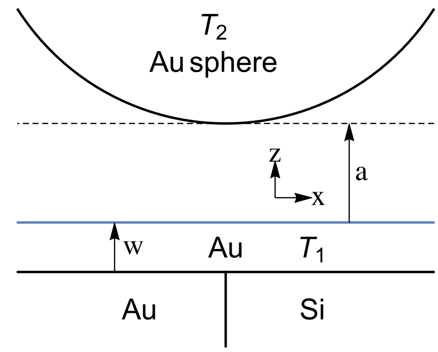

FIG. 1. (Color online) The setup consists of a gold sphere at temperature T_2 above a planar slab at temperature T_1. The planar slab is divided in two regions made of gold and (high-resistivity) silicon, respectively, and is fully covered with a plane-parallel gold overlayer of uniform thickness $w = 100$ nm.

III. COMPUTING THE NONEQUILIBRIUM FORCE DIFFERENCE

Over the past fifty years, the theory of dispersion forces has reached a remarkable degree of generality, which now allows us to describe these forces in a wide variety of physical situations. We recall that in the original version of Lifshitz theory, the material boundaries of the system were considered to be planar dielectrics in thermal equilibrium, fully described by the respective frequency-dependent (complex) permittivity $\epsilon(\omega)$. The extension of Lifshitz theory to plane-parallel layered slabs consisting of an arbitrary number of layers made of different magnetodielectric materials was developed in [31–33] (see also Ref. [2], p. 290). Lifshitz theory was later generalized to surfaces of arbitrary shapes [31,34,35]. The theory of dispersion forces between two plane-parallel dielectric slabs at different temperatures was developed in [6–8]. This nontrivial generalization of Lifshitz theory is based on the assumption that the correlator of the fluctuating dipole moments within two plates that are in local thermal equilibrium has the same expression as the one that holds in systems that are in global thermal equilibrium [4,5]. It should be stressed that this assumption, while plausible because of the local character of the source correlators, has a hypothetical character and as such it needs further theoretical and experimental validation [7]. The same assumption was used by Polder and Van Hove [36] to calculate the radiative heat transfer between two bodies at different temperatures and since then it has been universally adopted in theoretical and experimental investigations of radiative heat transfer (see [15], and references therein). On the basis of a scattering approach, the out-of-equilibrium theory of [6–8] was generalized to (possibly layered) plates of arbitrary shapes and constitutions in [12,15].

We can turn now to the computation of ΔF for our apparatus. In [8] it was shown that the Casimir pressure $F^{(\mathrm{PP})}(T_1,T_2)$ between two homogeneous plane-parallel plates at different temperatures is equal to the average of the equilibrium Casimir pressures corresponding to the two temperatures plus an extra term $\bar{F}^{(\mathrm{neq})}(T_1,T_2)$, which is antisymmetric in the temperatures. By using scattering methods, it was shown in [12,15] that an analogous formula holds for one or two layered plates (of any shapes) provided the appropriate reflection coefficients of the layered plates are used. We will see below that the sphere-plate force difference ΔF for our setup has a similar structure. To make the computation simple, we make two assumptions. First, we assume that both points p and q at which the force is measured are at a horizontal distance s from the Au-Si boundary of the plate, which is much larger than the typical radius $\rho = \sqrt{Ra}$ of the circular region around the sphere tip that contributes significantly to the Casimir force. The force $F_{\mathrm{Au}}(T_1,T_2)$ can then be identified with the force $\tilde{F}_{\mathrm{Au}}(T_1,T_2)$ between a gold sphere and a homogenous gold plate, while $F_{\mathrm{Si}}(T_1,T_2)$ becomes identical to the force $\tilde{F}_{\mathrm{Si}}(T_1,T_2)$ between the same gold sphere and a two-layer plane-parallel slab, consisting of a gold layer of thickness w deposited over a uniform Si slab. The forces $F_{\mathrm{Au/Si}}(T_1,T_2)$ can in principle be computed using the scattering formalism described in [12,15]. Here, for simplicity, we avoid the mathematical complexities of the scattering formulas and assume, as it is usually the case in Casimir experiments, that the sphere radius R is much larger than the separation a, $R \gg a$. Under this assumption it is possible to use the proximity force approximation (PFA) [1,2] to estimate both $\tilde{F}_{\mathrm{Au}}(T_1,T_2)$ and $\tilde{F}_{\mathrm{Si}}(T_1,T_2)$. According to the PFA, the force $F^{(\mathrm{sp})}$ between a large sphere and a plate can be expressed in terms of the potential $\mathcal{U}^{(\mathrm{PP})}$ for the unit-area force $F^{(\mathrm{PP})} = -\partial\mathcal{U}^{(\mathrm{PP})}/\partial a$ of the corresponding plane-parallel system:

$$F^{(\mathrm{sp})} = 2\pi R\,\mathcal{U}^{(\mathrm{PP})}. \tag{4}$$

The PFA formula (4) holds for any short-range interaction between gently curved surfaces and is valid also for the Casimir force out of thermal equilibrium. The PFA has been widely used to interpret Casimir experiments [2] (see [37] for more applications of the proximity approximation). It is now known that the PFA represents the leading term in a gradient expansion of the Casimir force, in powers of the slopes of the bounding surfaces [38–40]. By the PFA equation (4) one gets

$$\Delta F(T_1,T_2) = 2\pi R\big[\mathcal{U}^{(\mathrm{PP})}_{\mathrm{Si}}(T_1,T_2) - \mathcal{U}^{(\mathrm{PP})}_{\mathrm{Au}}(T_1,T_2)\big], \tag{5}$$

where $\mathcal{U}^{(\mathrm{PP})}_{\mathrm{Au}}(T_1,T_2)$ is the Casimir potential for two Au slabs at temperatures T_1 and T_2, respectively, and $\mathcal{U}^{(\mathrm{PP})}_{\mathrm{Si}}(T_1,T_2)$ is the potential for a Au slab at temperature T_2 in front of a two-layer Au-Si slab at temperature T_1. The potential $\mathcal{U}^{(\mathrm{PP})}(T_1,T_2)$ for two plane-parallel dielectric slabs at different temperatures can be found easily by integrating the formula for the nonequilibrium unit-area force $F^{(\mathrm{PP})}(T_1,T_2)$ provided in [8]:

$$\mathcal{U}^{(\mathrm{PP})}(T_1,T_2) = \tfrac{1}{2}[\mathcal{F}(T_1) + \mathcal{F}(T_2)] + \bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2). \tag{6}$$

In this formula, $\mathcal{F}(T)$ denotes the well-known Lifshitz formula for the equilibrium unit-area Casimir free energy:

$$\mathcal{F}(T) = \frac{k_B T}{2\pi}\sum_{l=0}^{\infty}\left(1-\frac{1}{2}\delta_{l0}\right)\int_0^{\infty} dk_\perp k_\perp \times \sum_{j=\mathrm{TE,TM}} \ln\left[1-e^{-2aq_l}R_j^{(1)}(\mathrm{i}\xi_l,k_\perp)R_j^{(2)}(\mathrm{i}\xi_l,k_\perp)\right], \tag{7}$$

while $\bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2)$ has the expression

$$\bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2) = \frac{\hbar}{4\pi^2}\int_0^{\infty} d\omega[n(\omega,T_1)-n(\omega,T_2)]\int_0^{\infty} dk_\perp k_\perp \sum_{j=\mathrm{TE,TM}} \mathrm{Im}\big[\ln\big(1-e^{2\mathrm{i}ak_z}R_j^{(1)}R_j^{(2)}\big)\big] \times\left[\theta(\omega/c-k_\perp)\frac{|R_j^{(2)}|^2-|R_j^{(1)}|^2}{1-|R_j^{(1)}R_j^{(2)}|^2} + \theta(k_\perp-\omega/c)\frac{\mathrm{Im}\big(R_j^{(1)}R_j^{(2)*}\big)}{\mathrm{Im}\big(R_j^{(1)}R_j^{(2)}\big)}\right]. \tag{8}$$

In Eqs. (7) and (8) $R_j^{(1)}$ and $R_j^{(2)}$ denote the reflection coefficients of the slabs for polarization j, k_B is Boltzmann constant, $\xi_l = 2\pi l k_B T/\hbar$ are the (imaginary) Matsubara frequencies, $k_\perp$ is the modulus of the in-plane wave vector, $q_l = \sqrt{\xi_l^2/c^2 + k_\perp^2}$, $k_z = \sqrt{\omega^2/c^2 - k_\perp^2}$, $\theta(x)$ is the unit step function [$\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x > 0$], and $n(\omega,T) = [\exp(\hbar\omega/k_B T) - 1]^{-1}$ is the Bose-Einstein distribution. According to Eq. (6), out of equilibrium the potential $\mathcal{U}^{(\mathrm{PP})}(T_1,T_2)$ is equal to the average of the equilibrium Casimir free energies $\mathcal{F}(T)$ at temperatures T_1 and T_2 plus a genuinely nonequilibrium contribution $\bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2)$. The latter term is antisymmetric in the temperatures T_1 and T_2 and vanishes identically if the slabs have identical reflection coefficients. Remarkably, this structure of the Casimir-Lifshitz force out of thermal equilibrium has been shown to be valid also for nonparallel plates of arbitrary shapes and constitution [12–15]. In order to evaluate Eq. (5), one substitutes in Eqs. (7) and (8) the reflection coefficient $R_j^{(2)}$ by the reflection coefficient $R_j^{(\mathrm{Au})}$ of a Au slab and $R_j^{(1)}$ by either $R_j^{(\mathrm{Au})}$ or the reflection coefficient $R_j^{(\mathrm{Si})}$ of a Si slab covered by a gold layer of thickness w.

The reflection coefficient $R_j^{(\mathrm{Au})}$ is equal to the Fresnel coefficient $r_j^{(0\mathrm{Au})}$ given in Eqs. (10) and (11) below with $a = 0$ and $b = \mathrm{Au}$, while $R_j^{(\mathrm{Si})}$ is provided by the following formula:

$$R_j^{(\mathrm{Si})}(\omega,k_\perp) = \frac{r_j^{(0\mathrm{Au})} + e^{2\mathrm{i}wk_z^{(\mathrm{Au})}} r_j^{(\mathrm{AuSi})}}{1 + e^{2\mathrm{i}wk_z^{(\mathrm{Au})}} r_j^{(0\mathrm{Au})} r_j^{(\mathrm{AuSi})}}. \tag{9}$$

Here $r_\alpha^{(ab)}$ are the Fresnel reflection coefficients for a planar interface separating medium a from medium b:

$$r_{\mathrm{TE}}^{(ab)} = \frac{k_z^{(a)} - k_z^{(b)}}{k_z^{(a)} + k_z^{(b)}}, \tag{10}$$

$$r_{\mathrm{TM}}^{(ab)} = \frac{\epsilon_b(\omega)k_z^{(a)} - \epsilon_a(\omega)k_z^{(b)}}{\epsilon_b(\omega)k_z^{(a)} + \epsilon_a(\omega)k_z^{(b)}}, \tag{11}$$

where $k_z^{(a)} = \sqrt{\epsilon_a(\omega)\omega^2/c^2 - k_\perp^2}$, ϵ_a denotes the electric permittivity of medium a, and we define $\epsilon_0 = 1$. In our computations, we used the tabulated optical data of Au and Si [41]. The data of Au were extrapolated towards zero frequency via the Drude model $\epsilon_{\mathrm{Dr}} = 1 - \omega_p^2/[\omega(\omega + \mathrm{i}\gamma)]$, with $\omega_p = 8.9$ eV$/\hbar$ and $\gamma = 0.035$ eV$/\hbar$.[2] We are now in a position to better justify Eq. (3), showing that ΔF measures the nonequilibrium thermal Casimir-Lifshitz force. We remarked earlier that if the thickness w of the gold overlayer is chosen in the range in Eq. (2), the $T = 0$ component of the Casimir force, which is included in the first two terms on the right-hand side of Eq. (6), is filtered out from ΔF. As to the thermal component of the Casimir-Lifshitz force, a distinction has to be made between the thermal correction to the equilibrium force, which is again included in the first two terms of Eq. (6), and the truly nonequilibrium contribution provided by the last term on the right-hand side of Eq. (6). The equilibrium thermal correction has a characteristic frequency of the order of the first Matsubara mode $\xi_1 = 2\pi k_B T/\hbar = 2.5 \times 10^{14}$ rad/s at room temperature. Since this radiation has a penetration depth in Au $\delta_T^{(\mathrm{eq})} \simeq 20$ nm $\ll w$, it is clear that the equilibrium thermal component of the force is filtered out as well by the overlayer. The situation with the nonequilibrium force proportional to $\bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2)$ is remarkably different. When the sphere tip is above the gold sector of the plate this contribution is zero because $\bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2)$ vanishes identically for two surfaces made of the same material. When the sphere tip is instead above the Si sector of the plate, the nonequilibrium contribution $\bar{\mathcal{U}}^{(\mathrm{neq})}(T_1,T_2)$ is different from zero and by inspection of its spectrum we estimated that for submicron separations it receives its main contribution from evanescent waves with TE polarization in the frequency range around $0.05(k_B T/\hbar) \simeq 2 \times 10^{12}$ rad/s for temperatures T around 300 K. Since the penetration depth δ_T of such a radiation in Au, around 160 nm, is much larger than $w = 100$ nm, this contribution to the thermal force is strongly affected by the Au-Si interface. The conclusion of all these considerations, fully confirmed by numerical computations, is that for our setup:

$$\Delta F \simeq 2\pi R \bar{\mathcal{U}}_{\mathrm{Si}}^{(\mathrm{neq})}(T_1,T_2). \tag{12}$$

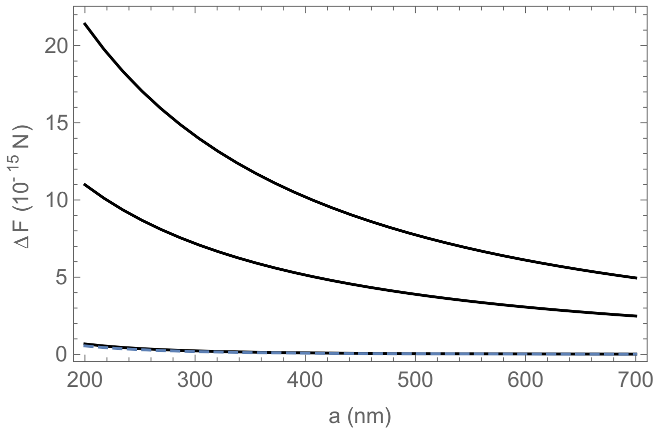

FIG. 2. (Color online) Force difference ΔF (in fN) versus separation a in nm, for a Au sphere of radius $R = 150$ μm. The three solid curves, from top to bottom, correspond to a fixed sphere temperature $T_2 = 300$ K and to three temperatures of the Au-Si plate $T_1 = 350$, 325, and 300 K, respectively. The dashed line represents the equilibrium force difference for $T_1 = T_2 = 350$ K.

In Fig. 2 we show a plot of ΔF (in fN) versus separation a in nm, for a sphere radius $R = 150$ micron and a Au overlayer of thickness $w = 100$ nm. The three solid curves, from top to bottom, correspond to a fixed sphere temperature $T_2 = 300$ K and to three temperatures of the Au-Si plate $T_1 = 350$, 325, and 300 K, respectively. The plot displays also (dashed line) the equilibrium force difference for $T_1 = T_2 = 350$ K.

The close proximity of the two equilibrium curves confirms that the force difference ΔF seen for $T_1 \neq T_2$ arises entirely from the nonequilibrium thermal force proportional to $\bar{\mathcal{U}}_{\mathrm{Si}}^{(\mathrm{neq})}(T_1,T_2)$, in accordance with Eq. (12). The isoelectronic "Casimir-less" experiments by López and co-workers [28,29] searching for non-Newtonian gravity in the submicron range measured dynamically the differential force between a Au sphere glued to a microtorsional oscillator and a rotating disk consisting of alternating Au and Si regions covered by a Au overlayer. A sensitivity better than 0.3 fN in force differences was reported in the separation range from 200 to 1000 nm for an integration time of 3000 s. If this level of sensitivity can be preserved in the presence of a temperature difference between the sphere and the disk of a few tens of degrees, it should be easily possible to adapt the setup by López and co-workers to measure precisely the out-of-equilibrium thermal force displayed in Fig. 2.

An ongoing controversy in Casimir physics concerns the influence of relaxation processes of conduction electrons on the thermal Casimir force [2,42–45]. Surprisingly, several Casimir experiments appear to be in agreement with Lifshitz theory only if conduction electrons are modeled by the dissipationless plasma model of infrared optics, while inclusion of dissipation via the plausible Drude model results in predictions of the Casimir force that are inconsistent with the data. In Fig. 3 we show the plasma model prediction of ΔF (in fN) versus separation a (in nm) computed for the same temperatures

[2]For the temperatures that we consider, the temperature variation of γ has a negligible effect.

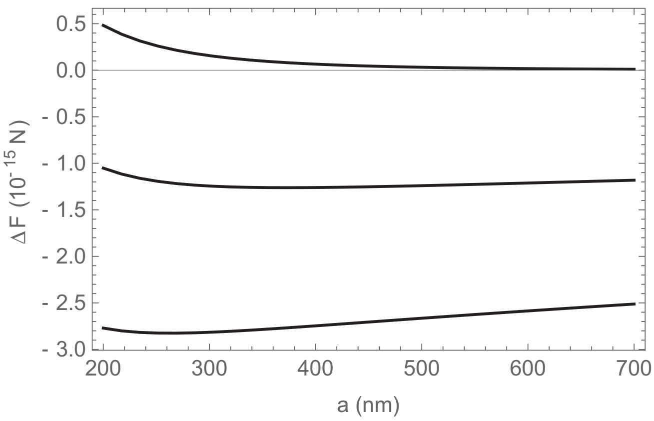

FIG. 3. Force difference ΔF (in fN) versus separation a in nm, for a Au sphere of radius $R = 150\,\mu$m at a fixed temperature $T_2 = 300$ K. The three curves, from top to bottom, correspond to temperatures of the Au-Si plate equal to $T_1 = 300$, 325, and 350 K, respectively. For this plot the conduction electrons of Au have been modeled as a dissipationless plasma.

T_1 and T_2 as in Fig. 2. Contrary to the Drude model result shown in Fig. 2, the plasma model predicts that ΔF should shift towards negative values as the temperature of the plate is increased. The widely different values of ΔF predicted by the two prescriptions should be easily detectable. Our apparatus should thus allow for a definitive experimental resolution of the Drude model versus plasma model conundrum.

IV. CONCLUSION

We have described an apparatus by which it should be possible to observe the thermal Casimir-Lifshitz force between two macroscopic surfaces out of thermal equilibrium. The sensitivity achieved by recent isoelectronic Casimir-less experiments should allow for a precise measurement of the thermal Casimir force in the submicron region and for moderate temperature differences between the plates. Apart from shedding light on the elusive thermal Casimir force, such an experiment might also allow us to resolve a long-standing controversy regarding the role of dissipation in the Casimir effect.

[1] V. A. Parsegian, *Van der Waals Forces* (Cambridge University Press, Cambridge, 2005).

[2] M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, Oxford, 2009).

[3] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1955) [Sov. Phys. JETP **2**, 73 (1956)].

[4] S. M. Rytov, *Theory of Electrical Fluctuations and Thermal Radiation* (Academy of Sciences, Moscow, 1953).

[5] S. M. Rytov, Y. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics* (Springer, Berlin, 1989), Vol. 3.

[6] M. Antezza, L. P. Pitaevskii, and S. Stringari, Phys. Rev. Lett. **95**, 113202 (2005).

[7] M. Antezza, L. P. Pitaevskii, S. Stringari, and V. B. Svetovoy, Phys. Rev. Lett. **97**, 223203 (2006).

[8] M. Antezza, L. P. Pitaevskii, S. Stringari, and V. B. Svetovoy, Phys. Rev. A **77**, 022901 (2008).

[9] J. M. Obrecht, R. J. Wild, M. Antezza, L. P. Pitaevskii, S. Stringari, and E. A. Cornell, Phys. Rev. Lett. **98**, 063201 (2007).

[10] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, Nat. Phys. **7**, 230 (2011).

[11] G. L. Klimchitskaya, M. Bordag, and V. M. Mostepanenko, Int. J. Mod. Phys. A **27**, 1260012 (2012).

[12] G. Bimonte, Phys. Rev. A **80**, 042102 (2009).

[13] R. Messina and M. Antezza, Phys. Rev. A **84**, 042102 (2011).

[14] R. Messina and M. Antezza, Europhys. Lett. **95**, 61002 (2011).

[15] M. Krüger, G. Bimonte, T. Emig, and M. Kardar, Phys. Rev. B **86**, 115423 (2012).

[16] G. Bimonte, T. Emig, M. Krüger, and M. Kardar, Phys. Rev. A **84**, 042503 (2011).

[17] M. Krüger, T. Emig, G. Bimonte, and M. Kardar, Europhys. Lett. **95**, 21002 (2011).

[18] G. Bimonte, Phys. Rev. Lett. **112**, 240401 (2014).

[19] G. Bimonte, Phys. Rev. Lett. **113**, 240405 (2014).

[20] G. Bimonte, Phys. Rev. B **91**, 205443 (2015).

[21] G. Bimonte, J. Phys.: Condens. Matter **27**, 214021 (2015).

[22] Q. Wei and R. Onofrio, Phys. Lett. A **374**, 2230 (2010).

[23] Q. Wei, D. A. R. Dalvit, F. C. Lombardo, F. D. Mazzitelli, and R. Onofrio, Phys. Rev. A **81**, 052115 (2010).

[24] W. J. Kim, M. Brown-Hayes, D. A. R. Dalvit, J. H. Brownell, and R. Onofrio, Phys. Rev. A **78**, 020101 (2008).

[25] R. S. Decca, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, D. López, U. Mohideen, and V. M. Mostepanenko, Phys. Rev. A **79**, 026101 (2009).

[26] S. de Man, K. Heeck, and D. Iannuzzi, Phys. Rev. A **79**, 024102 (2009).

[27] R. O. Behunin, Y. Zeng, D. A. R. Dalvit, and S. Reynaud, Phys. Rev. A **86**, 052509 (2012).

[28] R. S. Decca, D. López, H. B. Chan, E. Fischbach, D. E. Krause, and C. R. Jamell, Phys. Rev. Lett. **94**, 240401 (2005).

[29] Y.-J. Chen, W. K. Tham, D. E. Krause, D. López, E. Fischbach, and R. S. Decca, arXiv:1410.7267.

[30] R. O. Behunin, D. A. R. Dalvit, R. S. Decca, and C. C. Speake, Phys. Rev. D **89**, 051301(R) (2014).

[31] D. Langbein, *Theory of Van der Waals Attraction* (Springer, Berlin, 1974).

[32] S. Y. Buhmann, D.-G. Welsch, and T. Kampf, Phys. Rev. A **72**, 032112 (2005).

[33] M. S. Tomaš, Phys. Lett. A **342**, 381 (2005).

[34] A. Lambrecht, P. A. Maia Neto, and S. Reynaud, New J. Phys. **8**, 243 (2006).

[35] S. J. Rahi, T. Emig, N. Graham, R. L. Jaffe, and M. Kardar, Phys. Rev. D **80**, 085021 (2009).

[36] D. Polder and M. Van Hove, Phys. Rev. B **4**, 3303 (1971).

[37] M. Kruger, V. A. Golyk, G. Bimonte, and M. Kardar, Europhys. Lett. **104**, 41001 (2013).

[38] C. D. Fosco, F. C. Lombardo, and F. D. Mazzitelli, Phys. Rev. D **84**, 105031 (2011).

[39] G. Bimonte, T. Emig, R. L. Jaffe, and M. Kardar, Europhys. Lett. **97**, 50001 (2012).

[40] G. Bimonte, T. Emig, and M. Kardar, Appl. Phys. Lett. **100**, 074110 (2012).
[41] *Handbook of Optical Constants of Solids*, edited by E. D. Palik (Academic, New York, 1995).
[42] J. S. Hoye, I. Brevik, J. B. Aarseth, and K. A. Milton, Phys. Rev. E **67**, 056116 (2003).
[43] I. Brevik, J. B. Aarseth, J. S. Hoye, and K. A. Milton, Phys. Rev. E **71**, 056101 (2005).
[44] I. Brevik, S. Å. Ellingsen, and K. A. Milton, New J. Phys. **8**, 236 (2006).
[45] J. S. Hoye, I. Brevik, S. Å. Ellingsen, and J. B. Aarseth, Phys. Rev. E **75**, 051127 (2007).